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THEORETICAL AND EXPERIMENTAL ASSESSMENT OF PARAMETERS FOR THE NON-LINEAR VISCOELASTIC MODEL OF STRUCTURAL POUNDING

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Impacts between adjacent structures during earthquakes have been recently intensively studied with the help of different models of the impact force. It has been verified through comparisons that the non-linear viscoelastic model is the most accurate one among them. One of the aims of the present paper is to derive a formula relating the impact damping ratio, as a parameter of the non-linear viscoelastic model, with a more widely used coefficient of restitution. Another aim is to determine the range of the coefficients of restitution and the impact stiffness parameters for different building materials, such as: steel, concrete, timber and ceramics, based on the results of an impact experiment. Both aims are new and original elements of the study in the field of earthquake-induced structural pounding. The results of the analysis show a wide range of the model parameters obtained for various prior-impact velocity and mass values. The use of these parametric values in numerical simulations allows us to study the behaviour of colliding structures with the increased accuracy.

Key words: structural pounding, earthquakes, non-linear viscoelastic model

1. Introduction

Interactions between insufficiently separated structures with different dynamic characteristics have been repeatedly observed during earthquakes. This phenomenon, often referred as the earthquake-induced structural pounding, may lead to some minor damage at contact locations in the case of moderate ground motions (see, for example, Zembaty *et al.*, 2005) and may result in substantial destruction or even collapse of interacting structures during severe earthquakes (see Rosenblueth and Meli, 1986; Kasai and Maison, 1997). The problem of earthquake-induced structural pounding has been recently intensively studied with the use of different models of the impact force. The simplest model applies a linear elastic spring (see, for example, Maison and Kasai, 1992) and does not take into consideration energy dissipation during impact due to plastic deformations, local cracking or crushing, fracturing, friction, etc. A more precise linear viscoelastic model (see Anagnostopoulos and Spiliopoulos, 1992; Jankowski *et al.*, 1998) accounts for some energy loss, but the force-deformation relation is still simplified. In order to simulate this relation more realistically, a non-linear elastic model, which follows the Hertz law of contact, has been adopted by a number of researchers (see Jing and Young, 1991; Chau and Wei, 2001). This model, however, does not account for the energy dissipation during contact.

In order to overcome the disadvantages of the models mentioned, a non-linear viscoelastic model of earthquake-induced structural pounding has been proposed by Jankowski (2005c). It has been verified through experiments that the model is the most precise one in simulating the impact force time history during an impact as well as in simulating the pounding-involved structural response during earthquakes. The model has been successfully used for studying earthquake-induced pounding between two adjacent multi-storey buildings (Jankowski, 2005b) as well as for the analysis of the pounding force response spectrum under earthquake excitation (Jankowski, 2005a).

According to the non-linear viscoelastic model, the impact force, F, between two structural members with masses m_1 and m_2 is expressed by the following formula (Jankowski, 2005c)

$$F = \begin{cases} 0 & \text{for} \quad \delta \leqslant 0 & \text{(no contact)} \\ \overline{\beta}\sqrt{\delta^3} + \overline{c}\dot{\delta} & \text{for} \quad \delta > 0 \text{ and } \dot{\delta} > 0 & \text{(contact – approach period)} \\ \overline{\beta}\sqrt{\delta^3} & \text{for} \quad \delta > 0 \text{ and } \dot{\delta} \leqslant 0 & \text{(contact – restitution period)} \end{cases} \tag{1.1}$$

$$\delta = x_1 - x_2 - d \qquad\qquad \overline{c} = 2\overline{\xi}\sqrt{\overline{\beta}\sqrt{\delta}\frac{m_1 m_2}{m_1 + m_2}}$$

where $\overline{\beta}$ is the impact stiffness parameter, $\overline{\xi}$ denotes the impact damping ratio, which accounts for the energy dissipation during the impact, x_1, x_2 are displacements of the structural members and d is the initial in-between separation gap. Although the above model has been proposed for earthquake-induced structural pounding, due to its general form it can be also successfully used to study impacts between other types of colliding bodies.

