

# A method for analysing ram pressure characteristics of impeller pump rotor

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## Abstract

This paper presents a method in which typical tests of centrifugal pump are used to obtain information on real value of discharge angle of flow leaving the rotor. The method can be applied to properly choose inlet angle to blade palisade of centrifugal guide vanes in the case when to perform measurements of velocity fields behind the rotor more precisely is not possible.

**Keywords:** impeller pumps, characteristics, tests

## Introduction

Characteristics of impeller machines such as efficiency or ram pressure of experimental stages, are determined on the basis of certain definitions and measurements of relevant quantities, performed on a tested machine. This work deals with the following characteristics: ram pressure of radial pump rotor, as well as liquid flow torque generated by pump rotor, determined on an experimental test stand. The characteristics were determined either at constant rotational speed or at constant volumetric flow rate and varying rotational speed. The problem has been formulated as follows: if and in which way the rotor channel characteristics in the form of triangle of velocity vectors at outlet from the rotor can be determined on the basis of experimentally obtained ram pressure and torque characteristics. An important parameter belonging to the rotor characteristics is the outlet angle of flow leaving the rotor, which is usually much different from the rotor blade outlet geometrical angle. For this reason the erroneous mating of rotor to blades of guide vanes channels often occurs. The correct mating is conditioned by information on the outlet angle from rotor and inlet angle to guide vanes channel behind the rotor. This paper presents a method to obtain such information by measuring the rotor characteristics on the tested pump. The problem is in opposition to the method commonly used for determining the pump rotor characteristics on the basis of the characteristics of rotor channels.

## Basic relations

Fig. 1 presents schematic diagram of a test stand on which tests of a model pump were performed. The stand has made it possible to measure basic quantities necessary for determination of the characteristics and to perform their subsequent analysis.

The basic relation intended to be used in analysing measured quantities is the energy conservation equation

$$\frac{c_1^2}{2} + \frac{p_1}{\rho} + e_1 + a = \frac{c_2^2}{2} + \frac{p_2}{\rho} + e_2 \quad (1)$$

in the absolute coordinate system:

where:

$a$  -energy delivered to rotor,

$\frac{c^2}{\rho} + \frac{p}{2} + e$  -kinetic, pressure and internal energy at inlet (index '1') and at outlet (index '2'), respectively.

For single rotor the potential energy differences  $gz_1 \cong gz_2$  are usually very small or can be taken into account by introducing the piezometric pressure  $p + \rho gz$  to Eq. (1). The applied notation complies with that used in classical technical literature on pumps [1], [2].

If to introduce the total pressure increase measured on the model machine (marked 'm'):

$$\Delta p_m = \left( p_2 + \rho \frac{c_2^2}{2} \right) - \left( p_1 + \rho \frac{c_1^2}{2} \right) \quad (2)$$

then Eq. (1) can be expressed as follows

$$a = \frac{\Delta p_m}{\rho} + \Delta e \quad (3)$$

In Eq. (3) energy dissipation (losses) is determined by the increase of internal energy:

$$\Delta e = e_2 - e_1 > 0$$

Eq. (3) states that the energy delivered by the rotor is transformed to the total pressure increase and energy losses inside the rotor.

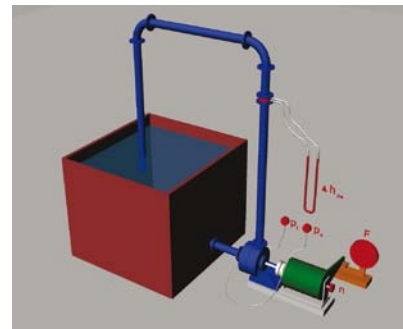


Fig. 1. Schematic diagram of model pump test stand (throttle valve not shown).

Notation:  $p_t$  – forcing pressure,  $p_s$  – suction pressure,  $h_{zw}$  – pressure difference on measuring orifice plate,  $F$  – force of action of driving motor pivotally fixed stator,  $n$  – rotational speed.

