# PROCESS OF SELF-IGNITION ENGINE LOADS AND ITS PROPERIES

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***Abstract***

*The paper presents a probabilistic interpretation of self-ignition combustion engine load, with regards to known parameters (rates) of engine operation. It has been shown that load of engines of this kind, considered in any moment, can be regarded as a multi-dimensional random variable. Changes of engine load in time of operation are considered herein as a process of loads and presented in a form of multidimensional stochastic process, of which states are loads as random variables. The process of loads can be regarded as a stochastic process with gains asymptotically independent, being stationary and ergodic. The researches on self-ignition engine loads should consider the stochastic dependence between thermal and mechanical loads. Load considered in arbitrary moment of work of each engine has been accepted as a random variable. Loads considered in successive moments make the process of loads.. The moments, however, are not random variables, but parameters of the process. Apart from the hypothesis specified above also other ones have been formulated. For verification of the hypotheses there have been proposed: the method of non-deductive (inductive) inference, called reductive inference and the method of deductive inference, called the „modus tollens" rule. The proposed approach to analysis of self-ignition engine loads is essential because the loads are ones of the most significant causes of surface (linear) as well as volume wear of the engines, what in consequence leads to engine failures. The engine failures happening during operation are random events. The failures considered in successive moments of their occurrence make the process of failures, which is a random process.*

***Keywords:*** *transport, combustion engine, engine load, hypothesis, stochastic process*

# PROCES OBCIĄŻEŃ SILNIKA O ZAPŁONIE SAMOCZYNNYM I JEGO WŁASNOŚCI

***Streszczenie***

*Zaproponowano probabilistyczną interpretację obciążenia silników spalinowych o zapłonie samoczynnym z uwzględnieniem znanych parametrów (wskaźników) ich pracy. Wykazano, że obciążenie tego rodzaju silników, rozpatrywane w dowolnej chwili, może być uważane za zmienną losową wielowymiarową. Zmiany obciążenia silnika w czasie jego pracy uznane zostały za proces obciążeń i przedstawione w formie wielowymiarowego procesu stochastycznego, którego stanami są obciążenia jako zmienne losowe. Wykazano, że proces obciążeń może być uznany za proces stochastyczny o przyrostach asymptotycznie niezależnych, stacjonarny i ergodyczny. Wykazano także, że w badaniach obciążenia silnika o zapłonie samoczynnym należy uwzględnić zależność stochastyczną między ich obciążeniem cieplnym a mechanicznym. Obciążenie rozpatrywane w dowolnej chwili pracy każdego silnika uznano za zmienna losową dlatego, że może być uważana za prawdziwą następująca hipoteza: „obciążenie silnika jest zmienną losową, dlatego ponieważ jego wartości, w kolejno wykonywanych pomiarach, można przewidzieć jedynie z określonym prawdopodobieństwem". Obciążenia rozpatrywane w kolejnych chwilach tworzą proces obciążeń. Zatem istnieje potrzeba opracowania koncepcji analizy i oceny tych obciążeń w aspekcie probabilistycznym z uwzględnieniem faktu, że zmiany obciążeń, następujące kolejno po sobie we wspomnianych chwilach, tworzą proces stochastyczny. Chwile te nie są jednak zmiennymi losowymi, lecz parametrami tego procesu. Oprócz wymienionej hipotezy sformułowano także inne. Do weryfikacji przedstawionych hipotez zaproponowano metodę wnioskowania niededukcyjnego (indukcyjnego) nazywaną wnioskowaniem redukcyjnym oraz metodę wnioskowania dedukcyjnego nazywaną regułą „modus tollens". Zaproponowane podejście do analizy obciążeń silników o zapłonie samoczynnym jest istotne dlatego, że należą one do najistotniejszych przyczyn zużycia powierzchniowego (liniowego) jak również*

*objętościowego tych silników a w konsekwencji do ich uszkodzeń. Zaś zachodzące w czasie eksploatacji uszkodzenia silników są zdarzeniami losowymi. Uszkodzenia te rozpatrywane w kolejnych chwilach ich pojawiania się tworzą proces uszkodzeń, który jest procesem losowym.*

