Journal of POLISH CIMAC
Faculty of Ocean Engineering & Ship Technology
GDAŃSK UNIVERSITY OF TECHNOLOGY

DRAWING CONCLUSIONS ABOUT RELIABILITY OF POWER SYSTEMS FROM SMALL NUMBER OF STATISTICAL DATA

Roman Liberacki

Gdansk University of Technology
ul. Narutowicza 11/12, 80-950 Gdańsk, Poland
tel.: +48 58 3471850, fax: +48 58 3472430
e-mail: romanl@pg.gda.pl

Abstract

Gathering reliability data is quite difficult. Potential sources of such data are: reliability guides and handbooks, data collecting during operation of technical devices, expert judgments. Practice shows that very often the statistical sample we have to our disposal is small and non – homogenous. The goal of the author was to verify, if it is possible to draw out useful in practice conclusions about reliability of technical items from small statistical sample and how to do it. In the article two potential ways to deal with the problem of small reliability data sample have been presented. First way is based on using mathematical statistics methods, the second way uses the fuzzy sets theorem methods.

Key words: *power system, reliability data, statistics, fuzzy sets.*

1. Introduction

The key problem in making reliability analysis of power systems, is getting credible reliability data, which are the entrance data to created mathematical models. The probabilities of suitable technical elements failures are on generality these data. Problem in this place appears - where from to draw these data. The sources of reliability data for technical structures can be: reliability guides and handbooks, data collecting during operation of technical devices, reliability tests, experts' opinions. Practice shows that very often the statistical sample we have to our disposal is small and non – homogenous.

The goal of the author was to verify, if it is possible to draw out useful in practice conclusions about reliability of technical items from small statistical sample and how to do it.

At first some assumptions have been made: non repairable items have been considered, the statistical distributions of time to failure of items and parameters of those distributions have been objects of interest as well as probability of failure in given period of time.

The two potential ways to deal with the problem of small reliability data sample have been considered. First way was to use mathematical statistics methods, the second way, to solve the problem, was to use the fuzzy sets theorem methods.

2. Mathematical statistics methods

Mathematical statistics methods are based on statistical hypothesis testing. We need to remember that there are two possible errors made in a statistical decision process. Error of the first kind known as an α error – which is the error of rejecting a hypothesis when it is actually true. Error of the second kind known as a β error – which is the error of failing to reject a hypothesis when it is in fact false.

In practical applications it is very hard to determine the probability value β of the error of the second kind. When the statistical sample is small and non – homogenous it is just impossible. That is why our care is focused only on the error of the first kind. In advance we impose the probability value of the error α, which in reliability analysis applications is mostly equal 5 % and is called the level of significance.

First of all we have to state the relevant hypotheses to be tested. Than we carry out the test of significance to reject our hypothesis, or to say that there is no reason to reject the hypothesis. The very important thing is to remember that we cannot say the hypothesis is true, in the best case we can only establish that there is no reason to reject the hypothesis.

2.1. Overview of parametric statistics models

Three typical models, described in handbooks [2, 3], have been studied to answer a question: if it is possible to draw out useful in practice conclusions about reliability of technical items from small statistical sample using those models.

Model 1

Let's make an assumption that our statistical parameter is time to failure of an item. The time to failure has the normal distribution N (μ, σ) with the mean value μ and the standard deviation σ. Moreover the standard deviation value is known for the population of such items.
The hypothesis about the mean value (H_0: $\mu = \mu_0$) is tested against the alternative Hypothesis H_a ($\mu \neq \mu_0$) with the significance level of a test α.

We can see at once, that such model is not useful for drowing conclusions about reliability of technical items if the standard deviatin value σ for population is not known. At our considerations the σ value is not known, so the model 1 is useles.

Model 2

Let's make an assumption that our statistical parameter is time to failure of an item. The time to failure has the normal distribution N (μ, σ) with the mean value μ and the standard deviation σ. Moreover both: the mean value μ and the standard deviation value σ are not known for the population of such items. The hypothesis about the mean value (H_0: $\mu = \mu_0$) is tested against the alternative Hypothesis H_a ($\mu \neq \mu_0$) with the significance level of a test α.

To verify the hypothesis H_0 in such case, we can use the hypothesis testing model based on the Student's t distribution. The σ value is estimate than from the sample we have in our disposal.

The model can be useful for us on one condition: we have to be convinced, that the time to failure of technical items has a normal distribution.

