Cent. Eur. J. Eng. • 1(1) • 2011 • 59-80
DOI: 10.2478/s13531-011-0005-x

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Central European Journal of **Engineering**

A literature review on computational models for laminated composite and sandwich panels

Review Article

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Received 27 January 2011; accepted 23 February 2011

Abstract: The present paper is devoted to a state-of-the-art review on the computational treatment of laminated composite and sandwich panels. Over two hundred texts have been included in the survey with the focus put on theoretical models for multilayered plates and shells, and FEM implementation of various computational concepts. As a result of the review, one could notice a lack of a single numerical model capable for a universal representation of all layered composite and sandwich panels. Usually, with the increase of the range of rotations considered in the particular model, one can observe the decrease of the degree of complexity of the through-the-thickness representation of deformation profiles.

Keywords: Composite laminates • Sandwich panels • Multi-layered shells • Computational models • Finite elements • Literature Survey

1. Introduction

Composites are made of two or more materials, combined together to obtain a new matter with properties that are superior to those of individual components. It is quite obvious that mechanical properties of composites depend mainly on the choice of material components used for the composite but they are also considerably influenced by the applied fabrication technique. Probably the most suitable for structural applications among all composite materials there are *fiber reinforced composites* (FRC), with the reinforcement taking the form of either *continuous (long) fibers* or *whiskers* (*short fibers*), [1–5].

Composites reinforced with continuous fibers frequently appear as *fiber reinforced composite laminates,* see Fig. 1.

A typical *fiber reinforced composite laminate* is made of a number of unidirectional fiber reinforced composite

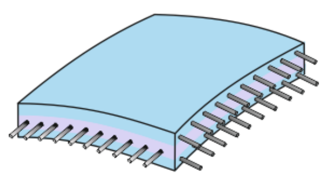

Figure 1. An example of a FRC three-layer laminate.

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layers, [1–3]. A composite layer with a parallel system of reinforcement fibers represents an orthotropic medium with three mutually orthogonal planes of symmetry. Usually a composite layer consists of high modulus fibers (typically they are glass, boron or graphite fibers) embedded in a matrix (epoxy or polyamide). Considering its light weight, a lamina of fiber reinforced composite is remarkably strong along the fiber direction. However, the same lamina is considerably weaker in all off-fiber directions. To address this issue and withstand loadings from multiple angles, one would use a laminate constructed by a number of laminas oriented at different directions. A laminated composite panel can be considered as an optimal structure with effective utilization of composite material directional properties [3]; however, fiber reinforcement can be applied also in a three-dimensional layout (compare Tong *et al.* [6]).

A *sandwich shell* and a *thin composite laminate* take a form of a *layered shell* that is built of several laminas bonded together with some adhesives. A typical *sandwich shell* consists of a light core and two thin outer faces, which can take a form of either isotropic or laminated composite panels. The light core does not transfer significant forces; the most important for the core is its low weight. When the core is made as a composite then the requirement for reinforcement is minimal and the contribution of light fillers significantly increases. Very often the fillers are simple empty voids, and then one can talk about *porous cores* or *foams*. Another option is a core build as a 3D structure, *e.g.* corrugated panels and spatial lattices of beams or plates. Probably the most popular type of a sandwich core is based on two-dimensional cellular geometries with large-scale cells (see *e.g.* [7–9]); here the flagship example is a hexagonal sheet structure that visually resembles a product of the apiarian industry, therefore is commonly known as a *honeycomb* (Fig. 2). Other examples of multilayered anisotropic structures are laminates made of different isotropic layers *e.g.* employed for better thermal insulation or noise suppression. Considering the application of layered structures, one should not forget also smart thin-walled sandwich structures with piezoelectric layers embedded for passive or active vibration or sound control (see *e.g.* [10, 11]).

2. Theoretical models for multilayered thin-walled structures

The increasing use of laminated composites and sandwich panels demands a better understanding of the behavior of multilayered thin-walled structures (see *e.g.* [11, 12]).

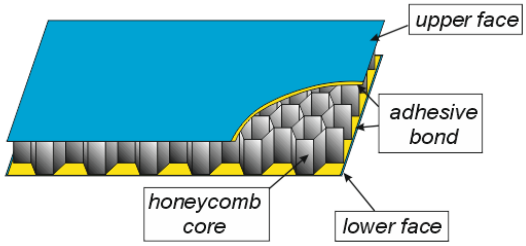

Figure 2. Sandwich plate with a honeycomb core.

Generally, analysis of such complex structure as a *fiber reinforced composite laminate* can be performed either from the *micro-mechanical* or *macro-mechanical* point of view, [1, 2, 11, 13]. It is quite obvious that a precise study of interaction between the fibers and the matrix can be examined in detail only in the *micro-mechanical* scale. However, costs of *micro-mechanical scale* calculations of any real structure are still far too high for practical applications; a much more realistic option but still quite costly is multi-scale modeling (see *e.g.* [14–17] and the review paper by Ladevéze [18]). If one is interested in an overall performance of a thin-walled structure made of *fiber reinforced composite laminates*, then the *macro-mechanical* modeling can be applied, where all micro-scale effects are smeared in a phenomenological material model.