***Słowa kluczowe:*** *transport, silnik spalinowy, obciążenie silnika, hipoteza, proces stochastyczny*

## 1. Introduction

Loads of self-ignition engines, mechanical as well as thermal, are among the most significant causes of surface (linear) and capacity wear, what leads in result to their failures. Excessive thermal loads of components making the engine working spaces (combustion chamber) have extremely unfavourable effect on wear of the engines. Currently, loads of combustion engines are analyzed generally in deterministic aspect [1, 7, 10, 12, 13, 14]. Such analyses, in probabilistic aspect, are made in a simplified way and, in principle, are limited to stochastic methods. Combustion engine loads should be, however, mapped in the space-time as random variables, assigned to random events, in which measurement of load of proper value is taken. The fact, that the taken measurement of load is a random event means that it is the event which determined probabilities should be assigned to. Moreover, the loads should be analysed in successive moments of combustion engines operation. Loads considered in these moments create the process of data. That is why, there is a need to elaborate a conception of analysis and estimation of the loads, in probabilistic aspect, with regards to the fact that the changes of loads following one after one in the mentioned moments create the stochastic process. The moments, however, are not random variables but parameters of the process.

To elaborate such conception, the probabilistic properties of the process of combustion engine load should be established first, beginning from analysis of states of the process, which are the loads of the considered engines, understood as random variables..

## 2. Engine load as a random variable

Empirical researches show that the values of engine loads are not predictable to the end [3, 5, 6, 9]. Thus, the following hypothesis $H_1$ can be formulated: **„engine load is a random variable, because its values, in measurements taken successively, can be predicted only with determined probability**”.

Quality interpretation of combustion engine load in any moment *t* can be presented in a form of the following dependence:

$$Q_D(t) = Q_C(t) + Q_M(t), \tag{1}$$

where:

$$Q_C(t) = \Delta U(t) + Q_O(t),$$

at:

$Q_D(t)$ – thermal energy supplied to engine in time *t*,
$Q_C(t)$ – thermal energy transmitted by engine components during operation (thermal load) in time *t*,
$Q_M(t)$ – mechanical energy connected with existing terrestrial gravity force and arising gas forces, friction and inertia (mechanical; load) in time *t*,
$\Delta U(t)$ – increase of material internal energy of components which a part of thermal energy $Q_C(t)$, marked as $Q_O(t)$ go through, in time *t*,
$Q_O(t)$ – energy transmitted to the environment in time *t* through walls of engine components forming ex. working spaces (combustion chamber) and the others.

The mentioned engine loads can be considered as random variables. In the case when one parameter (one random value) is chosen to describe the load, the load (as a value of the loading process) is a one-dimension random variable. But, when more parameters are applied to describe the loads, the load is a multidimensional random variable. Then, the load will be described by system of many (*n, in general*) random variables. The load can be, then, regarded as a *n*-dimensional random variable, so a system of many (*n*) real-valued functions assigning unequivocally many numerical values to each random event (which is measurement of load) [2, 5, 6, 8, 11].

In operation practice, self-ignition engine load is expressed, the most often, by the following parameters: $p_{\max}$ – maximal combustion pressure, $t_{\max}$ – maximal combustion temperature, $p_e$ – average effective pressure, $c_{śr}$ – average piston speed, $\Delta\varphi_{pśr}$ –average rate of pressure increase. The parameters can be regarded as values of random variables which can be designated adequately with: $P_{\max}$, $T_{\max}$, $P_e$, $C_{śr}$ and $\Delta\Phi_{pśr}$.

