MODELING OF NONLINEAR GENERATION IN A BUBBLE LAYER

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The paper presents a numerical approach to modelling the linear and nonlinear scattering at a layer of bubbles. Numerical studies of noncoherent scattering using a Monte Carlo method that can cope with various effects of propagation inside of a bubbles layer such as boundary reflections, scattering anisotropy, sound dispersion, sound attenuation and time dependence are performed. Based on Rayleigh-Plesset bubble's oscillation equation, the propagation and backscattering of signals at bubble population of different size spectra is analyzed. Propagated through the layer and backscattering changes in the signal spectra are studied and modeled according to the properties of bubble populations similar as in the sea.

INTRODUCTION

The backscattering and transmission properties of high amplitude waves propagating through the bubble medium are of interest in a wide range of technical applications. For example, nonlinearity of water, however small, is commonly exploited in parametric sonars for the generation of nonlinear components.

In their paper Druzhinin *et al.*, [2] suggested that the efficiency of nonlinear generation by bubbly liquid enclosed in a homogenous layer could be improved using the layer's resonances at basic and difference frequencies. The bubble population size distribution and/or the operating frequencies had been chosen so that the largest part of individual bubble's radial resonances were much higher than the driven signal frequencies.

In the further more accomplished analysis, performed by Karpov *et al.* [4], on the basis of the assumption on adiabatic process of a single bubble oscillations (the isothermal process was included in the model of [2]) , the increased effectiveness of low frequency generation was also predicted.

However Karpov *et al*. were less eager than their predecessors [2] on the success in use of the layer resonances as the remarkably efficient source of nonlinearity, predicting that a difference wave power generation is only of the order of 1% of the incident power for incident wave amplitudes of the order 50 kPa.

Ostrovskii and others [6] had carried out an experimental work using the layer resonance limited to a low-frequency signal and combined with the highly nonlinearity of the medium by the bubble population widely distributed in radii. The nonlinearity of the medium was owed to bubbles at resonance with primary frequencies. The outcome of the experiment was not comparable to the results of the both mentioned theoretical works.

The objectives of the presented work are :

(i) to construct a model which could carry out evaluation of nonlinear generation in a bubbly liquid layer with different form of bubble size spectra, at high acoustic pressure amplitudes, when the bubble oscillation is essentially nonlinear.

(ii) to test theoretically the method in order to demonstrate that layered medium provides an efficient generator of nonlinear components.

(iii) to estimate the output of nonlinear component pressure in the regime of nonlinear oscillations, using Rayleigh-Plesset equation at different acoustic pressure amplitudes.

In the presented paper we make an assumption that there is no interaction between bubbles, and the reradiated acoustic field is merely the sum of primary waves energy scattered (reradiated) by all the bubbles; what is referred to as the single-scattering approximation.

Transmission and reflection coefficients for the layer as well as acoustic pressure inside of the layer were calculated for a variety of bubble size spectra and their concentrations.

1. SOUND VELOCITY AND ATTENUATION IN BUBBLY MEDIA

It is well known that the presence of bubbles in liquid introduces sound speed dispersion and significantly reduces the wave speed, at higher concentrations making it much lower than the sound speed either in pure water and even below the sound speed in the air. In literature there exist a few known formulae for prediction of sound speed in a bubbly medium. Among them a used commonly by many authors (Hall [3] as the example) procedure proposed by Commander and Prosperetti [1]. The Commander-Prosperetti formula allows to compute the phase sound speed and attenuation in a bubbly liquid on the basis of a concentration of bubbles and their size spectrum.

In the presented model, more complex, including second-order interactions of bubbles formula according to Ye and Ding [8] was used in computations.

The phase speed (c_{eff}) and amplitude attenuation of acoustic waves are calculated from the effective complex wave number κ, in the gas-liquid mixture:

$$\kappa^2 = k^2 + 4\pi A\left(1 - i\frac{2\pi B}{k}\right) \tag{1}$$

where $\frac{\tilde{k}}{k}=\frac{c_0}{c_{eff}}$; c_{eff} – complex sound speed in a bubbly liquid, c_0 - sound speed in the pure liquid, and the real part of c_{eff} – phase sound speed in the bubbly mixture and imaginary part represents the sound attenuation in Np/m,

$$A=\int\frac{n(a)a\,da}{\omega_0^2/\omega^2-1+i\delta} \quad \text{and} \quad B=\int\frac{n(a)a^2\,da}{\left(\omega_0^2/\omega^2-1+i\delta\right)^2};$$

