Acta of Bioengineering and Biomechanics
Vol. 12, No. 1, 2010

Original paper

Analysis of muscles' behaviour. Part II. The computational model of muscles' group acting on the elbow joint

WIKTORIA WOJNICZ*, EDMUND WITTBRODT*

Mechanics and Strength of Materials Department,
Mechanical Engineering Faculty, Gdańsk University of Technology, Gdańsk, Poland.

The purpose of this paper is to present the computational model of muscles' group describing the movements of flexion/extension at the elbow joint in the sagittal plane of the body when the forearm is being kept in the fixed state of supination/pronation. The method of evaluating the muscle forces is discussed in detail. This method is the basis for the quantitative and qualitative verification of the proposed computational model of muscles' group. Applying this computational model, the forces of real muscles belonging to the muscles' group can be evaluated without using any optimization technique.

Key words: *muscle, modelling, elbow joint, flexion/extension, verification*

1. Introduction

The modelling of the behaviour of complex biological system consists in describing and combining at the time the series of cause-and-effect phenomena that happen therein. However, the kind of phenomena taken into account and the method of modelling their behaviour depend exclusively on the experience and skill of a researcher-modeller. Furthermore, it is worth remembering that in most cases the excessive complication of model leads to the lack of its solution and does not allow us to elucidate the causes and effects of the phenomenon under examination.

The modelling of the action of upper/lower limb muscles' group is based on anatomical data describing spatial positions of muscles that can be collected from cadaver measurements (KLEIN BRETELER et al. [9], LANGENDERFER et al. [11], MURRAY et al. [15], VEEGER et al. [20], VEEGER et al. [21]) or using modern medical imaging techniques, e.g., the computed tomography or the magnetic resonance imaging (DANIEL et al. [3], KOO et al. [10]). Among these data there are: the origins of coordinates and directions of axes of coordinate systems used to describe the motions of the joints examined; the coordinates of muscle origins and insertions; the moment arms of muscle forces, etc. Based on these anatomical data and the location of muscles in two-dimensional (2D) or three-dimensional (3D) space, different computational models of muscles' group are proposed.

In the computational model of muscles' group, the action of each composed muscle can exclusively be treated as a force (AIT-HADDOU et al. [1], RAIKOVA [18]) or as a result of the influence of the Hill-type muscle model (CAMILLERI and HULL, [2], HERZOG [6], KOO et al. [10], REHBINDER and MARTIN [19]). Therefore, solving forward or inverse dynamic task, one can perform numerical simulations of movements at the joint examined. In a forward dynamic task, the mus-

* Corresponding authors: Wiktoria Wojnicz, Edmund Wittbrodt, Mechanics and Strength of Materials Department, Mechanical Engineering Faculty, Gdańsk University of Technology, ul. Narutowicza 11/12, 80-233 Gdańsk, Poland. E-mail: wiktoria.wojnicz@pg.gda.pl, e.wittbrodt@pg.gda.pl

Received: June 15th, 2009
Accepted for publication: January 11th, 2010

cles' forces are the causes, thus the trajectory of limb movement is the effect. Since in an inverse dynamic task the trajectory of limb movement is the cause, thus the muscles' forces are the effects. It should be noted that nowadays one uses exclusively optimization techniques to reach a unique solution of inverse dynamic tasks (AIT-HADDOU et al. [1], CAMILLERI and HULL [2], van der HELM and CHADWICK [5], KOO et al. [10], MAUREL [13]). Such approach does not still have any physiological explanation. It results from the lack of cause-and-effect relationships that are necessary to obtain a unique solution of inverse dynamic task.

Considering the problems mentioned, we can see that there is still the demand for computational model of muscles' groups that permits one to evaluate precisely and uniquely the force of each alive muscle belonging to real muscles' group. Moreover, one must formulate such a method of identification of its parameters that might be used for alive muscles belonging to the real muscles' group (because the parameters obtained from cadaver measurements do not permit the true behaviour of real muscles' group to be modelled). In addition, it also seems that during the movements of the joint examined the evaluation of shares of particular muscles should not be based exclusively on the optimization approach.

The main goal of this paper is to present a new approach to modelling the behaviour of muscles' group acting on the elbow joint. For this approach there have been elaborated: 1. The computational model of muscle and a comparatively simple method of identifying its parameters, which can be used to examine alive muscles (for details see part I of the paper). 2. The method of evaluating the shares of particular muscles in movements of the joint examined without using any optimization techniques. 3. The method of verifying the computational model of muscles' group.