According to Euler formula, for axial inflow direction to rotor the delivered energy is expressed by the relation:

$$a = u c_u \quad (4)$$

determined by the rotor discharge parameters (index '2' assigned to the circumferential velocity  $u_2$  and its projection to circumferential direction,  $c_{u2}$ , has been here omitted).

The energy conservation equation:

$$u c_u = \frac{\Delta p_m}{\rho} + \Delta e \quad (5)$$

is expressed in the form in which appear two measured quantities:

$$\Delta p_m \quad \text{and} \quad u = \frac{\pi D_2 n_m}{60}$$

where the rotational speed  $n_m$  is measured. In Eq. (5) the quantities  $c_u$  and  $\Delta e$  are unknown. For the torque  $M_m$  measured at the pivotally fixed stator of the electric driving motor of the pump, the following relation can be written:

$$M_m = \rho \frac{D_2}{2} \cdot c_u \cdot Q_m + M_t \quad (6)$$

where:

$Q_m$  - measured volumetric flow rate;  
 $M_t$  - moment of friction of rotor disk and cover against liquid.

In Eq. (6) the quantities  $c_u$  and  $M_t$  are unknown.

By analysing the following characteristics:

$$\Delta p = p(Q), \quad \text{where } n = \text{const} \quad (7)$$

$$M_m = M(Q_m), \quad \text{where } n_m = \text{const} \quad (8)$$

and

$$\Delta p_m = \Delta p(n_m), \quad \text{where } Q_m = \text{const} \quad (9)$$

$$M_m = M(n_m), \quad \text{where } Q_m = \text{const} \quad (10)$$

it is possible to determine parameters of the velocity triangle, under additional assumptions as to the velocity triangle at outlet from the rotor. Like in the above mentioned case, with exception

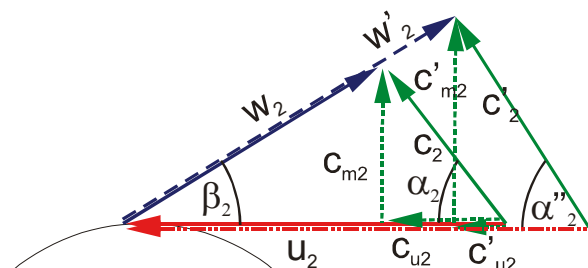


Fig. 2 Velocity triangles at rotor inlet; for  $n_m = \text{const}$  and  $Q_m = \text{var}$ .

of the characteristics (10), changes of  $M_t$  values proportional to  $n_m^2$  with an unknown proportionality coefficient, will be equivalent to changes of the rotational speed  $n_m$ . For  $n_m = \text{const}$  and varying  $Q_m$  the velocity triangle shown in Fig. 2.

The meridional velocity  $c_m$  in the rotor outlet cross-section of the breadth  $h_2$  at the diameter  $D_2$ , is determined by the formula:

$$c_m = \frac{Q_m}{\pi D_2 h_2} \quad (11)$$

If to assume that change of  $Q_m$  does not cause any significant change of the angle  $\beta$  then the differentiation of the relation:

$$\text{tg } \beta = \frac{Q_m}{\pi D_2 h_2 (u - c_u)} = \text{const} \quad (12)$$

with respect to  $Q_m$  yields:

$$\frac{dc_u}{dQ_m} = \frac{c_u - u}{Q_m} \quad (13)$$

For  $Q_m = \text{const}$  ( $c_m = \text{const}$ ) and varying  $u$  ( $n_m = \text{var}$ ) the velocity triangles are such as shown in Fig. 3.

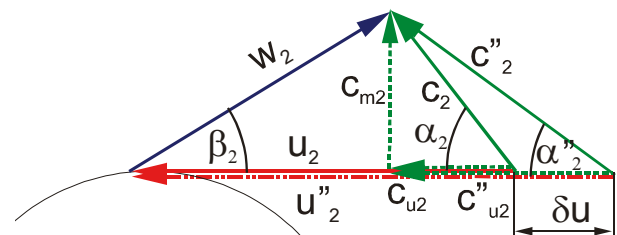


Fig. 3 Velocity triangles at rotor outlet; for  $Q_m = \text{const}$  and  $n_m = \text{var}$

It is easy to observe that in this case the following yields from Eq. (12):

$$\frac{dc_u}{du} = 1 \quad (14)$$

Eqs. (13) and (14) will be further used for analysis of the characteristics.