The precise determination of the parameters of the non-linear viscoelastic model: $\overline{\beta}$ and $\overline{\xi}$ is essential in order to enhance the accuracy of the numerical analysis. Therefore, one of the aims of this paper is to derive a formula relating the impact damping ratio with a coefficient of restitution, which is a parameter widely used and studied in the literature (Goldsmith, 1960). The analogous formula defined for the linear viscoelastic model has confirmed its applicability (see Anagnostopoulos and Spiliopoulos, 1992). Another aim of the paper is to assess the range of the coefficients of restitution and the impact stiffness parameters for different building materials, such as: steel, concrete, timber and ceramics, based on the results of an impact experiment. Both aims are new and original elements of the study conducted by the author in the field of earthquake-induced structural pounding.

2. The formula between the impact damping ratio and coefficient of restitution

The coefficient of restitution is a well-known parameter used in the classical theory of impact. It defines the relation between the post-impact relative velocity, $\dot{\delta}_f$ ($\dot{\delta}_f \leqslant 0$), and the prior-impact relative velocity, $\dot{\delta}_0$ ($\dot{\delta}_0 > 0$), of two colliding bodies (Goldsmith, 1960)

$$e = \frac{|\dot{\delta}_f|}{\dot{\delta}_0} \tag{2.1}$$

The formula for the relation between the impact damping ratio, $\overline{\xi}$, and the coefficient of restitution, e, for the non-linear viscoelastic model can be obtained by equating the loss in the kinetic energy (see Goldsmith, 1960) with the energy loss through the work done by the damping force during the impact

$$\frac{m_1 m_2}{2(m_1+m_2)}(1-e^2)(\dot{\delta}_0)^2 = \int\limits_0^{\delta_{max}} \overline{c}\dot{\delta}\, d\delta = 2\overline{\xi}\sqrt{\beta\frac{m_1 m_2}{m_1+m_2}} \int\limits_0^{\delta_{max}} \sqrt[4]{\delta}\dot{\delta}\, d\delta \tag{2.2}$$

where $\dot{\delta}$ is the relative velocity between colliding structures during the approach period ($\dot{\delta} > 0$) and δ_{max} denotes the maximum deformation. In order to determine the formula for $\dot{\delta}$ during the approach period (required to evaluate the integral of equation (2.2)) let us first look at the energy balance during the restitution period of collision, which is considered to be elastic (see equation (1.1)). Due to the energy transfer from the accumulated elastic strain energy at the beginning of the period to the kinetic energy at the end of it, the following condition holds for each value of deformation $\delta \in \langle 0, \delta_{max} \rangle$ in the restitution period

$$\int\limits_0^{\delta} \overline{\beta}\sqrt{\delta^3}\,d\delta + \frac{m_1 m_2}{2(m_1+m_2)}\dot{\delta}^2 = \frac{m_1 m_2}{2(m_1+m_2)}(\dot{\delta}_f)^2 \tag{2.3}$$

Solving the above equation allows us to determine the formula for the relative velocity, $\dot{\delta}$, during the restitution period $(\dot{\delta} \leqslant 0)$ as equal to

$$\dot{\delta} = -\sqrt{(\dot{\delta}_f)^2 - \frac{4\overline{\beta}(m_1+m_2)}{5m_1 m_2}\sqrt{\delta^5}} \tag{2.4}$$

Moreover, for the point of maximum deformation, when $\delta = \delta_{max}$ and $\dot{\delta} = 0$, from equation (2.3) we obtain

$$\delta_{max} = \sqrt[5]{\left(\frac{5m_1 m_2(\dot{\delta}_f)^2}{4(m_1+m_2)\overline{\beta}}\right)^2} \tag{2.5}$$

Assuming that a similar expression as equation (2.4) concerns also the approach period of collision and ensuring that the relation between the post-impact and prior-impact relative velocities, defined by equation (2.1), is satisfied, we can express the formula for the relative velocity, $\dot{\delta}$, during the approach period $(\dot{\delta} > 0)$ as

$$\dot{\delta} = \frac{1}{e}\sqrt{(\dot{\delta}_f)^2 - \frac{4\overline{\beta}(m_1+m_2)}{5m_1 m_2}\sqrt{\delta^5}} \tag{2.6}$$