2. The computational model

The computational model of muscles' group describes the movements of flexion/extension of forearm in the sagittal plane with respect to the unmoving arm (figure 1) (WOJNICZ [24]). The elbow joint has been treated as the hinge. In its geometric middle O (that was defined in accordance with LI et al. [12]), the origin of the global immovable coordinate system XYZ, whose axes are parallel to the main axes of the body, was placed (PLATZER [17]). The *X*-axis is parallel to the sagittal axis directed from the anterior to

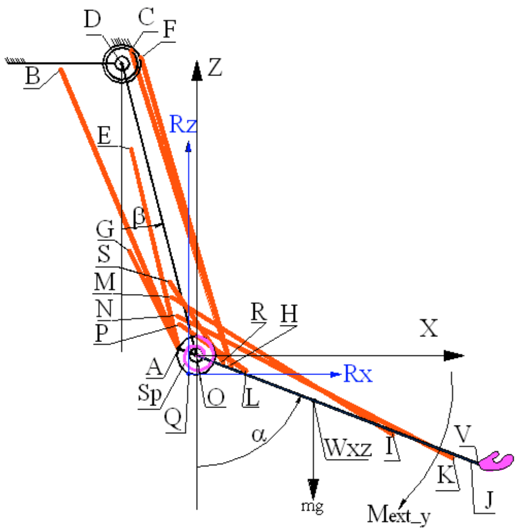

Fig. 1. The model of muscles' group acting on the elbow joint (2D view)

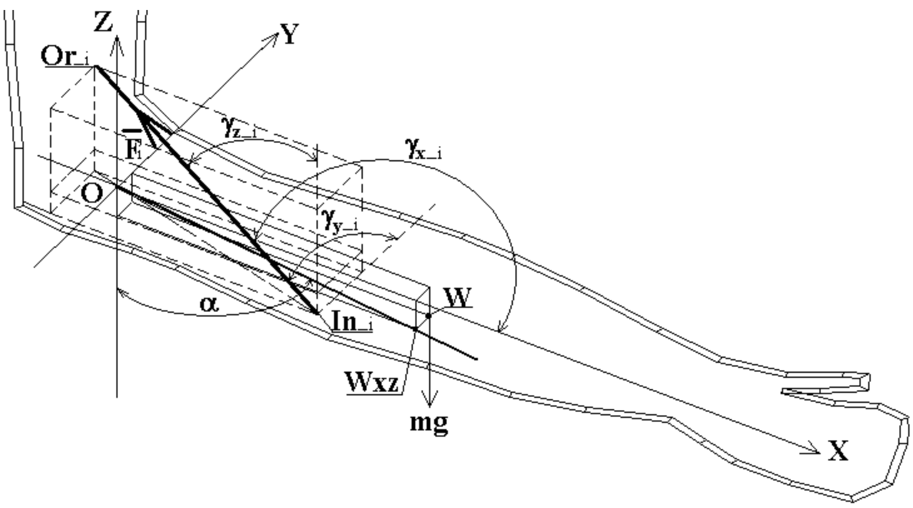

Fig. 2. The location of single muscle of model in the arm–forearm structure (left forearm)

posterior body surface. The Y-axis is parallel to the transversal axis oriented from the medial to the lateral body surface. The Z-axis is parallel to the longitudinal axis of the body directed from the coccyx to the cranial part of the body.

