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Simulation of incremental encoder signals

Abstract. *In this paper the pulse signal generator for the simulation of signals of incremental encoder in a transient state is presented. The algorithm for determining the intervals of pulses for three modes variation of rotational speed: linear, exponential and sinusoidal is discussed. Counting errors due to digital realization of the generator is analyzed.*

Streszczenie. *W artykule przedstawiono generator sygnału impulsowego do symulacji sygnału z przetwornika obrotowo-impulsowego w stanach przejściowych. Omówiono algorytmy wyznaczenia przedziałów międzyimpulsowych dla trzech rodzajów zmian prędkości obrotowej: liniowej, wykładniczej oraz sinusoidalnej. Przeanalizowano błędy kwantowania wynikające z cyfrowej realizacji generatora.* ***(Symulacja sygnałów z przetwornika obrotowo-impulsowego)****.*

Keywords: rotational speed, incremental encoder, pulse signal, counting error.
Słowa kluczowe: prędkość obrotowa, przetwornik obrotowo-impulsowy, sygnał impulsowy, błąd kwantowania.

Introduction

Pulse frequency modulation (PFM) is often used in measuring channels [1]. In this case signals are more resistant against disturbances and their conversion into digital form is easy and simple in realization. It is possible to use different kinds of transducers with the direct conversion x/f and indirect conversion x/U/f or x/P/f (where P is another quantity than voltage, e.g. inductance or capacitance) [2]. An incremental encoder belongs to the group of transducers with direct conversion.

Incremental encoders are commonly used in the measuring channels to define a rotational speed or to control driving systems with rotating machines. In the case of investigating driving systems it is necessary to use the signals of incremental encoder in defined modes of work. However it is not recommended to use the control signals directly from the incremental encoder because it is very difficult to form these signals in a strictly determined mode.

Therefore the special method was prepared by the authors. It enables the generation of pulse signals similar to the signal of incremental encoder in the case of rotational speed changes according to the characteristic often existed in practice.

Assumption of method

The digital measurements of rotational speed most frequently use optical transducers which convert rotation speed into pulses [3, 4]. The transducer consists of a disc fixed to a shaft. The disc has many slots positioned around its circumference [5]. On one side of the disc is a source of light on the other is a photosensitive element. The rotating disc periodically interrupts luminous flux, therefore the electrical output signals of photoelectric transducer depend on the changes of velocity of the disc.

The number of pulses is equal to the number of slots, which cut across the field of luminous flux. As the photoelectric sensor demands a great deal of luminous energy, the individual slots must be adequately large. Between the source of light and the disc an additional mask with slots of the same pitch as the slots on the disk is fixed, in order to prevent the reduction of the sensor's resolution. In this case the luminous flux passes simultaneously through several slots and enables the width of individual slot to diminish. Then it is possible to increase the number of slots along the disc circumference.

The generation of pulse signals of the incremental encoder for registration rotational speed was presented in [6]. The instant t_i of successive pulse N_i can be calculated from the relation of the mean value of rotational speed ω_{mi} in the interval of t_i - t_{i-1} (Fig. 1).

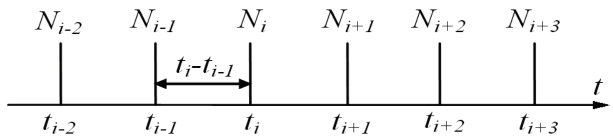

Fig.1. Determination instant t_i of pulse N_i

The mean value of rotational speed is calculated as the ratio of the angel displacement of the encoder's disc and the time of this displacement. In the interval $t_i - t_{i-1}$ of two pulses N_{i-1} and N_i the disc of encoder rotates about $1/C_e$ of revolution, where C_e is the sensitivity of incremental encoder defined by the number of pulses corresponding to one revolution of the disc. The mean value of rotational speed in this interval is determined by equation

$$\omega_{mi} = \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \omega(t)dt = \frac{\frac{1}{C_e}}{t_i - t_{i-1}} \tag{1}$$

Substituting for $\omega(t)$ functional relation of rotational speed, after transforming, it is possible to determine the instant t_i of pulse N_i.