Model 3

Let's make an assumption that our statistical parameter is time to failure of an item. The time to failure has the unfounded statistical distribution with the mean value μ and the standard deviation σ. Both parameters μ and σ are not known. The hypothesis about the mean value (H_0: $\mu = \mu_0$) is tested against the alternative Hypothesis H_a ($\mu \neq \mu_0$) with the significance level of a test α. To verify the hypothesis H_0 in such case, we can use the hypothesis testing model based on the zero – one standarised normal distribution N(0,1) . The μ and σ values are estimate from the sample we have in our disposal.

The model can be used only when we have at our disposal a large statistical sample number. The large sample number means at least 30 observations [2] . Some authors say 100 or more observations [3]. Using this model we can estimate the mean value and standard deviation of time to failure of item but we cannot to find out the time to failure distribution density function shape.

Similarly, like it was done in the above models for the mean value, we can also make statistical significance tests, using the above models, for the standard deviation values. However there are some conditions to choose the model [2]:
When we have the normal distribution N (μ, σ) of time to failure, μ and σ values are not known and sample number $n \leq 50$ then we choose model 1.

When we have the normal distribution N (μ, σ) of time to failure, μ and σ values are not known and sample number $n > 50$ than we choose model 2.

When we have the normal distribution N (μ, σ) of time to failure, μ and σ values not known and sample number $n \leq 50$ than we choose model 3.

It's worth to notice that we can use those three models for statistical significance tests only if we are convinced, that the time to failure of the item has normal distribution.

2.2. Overview of nonparametric statistics models

Nonparametric hypothesis testing models are used when we do not know the random variable probability density function shape as well as parameters of the function. We have face to such situation very often in reliability analysis. However using data set, we have in our disposal, we are able to build a histogram of time to failure (which is our random variable) first, and then make density estimation based on the data to evaluate theoretical probability density function of time to failure.

Statistical hypothesis testing in this case let us only to find out how much our theoretical model (theoretical density function) is adequate to our data distribution in the sample we have. Unfortunately, we never can be sure the model will be adequate to whole population of technical items being the object of our interest.

The most popular nonparametric tests in statistics are: the Pearson's chi - square test and the Kolmogorov – Smirnov test, in that case used as a nonparametric tests of equality of one-dimensional probability distributions used to compare a sample with a reference probability distribution.

The most suitable, to solve the problem given in this article, seems to be the Kolmogorov – Smirnov test. The Pearson's chi - square test needs a large sample (at least 100 items). For the Kolmogorov – Smirnov test a much smaller sample will be enough. Of course the test is adequate for different density functions of the random variable. Moreover the Kolmogorov–Smirnov test is the only test usually used to test whether a given distribution function $F(x)$ is the underlying probability distribution of $F_n(x)$, the procedure may be inverted to give confidence limits on $F(x)$ itself. If one chooses a critical value of the test statistic D_α such that $P(D_n > D_\alpha) = \alpha$, then a band of width $\pm D_\alpha$ around $F_n(x)$ will entirely contain $F(x)$ with probability $1 - \alpha$ [4].

The last feature is very important in reliability analysis, because if the random variable is time to failure of the item, then the density function is just the unreliability function. So we are able not only to estimate the unreliability function but also to find the confidence limits for estimated unreliability function.

If the density function of time to failure of technical items is normal, then we can replace the Kolmogorov – Smirnov test by the Lilliefors test (sample size $n \leq 500$) or the Shapiro Wilk test (sample size $n > 30$) or Anderson - Darling test. The last test is more powerful [4].

3. Methods based on fuzzy logic

At the beginning let’s make an assumption, that we have no statistical data of time to failure of technical items. The only way to collect needed reliability data is to use experts judgments. It is very difficult for experts to give answers in the form of numerical values, for example: the time to failure of the item is 5000 hours or the failure probability per year is 0,003. Much easier is to ask them using linguistic values. The problem arise - how to converse the linguistic values into numerical values. The fuzzy logic can be helpful to deal with the problem.
Assume that the object of our interest is the probability of failure of technical item during one year period of time. The idea of the author is to test the truthfulness of the sentence given in such form: “The item will fail in the one year period of time”. Experts give their opinions shown in Tab. 1.

Tab. 1. An example of using expert opinions in reliability analysis of technical items

Tested sentence:	**Possible experts opinions:**
The item will fail during one year period of time.	(1) Sentence is absolutely truth. (2) Sentence is very truth. (3) Sentence is truth. (4) Sentence is rather truth. (5) Sentence is truth or false. (6) Sentence is rather false. (7) Sentence is false. (8) Sentence is very false. (9) Sentence is absolutely false.

Now we can transform expert opinions into fuzzy numbers according to so called “standard degrees of truth” proposed by Baldwin in [1] with the membership functions $\mu(v)$ given in Fig. 1.