During flexion/extension at the elbow joint, the arm DO is kept immovably at a constant angle of abduction β with respect to the Z-axis. The forearm and the hand are the assembly treated as the forearm–hand structure OV, whose moment of inertia with respect to the Y-axis is J_y and its gravity force mg is applied at the point W (figure 2). At this stage of modelling it has been assumed that the forearm–hand structure OV does not make any movements of pronation/supination at the elbow joint and it exclusively produces the movements in the sagittal XZ-plane with respect to the steady Y-axis of rotation (this means that the y-coordinates of all the points of the model of muscles' group are constant). The angle $\alpha = \alpha(t)$ is measured between the Z-axis and the segment OW_{xz} (the point W_{xz} is the projection of the point W on the XZ-plane) at the time t. This angle α is the measure of the movements of flexion/extension at the elbow joint. In the model of muscles' group, the influences of nine muscles are taken into consideration: *caput laterale musculi tricipitis brachii AE* (i = 1), *caput longum tricipitis brachii AB* (i = 2), *caput mediale tricipitis brachii AG* (i = 3), *musculus extensor carpi radialis longus KN* (i = 4), *musculus brachioradialis IM* (i = 5), *musculus pronator teres RS* (i = 6), *musculus brachialis OP* (i = 7), *caput longum musculi bicipitis brachii HC* (i = 8) and *caput breve musculi bicipitis brachii HF* (i = 9). Due to a lack of data about the displacements of each muscle's origin and insertion during the movement of forearm, it has been supposed that all muscles are permanently fixed on the surfaces of bones. The model of muscles' group can be subjected to the action of external moment $M_{\text{ext}} = M_{\text{ext}}(t)$, whose components are as follows: $M_{\text{ext}\_x} = M_{\text{ext}\_x}(t)$, $M_{\text{ext}\_y} = M_{\text{ext}\_y}(t)$, $M_{\text{ext}\_z} = M_{\text{ext}\_z}(t)$, in respect of the axes of the coordinate system XYZ.

As a result of friction between the components of elbow joint (the humerus, ulna, radius, articular capsule and ligaments), dynamic reactions appear and influence the dynamics of the flexion/extension movements of forearm. Nevertheless, due to a lack of experimental data that could describe precisely this phenomenon, it has been assumed that the resultant of the dynamic reactions R is applied at the point Q (figure 1), whose coordinates are (r_x, r_y, r_z), and its components $R_x = R_x(t)$, $R_y = R_y(t)$, $R_z = R_z(t)$ produce reducing moments with respect to X-, Y-, Z-axes:

$$(R_y \cdot r_z - R_z \cdot r_y = 0) \wedge (R_x \cdot r_z - R_z \cdot r_x = 0) \wedge (R_x \cdot r_y - R_y \cdot r_x = 0). \quad (1)$$

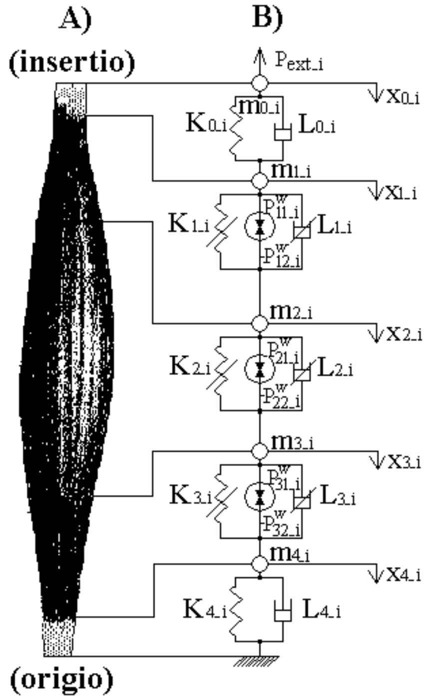

Fig. 3. The fusiform muscle: A) the real form, B) the rheological model

The modelling of these movements is based on the assumption that muscles exerting an influence at the elbow joint are treated as fusiform muscles and they are not influencing transversely themselves while their shapes are changing. The behaviour of the i-th muscle reflects the reological model presented in figure 3 (its work is described in detail in part I of the paper). This model reflects: the muscle elastic properties described by the stiffness coefficients $K_i \in \{K_{0\_i}, K_{1\_i}, K_{2\_i}, K_{3\_i}, K_{4\_i}\}$; the muscle viscous properties described by the damping coefficients $L_i \in \{L_{0\_i}, L_{1\_i}, L_{2\_i}, L_{3\_i}, L_{4\_i}\}$; the muscle mass properties described by the reduced masses of defined parts of muscle $m_i \in \{m_{0\_i}, m_{1\_i}, m_{2\_i}, m_{3\_i}, m_{4\_i}\}$; the muscle forcible properties (modelled as force elements generating two internal forces and having opposite directions: $(P^w_{11\_i} = P^w_{11\_i}(t)) \wedge (-P^w_{12\_i} = -P^w_{12\_i}(t))$, $(P^w_{21\_i} = P^w_{21\_i}(t)) \wedge (-P^w_{22\_i} = -P^w_{22\_i}(t))$, $(P^w_{31\_i} = P^w_{31\_i}(t)) \wedge (-P^w_{32\_i} = P^w_{32\_i}(t))$); the action of the external force $P_{\text{ext}} = P_{\text{ext}}(t)$ directed always towards the elongation of the muscle model. The displacements of reduced mass of the muscle model $x_{0\_i}(t) \wedge [x_i = x_i(t) \in \{x_{1\_i}(t), x_{2\_i}(t), x_{3\_i}(t), x_{4\_i}(t)\}]$ reflect the displacements of the points that define borders between distinguished parts of the real muscle examined. However, the displacement of insertion tendon x_{0_i} is a non-linear function of the angle of rotation $\alpha(t)$ at the elbow joint:

$$x_{0\_i}(t) = l_{0\_i} - l_i[\alpha(t)] \ , \tag{2}$$

where l_{0_i} and $l_i[\alpha(t)]$ are the lengths of the i-th muscle model at the initial time t_0 and the time t.

Taking into account the spatial arrangement of muscles in 3D space, the equation of motion of flexion/extension movements at the elbow joint has been derived. In this equation, the force $F_i = F_i(t)$ reflects the action of each i-th muscle (figure 3). Its line of action $InOr$ is directed from the insertion In_i (its coordinates $(x_{in\_i}, y_{in\_i}, z_{in\_i}) = (x_{in\_i}(t), y_{in\_i}, z_{in\_i}(t))$ are on the forearm) towards the origin On_i (its coordinates $(x_{or\_i}, y_{or\_i}, z_{or\_i})$ are on the arm). In figure 2, there are also presented: the global coordinate system XYZ with its origin O; the gravity force mg of the forearm–hand structure OV applied at the point W and the projection of this point on the XZ-plane – W_{xz}; the angle $\alpha(t)$ of flexion/extension at the elbow joint; the angle $\gamma_{x\_i} = \gamma_{x\_i}(t)$ between the line of force F_i and the X-axis; the angle $\gamma_{y\_i} = \gamma_{y\_i}(t)$ between the line of force F_i and the Y-axis; the angle $\gamma_{z\_i} = \gamma_{z\_i}(t)$ between the line of force F_i and the Z-axis. Taking into consideration the influences of nine muscles of muscles' group, the equation of motion is defined as follows:

$$J_y \cdot \frac{d^2\alpha}{dt^2} = \sum_{i=1}^{9} F_i \cdot (\cos(\gamma_{x\_i}) \cdot z_{in\_i} - \cos(\gamma_{z\_i}) \cdot x_{in\_i})$$

$$+ mg \cdot OW_{xz} \cdot \sin(a) - M_{ext\_y}. \tag{3}$$

The equation of motion (equation (3)) does not take into account that muscles have initial forces caused by their introductory stretching.

Ligaments protecting the elbow joint from damage restrict the flexion/extension movements at this joint. The range of admissible movements in the sagittal XZ-plane is $\alpha \in [5°; 150°]$, where the position of a full extension of the forearm–hand structure OV is the low border, thus its position in the full flexion is the high border. In the computational model of muscles' group, those restrictions are assured by the non-linear torsion spring Sp (figure 1), which models the influences of ligaments. It generates the compensation moment $M_{y\_\text{com}} = M_{y\_\text{com}}(t)$ when the forearm–hand structure OV emerges on the border of the defined range. Due to the action of the compensation moments $M_{z\_\text{com}} = M_{z\_\text{com}}(t)$ and $M_{x\_\text{com}} = M_{x\_\text{com}}(t)$ caused by ligaments in the coronal YZ-plane and transversal XY-plane, the movements of the elbow joint are performed exclusively in the sagittal XZ-plane:

$$M_{x\_\text{com}} = \sum_{i=1}^{9} F_i \cdot (\cos(\gamma_{z\_i}) \cdot y_{in\_i} - \cos(\gamma_{y\_i}) \cdot z_{in\_i})$$

$$- mg \cdot WW_{xz} \cdot \sin(a) + M_{\text{ext}\_x}, \tag{4}$$

$$M_{z\_\text{com}} = \sum_{i=1}^{9} F_i \cdot (\cos(\gamma_{y\_i}) \cdot x_{in\_i}$$

$$- \cos(\gamma_{x\_i}) \cdot y_{in\_i}) + M_{\text{ext}\_z}, \tag{5}$$

where WW_{xz} is the a straight line between the point W and the point W_{xz}.

Additionally, in the proposed model of muscles' group the compression of non-excited muscles (that do not generate any internal forces) has been neglected.