Determining the position of pulses for linear changes of rotational speed

In this case let's assume that the rotational speed changes in linear mode, from ω_1 to ω_2 in the time T_a

$$\omega = \omega_1 + \frac{t}{T_a}\left(\omega_2 - \omega_1\right) \tag{2}$$

The mean value of rotational speed in the interval of two output pulses of incremental encoder from t_{i-1} to t_i, according to (1) is equal

$$\frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \left[\omega_1 + \frac{t}{T_a}\left(\omega_2 - \omega_1\right)\right]dt = \frac{\frac{1}{C_e}}{t_i - t_{i-1}} \tag{3}$$

After transformation one obtains the equation of second order

$$\frac{\omega_2 - \omega_1}{2 \cdot T_a} \cdot t_i^2 + \omega_1 \cdot t_i - \omega_1 \cdot t_{i-1} - \frac{\omega_2 - \omega_1}{2 \cdot T_a} \cdot t_{i-1}^2 - \frac{1}{C_e} = 0 \tag{4}$$

This equation has two resolutions, but only one t_i has positive value

$$t_i = \frac{-\omega_1 + \sqrt{\omega_1^2 + 4\frac{\omega_2 - \omega_1}{2 \cdot T_a}\left(\omega_1 t_{i-1} + \frac{\omega_2 - \omega_1}{2 \cdot T_a} t_{i-1}^2 + \frac{1}{C_e}\right)}}{\frac{\omega_2 - \omega_1}{T_a}} \tag{5}$$

Assuming the position of the first pulse in the instant $t = 0$, according to the equation (5), one determines the position of the next pulse t_i, which in the successive calculation is treated as the previous position t_{i-1}.

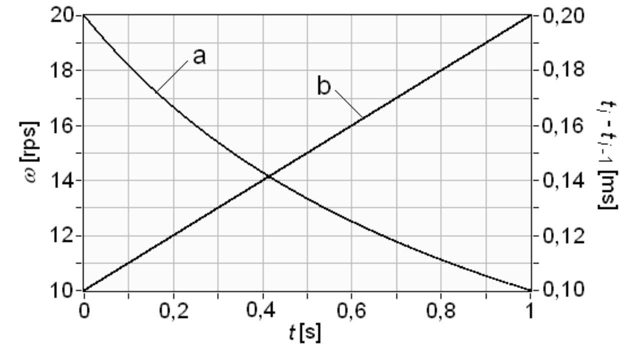

Fig.2. Values of intervals (t_i-t_{i-1}) between pulses t_i and t_{i-1} (a) and determined on their base rotational speed ω versus time in the case of linear changes of rotational speed (b)

Figure 2 presents hypothetical results of the simulation measurement using the incremental encoder with sensitivity C_e = 500 pulses for 1 revolution, assuming the linear changes of rotational speed from the value of ω_1 = 10 rps to ω_2 = 20 rps in the time T_a = 1 s. The diagram presents values of intervals between pulses t_i - t_{i-1} and evaluated on their base rotational speed ω as a function of time in the case of linear changes of rotational speed.

Determining the position of pulses for exponential changes of rotational speed

Often the rotational speed changes exponentially. It was assumed the changes of rotational speed from the value ω_1 to ω_2 were in accordance to the exponential function at base e

$$\omega = \omega_1 + (\omega_2 - \omega_1) \cdot \left(1 - e^{-\frac{t}{T_w}}\right) \tag{6}$$

Substituting (6) to formula (1) for the mean value of rotational speed between two successive output pulses of incremental encoder (from t_{i-1} to t_i), after transformation one obtains

$$\omega_2 (t_i - t_{i-1}) + (\omega_2 - \omega_1) \cdot \left(e^{-\frac{t_i}{T_w}} - e^{-\frac{t_{i-1}}{T_w}}\right) T_w - \frac{1}{C_e} = 0 \tag{7}$$

For specified t_{i-1}, one calculates the position of successive pulse t_i from formula (7). Hypothetical results of simulation are presented in figure 3. The results are obtained using the incremental encoder with C_e = 500 pulses/revolution, assuming the exponential changes of rotational speed from ω_1 = 10 rps to ω_2 = 20 rps and T_w = 0,1 s.