### Analysis of the ram pressure characteristics $\Delta p_m = \Delta p(Q_m)$ .

In Fig. 4 is presented the pump rotor characteristics  $\Delta p = f(Q)$  where:  $\Delta p$  [Pa] - ram pressure rise in function the volumetric flow rate  $Q$  [ $\text{m}^3/\text{h}$ ] at the rotational speed  $n = \text{const}$ , obtained by means of the measurements carried out on the model test stand. In Fig. 4 the indices 'm' appearing in the notation of coordinates, are omitted.

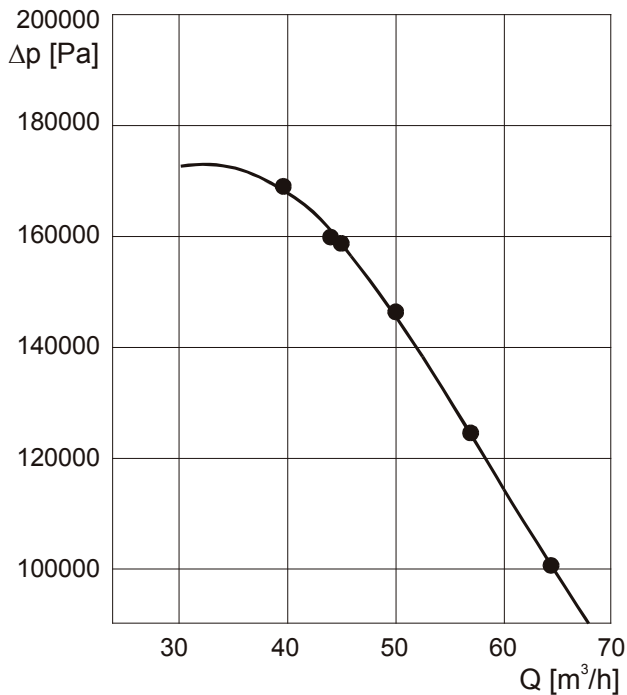


Fig. 4 The measured pump rotor characteristics  $\Delta p = f(Q)$

The measurement points were interpolated by using a 3rd order curve which yielded a satisfactory approximation within the range of flow rate of 40÷70 [m³/h].

Eq. (5) in the form:

$$\rho \cdot u \cdot c_u = \Delta p + \rho \cdot \Delta e \quad (15)$$

after differentiation with respect to Q, yields:

$$\rho u \frac{dc_u}{dQ} = \frac{d\Delta p}{dQ} + \rho \frac{d\Delta e}{dQ} \quad (16)$$

By making use of Eq. (13) the following equation is obtained:

$$\rho u c_u = \rho u^2 + Q \frac{d\Delta p}{dQ} + \rho Q \frac{d\Delta e}{dQ} \quad (17)$$

which, after dividing by  $\rho u c_u$ , makes it possible to estimate the following parameter crucial for the velocity triangle:

$$\frac{c_u}{u} = \frac{1}{1 - \frac{Q}{\rho u c_u} \frac{d\Delta p}{dQ} - \frac{\rho Q}{\rho u c_u} \frac{d\Delta e}{dQ}} \quad (18)$$

By introducing the approximations:

$$\rho u c_u \cong \Delta p \quad (19)$$

and:

$$\frac{d\Delta e}{dQ} \approx 0 \quad (20)$$

the following formula is achieved:

$$\frac{c_u}{u} \cong \frac{1}{1 - \frac{Q}{\Delta p} \frac{d\Delta p}{dQ}} \quad (21)$$

Eq. (21) makes use of information taken from the characteristics, inclusive of derivative in a selected point of it.