3. The calculation of forces of muscles' model

The muscles' model presented can be used to solve forward and inverse dynamic tasks. A forward dynamic task consists in inputting the internal forces $P^w_{11\_i}, P^w_{12\_i}, P^w_{21\_i}, P^w_{22\_i}, P^w_{31\_i}, P^w_{32\_i}$ that are generated in force elements of each composed i-th muscle at the time t and outputting the angle of rotation of arm–forearm structure $\alpha(t)$. An inverse dynamic task consists in inputting the angle $\alpha(t)$ of rotation of arm–forearm structure and outputting these internal forces generated by all the muscles of the model at the time t. Additionally, in the latter case, the displacements of chosen points located on each composed i-th muscle x_i, their velocities $\dot{x}_i$ and their accelerations $\ddot{x}_i$ can also be calculated.

Solving a forward/inverse dynamic task, the share of each i-th muscle belonging to the muscles' model at the time t can be evaluated by calculating its force $F_i(t)$:

$$F_i(t) = P_{a\_i}(t) - P_{p\_i}\left[\alpha(t), \frac{d\alpha(t)}{dt}, \frac{d^2\alpha(t)}{dt^2}\right], \tag{6}$$

$$P_{p\_i}(t) = A_{2\_i} \cdot \frac{d^2(l_{0\_i} - l_i[\alpha(t)])}{dt^2}$$

$$+ A_{1\_i} \cdot \frac{d(l_{0\_i} - l_i[\alpha(t)])}{dt}$$

$$+ A_{0\_i} \cdot (l_{0\_i} - l_i[\alpha(t)])$$

$$+ \int_o^t \left(\sum_{j=0}^{7} B_{j\_i} \cdot e^{s_{j\_i} \cdot (t-\tau)} \right) \cdot (l_{0\_i} - l_i[\alpha(\tau)])\, d\tau, \quad (7)$$

$$P_{a\_i}(t) = \int_0^t \left(\sum_{j=0}^{7} D_{j1\_i} \cdot e^{s_{j\_i} \cdot (t-\tau)} \right) \cdot P^w_{11\_i}(\tau)\, d\tau$$

$$+ \int_0^t \left(\sum_{i=0}^{7} D_{i2\_i} \cdot e^{s_{j\_i} \cdot (t-\tau)} \right) \cdot P^w_{12\_i}(\tau)\, d\tau$$

$$+ \int_0^t \left(\sum_{j=0}^{7} D_{j3\_i} \cdot e^{s_{j\_i} \cdot (t-\tau)} \right) \cdot P^w_{2\_i}(\tau)\, d\tau$$

$$+ \int_0^t \left(\sum_{j=0}^{7} D_{j4\_i} \cdot e^{s_{j\_i} \cdot (t-\tau)} \right) \cdot P^w_{31\_i}(\tau)\, d\tau$$

$$+ \int_0^t \left(\sum_{j=0}^{7} D_{j5\_i} \cdot e^{s_{j\_i} \cdot (t-\tau)} \right) \cdot P^w_{32\_i}(\tau)\, d\tau\,, \quad (8)$$

where:

$P_{p\_i}\left[\alpha(t), \frac{d\alpha(t)}{dt}, \frac{d^2\alpha(t)}{dt^2}\right]$ – the passive component of the i-th muscle that depends on the angle of rotation $\alpha(t)$, the angular velocity $\frac{d\alpha(t)}{dt}$ and the angular acceleration $\frac{d^2\alpha(t)}{dt^2}$;

$P_{a\_i}(t)$ – the active component of the i-th muscle that in the model of muscle generating unbalanced forces (see part I of the paper) depends on the calculated five internal forces $P^w_{11\_i}(t), P^w_{12\_i}(t), P^w_{2\_i}(t), P^w_{31\_i}(t), P^w_{32\_i}(t)$, where $P^w_{21\_i}(t) = P^w_{22\_i}(t) = P^w_{2\_i}(t)$;

A_{0_i}, A_{1_i}, A_{2_i}, B_{j_i}, s_{j_i}, D_{j1_i}, D_{j2_i}, D_{j3_i}, D_{j4_i}, D_{j5_i}, (j = 0, 1, 2, 3, 4, 5, 6, 7) – the coefficients of the i-th muscle dependent on its mass coefficients m_i, stiffness coefficients K_i, damping coefficients L_i and the kind of work of muscle parts (lengthening or shortening).

