

## PASSIVE SONAR WITH CYLINDRICAL ARRAY

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*(received June 15, 2006; accepted September 30, 2006)*

The article presents a method of using active sonar with cylindrical array for passive monitoring of underwater sound sources. It is demonstrated that detecting a sound source and determining its bearings may be carried out with no changes in the construction of the receiver. Changing signal processing algorithms in the beamformer is all that is required. It is also demonstrated that in order to increase the accuracy of determining bearing it is possible to use high-resolution methods of spatial spectrum estimation. The methods are effective for angles of wave incidence contained in sonar's beam width. Inaccuracies for larger angles result from uneven sampling of acoustic field distribution.

**Key words:** passive sonar, cylindrical array, signal processing, spatial spectrum.

### 1. Introduction

Active sonars designed for detection of submarines and other objects located underwater and at sea bottoms are usually equipped with cylindrical arrays, allowing for simultaneous observation of targets at round angle. Receiver of the array equipped sonar contains a beamformer which most frequently produces a set of narrow beams covering a  $360^\circ$  or slightly smaller sector of observation. The number of beams is as a rule equal to the number of transducer columns making up the array. The columns are placed at equal angular distances, with the angle between the centres of neighbouring columns being approximately equal to the width of beam.

Users of the described sonars every so often call for their usage in detection and determining of location of underwater sound sources. Requests for such applications may be met in two ways, either through changing of sonar construction or solely through providing additional software for receiver's computers. The first solution involves in practice construction of new analogue blocks of the receiver with transmission band ranging from a few to a few thousand cycle per second. It is a high-cost solution, taking

into account the fact that an analogue block of a receiver contains as many independent channels as there are columns in the array. However, it does offer good detection conditions since overwhelming capacity of signals is located then within the receiver's transfer band. On the other hand, small dimensions of cylindrical array do not ensure desired directivity at such low frequencies. Therefore, one should not expect to obtain high accuracy in detecting target's location. Low directivity of the array also has a negative effect on the signal to noise ratio, in consequence impairing conditions of detection. As a result, despite significant increase in the cost of the sonar, satisfying operational parameters are not obtained.

In the other solution, it is assumed that the actual construction of the receiver is not in any way modified. Monitoring of signals emitted by underwater sound sources is therefore carried out solely within the transmission width of the receiver around carrier frequency. In the majority of the described sonars the carrier frequency ranges from a few to a dozen or so kilocycle per second. Within this frequency width the capacity of emitted signals is clearly smaller than capacity in the band of low acoustic frequencies. This is the fundamental shortcoming of the solution. However, despite the low level of received signals the range of the sonar may be quite high, which is supported by good directivity of the array improving the signal to noise ration. Advantage of the solution lies in its high accuracy of detection – similar to the accuracy obtained in the active working mode, as it will be demonstrated further on in the article.

## 2. Principle of operation of the sonar in passive working mode

Let us assume, that one sonar beam is generated in the beamformer through processing of signals occurring at the output points  $P = 2N + 1$  of cylindrical array's column ultrasound transducers. The column centres have been placed at fixed angle distances equal to  $\alpha$ . For the sake of simplification we assume that the columns may be replaced by point hydrophones of omni-directional directivity pattern. Angle sector of the array producing a single beam is therefore equal to  $(P - 1)\alpha$ . Cylinder radius  $R$  and angle  $\alpha$  are selected so that the distance between the centres of neighbouring columns is equal to  $\lambda_0/2$ , where  $\lambda_0$  is the length of the acoustic wave of mid-band frequency  $f_0$  of receiver's transmission band.

Let  $s(t)$  represent the signal received by the virtual hydrophone placed in the centre of the cylinder. If a section of the cylinder receives a plane wave at the angle  $\theta$ , then the signal at the hydrophone output numbered  $n$  is equal to:

$$s_n(t, \theta) = s[t + \tau_n(\theta)], \quad (1)$$

In the above presented case of cylindrical array geometry, the delay  $\tau_n(\theta)$  is equal to:

$$\tau_n(\theta) = \frac{R}{c} \cos(n\alpha - \theta), \quad (2)$$

where  $c$  stands for the speed of acoustic wave in water.