After determination of the quantities:

$$u = \frac{\pi D_2 n}{60} \quad \text{and} \quad c_m = \frac{Q}{\pi D_2 h_2} \quad (22)$$

for known values of: the flow rate Q, rotational speed n, rotor outlet diameter  $D_2$ , and breadth  $h_2$ , the following angles appearing in the velocity triangle, can be determined:

$$\alpha = \text{ArcTan} \frac{c_m}{c_u} \quad \text{and} \quad \beta = \text{ArcTan} \frac{c_m}{u - c_u} \quad (23)$$

### Analysis of the rotor torque characteristics

$$M_m = M(Q)$$

As the pump motor stator is pivotally fixed there is possible to measure the rotor - generated torque. Example results of the torque measurements are depicted in Fig. 5 where the experimental points has been interpolated by using a 2nd order curve. For their analysis the relation (6) will serve as the basis. Let's observe that at the constant rotational speed n and only the flow rate Q variable the following approximation seems to be good:

$$\frac{dM_t}{dQ} \approx 0 \quad (24)$$

Hence by differentiating the relation (6) the following expression is obtained:

$$\frac{dM}{dQ} = \rho \frac{D_2}{2} \left( c_u + Q \frac{dc_u}{dQ} \right) \quad (25)$$

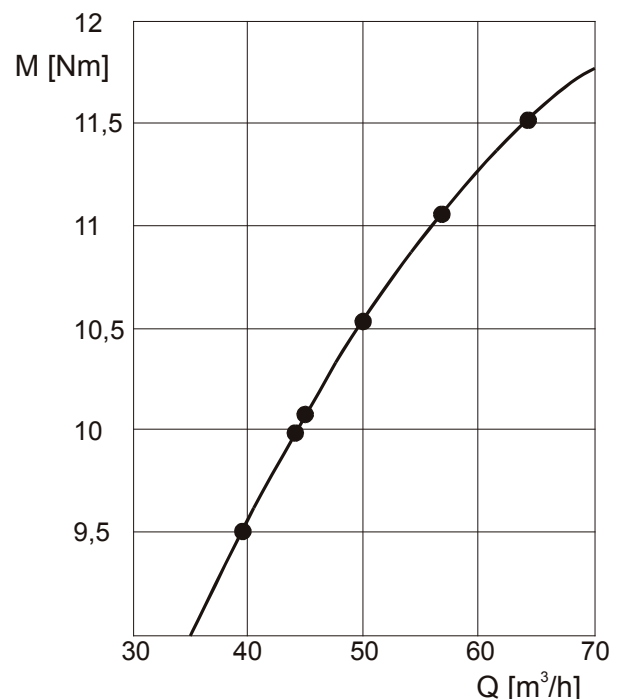


Fig. 5 The measured pump rotor characteristics  $M = f(Q)$

From Eq. (25), by making use of Eq. (13), the following formula can be obtained:

$$\frac{dM}{dQ} = \rho \frac{D_2}{2} (2 c_u - u) \quad (26)$$

from which it yields:

$$\frac{c_u}{u} = \frac{1}{2} + \frac{1}{\rho D_2 u} \frac{dM}{dQ} \quad (27)$$

The right hand side of the relation is determined on the basis of the experimental characteristics  $M(Q)$  and measured values of the rotational speed  $n_m$ . The angles are determined from the expressions (23).

### Analysis of the ram pressure characteristics $\Delta p_m = \Delta p(u)$ .

A similar analysis can be performed for the pump rotor characteristics  $\Delta p = f(u)$  at  $Q = \text{const.}$  The characteristics are shown in Fig. 6 where the experimental points were interpolated by using a 2nd order curve which provides a good approximation.

$$c_u + u \frac{dc_u}{du} = \frac{1}{\rho} \frac{d\Delta p}{du} + \frac{d\Delta e}{du} \quad (28)$$

Differentiation of the relation (5) with respect to  $u$  gives:

$$1 + \frac{u}{c_u} = \frac{u}{\rho c_u} \frac{d\Delta p}{du} + \frac{\rho u}{\rho c_u} \frac{d\Delta e}{du} \quad (29)$$