Substituting equation (2.6) as well as the formula for $(\dot{\delta}_f)^2$ obtained from equation (2.5) into equation (2.2), yields

$$\frac{m_1 m_2}{2(m_1+m_2)}(1-e^2)(\dot{\delta}_0)^2 = \frac{4\sqrt{5}}{5}\frac{\overline{\xi}\,\overline{\beta}}{e}\int\limits_0^{\delta_{max}} \sqrt[4]{\delta}\,\sqrt{\sqrt{\delta_{max}^5} - \sqrt{\delta^5}}\,d\delta \tag{2.7}$$

The detailed evaluation of the integral of equation (2.7) has been presented in Appendix. Substituting equation (A.10), for $b = \delta_{max}$ and $c = \sqrt[4]{\delta_{max}^5}$, into equation (2.7) leads to

$$\frac{m_1 m_2}{2(m_1+m_2)}(1-e^2)(\dot{\delta}_0)^2 = \frac{4\sqrt{5}}{25}\pi\frac{\overline{\xi}\,\overline{\beta}}{e}\sqrt{\delta_{max}^5} \tag{2.8}$$

Substituting equations (2.5) and (2.1) into equation (2.8) and solving for $\overline{\xi}$ gives

$$\overline{\xi} = \frac{\sqrt{5}}{2\pi}\frac{1-e^2}{e} \tag{2.9}$$

3. Experimental determination of the coefficients of restitution and impact stiffness parameters for different building materials

The experimental study has been carried out in order to determine the range of the coefficients of restitution and the impact stiffness parameters for the most commonly used building materials, such as: steel, concrete, timber and ceramics. The experiment has been conducted by dropping balls of different masses, m_1, on a rigid surface $(m_2 \to \infty)$ of the same material and observing the impact force time histories as well as recording the prior-impact and the post-impact velocities. The properties of balls used in the experiment are specified in Table 1. The experimental setup is shown in Fig. 1.

Table 1. Properties of balls used in the experiment

Material	Type/grade/ class	Ball diameter [mm]	Ball mass, m_1 [kg]	No. of balls tested
Steel	18G2A	21	0.053 - 0.054	2
		50	0.538 - 0.541	2
		83	2.013	2
Concrete	C30/37	103	1.329 - 1.350	5
		114	1.763 - 1.835	5
		128	2.531 - 2.636	5
Timber	pinewood	55	0.065 - 0.066	2
		71	0.109 - 0.112	2
		118	0.493 - 0.497	2
Ceramics	25	58	0.243 - 0.247	2
		69	0.372 - 0.377	2
		80	0.538 - 0.572	2

From the experimental results concerning the prior-impact and the post-impact velocities, the coefficients of restitution, e, have been first calculated with the help of equation (2.1). The values of e for different prior-impact velocities are summarised in Table 2. It should be mentioned that due to exceeding the allowable acceleration limit, the steel balls could not be tested for the impact velocities higher than 2 m/s, and therefore these results are not given in the table. A graphical presentation of the relation between the mean value of the coefficient of restitution and the prior-impact velocity for different materials is also shown in Fig. 2. The results obtained indicate that

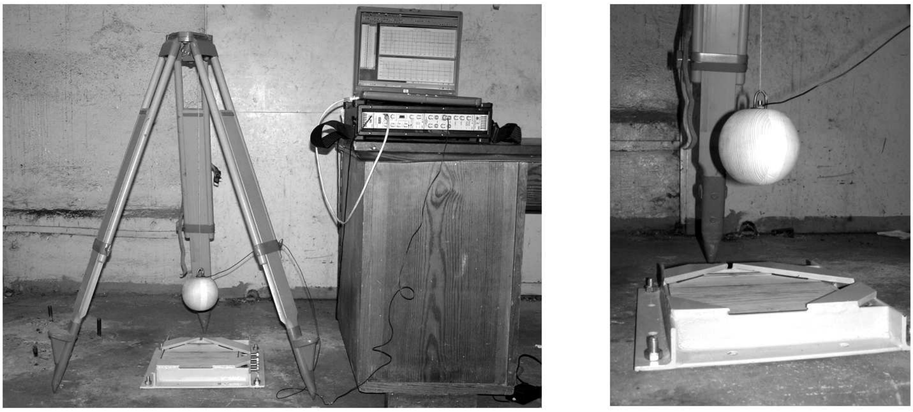

Fig. 1. Experimental setup

the value of e does not depend on the mass of the balls tested but it is much sensitive to the prior-impact velocity. The highest values of the coefficient of restitution have been obtained for ceramic balls, whereas the lowest for timber ones. Moreover, the general trend for all materials shows a decrease in the coefficient of restitution with an increase in the prior-impact velocity.