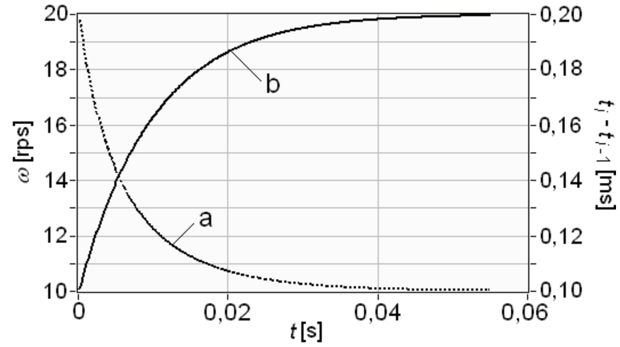

Fig.3. Values of intervals (t_i-t_{i-1}) between pulses t_i and t_{i-1} (a), and determined on their base rotational speed ω (b) versus time in the case of exponentials variations of rotational speed

Determining the position of pulses for sinusoidal variation of rotational speed

Very often the sinusoidal signal is used as the test signal. In this case measured rotational speed is defined as

$$\omega = \omega_0 + \omega_m \sin(2\pi f_p t) \tag{8}$$

where the steady rotational speed ω_0 is modulated by the signal with amplitude ω_m and frequency f_p.

The mean value of the speed between two successive output pulses of rotational-pulses sensor in the interval from t_{i-1} to t_i is equal [7]

$$\frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \left[\omega_0 + \omega_m \sin(2\pi f_p t)\right] dt = \frac{\frac{1}{C_e}}{t_i - t_{i-1}} \tag{9}$$

After transformation it is possible to obtain the following equation

$$\omega_0 t_i - \omega_0 t_{i-1} + \frac{\omega_m \cos(2\pi f_p t_{i-1})}{2\pi f_p} - \frac{\omega_m \cos(2\pi f_p t_i)}{2\pi f_p} - \frac{1}{C_e} = 0 \tag{10}$$

From equation (10) one calculates the instances of appearance for the output pulses from the incremental encoder.

Hypothetical results of simulation are presented in figure 4. The results were obtained using the incremental encoder with C_e = 500 pulses/revolution in the case of changes of rotational speed in sinusoidal mode, where the constant component ω_0 = 10 rps is modulated by the component with amplitude ω_m = 2 rps and frequency f_p = 50 Hz.

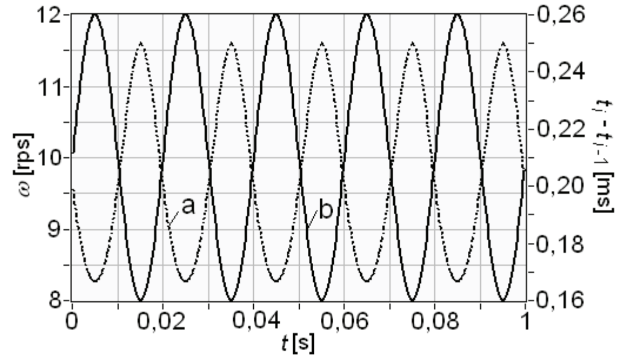

Fig.4. Values of intervals (t_i-t_{i-1}) between pulses t_i and t_{i-1} (a) and calculated on their base rotational speed ω (b) versus time in case of sinusoidal changes of rotational speed

Counting errors

Simulated pulse signals are generated by a digital system. The intervals between successive pulses are created by counting the periods of the standard generator.