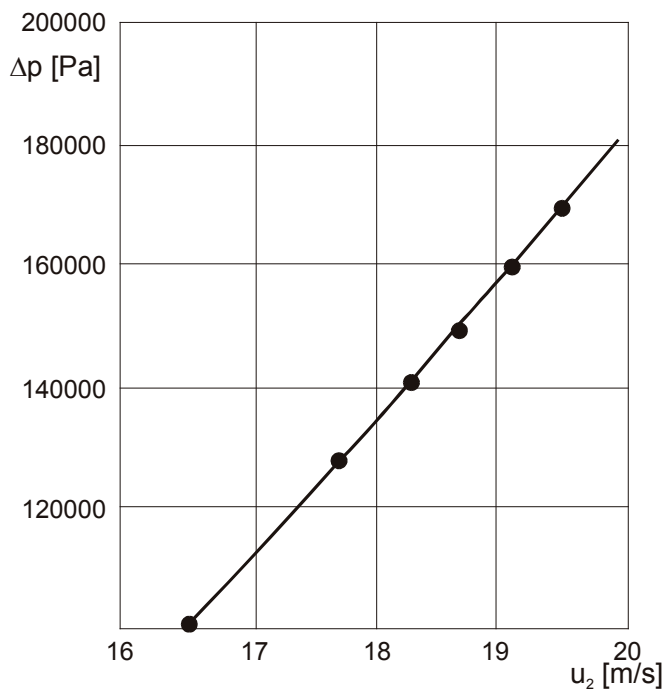


Fig. 6 The measured pump rotor characteristics  $\Delta p = f(u)$

By making use of Eq. (14) the above given relation can be transformed into the following:

$$\frac{c_u}{u} \cong \frac{1}{\frac{u}{\Delta p} \frac{d\Delta p}{du} - 1} \quad (30)$$

Finally the approximation (19) and the assumption on a small change of losses resulting from  $u$  (see Eq.(20) yields the formula:

whose right-hand-side values can be determined from the characteristics such as given in Fig. 6. Next from the expressions (23) the angles can be determined.

### Discussion of results of analysis of experimental characteristics

The calculation results of the outlet angles  $\alpha$  and  $\beta$  for particular characteristics, are presented in Figs. 8 through 10. The values obtained from the calculations according to Eqs. (21), (27), (30) are burdened both by measurement systematic errors and simplifying assumptions.

The characteristics  $\Delta p = \Delta p(Q)$  are „contaminated” by backflow drag behind the rotor and guide vanes drag in the centripetal channel behind the rotor. In the conditions of the performed experiment it was not possible to obtain separate characteristics for the rotor only. Drag rise behind the rotor at greater values of  $Q$  leads to lower values of measured ram pressures, that - in consequence - gives greater values of the angle  $\alpha$ . If behind-the-rotor drags were reduced and

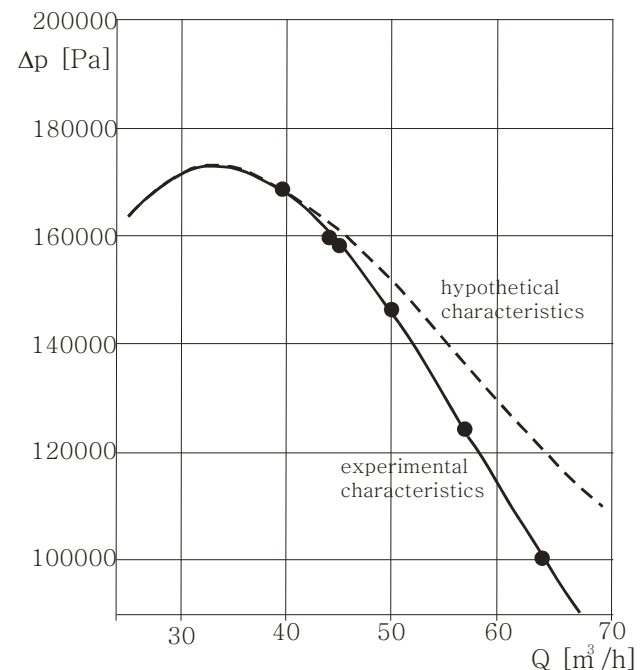


Fig. 7 Comparison of the experimental and hypothetical pump rotor characteristics