Signals from the output of array are sampled, and then out of each signal a discrete Fourier transforms are determined. Continuing for simplification reasons with continuous functions, we have:

$$S_n(f, \theta) = \mathfrak{F}\{s_n(t, \theta)\} = \exp[j2\pi f\tau_n(\theta)]S(f), \tag{3}$$

where for each  $n$

$$S(f) = \mathfrak{F}\{s(t)\}. \tag{4}$$

In the beamformer each spectral line  $S_n(f)$  is multiplied by a complex number having the value of:

$$W_n(f) = \exp(-j2\pi f\tau_n), \tag{5}$$

where

$$\tau_n = \frac{R}{c} \cos(n\alpha). \tag{6}$$

Calculated products are then summed up, resulting in:

$$B(f, \theta) = \sum_{n=-N}^N S_n(f, \theta) W_n(f) = S(f) \sum_{n=-N}^N \exp\{j2\pi f[\tau_n(\theta) - \tau_n]\}. \tag{7}$$

The expression

$$b(f, \theta) = \frac{1}{2N + 1} \sum_{n=-N}^N \exp\{j2\pi f[\tau_n(\theta) - \tau_n]\} \tag{8}$$

describes directivity pattern of a cylinder sector for spectral line of frequency  $f$ . The maximum value of directivity pattern occurs for the angle  $\theta = 0^\circ$ . Beam centre is therefore perpendicular to the chord of cylinder sector and passes through the middle of the central column numbered as  $n = 0$ . Figure 1 presents in graphic form an example

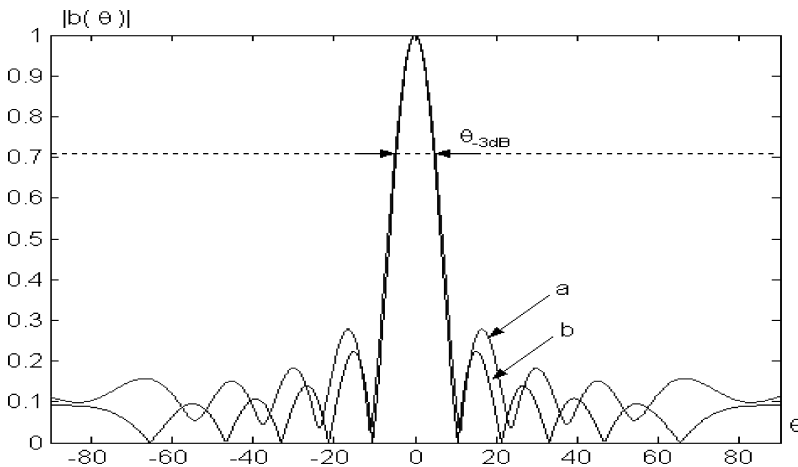


Fig. 1. Directivity pattern: a – sector of cylindrical array, b – chord of cylindrical array sector.

of a directivity pattern determined for the following parameters of an array:  $P = 11$ ,  $R/\lambda_0 = 3.2$ ,  $\alpha = 9^\circ$ . Presented in the same diagram is a directivity pattern of a chord of a cylinder section with the angle  $(P - 1)\alpha = 90^\circ$ . As it can be seen, the directivity pattern of the cylinder sector has for all practical purposes the same width and slightly higher level of side lobes.

Succeeding beams are generated using signals received by the transducers located in subsequent identical cylinder sectors, shifted by a multiple of the  $\alpha$  angle. Acoustic axes of these beams are also shifted by a multiple of the  $\alpha$  angle. Using the parameters referred to in Fig. 1, cylindrical array is composed of 40 columns and produces 40 beams covering round angle  $(40 \cdot 9^\circ = 360^\circ)$ . The width of each beam amounts to approximately  $10^\circ$ , thus they intersect each other at a level, which is slightly higher than  $-3$  dB.

Spectra  $B(f, \theta)$  in each beam are used for detecting received signals. Spectrogram  $|B(f, \theta)|^2$  are determined on their basis. Appearance of spectral lines in a beam of value significantly exceeding the average height of the lines of noise is interpreted as detection of an underwater sound source. Figure 2 presents an example of a spectrogram for the case of receiving two sinusoidal signals. Accepted, as a sound location bearing is the angle of beam axis in which the selected spectral line of the spectrogram is of the biggest height.