In case when the load is determined by such random variables as: maximal pressure ($P_{\max}$) and maximal temperature ($T_{\max}$), the two random variables ($P_{\max}$ i $T_{\max}$) can be considered at the same time. The variables can be regarded as jump variables. Then, arbitrary realizations of the random variables are the quantities $p_{i\max}$ and $t_{j\max}$. So the pair ($p_{i\max}$ and $t_{j\max}$) is realization of a two-dimensional random variable ($P_{\max}$ and $T_{\max}$). Occurring the events $P_{\max} = p_{i\max}$ and $T_{\max} = t_{j\max}$ in the same time is determined by the probability $q(p_{i\max}, t_{j\max})$. In case of considering values of all $p_{i\max}$ and $t_{j\max}$ which may occur, the following dependence is effective [2, 8]:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} q(p_{i\max}, t_{j\max}) = 1. \tag{2}$$

Set of probabilities $p(p_{i\max}, t_{j\max})$ is two-dimensional distribution of random variable ($P_{\max}$, $T_{\max}$).

The probability $q(p_{i\max})$ of the random event $P_{\max} = p_{i\max}$ without consideration of the value of random variable $T_{\max}$ is equal to the sum of probabilities $q(p_{i\max}, t_{j\max})$ including all possible values $t_{j\max}$, , so:

$$q(p_{i\max}) = \sum_{j=1}^{r} q(p_{i\max}, t_{j\max}). \tag{3}$$

Set of probabilities $q(p_{i\max})$ determined according to the formula (3) is a marginal distribution of random variable $P_{\max}$.

In practice it is essential to determine the conditional probability $q(p_{i\max}/t_{j\max})$ of the random event $P_{\max} = p_{i\max}$ under condition (at the assumption), that $T_{\max} = t_{j\max}$. This follows from that the engine is the most loaded mechanically and thermally when the maximal pressures and temperatures occur in the engine working spaces (cylinders). The probability can be determined from the formula:

$$q(p_{i\max}/t_{j\max}) = \frac{q(p_{i\max}, t_{j\max})}{q(t_{j\max})}. \tag{4}$$

Set of conditional probabilities $q(p_{i\max}/t_{j\max})$ for the same condition (assumption ) $T_{\max} = t_{j\max}$ is a conditional distribution of random variable $P_{\max}$ under the condition $T_{\max} = t_{j\max}$. The sum of conditional probabilities $q(p_{i\max}/t_{j\max})$ including all possible $p_{i\max}$ values is equal to 1, so:

$$\sum_{i=1}^{r} q(p_{i\max}/t_{j\max}) = 1. \tag{5}$$

In practice it may happen that random variables $P_{\max}$ and $T_{\max}$ will be independent variables, ex. in the result of incorrect performance of engine regulation. Then, the following dependences are effective:

$$q(p_{i\max}/t_{j\max}) = q(p_{i\max}), \tag{6}$$

$$q(t_{j\max}/p_{i\max}) = q(t_{j\max}). \tag{7}$$

Taking into account the dependences (6) and (7) in the equation (4) the following formula is obtained:

$$q(p_{i\max}, t_{j\max}) = q(p_{i\max})q(t_{j\max}). \tag{8}$$

Random variables $P_{\max}$ and $T_{\max}$ satisfying the condition (8) are stochastically independent. In case when the condition is not satisfied, the random variables $P_{\max}$ and $T_{\max}$ are random variables stochastically dependent (so, correlated). From the dependence (8) results that researches should consider the fact that in case of independent random variables $P_{\max}$ and $T_{\max}$, conditional distributions of the variables do not differ from their marginal distributions.

In a similar way it can be considered the load which is described by three parameters ex. maximal pressure $P_{\max}$, maximal temperature $T_{\max}$ and average effective pressure $p_e$.

The presented proposal of description of combustion engine load is significant because the load effects the wear of engine components. From this reason the wear in a chosen moment of engine operation time is also a random variable and being analyzed in successive moments of the time it should be regarded as a random process.

## 3. Engine load as a stochastic process

Engine loads (mechanical and thermal) can be considered in dynamic (short) time measured in [ms] (this is the time in which there exists one or a few thermodynamic cycles of engine) and in quasi-statistic (long) time being the time of correct work of engine, measured the most often in w [hours].