$n(a)$ - the number of bubbles per unit volume (m^{-3}) with radii a in da=1 μm range;

ω_0 - the resonant (angular) frequency of the bubble with the radius a_r, $\omega=2\pi f$, f- sounding frequency. For the weak nonlinear case the simplest relationship between resonance frequency of a bubble and its radius could be used in the Minnaert form - $f_r=\frac{1}{2\pi a_r}\sqrt{3\gamma 10^5(1+0.1z)/\rho}$; here κ-polytropic exponent of gas, z - depth,

δ - the damping coefficient for the bubble, generally a function of the ambient pressure, frequency and bubble radius defined as a sum of the three terms:

$$\delta=\delta_{rad}+\delta_{visc}+\delta_{th}$$

where δ_{rad} - is the damping coefficient component caused by reradiation of acoustical energy by excited bubble; δ_{visc} is the damping coefficient due to the viscosity of the surrounding liquid; δ_{th} - the thermal damping coefficient due to heat conduction.

When in Eq. 1 we put B=0, we have the Commander - Prosperetti (C-P) case. Computations show that for some relations between sounding signal frequencies and bubble size spectra (characteristic for bubble plumes under breaking wind waves or near-sea-surface bubble layer) the higher-order correction could significantly alter the results comparing to C-P procedure for both sound speed dispersion and attenuation.

The above formula should be used warily for the case of the high amplitude signals, when the sound propagation at the frequencies under the consideration in the medium is far from the linear waves theory.

Below, the comparison of results given by formula Ye and Ding versus Commander and Prosperetti is presented in Fig.1. The evaluation of the sound speed in bubble medium and attenuation was accomplished here for continuous bubbles size spectrum presented by functional dependence $n(a)=N a^{-m}$ in some range of bubble radii $a>a_{min}$ and $a<a_{max}$. N – normalization factor follows from the void gas fraction.

As expected, the results show that the sound speed decreases with frequency, however in different manner and at low frequencies the rather large discrepancy is observed. It is also worth noting that in the frame of the Ye and Ding theory, there exists a negative spike near 8 kHz.

At high frequencies, the sound speed curves for the both formula, with only a slight difference are close to each other.

The Ye and Ding's model, in principle, should be superior to the simpler, not accounting between bubbles interactions, model of Commander and Prosperetti. However, it yields quite different, and sometimes non-physical results, especially, for not smooth bubble size distribution function. It is important to note, that the comparison with low frequency approximation for sound speed proposed by Wood or Clay and Medwin formula [5] also gives evident differences.

The computation of the sound speed and attenuation coefficients have shown that the variation of bubble size spectra inside the layer may have a great influence on the both parameters. Even only small changes in the spectra shape, produce a discernible difference in the results. This indicates that the proper use of the bubbles size function is important in the modeling of the prediction of nonlinear generation inside the layer.

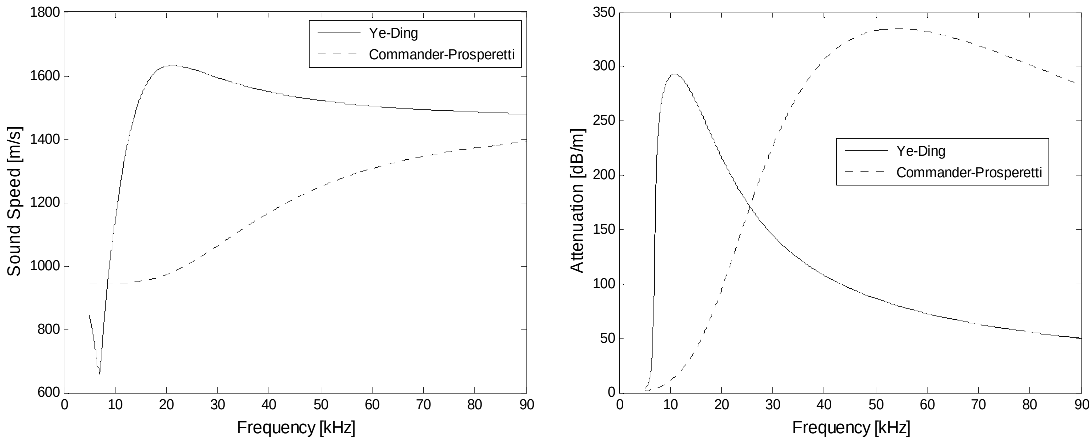

Fig.1. Theoretical estimations of phase sound speed dispersion and attenuation in bubbly water estimated on the basis of the Commander and Prosperetti theory (single scattering approximation) and on the basis of multiple scattering theory by Ye and Ding. Void fraction $V=10^{-5}$. Bubble size spectrum in the form $n(a) \sim a^{-m}$, m=3; a_{min}=50 µm, a_{max}=500 µm