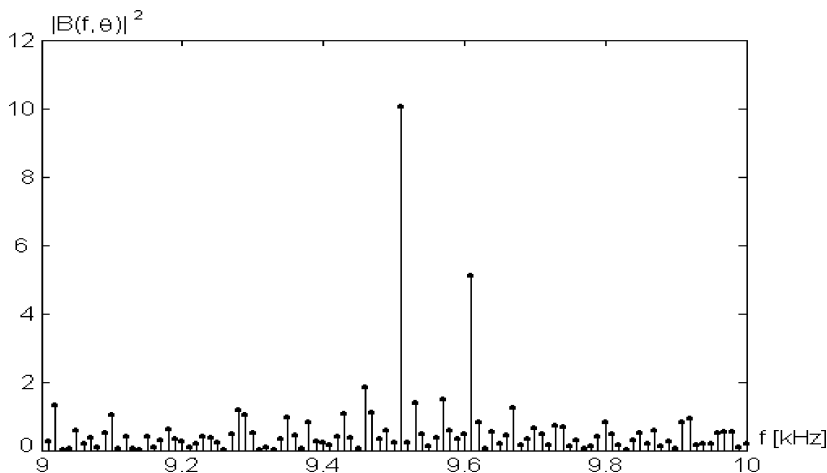


Fig. 2. Spectrogram of sinusoidal signals of 9.5 kHz and 9.6 kHz frequencies received with Gaussian noise.

### 3. Spatial spectrum estimation method

Employing highresolution methods of spatial spectrum estimation may increase accuracy of determining bearings, as well as angular resolution of sonar. However, methods described in the literature and used in practice, refer to linear arrays in which hydrophones are placed at equal distances. Such arrays ensure even distribution of acoustic field sampling on the surface of the array. Such way of sampling is employed both in

relation to estimation of spatial spectrum, as well as the spectrum of regular signals in time domain.

In cylindrical arrays hydrophones are placed on a circle, therefore the distribution of plane wave is not sampled uniformly from the point of view of linear spatial coordinates. The unevenness of spatial sampling may be decreased to some degree by using the beamforming described above. This is due to the fact that it introduces delays, which compensate the time of wave's transition from a given hydrophone to its perpendicular projection tangent on cylindrical surface passing through a central transducer. In this way we achieve a virtual linear array in which, however, the virtual hydrophones are still not distributed evenly. Nevertheless, with a small angular width of the active sector of the cylindrical array the differences in the distances between neighbouring virtual hydrophones are slight and one may hope for effective functioning of high-resolution methods of spectral estimation. This is proven by the numerical experiments presented below.

The signal out of which the spatial spectrum is determined consists of a series of spectral lines:

$$x(n, \theta_k) = \sum_{k=1}^K S_n(f_0, \theta_k) W_n(f_0) \quad \text{for } -N \leq n \leq N, \quad (9)$$

where  $\theta_k$  represents wave's angle of incidence, and the  $f_0$  stands for the frequency of selected spectral line whose size indicates detection of a sound source. It is most often the case that at a single frequency acoustic wave is emitted by just one source, therefore as a rule  $k = 1$ . Nevertheless, the spatial spectrum estimation methods allow for determining the bearings of a larger number of sound sources emitting a wave at the same frequency.

Based on the formulas (2), (6) and (7) it is seen that in receiving waves from a single sources, the sequence  $\{x(n, \theta_1)\}$  is a series of samples of a nearly sinusoidal distribution. The frequency of such distribution (spatial frequency) is dependant on the angle of incidence of waves and changes from 0 at perpendicular incidence to 1 at parallel incidence, provided that the distance  $d$  between neighbouring linear array hydrophones is equal to  $\lambda_0/2$  ( $\lambda_0 = c/f_0$ ). High-resolution methods of spatial estimation allow for precise determination of this frequency, and in consequence for precise determination of the direction of waves coming into the array.

Figure 3 presents a diagram of spatial spectrum determined in accordance with the Burg method with  $N = 11$  samples  $x(n, \theta_1)$ , where  $\theta_1 = 2^\circ$ . The spectrum is shown against the background of directivity pattern of cylinder sector out of which the  $\theta_1$  samples were obtained. Substantial increase in the accuracy of determined bearings is visible. It is more clearly demonstrated in Fig. 4, where the fragment of spatial spectrum in question is shown in magnification. The spatial spectrum was determined for the parameters of arrays given in the description of Fig. 1. In the Burg method, line of model 4 was accepted, and the spatial spectrum contains 1801 samples (on average  $0.1^\circ$ ).