It should be noted that while an inverse dynamic task is being solved, internal forces of muscles' model are being calculated by using the identification of internal forces (see part I of the paper). Furthermore, input/calculating internal forces of each muscle of the model must guarantee "the admissible state" of this model.

4. Evaluating the forces of muscles that belong to a real muscles' group

Using the computational model of muscles' group, the force of each real muscle belonging to the alive muscles' group acting on the elbow joint can be evaluated. This evaluation is carried out in two stages.

4.1. The first stage

The first stage consists in identifying the mechanical properties of composed muscles belonging to the muscles' group examined. It is assumed that the mechanical properties of unexcited muscles are the same as those of excited muscles.

Using the imaging technique (e.g., the computed tomography, magnetic resonance imaging or ultrasonography), at the beginning we obtain 3D images of muscle and the data of dimensions/mass/volume/density of its parts (NARICI [16]). These data are the basis for the virtual dividing-up of each composed i-th muscle into parts, the evaluation of their masses m_i and the points marking the limits between the chosen parts. The displacements x_i of these points, their velocities $\dot{x}_i$ and accelerations $\ddot{x}_i$ must be recorded during the first and the second stages of the evaluation of each real i-th muscle force. In order to record timing, the displacements of chosen points' markers (e.g. fluorescent polystyrene spheres or sonographic crystals) can be used – they are glued to the muscle surface (van DONKELAAR et al. [4], HUIJING [7]). In the next step, the stiffness coefficients K_i and the damping coefficients L_i are evaluated in accordance with the method described in part I of this paper.

For the purpose of precise identification of the viscoelastic properties of the muscle examined one needs to detach the examined muscle insertion tendon from the bone and then to measure the true value of external force $P_{\text{ext}}(t)$ acting on it. This invasive procedure is used because of two reasons. First, there is no possibility of evaluating the external moment M_{ext} share, because this moment is applied to all muscles of the group. Second, the function of fascia that couples all adjacent muscles and induces their interaction is disregarded.

4.2. The second stage

The second stage consists in evaluating the forces of muscles belonging to the real muscles' group. At the beginning of this stage the forearm must be established in a boundary position (in the full extension or the maximal flexion at the elbow joint) treated as the initial position. Using the chosen imaging technique, the coordinates of inserts/origins of muscles are determined on the humerus, radius and ulna, when the forearm is being kept at the fixed state of supination/pronation. If the displacements of muscles' inserts/origins have to be taken into account during movements, we need to determine those coordinates in all the positions of arm–forearm system. Also, the moment of forearm–hand structure inertia with respect to the Y-axis, its gravity force mg and the coordinates of gravity centre must be known (WINTER [22]).

After that, beginning from the initial state, the forearm begins to rotate. During movements at each time t there are recorded: the angle of rotation $\alpha(t)$; the angular velocity $\omega(t)=\dfrac{d\alpha(t)}{dt}$; the angular acceleration $\varepsilon(t)=\dfrac{d^2\alpha(t)}{dt^2}$; the displacements of points placed on each i-th examined muscle x_i, their velocities $\dot{x}_i$ and accelerations $\ddot{x}_i$; the external moments $M_{\text{ext}\_y}(t)$ that act on what happens in the sagittal XZ-plane.

Based on the above mentioned data it can be inferred that at each time t the force of each i-th muscle under examination is evaluated as follows: At first the external force $P_{\text{ext}\_i}(t)$ influencing the i-th muscle is calculated from the following equation:

$$m_{0\_i}\cdot\frac{d^2(l_{0\_i}-l_i[\alpha(t)])}{dt^2}+L_{0\_i}\cdot\left(\frac{d(l_{0\_i}-l_i[\alpha(t)])}{dt}-\dot{x}_{1\_i}\right)+K_{0\_i}\cdot((l_{0\_i}-l_i[\alpha(t)])-x_{1\_i})=-P_{\text{ext}\_i(t)}. \quad (9)$$