a greater ram pressure rise were this way obtained as it indicate the hypothetical characteristics shown in Fig. 7, then good conformity between the angles  $\alpha$  calculated from Eqs. (21) and those obtained from torque measurements, acc. Eq. (27), would be achieved. The comparison of the values calculated according to the formula (21) for the experimental and hypothetical characteristics demonstrates influence of the error residing in the characteristics on the determined values characterizing the velocity triangle. In Fig. 8 the point determined on the basis of the calculations performed by using a 3D code, is presented. This is the value of the angle  $\alpha$ , calculated as the surface average. The value relatively well fits those determined from the characteristics.

The determined angle  $\alpha$  makes it possible to calculate value of the rotor outlet angle  $\beta$  which constitutes an important feature of rotor blades palisade. Fig. 9 presents the calculation results. The change of the angle  $\beta$ , observed in the diagram, demonstrates an approximate character of the assumption on its invariance. Only variability ranges for  $Q$  – values, within which the changes would be sufficiently small, can be indicated.

Analysis results of the characteristics  $\Delta p = \Delta p(u)$  are given in Fig. 10 and 11. The experimental point shown in Fig. 10 is situated relatively close to the point obtained from the analysis of experimental characteristics. The small changes of the angle  $\beta$ , seen in Fig. 11, seem to satisfy the assumption on its invariance, better. However, by comparing

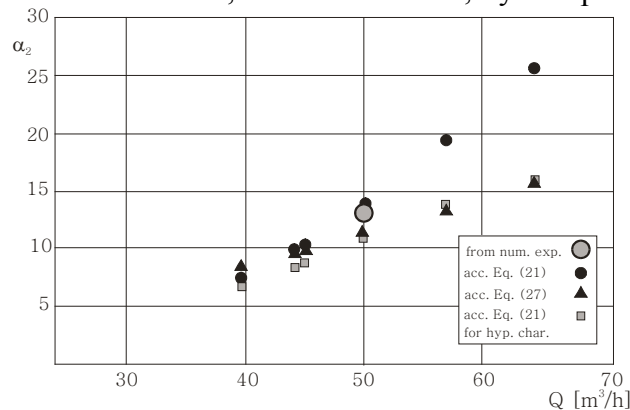


Fig. 8 Comparison of calculation results of the discharge angle  $\alpha_2$ , obtained from experiment with those obtained from 3D-calculations