Comparing the loads existing in successive thermodynamic cycles, so in short time ($t_d$), it can be said that they are different from different reasons [5, 9, 10, 12, 13]. With increase of the value of time $t_d$ of engine operation the differences grow. Thus, at the fixed value of dynamic time and investigated load in particular engine cycles in moments $t_{d1}$, $t_{d2}$,…, the different values of load can be obtained. Obtaining a specific value of the load is a random event. That means that to each moment of the time $t_d$ it can be assigned a random variable indicating the load assigned just to this moment. Similarly, to each moment of quasi-statistic time ($t_q$) it can be assigned a random variable indicating the engine load just in this moment.

Engine load as a random variable in any moment $t$ is a value of the process of loading. This process forms the run of loads occurring one after one and being connected casually, during changes of load. The load can be described by different random quantities of which values can be predicted only with determined probability. Thus, the process of loading is a stochastic process so function of which values are random variables assigned to defined moments of engine operation time $t$ [2, 4, 15].

Therefore, the following hypothesis $H_2$ can be formulated: „**process of loading an engine is a stochastic process, because the values of engine loads, assigned to arbitrary moment, are random variables**". From the theory of stochastic processes follows that a set of such moments is a set of parameters of the process [4, 15].

Engine load can be characterized by different quantities (parameters, rates). Generally, the load can be expressed in a form of the following dependence (1):

$$Q(t) = f[Q_M(t), Q_C(t)], \tag{9}$$

where:

$Q$ – engine load,
$Q_M$ – mechanical load of engine,
$Q_C$ – thermal load of engine,
$t$ – engine operation time.

Thus, in empirical researches on load there can be considered at least two stochastic processes $\{Q_M(t): t \geq 0\}$ and $\{Q_C(t): t \geq 0\}$. The processes are components of the vector process $\{Q(t: t \geq 0\}$ [4, 11, 15].

As for the loads $Q_M$ i $Q_C$ it can be said that they are vectors of the following components in arbitrary moment $t$:

$$Q_M : [p_{\max}, p_z, p_e, c_{śr}, \varphi, \varphi_p, n, P_g, P_b, \ldots], \tag{10}$$

$$Q_C : [\dot{q}, \nabla T, \rho, p_e, c_{śr}, T_{\max}, T_z, T_{sw}, \dot{Q}, \ldots], \tag{11}$$

where:
$p_{\max}$ – maximal combustion pressure,
$p_z$ – combustion pressure,
$p_e$ – mean effective pressure ($p_e = \eta_m p_i$,
$\eta_m$ – mechanical efficiency,
$p_i$ – average indicated pressure),
$c_{śr}$ – average piston speed,
$\varphi$ - (isochoric) pressure increase rate,
$\varphi_p$ – moment rate of pressure increase,
$n$ – (engine) shaft speed rate,
$P_g$ – force coming from gases pressure,
$P_b$ – force of inertia,
$\dot{q}$ – thermal energy flux density,
$\nabla T$ – temperature gradient,
$\rho$ – rate of preliminary (isobaric) expansion,
$T_{\max}$ – maximal combustion temperature,
$T_z$ – combustion temperature,
$T_{sw}$ – exhaust gases temperature,
$\dot{Q}$ – thermal flux,
$T_{ol}$ – temperature of oil,
$T_w$ – temperature of cooling water.

From the dependences (9) - (11) follows that the engine load depends on many quantities (parameters, indexes), so the load can be considered as a stochastic process including superposition (composition) of particular individual processes ($Q_M$ and $Q_C$, in the most simple case). This understanding of load is convenient for general considerations, but may be of a little usability for practical needs.

Generally, load can be understood as a process in which its states can be considered in the form of random variables. The process, as it is seen from the dependences (10) and (11) is a multidimensional process. Thus, the following $n$ random variables: $Q_{t_{d1}}$, $Q_{t_{d2}}, \ldots, Q_{t_{dn}}$ can be considered in any moment $t_{dl}$($l$ = 1, 2, …,$n$) of the dynamic time $t_d$. The variables, for further considerations, are designated as $Q_1, Q_2, \ldots, Q_n$). The variables can be $n$ different quantities (parameters, rates) of the examined load (ex. $Q_1 = p_{\max}$, $Q_2 = T_{\max}$, $Q_3 = p_e$, $Q_4 = \dot{q}$, etc.).