Then from equation (7) the passive component $P_{p\_i}[\alpha(t),\omega(t),\varepsilon(t)]$ is evaluated. In the next step, relation (8) is inserted into the first equation of the following system of equations which allows its solution:

$$\begin{cases} P_{a\_i}(t)=P_{a\_i}(P^w_{11\_i}(t),P^w_{12\_i}(t),P^w_{2\_i}(t),P^w_{31\_i}(t), \\ \qquad P^w_{32\_i}(t))=P_{p\_i}(t)-P_{\text{ext}\_i}(t), \\ P^w_{11\_i}=m_{1\_i}\cdot\ddot{x}_{1\_i}+L_{0\_i}\cdot\left(\dot{x}_{1\_i}-\dfrac{\mathrm{d}(l_0-l[\alpha(t)])}{dt}\right) \\ \qquad +K_{0\_i}\cdot(x_{1\_i}-(l_0-l[\alpha(t)])) \\ \qquad +L_{1\_i}\cdot(\dot{x}_{1\_i}-\dot{x}_{2\_i})+K_{1\_i}\cdot(x_{1\_i}-x_{2\_i}), \\ P^w_{2\_i}-P^w_{12\_i}=m_{2\_i}\cdot\ddot{x}_{2\_i}+L_{1\_i}\cdot(\dot{x}_{2\_i}-\dot{x}_{1\_i}) \\ \qquad +K_{1\_i}\cdot(x_{2\_i}-x_{1\_i})+L_{2\_i}\cdot(\dot{x}_{2\_i}-\dot{x}_{3\_i}) \\ \qquad +K_{2\_i}\cdot(x_{2\_i}-x_{3\_i}), \\ P^w_{31\_i}-P^w_{2\_i}=m_{3\_i}\cdot\ddot{x}_{3\_i}+L_2\cdot(\dot{x}_{3\_i}-\dot{x}_{2\_i}) \\ \qquad +K_{2\_i}\cdot(x_{3\_i}-x_{2\_i})+L_{3\_i}\cdot(\dot{x}_{3\_i}-\dot{x}_{4\_i}) \\ \qquad +K_{3\_i}\cdot(x_{3\_i}-x_{4\_i}), \\ P^w_{32\_i}=-m_{4\_i}\cdot\ddot{x}_{4\_i}-L_{3\_i}\cdot(\dot{x}_{4\_i}-\dot{x}_{3\_i}) \\ \qquad -K_{3\_i}\cdot(x_{4\_i}-x_{3\_i})-L_{4\_i}\cdot\dot{x}_{4\_i}-K_{4\_i}\cdot x_{4\_i}. \end{cases} \quad (10)$$

Thereafter, the active component $P_{a\_i}(t)$ of equations (10) and the internal forces $P^w_{11\_i}(t), P^w_{12\_i}(t)$, $P^w_{2\_i}(t), P^w_{31\_i}(t)$, and $P^w_{32\_i}(t)$ can be defined. Then the force $F_i(t)$ of the i-th muscle is evaluated from equation (6). Assuming that all measurements have been carried out with a high accuracy, the deviation is given by:

$$\Delta M(t)=M_{\text{ext}\_y}(t)-\left\{\sum_{i=1}^{9}F_i(t)\cdot(\cos(\gamma_{x\_i})\cdot z_{in\_i}-\cos(\gamma_{z\_i})\cdot x_{in\_i})+mg\cdot OW_{xz}\cdot\sin(a(t))-J_y\cdot\frac{d^2\alpha(t)}{dt^2}\right\}. \quad (11)$$

The deviation $\Delta M(t)$ is the basis for the quantitative verification of the proposed model of muscles' group.

5. Conclusions

In our computational model of muscles' group, which represents planar movements of flexion/extension at the elbow joint when the forearm is being kept in the fixed state of supination/pronation, the following assumptions are accepted: 1) the motion of each muscle in a 3D space has a rectilinear character; 2) complex muscles have different mechanical properties that can be determined during the parametric identification; 3) the influence of ligaments is observed at the boundaries of admissible range and their parts guarantee the exclusively planar movements in the sagittal plane; 4) the compression of non-excited muscles is neglected.

tors causing the rotation of the hand–forearm structure have been modelled. On the other hand, the qualitative verification consists in comparing the EMG-signals measured at the defined place of each examined muscle surface with internal forces calculated during the second stage of evaluating real muscle forces.