2. BUBBLE SCATTERING MODEL

For prediction of radius changes of a bubble, driven by a two frequency signal, the version of Rayleigh-Plesset equation is used in the form -

$$\rho R\ddot{R} + \tfrac{3}{2}\rho\dot{R}^2 = p_g\left(\frac{a_\circ}{R}\right)^{3\kappa} + p_v - p_{stat} - \frac{2\sigma}{R} - \delta\omega R\dot{R} - p(t)\,, \qquad (2)$$

where: R=R(t) instant bubble radius, ρ - mean fluid density, p_{stat} - ambient static pressure, σ-surface tension coefficient, p_v - gas and vapor pressure inside a bubble, p(t) - incident signal acoustic pressure; $p_g = 2\sigma/R_n + p_{stat} - p_v$. The thermal, acoustical and viscous effects are included into the Rayleigh equation through the δ.

The following steps were performed using MATLAB routines :

- Firstly the time series of $R(t)$ and $\dot{R}(t)$ was calculated using the adaptive Runge-Kutta algorithm of order 4/5.
- Using the interpolation procedure the new time series of R(t) and $\dot{R}(t)$ at the equidistant time intervals were computed. Where dt's intervals should fulfil the Nyquist's frequency.
- The new $R(t_i)$ and $\dot{R}(t_i)$ time series after interpolation are used to calculate the spectra.

The emitted acoustic pressure as the function of the time series could be obtained from numerical solution of the R-P equation through radius, velocity of the bubble wall and the acceleration [9].

$$p_s(r,t) = \frac{\rho_0}{r} R(t)\left(2\dot{R}(t)^2 + R(t)\ddot{R}(t)\right) \tag{3}$$

The scattered field is represented by a sum of spherical waves:

$$p(r;\omega_1,\omega_2.|\omega_1 - \omega_2|) = \sum_j \frac{p_s(\omega_1,\omega_2.|\omega_1 - \omega_2|)}{r_j} \exp\left(i\left(\tilde{k}r_j - \omega_{1,2,d}t + \varphi_j\right)\right); \tag{4}$$

where $\tilde{k}$ is the complex wave vector, $\tilde{k} = k + i\alpha$, inside of the layer, and $\tilde{k} = k$ in the liquid; the spherical wave origin is placed at the position of a bubble, r_j is a length of a position vector.

The spectra of the emission from the bubble are obtained using a FFT analysis of the time series of acoustic emission. In the FFT analysis, no windowing was employed.

3. LAYER ACOUSTICS

The bubble population with the different size spectra, uniformly distributed in the layer are considered in the model, and the incoherent scattering which is the subject of the investigations is appropriate only for the case of the random scatterers distribution.

The bubbles pulsations are assumed to be small and only at the zero radial mode (volume oscillations, of individual bubbles). The compressibility of the liquid has been neglected.

The adiabatic process of gas oscillations inside a bubble and the influence of the surface tension on the process are included in the R-P equation.

When, reflections between boundaries of the layer are incorporated, the driving signal is increasing in time at the beginning of the signal and with tail at the end, when straight signal is zeroing. Here, we put forth also that inside the bubble medium the multiple interactions of the acoustic waves with a single bubble be considered only for a wave at the primary frequencies reflected between walls of the layer. For this assumption to be valid, should be $n^{2/3}\sigma_S << 1$ and $n\sigma_S/k << 1$ where σ_S is the scattering cross-section of a bubble and n is bubble density [1].

The nonlinearity of the acoustic field is only a result of the nonlinear behavior of oscillating bubbles driven by high pressure amplitude signals. To avoid appearing of shock waves only relatively thin bubble layer and moderately amplitude pressures of the order of 10 kPa at the each of the primary frequency excitations are considered in the calculations. The primary waves are regarded to be linear in the absence of bubbles.