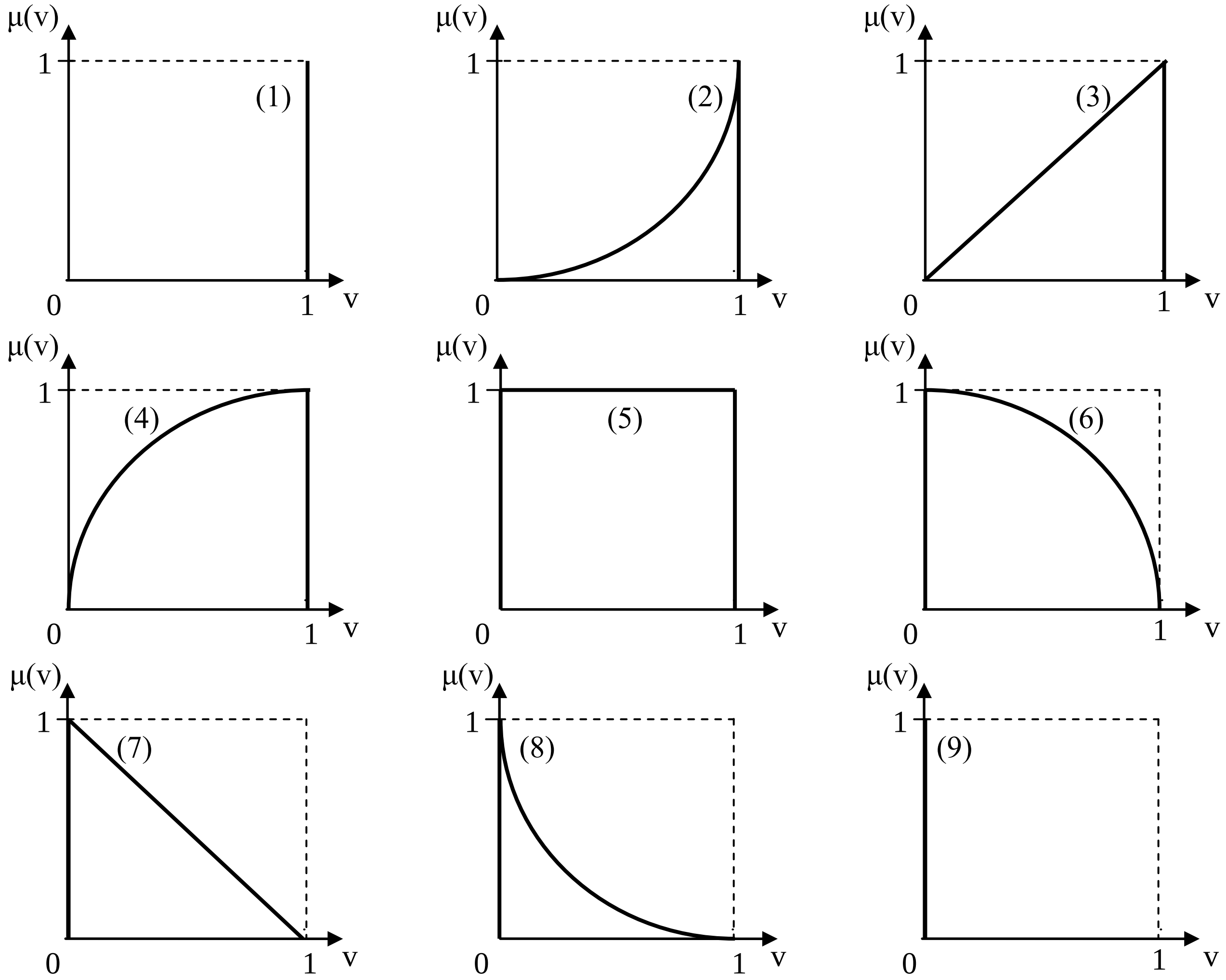
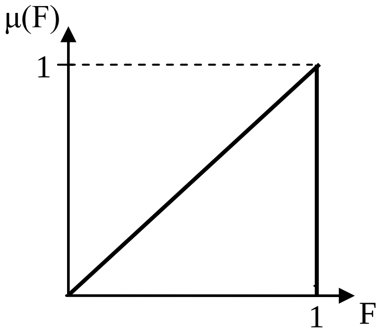
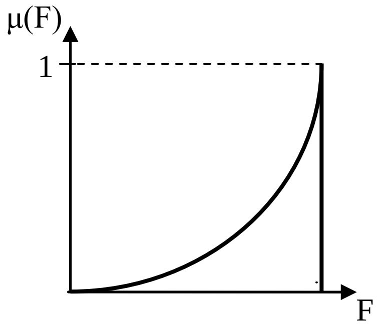


Fig. 1. Standard degrees of truth by Baldwin given as the fuzzy numbers

As the result we receive unreliability value F, for given period of time, as a fuzzy number. For instance:



Unreliability of item for given period of time is equal "about 1".