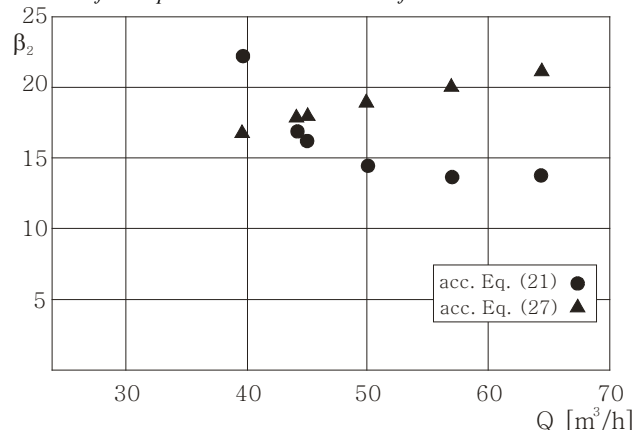


Fig. 9 The discharge angle  $\beta_2$  calculated on the basis of experiment

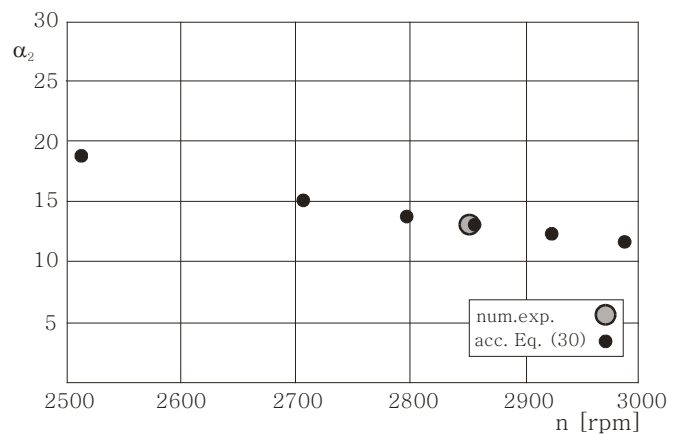


Fig. 10 Comparison of calculation results of the discharge angle  $\alpha_2$  acc. Eq. (30) with those obtained from numerical experiment

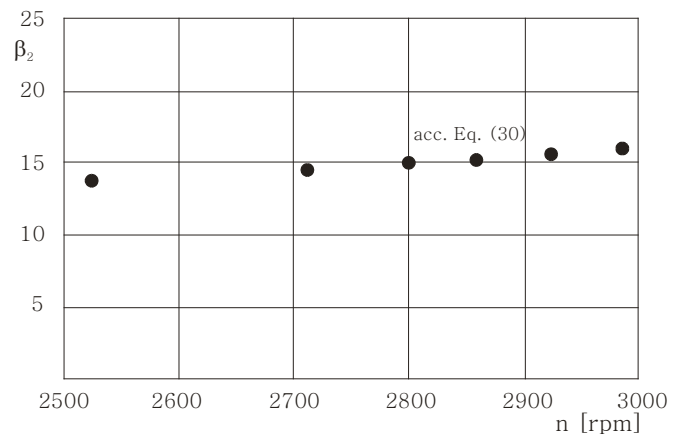


Fig. 11 Values of the discharge angle  $\beta_2$  calculated acc. Eq. (30)

Fig. 9 with Fig. 11, the discrepancy in the angle  $\beta$  values of the range from  $14^\circ$  to  $20^\circ$ , was revealed. The value of the geometrical angle of the rotor outlet edge amounted to about  $30^\circ$ .

The angle  $\alpha$  determined from the analysis makes it possible to more properly design the inlet to guide vanes channel behind the rotor for a given nominal value of  $Q$ .

### Final remarks

The presented method for determining the velocity triangle parameters on the basis of the pump rotor characteristics can be used in the case if other possibilities to measure flow parameters are lacking. Information on the angle of flow discharge from the rotor is especially important. It much differs from the rotor blade geometrical outlet angle. This is an important parameter for correct forming the inlet to guide vanes ring behind the rotor. In this paper attention has been also drawn to the necessity of exact determination of discharge pressure from the rotor. In the described experiment the pressure was „contaminated” by flow drag occurring in the space between the rotor outlet and the forcing pressure measurement point. For this reason the experiment was not sufficiently „pure” for the presented method. Nevertheless its relatively good conformity with the experiment was achieved at some scatter of values of the determined angles.

The analysis demonstrates in which sense the rotor characteristics constitute „carriers” of information on flow parameters.

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### Nomenclature

- a – work determined by Euler equation;
- c – absolute velocity;
- e – internal energy;
- h – rotor breadth;
- n – rotational speed;
- p – pressure;
- u – circumferential velocity;

- z – location height;
- D – rotor diameter;
- M – torque;
- Q – volumetric flow rate (volumetric flow capacity);
- $\alpha$  – angle between directions of the absolute velocity (c) and the circumferential velocity (u);
- $\beta$  – blade angle;
- $\Delta$  – rise of a quantity;
- $\rho$  – liquid density;

### Indices

- 1 – (stands for) parameters at inlet edge of rotor blade;
- 2 – parameters at outlet edge of rotor blade;
- m – parameters of model machine;
- u – projection towards direction of circumferential velocity;
- t – quantities associated with friction.