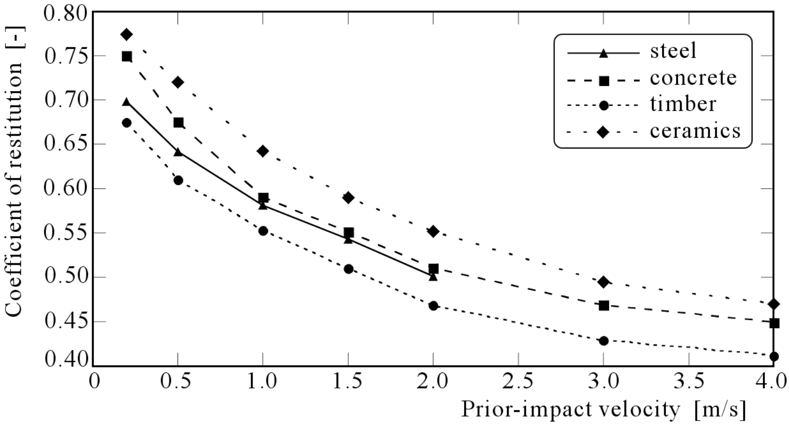

Fig. 2. Coefficient of restitution vs. prior-impact velocity

After determination of the coefficients of restitution, the impact stiffness parameters, $\overline{\beta}$, have been determined by fitting the experimentally obtained impact force time histories using the method of the least squares. The values of $\overline{\beta}$ for different masses of the balls tested are presented in Table 3. The results obtained indicate that the impact stiffness parameter does not depend

Table 2. Values of the coefficient of restitution, e, obtained from the experiment

Material	Prior-impact velocity, $\dot{\delta}_0$ [m/s]	Range of coefficient of restitution, e [–]
Steel	0.2	0.6415 - 0.7438
	0.5	0.5264 - 0.7142
	1.0	0.5140 - 0.6239
	1.5	0.4967 - 0.5652
	2.0	0.4863 - 0.5057
Concrete	0.2	0.7230 - 0.7840
	0.5	0.6580 - 0.6859
	1.0	0.5858 - 0.5981
	1.5	0.5378 - 0.5658
	2.0	0.4419 - 0.5659
	3.0	0.4311 - 0.5001
	4.0	0.4278 - 0.4826
Timber	0.2	0.6567 - 0.6934
	0.5	0.5852 - 0.6356
	1.0	0.5334 - 0.5797
	1.5	0.4851 - 0.5354
	2.0	0.4281 - 0.5347
	3.0	0.4076 - 0.4708
	4.0	0.4002 - 0.4428
Ceramics	0.2	0.7575 - 0.7996
	0.5	0.6870 - 0.7681
	1.0	0.6270 - 0.6697
	1.5	0.5770 - 0.6456
	2.0	0.5115 - 0.5642
	3.0	0.4737 - 0.5208
	4.0	0.4330 - 0.4921

on the prior-impact velocity but it shows a dependence on mass of the tested balls. The general trend for all tested materials shows a small increase in $\overline{\beta}$ with an increase in the ball mass. Moreover, in the case of the impact stiffness parameter, the highest values of $\overline{\beta}$ have been obtained for steel balls, whereas the lowest for timber ones.

Table 3. Values of the impact stiffness parameter, $\overline{\beta}$, obtained from the experiment