Unreliability of item for given period of time is "very close to 1".

Fig. 2. Examples of unreliability values given in the form of fuzzy numbers

Fuzzy logic is a very good tool when we have in our disposal very imprecise data. In contrast with binary logic, where the truth has only two values (1 – true, 0 – false) fuzzy logic variables of truth may vary form 0 to 1. It gives us possibility to deal with reasoning that is approximate rather than precise. And such situation is very typical in reliability analysis of technical system.

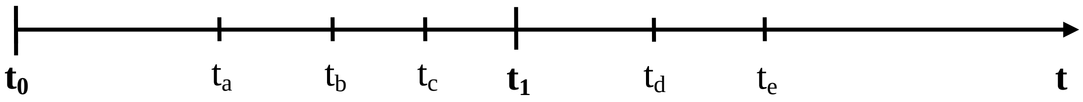
Another interesting way to solve the problem of reliability data evaluation, in author's opinion, seems to be the possibility theory. The main idea of possibility theory is to replace probability measure with possibility measure and necessity measure.

If we are not able to evaluate probability value of item failure, we can try to evaluate the possibility measure and necessity measure – what is much easier. Let assume A – is event of item failure. Between probability, possibility and necessity measure there is relationship given below [1]:

$$N(A) \leq P(A) \leq \Pi(A) \tag{1}$$

where:
N(A) – necessity measure of event A,
P(A) – probability measure of event A,
Π(A) – possibility measure of event A.

Now let's try to illustrate usage of the possibility and necessity measure in reliability analysis on example. Given: A – event of the item failure during one year period of time and 5 statistical data in form of time to failure of five items (a, b, c, d, e) presented in Fig. 3.



$t_0 - t_1$ – one year period of time,
$t_0 - t_a$ – time to failure of item a,
$t_0 - t_b$ – time to failure of item b,
$t_0 - t_c$ – time to failure of item c,
$t_0 - t_d$ – time to failure of item d,
$t_0 - t_e$ – time to failure of item e,

Fig. 3. Time to failure of items

Given the above set of data we can evaluate:
Possibility measure of event A: Π(A) = 1,
Necessity measure of event A: N(A) = 3/5 = 0,6.

So we can give a statement 1: Probability of item failure during one year period of time can range between (0,6 ; 1).

Imagine now that we have a new information about another (f) item time to failure.

First situation: the time to failure of a new item is less then one year – then a new necessity measure is N'(A) = 4/6 = 0,67 so we have a new statement 2: Probability of item failure during one year period of time can range between (0,67 ; 1). Conclusion: statement 2 is not contradicted to statement 1. Statement 1 is still truth.

Second situation: the time to failure of a new item is more then one year – then a new necessity measure is N''(A) = 3/6 = 0,5 so we have a new statement 3: Probability of item failure during one year period of time can range between (0,5 ; 1). Conclusion: statement 3 is contradicted to statement 1. Statement 1 is not truth, but statement 1 is more pessimistic than statement 3 what is consistent with the rule of the worst case which is often used in reliability analysis. Statement 1 then is still worthy for us.

4. Final remarks

In the authors opinion, it is possible to draw out useful in practice conclusions about reliability of technical items from small statistical sample, using statistical methods, fuzzy logic and possibility theory.

We should always to remember, that received results in reliability analysis are never sure. We have to deal with the problem of large uncertainty. Fuzzy logic and probability are the ways of expressing uncertainty.

With the very small data set it is very hard to talk about probability in the classical sense, even from statistical point of view. We should rather to use fuzzy numbers, fuzzy probability with the help of experts judgments or the possibility theory, as it has been shown in the article.

References

[1] Dubois, D., Prade, H., *Possibility Theory. An Approach to Computerised Processing of Uncertainty*, New York 1988.

[2] Kołowrocki, K., *Wybrane wykłady z rachunku prawdopodobieństwa i statystyki matematycznej*, Fundacja Rozwoju Wyższej Szkoły Morskiej w Gdyni, Gdynia 1994.

[3] Krysicki, W., Bartos, J., Dyczka, W., Królikowska, K., Wasilewski, M., *Rachunek prawdopodobieństwa i statystyka matematyczna w zadaniach, część II Statystyka matematyczna*, Wydawnictwo Naukowe PWN, Warszawa 1995.

[4] Stephens, M. A., *EDF Statistics for Goodness of Fit and Some Comparisons*". Journal of the American Statistical Association 69, 1974.