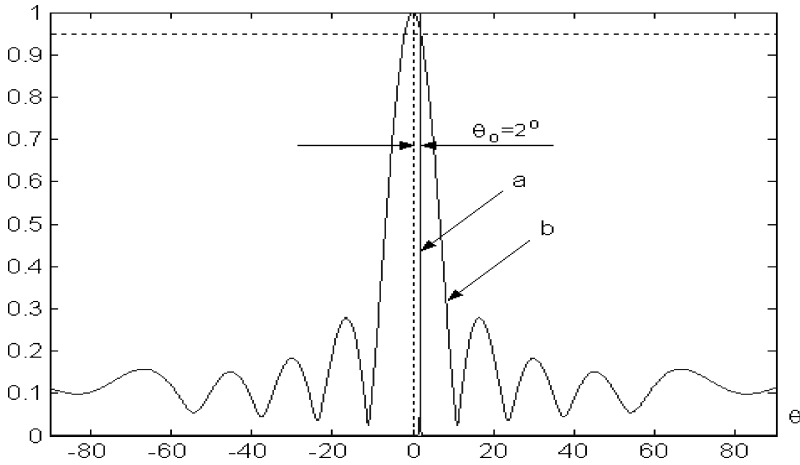


Fig. 3. Spatial spectrum determined using the Burg method (a) and the directivity pattern of cylinder sector (b).

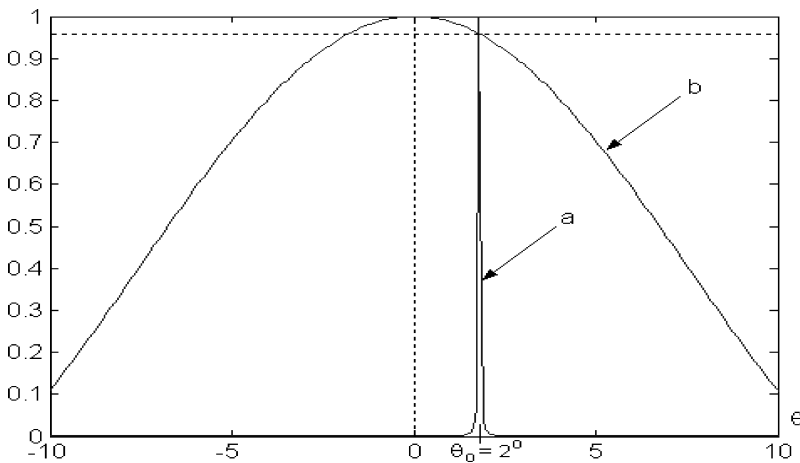


Fig. 4. Central fragment of spatial spectrum determined using the Burg method (a) and the directivity pattern of cylinder pattern (b).

High accuracy of determining the direction of wave's incidence is achieved only in the range of small angles covering the width of array beam. For larger angles there appears an error whose size increases with increase of the angle of incidence of waves – as illustrated in Fig. 5. In addition, the error increases when the ratio of signal to noise decreases, which is a typical effect in the methods of spatial spectrum estimation. With a small ratio of signal to noise and badly selected line of model false spectral lines appear. Their influence on estimating the direction of incidence of waves in the presented method is relatively insignificant since they are reduced by directivity pattern of the beamformer.

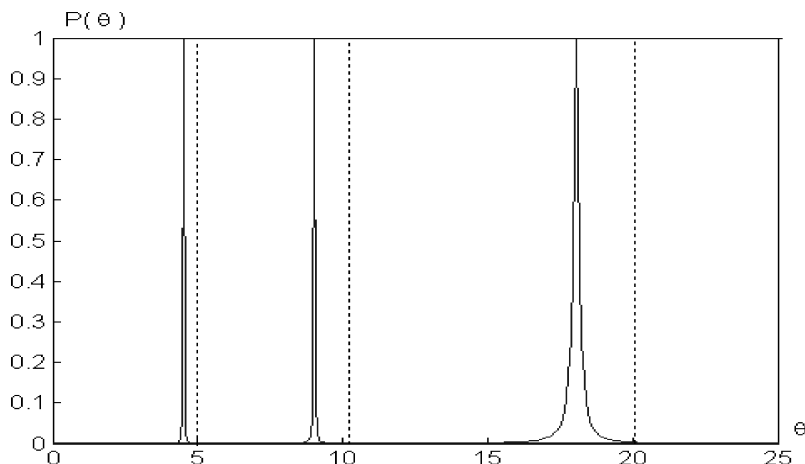


Fig. 5. Central fragment of spatial spectrum determined using the Burg method for wave incidence angles of  $5^\circ$ ,  $10^\circ$  and  $20^\circ$ .

#### 4. Conclusions

Receiver of sonar with a cylindrical array may be employed in detecting sound sources using a passive method. All that is required is a change in signal processing algorithms in the beamformer. Accuracy of determined bearings may be improved by using high-resolution algorithms of spatial spectrum estimation. They function properly within the width of sonar band without any significant modifications.

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