Taking into account simultaneously the particular properties of load (the load may be characterized by excessive combustion pressure, excessive combustion temperature, excessive thermal flux, etc.) it is obtained a system of *n* random variables, which makes *n*-dimensional random variable (designated later on as *Q*).

In general, (in order to simplify the further considerations) considerations can be limited by accepting the random variable *Q* as a two-dimensional random variable ($Q_M, Q_C$). Because measurements are taken periodically it can be assumed that random variables $Q_M$ and $Q_C$ are jump (discrete) random variables. Realizations of the variables are quantities of $q_{Mi}$ and $q_{Ci}$ accordingly. Thus, the pair ($q_{Mi}, q_{Ci}$) is realization of a random variable ($Q_M$, $Q_C$). The variable takes the values ($q_{Mi}, q_{Ci}$) with determined probability ($q_{Mi}, q_{Ci}$) which is the probability of occurring the events $Q_M = q_{Mi}$ and $Q_C = q_{Ci}$ at the same time.

Set of the mentioned probabilities $p(q_{Mi}, q_{Ci})$ makes a two-dimensional distribution of the random variable ($Q_M$, $Q_C$). Just like in the previous considerations which enabled formulation of dependences (2) and (3), the distribution $p(q_{Mi}, q_{Ci})$ satisfies the following condition:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} p(q_{M_i}, q_{C_j}) = 1. \tag{12}$$

Marginal distribution of the random variable $Q_M$ is as follows

$$p(q_{M_i}) = \sum_{j=1}^{r} p(q_{M_i}, q_{C_j}), \tag{13}$$

and distribution of the random variable $Q_C$ is of the following form:

$$p(q_{C_j}) = \sum_{i=1}^{r} p(q_{M_i}, q_{C_j}). \tag{14}$$

The above investigations shows that some quantities ex. $p_e$, $c_{śr}$ [10, 12, 13] characterize mechanical as well as thermal load. Thus, it is obvious that between mechanical load and thermal load there exist dependences. Because they are random processes, the stochastic connection should also be expected between them. In order to explain this connection the following hypothesis $H_3$ can be formulated: „**there is a stochastic dependence between the mechanical load $Q_M(t)$ and thermal load $Q_C(t)$ because the determined variants of one of the variables are accompanied by different variants of the second variable**". Hence there is a conclusion that the dependence between the loads ($Q_M$ (*t*) and $Q_C$ (t)) cannot be described in the result of applying a common method of algebraic equations. It seems to be true because the load depends on big number of factors, including these which cannot be measured [9, 10, 12, 13, 14]:

In case of combustion engine, in the main process of energy transition, thermal energy undergoes transition into mechanical energy and not the other way round. That is why, the thermal load ($Q_C$) can be accepted conventionally as independent variable and mechanical load ($Q_M$) – as dependent variable. By analogy, $Q_{C_1}$, $Q_{C_2}$,…, $Q_{C_n}$ can be independent variables and $Q_{M_1}$, $Q_{M_2}$,…, $Q_{M_n}$ .– dependent variables

Degree in which the load $Q_M$ is determined by the load $Q_C$, or $Q_{C_j}$ (*j*=1,2,…,*n*) as independent variables can be very different. In practice, it may happen that one independent variable ($Q_C$) almost totally determines dependent variable ($Q_M$). It also can happen that a few independent variables ($Q_{C_j}$) only in a small degree influence the dependent variable ($Q_M$). From this it follows that there is a need of taking into account the intensity (power) of stochastic connection between $Q_C$ and $Q_M$.