The amplitude of the acoustic oscillation inside a layer is determined by the net effects of the driving and damping mechanisms presented inside of the layer. Among damping mechanisms are an energy drop created by viscosity, thermal and reradiating effects.

As a pressure wave oscillates between the layer surfaces, it looks that acoustic energy should be increasing when the sound speed contrast is increasing. But on the other hand there are competing phenomena. When we have higher impedance mismatch which diminish the transfer of the acoustic energy outside of the layer, the acoustic pressure in the standing wave is increasing which is the consequence of the higher bubblers concentration. We should observe not only increase nonlinear output but on the other side theoretically predict the lessening of the nonlinear generation due to the dissipation of primary waves. So, here arises the necessity to perform thoroughly the analysis of those concurrent processes, remembering that the generation of a nonlinear component is proportional to a square of a acoustic pressure of the driven signal. The problem is illustrated in Fig.3 where the integral over the squares of amplitudes of the both primary frequencies over a layer thickness is presented as the function of the sound speed contrast between the bubbly medium and surrounding liquid.

The higher impedance contrast is accompanied by changes in the reflection of primary waves on the outer layer surface. Since the nonlinear component generation rate increasing with growing the bubbles concentration, the damping due to different form of dissipation mechanisms also increases.

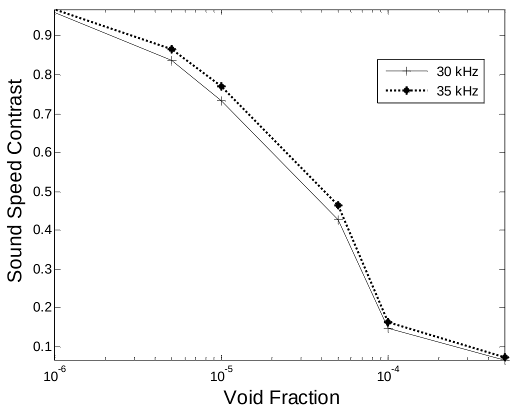

Fig.2. Example of the sound speed contrast as the function of the gas void fraction. Bubble size spectrum as in Fig.1. The algorithm is based on Ye and Ding formula

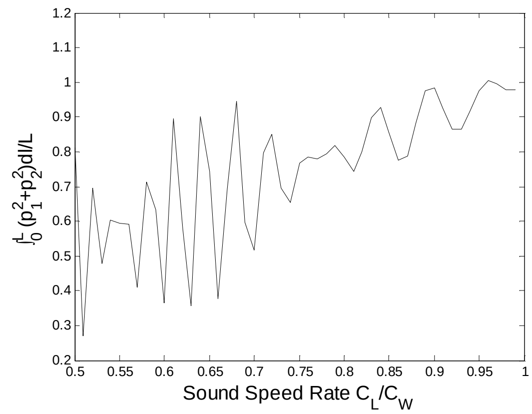

Fig.3. Examination of the viability of the nonlinear generation by the layer, as the dependence of the intensity of the acoustic field inside a layer on the sound speed contrast

The analysis of the Fig.3 reveals that the primary waves acoustic energy inside of the layer increases only for the lower sound speed contrasts (lower bubbles concentrations) when the reflection coefficient reaches the unity. To overcame this trend the other approach could be proposed - matching the impedance between the pure water and the gassy layer.

The resonance amplification a time-harmonic plane wave, implies that the oscillations of all the bubbles all over the layer must be almost in phase. The latter condition is satisfied only if the layer thickness is moderate and much less comparing to a wavelength of the difference frequency component. Another possibility to improve the effectiveness of the generation of the difference frequency component is possible when our layer is created as an ensemble of parallel bubbly sublayers. Every one should be reasonably thin and placed equidistantly at distances d. The d value should fulfill the condition - $r_x \cdot \Re e(\kappa) = n\frac{2\pi}{d}$. n= 1,2, 3, . .., r_x – co-ordinate.

Also we can propose the bubbly layer with continuously changing sound speed, with small changes at the distance comparable to the wavelength at the primary frequencies and much higher for the difference frequency. It is understandable that this kind of a complex geometry layer could be performed only inside of a porous rubberlike substance.