The proposed model of muscles' group allows the forces of all individual muscles to be evaluated without using any optimisation technique. That is why

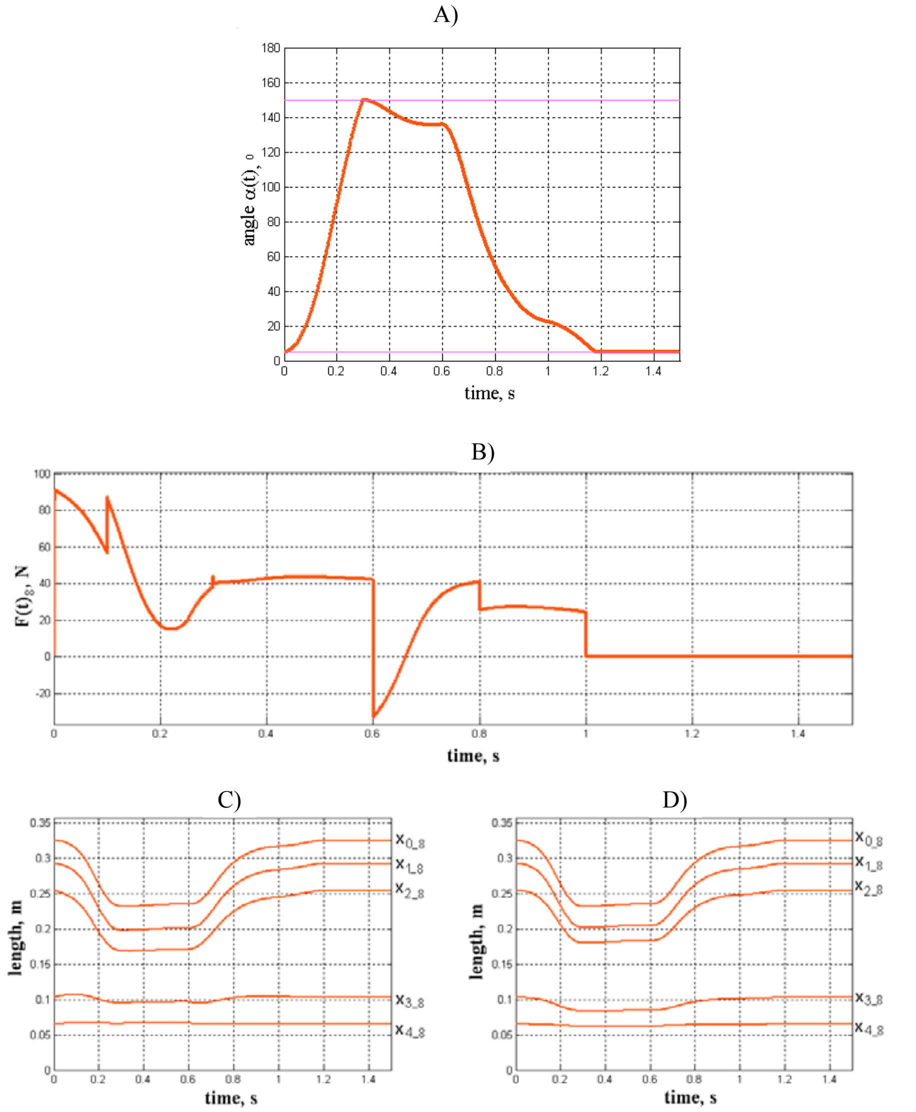

Fig. 4. Outcomes of the example of simulation: A) the angle of rotation of arm–forearm structure $\alpha(t)$, B) the *caput longum musculi bicipitis brachii* force $F_8(t)$, ($i = 8$), C) deformation of parts of exerted *caput longum musculi bicipitis brachii*, D) deformation of parts of non-exerted *caput longum musculi bicipitis brachii*

The quantitative verification consists in: 1) comparing the forces (measured in tendons) and displacements of markers (placed on the surfaces of muscles) with calculated ones; 2) evaluating the deviation $\Delta M(t)$, which allows us to check whether in the proposed model of muscles' group all the principal fac-

this model can be used: 1) to confirm a physiological correctness of optimisation criteria; 2) to achieve a better solution of steering problem in biological systems, namely, to explain clearly how the neural system controls all muscles that perform a given movement.

The outcomes of the example of the simulation of forearm rotation are presented in figure 4.

The authors are presently working on collecting complete anatomical data for the model proposed and the extension of modelling by taking into account: 1) the movements of supination/pronation at the elbow joint; 2) the action of fascia causing the interaction between adjacent muscles; 3) the influence of curved trajectories of muscle fibers.

Numerical simulations that were helpful during the development of the method presented in this paper had been performed using the computers of "Centrum Informatyczne Trójmiejskiej Akademickiej Sieci Komputerowej" in Gdańsk, Poland.

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