Intensity (power) of the stochastic connection between $Q_M(t)$ and $Q_C(t)$ can be established by applying for empirical researches the following dependences [8]:

$$T^2_{MC} = T^2_{CM} = \frac{\chi^2}{N\sqrt{(k-1)(l-1)}}\,, \tag{15}$$

where:
$k$ – number of variants of the variable $Q_M$,
$l$ – number of variants of the variable $Q_C$,
$N$ – marginal number of variants of the variable $Q_M$ or $Q_C$,
$\chi^2$ – value calculated from the chi-square formula,
$T_{(\cdot)}{}^2$ – convergence rate of Czuprow.

It can be proved [8], that $T_{MC}$ takes the values from the interval [0,1]. The rate equals zero ($T_{MC} = 0$) when there is no connection between values of the process ($Q_M$ i $Q_C$) and equals one ($T_{MC} = 1$) when functional dependence exists. General statement whether the random variables: $Q_M$ and $Q_C$ are dependent is possible after examining the probabilities defined by generalized formulas (14) ÷ (16) or detailed formulas (6) ÷ (8).

Comparing the loads occurring in successive cycles of engine, so in time $t_d$ (short), a conclusion can follow that they are different. So, at a fixed value of time $t_d$ and by testing the load in particular cycles in the moment $t_{d1}, t_{d2}, \ldots$ different values of load are obtained. Thus, the fact of obtaining a determined (expected) value of load is a random event. This is such event because in the result of establishing the same conditions of empirical researches the expected value of load may occur, but also may not occur. That means that gains in load are less and less dependent on each other with increase of time distance between the mentioned moments $t_{d1}, t_{d2}, \ldots$ [3, 6, 10, 12, 14].

On basis of the presented considerations the following hypothesis $H_4$: can be formulated: „**load is a process of gains asymptotically independent because with increase of the time distance between the time intervals in which the load undergoes testing (measurements of load are taken) its values become less and less dependent on each other**".

Another property of changes of the load consists in that observed quantities of load in dynamic time ($t_d$) as well as in quasi-statistic time ($t_q$) do not show any oriented (monotonic) variations. That is why it can be accepted that the peak quantities characterizing the load occur in a random way. Lack of monotony of changes of engine load enables formulating the hypothesis $H_5$ saying that: „**load is a stationary process, because in longer time there is no monotony of engine load changes**" [11].

From up-to-now researches on self-ignition engines follows that their load undergoes continuous changes in a way that its particular values measured after small time intervals are strongly correlated one with another. However, when the time distance between measurements of the loads grows the correlation between the loads decreases. Thus, values of the load measured in time intervals (or moments) considerably distant from each other, can be considered as independent. This property is called an asymptotic independence of the load value measured in the moment ex. $\tau_{i+1}$, from the value taken in the moment $\tau_i$ when the distance $\Delta\tau = \tau_{i+1} - \tau_i$ is sufficiently big [3]. Such understood asymptotic independence between the load values measured (or calculated) in the moments $\tau_i$ and $\tau_{i+1}$ reflects the fact that along with the growth of $\Delta\tau$ the dependence between the values decreases.

The presented view on engine load properties can lead to arising new possibilities of getting dependences of wear from load, via empirical researches.

## 4. Final conclusions

The presented analysis and synthesis of events show that load of each self-ignition engine, considered in any moment of its operation (work) time, can be regarded as a multi-dimensional random variable. Loads analyzed in successive moments of operation time of this kind of engines can be considered as realizations of the process of loading. Thus, the process of loading of each engine needs to be investigated as a multi-dimensional stochastic process. Herein, there have been proposed some hypotheses explaining why the process of loading of any self-ignition engine can be regarded as a stochastic process with gains asymptotically independent, stationary and ergodic, and that there is a stochastic dependence between its mechanical and thermal loads. Intensity of stochastic connection between the loads can be fixed during researches by applying the convergence test of Czuprow [8].

For verification of the presented hypotheses there have been proposed the method of non-deductive (inductive) inference called reduction inference and the method of deductive inference called the rule of „modus tollens".

Learning about the properties of the processes requires construction of applicable mathematical models on the way of the system modeling and performance of adequate empirical tests.

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