Material	Ball mass, m_1 [kg]	Impact stiffness parameter, $\overline{\beta}$ [N/m$^{3/2}$]	
		Range	Mean
Steel	0.053 - 0.054	$1.1 - 1.5 \cdot 10^{10}$	$1.30 \cdot 10^{10}$
	0.538 - 0.541	$2.4 - 4.4 \cdot 10^{10}$	$3.55 \cdot 10^{10}$
	2.013	$3.8 - 6.6 \cdot 10^{10}$	$5.44 \cdot 10^{10}$
Concrete	1.329 - 1.350	$3.9 - 10.0 \cdot 10^{9}$	$7.90 \cdot 10^{9}$
	1.763 - 1.835	$4.7 - 11.2 \cdot 10^{9}$	$8.13 \cdot 10^{9}$
	2.531 - 2.636	$6.4 - 13.0 \cdot 10^{9}$	$10.45 \cdot 10^{9}$
Timber	0.065 - 0.066	$0.7 - 1.8 \cdot 10^{8}$	$1.38 \cdot 10^{8}$
	0.109 - 0.112	$0.9 - 2.8 \cdot 10^{8}$	$2.16 \cdot 10^{8}$
	0.493 - 0.497	$1.0 - 5.2 \cdot 10^{8}$	$2.97 \cdot 10^{8}$
Ceramics	0.243 - 0.247	$1.1 - 2.3 \cdot 10^{9}$	$1.80 \cdot 10^{9}$
	0.372 - 0.377	$2.2 - 4.0 \cdot 10^{9}$	$3.13 \cdot 10^{9}$
	0.538 - 0.572	$2.8 - 5.8 \cdot 10^{9}$	$4.57 \cdot 10^{9}$

4. Concluding remarks

In this paper, the determination of parameters for the non-linear viscoelastic model of structural pounding has been carried out. The formula relating the impact damping ratio with a coefficient of restitution has been first derived. Then, values of the coefficients of restitution and impact stiffness parameters have been determined for different building materials based on the results of an impact experiment. The paper deals with new and original elements of the study conducted in the field of earthquake-induced structural pounding.

The results of the study show a wide range of parameters of the non-linear viscoelastic model determined for steel, concrete, timber and ceramics for various prior-impact velocities and masses of balls tested. The application of the obtained parametric values to numerical simulations allows us to study the behaviour of colliding structures with increased accuracy.

In this paper, the results of the experiment conducted by dropping relatively small balls on a rigid surface have been used. Further experimental studies involving larger elements with different contact surface geometries are therefore required to verify the results obtained. The confirmation of the range of parameters of the non-linear viscoelastic model should also be done through experiments of pounding between models of real structures conduced on a shaking table under real earthquake excitations.

The use of the results of the study presented in this paper does not have to be limited to simulation of pounding-involved behaviour of structures during earthquakes. They can be also applied to study impacts between different types of colliding bodies in other conditions.

A. Appendix

The present appendix shows the evaluation of the definite integral of the form

$$\int_0^b \sqrt[4]{\delta}\,\sqrt{c^2-\sqrt{\delta^5}}\;d\delta \tag{A.1}$$

where b and c are positive constants and $c^2 \geqslant \sqrt{\delta^5}$.

Let us start the evaluation by making a substitution, $\sqrt[4]{\delta}=y$. Then, we can write

$$\int_0^b \sqrt[4]{\delta}\,\sqrt{c^2-\sqrt{\delta^5}}\;d\delta = 4\int_0^{\sqrt[4]{b}} y^4\sqrt{c^2-y^{10}}\;dy \tag{A.2}$$

After making the second substitution, $y^5=t$, we receive (for $c>0$)

$$\int_0^b \sqrt[4]{\delta}\,\sqrt{c^2-\sqrt{\delta^5}}\;d\delta = \frac{4}{5}\int_0^{\sqrt[4]{b^5}} \sqrt{c^2-t^2}\;dt = \frac{4}{5}c\int_0^{\sqrt[4]{b^5}} \sqrt{1-\Big(\frac{t}{c}\Big)^2}\;dt \tag{A.3}$$

The next substitution, $t/c=z$, gives

$$\int_0^b \sqrt[4]{\delta}\,\sqrt{c^2-\sqrt{\delta^5}}\;d\delta = \frac{4}{5}c^2\int_0^{\frac{\sqrt[4]{b^5}}{c}} \sqrt{1-z^2}\;dz \tag{A.4}$$

Let us now try to evaluate the integral of the right-hand side of equation (A.4). Note that

$$\int_0^{\frac{\sqrt[4]{b^5}}{c}} \sqrt{1-z^2}\;dz = \int_0^{\frac{\sqrt[4]{b^5}}{c}} \frac{1-z^2}{\sqrt{1-z^2}}\;dz = \int_0^{\frac{\sqrt[4]{b^5}}{c}} \frac{1}{\sqrt{1-z^2}}\;dz - \int_0^{\frac{\sqrt[4]{b^5}}{c}} z\frac{z}{\sqrt{1-z^2}}\;dz \tag{A.5}$$