Examples of time series of scattered pressure by a bubble with radius a=100μm, driven by a two-frequency pulse length of τ=20 ms, with primary wave frequencies f_1=30kHz and f_2=35kHz are displayed in Fig.4. In the presented examples the amplitude at the source is the same, but the amplitude of the incident waves is computed with account of reflection at the layer walls, with the reflection coefficient R_{12}=0.8 (void gas fraction approximately equal $V=5\cdot 10^{-5}$). The increasing of nonlinear component is clearly visible. The other parameters of geometry are $L = 6\lambda_1 = 7\lambda_2$. The bubble is placed at the distance L/4 from the layer's front wall.

In order to authenticate the model's performance, the comparison of spectra of backward and forward scattered acoustic waves at the bubbles population by bi-harmonic pulse is shown in Fig. 5.

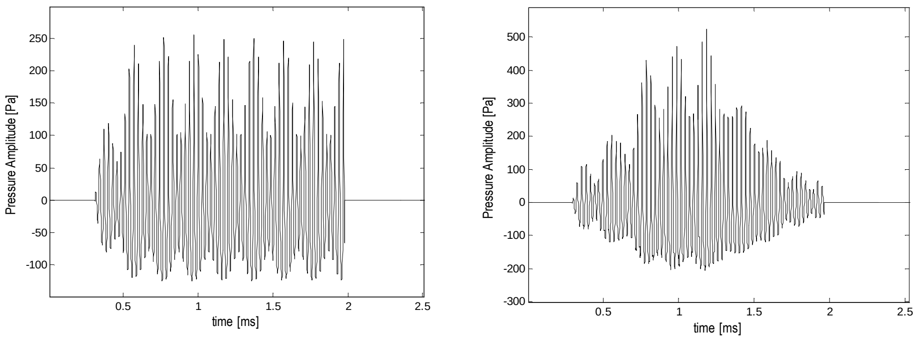

Fig.4. Example of time series of scattered pressure by a bubble with radius a=100μm, driven by a two-frequency pulse, τ=20 ms, f_1=30kHz and f_2=35kHz. On the left, with no resonances inside the two phase liquid layer, on the right, the reflections on the layer walls are incorporated

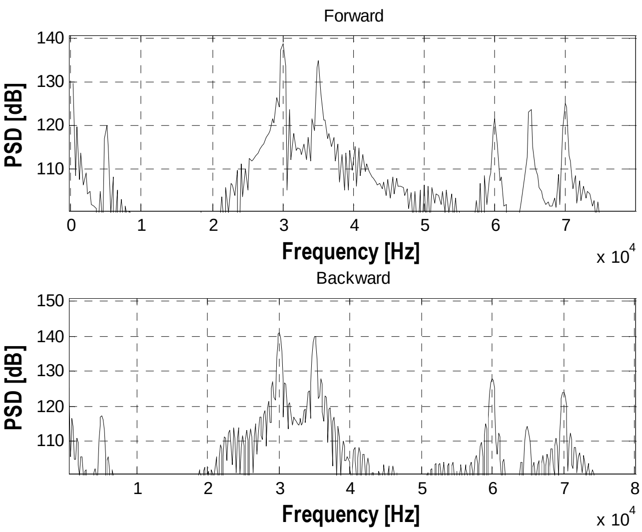

Fig.5. Power spectral density of noncoherent component of a signal backscattered and going through the bubbly layer, L=6*λ_1, N=500, T=30*(1/f_1); P_a=6 kPa, with no reflections inside the layer, n(a)~a^m , bubbles radius range between 186 and 37 μm, where m=-3

4. SUMMARY

The numerical algorithms in MATLAB are proposed for a design of the nonlinear scattering layer comprised of gas bubbles placed in the liquid or semiliquid medium. The study offers a realistic model for the bubbly environment, and is useful in the simulation of the efficiency of parametric echosounders on functioning on the different form of nonlinear layer.

The purpose of this paper was to design of numerical investigation of the nonlinear generation by a bubbly layer. However, the result of numerical modeling has provided a dissatisfied outcome because the strongly enhanced efficiency is not observed. An increase in the bubble concentration inside the bubbly liquid results in the reflection of the incident wave at the layer boundaries and in consequence decreasing the acoustic pressure inside the layer. Interesting problem arises with the design of the bubble population size spectrum function.

The use of a multilayered medium could diminish the dissipation of acoustic energy and makes it ideally suited for generation of the difference frequency component. Another possibility to make reflection of primary waves at the bubbles wall smaller, is the utilization of a medium with continuously changing bubble concentrations (as the example the Epstein symmetrical layer) or a multilayered medium or a special space distribution of scatterers which can considerably influence the output of the nonlinear scattering [7].

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