Therefore the real intervals between pulses can be distinguished from the values determined in the theoretical way. Relative error is calculated from the formula (11)

$$\delta_i = \frac{\dfrac{round\left(\left(t_i - t_{i-1}\right) f_g\right)}{f_g} - \left(t_i - t_{i-1}\right)}{t_i - t_{i-1}} \tag{11}$$

where $round(x)$ is rounding of number x to the closest integer.

The measurement of the interval of pulses starts in the instant of generation of the signal synchronized with pulses of standard generator. Therefore the counting error is not greater than the half of inverse of the minimal number of the counted periods of signal from the standard generator

$$\delta_{\max} = \frac{1}{2 \cdot C_{i\min}} = \frac{1}{2 \cdot \left(t_i - t_{i-1}\right)_{\min} \cdot f_g} = \frac{C_e \cdot \omega_{\max}}{2 \cdot f_g} \tag{12}$$

For linear and exponential changes of rotational speed maximal value of counting error is calculated from the formula

$$\delta_{\max} = \frac{C_e \cdot \max\left(\omega_1, \omega_2\right)}{2 \cdot f_g} \tag{13}$$

Figure 5 presents the distribution of the counting errors in the case of exponential changes of rotational speed. The value of maximal error $\delta_{\max}$, calculated according to formula (13) is 0,025%.

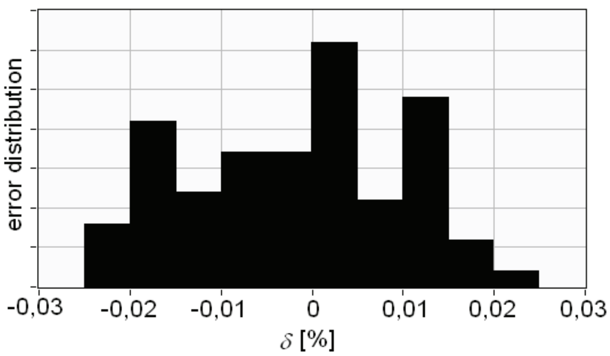

Fig.5. Distribution of counting error in the case of exponential changes of rotational speed

For sinusoidal changes of rotational speed maximal counting error δ_{max} is calculated from formula

$$\delta_{\max} = \frac{C_e \cdot \left(\omega_0 + \omega_m\right)}{2 \cdot f_g} \tag{14}$$

Pulse signal generator

The generator of pulse signal can be in the form of virtual instrument or as autonomic microprocessor device. Using the microprocessor instrument it is necessary to record the table with the values of periods in the memory. Therefore if the generator can be a stationary instrument, more convenient it is to apply the computer equipped with acquisition module and time-counter element. It enables the calculation of the table of periods in the normal course and sends its values to counting circuit.

In the performed investigation the acquisition module of National Instrument with LabVIEW software and controllers DAQmx were used [8].

The subVI *NI-DAQmx Channel Property Node* [9] controls the period of generated signal, whilst the program was being executed. Its two inputs are fed by following signals: the time of high state and low state in the definite period or the frequency of signal and it filling factor in the definite period. In practice the pulses intervals t_k are usually set from earlier established algorithms for the incremental encoder with two time greater resolution C_e. Then from two contiguous pulses intervals a single period for the time of high state t_{kh} and low state t_{kl} or frequency f_k and filling factor α_k are calculated

$$t_{kh} = t_{2k-1} \quad , \quad t_{kl} = t_{2k} \tag{15}$$

$$f_k = \frac{1}{t_{2k-1} + t_{2k}} \quad , \quad \alpha_k = \frac{t_{2i-1}}{t_{2k-1} + t_{2k}} \tag{16}$$

Conclusions

The presented method of simulation signals of the incremental encoder in the transient state:

- enables signals simulation for changes of rotational speed according to assumed functions,
- can be implemented into the system with the computer equipped with acquisition module of measured signals,
- can be used for the test of measuring systems or control of driving systems.

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