Indeed

$$\begin{aligned}\int\limits_0^{\frac{\sqrt[4]{b^5}}{c}} \sqrt{1-z^2}\,dz &= \arcsin z\Big|_0^{\frac{\sqrt[4]{b^5}}{c}} - \int\limits_0^{\frac{\sqrt[4]{b^5}}{c}} z\frac{z}{\sqrt{1-z^2}}\,dz = \\ &= \arcsin\frac{\sqrt[4]{b^5}}{c} - \int\limits_0^{\frac{\sqrt[4]{b^5}}{c}} z\frac{z}{\sqrt{1-z^2}}\,dz\end{aligned} \tag{A.6}$$

In order to evaluate the integral of the right-hand side of equation (A.6), let us apply the method of integrating by parts. It yields

$$\int\limits_0^{\frac{\sqrt[4]{b^5}}{c}} \sqrt{1-z^2}\,dz = \arcsin\frac{\sqrt[4]{b^5}}{c} - \Big(-z\sqrt{1-z^2}\Big|_0^{\frac{\sqrt[4]{b^5}}{c}} + \int\limits_0^{\frac{\sqrt[4]{b^5}}{c}} \sqrt{1-z^2}\,dz\Big) \tag{A.7}$$

So

$$\int\limits_0^{\frac{\sqrt[4]{b^5}}{c}} \sqrt{1-z^2}\,dz = \arcsin\frac{\sqrt[4]{b^5}}{c} + \frac{\sqrt[4]{b^5}}{c}\sqrt{1-\frac{\sqrt{b^5}}{c^2}} - \int\limits_0^{\frac{\sqrt[4]{b^5}}{c}} \sqrt{1-z^2}\,dz \tag{A.8}$$

Hence

$$\int\limits_0^{\frac{\sqrt[4]{b^5}}{c}} \sqrt{1-z^2}\,dz = \frac{1}{2}\Big(\arcsin\frac{\sqrt[4]{b^5}}{c} + \frac{\sqrt[4]{b^5}}{c^2}\sqrt{c^2-\sqrt{b^5}}\Big) \tag{A.9}$$

Substituting the above into equation (A.4), finally gives

$$\int\limits_0^b \sqrt[4]{\delta}\,\sqrt{c^2-\sqrt{\delta^5}}\,d\delta = \frac{2}{5}\Big(c^2\arcsin\frac{\sqrt[4]{b^5}}{c} + \sqrt[4]{b^5}\,\sqrt{c^2-\sqrt{b^5}}\Big) \tag{A.10}$$

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Analityczne i eksperymentalne szacowanie wartości parametrów nieliniowego lepkosprężystego modelu zderzeń pomiędzy konstrukcjami budowlanymi

Streszczenie

Zjawisko zderzeń pomiędzy sąsiednimi konstrukcjami budowlanymi podczas trzęsień ziemi jest w ostatnim czasie intensywnie badane z wykorzystaniem różnych modeli numerycznych siły zderzenia w czasie kontaktu. Wyniki badań eksperymentalnych pokazują, iż nieliniowy model lepkosprężysty jest najdokładniejszy wśród modeli stosowanych do analizy. Model ten zdefiniowany jest poprzez dwa parametry: liczbę tłumienia zderzenia oraz parametr sztywności zderzenia. Jednym z celów niniejszego artykułu jest wyprowadzenie wzoru na zależność pomiędzy liczbą tłumienia zderzenia a współczynnikiem odbicia, który jest parametrem często stosowanym i opisywanym w literaturze. Kolejnym celem jest wyznaczenie zakresu wartości współczynników odbicia i parametrów sztywności zderzenia dla różnych materiałów budowlanych (stali, betonu, drewna i ceramiki) na podstawie wyników badań eksperymentalnych. Wyniki analizy pokazują szeroki zakres wartości parametrów nieliniowego modelu lepkosprężystego zderzeń otrzymanych dla różnych wartości prędkości zderzenia oraz masy testowanych elementów. Zastosowanie tych wartości w symulacjach numerycznych prowadzi do zwiększenia dokładności uzyskiwanych wyników w analizie zjawiska zderzeń pomiędzy konstrukcjami budowlanymi podczas trzęsień ziemi.

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