The *macro-mechanical* models of laminated plates and shells are usually constructed according to an appropriate *lamination theory*, where it is assumed that the laminated panel is made up by a certain number of layers, which are supposed to be perfectly bonded together. In such a model a single layer is considered as an elementary and homogenous part of the structure. Therefore, even in a case of *fiber reinforced composite laminates* each lamina can be considered as a complete physical entity instead of a collection of isolated components. The effective properties of a layer made of any heterogeneous material can be obtained by the homogenization that may be understood as "*finding a homogeneous comparison material that is energetically equivalent to a given microstructured material*", Böhm [19] (see also [14, 16, 20]).

Basically, talking about 2D computational models for multilayered shells one can distinguish between two primary categories of lamination theories: the *Equivalent Single Layer* (ESL) model or *Discrete-Layer* (DL) theory, [21–25]. Discrete-layer models appear also very often in the literature as layer-wise theories, see *e.g.* [23, 26–29].

2.1. Equivalent Single Layer models

In the ESL model the entire laminate is represented by a single-layer panel with macro-mechanical properties estimated as a weighted average of the mechanical properties of each lamina. A single layer model replacement for the heterogeneous panel can also be constructed with a help of the homogenization technique, see *e.g.* [30–33].The *Equivalent Single Layer* model used in conjunction with the classical *Kirchhoff-Love* theory of thin shells/plates is commonly known as the *Classical Lamination Theory* (CLT) (see *e.g.* [1, 2, 25, 34]). Sun & Chin [35] presented the von Kármán-type CLT model for a geometrically non-linear analysis of laminated composite plates; a similar large displacement formulation for thin composite shells was given by Saigal *et al.* [36]. Composite multilayered shells are typically characterized by a large ratio of Young's modulus to shear modulus, even if they are relatively thin (see *e.g.* [1, 2, 5]). Therefore, the CLT formulation, which does not account for the transverse-shear deformation, is rather inadequate for accurate prediction of the elastic behavior of composites. In order to overcome that limitation a refined lamination theory is required that accounts for transverse shear deformation. An extension of the *First-Order Shear Deformation* (FOSD) model for laminated plates was described by Whitney & Pagano [37], whereas Dong & Tso [38] presented FOSD model for composite shells (see also [1, 2, 25, 39, 40]). Reissner [41] developed a FOSD model designated for sandwich shells. Various variants of CLT and FOSD shell theories were examined in a linear static analysis of cylindrical laminated shells by Chandrashekhara & Pavan Kumar [42, 43]. An interesting asymptotic formulation of the FOSD multilayered plate theory based on the mixed Hellinger-Reissner variational principle was presented by Tarn & Wang [44]. Large deformation formulations based on the FOSD model was presented *e.g.* by Reddy & Chandrashekhara [45], Schmidt & Reddy [46], Palmerio *et al.* [47], Kreja *et al.* [48].

It is well known that FOSD models require an appropriate transverse shear correction due to a constant shear distribution across the shell thickness resulting from the linear interpolation of the displacement field in that direction. One should realize, however, that the estimation of a shear correction for laminated composites is much more complicated than for homogeneous panels. In the literature one can find innumerable proposals of different formulas for appropriate shear correction factors, depending on material properties and also on such geometrical characteristics of the laminate as stacking sequence of the layers and their ply angles. Most of those formulas have been determined by matching the transverse shear strain energy predicted by the FOSD plate model with that obtained from the three-dimensional elasticity theory (see *e.g.* Dong & Tso [38], Whitney [49], Wittrick [50], Vlachoutsis [51], Jemielita [52]). Noor & Peters [53] as well as Sze *et al.* [54] calculated the transverse shear correction factors for multilayered cylindrical panels by means of the predictor-corrector approach. A similar methodology was applied by Auricchio & Sacco [55, 56], who determined the shear correction in an iterative manner. Both those tactics based on the comparison between the shear energy computed for the transverse shear stress obtained from constitutive relations and the shear energy calculated for transverse shear stress recovered from the three-dimensional equilibrium. Another approach was proposed by Pai in [57] where not only the transverse shear strain energy but also the shear stress resultants estimated with the FOSD model were balanced with those calculated by the layer-wise higher-order shear theory. Rolfes & Rohwer [58] introduced an "*improved*" transverse shear stiffness in the FOSD model as calculated with the assumption of a cylindrical bending mode and by utilizing the differential relation between the resulting transverse shear forces and bending moments. Similar procedure was incorporated also in the commercial FEA system MSC-Nastran [59]. Altenbach [60] estimated the shear stiffness of layered plates by comparing the forces and moments calculated from two-dimensional and three-dimensional models. Tanov & Tabiei [61] presented a *Corrected* FOSD model where they enforced a parabolic shear strain distribution across the shell thickness, what not only improved a profile of the transverse shear stress but also eliminated the need for using a shear correcting factor. Although the authors of [61] classified themselves their model as the "*displacement-based formulation*" it should be rather classified as the mixed formulation based on the Reissner partial-mixed variational principle [62]. Similar strategy was adopted by Fares & Youssif [63], Fares *et al.* [64], and Auricchio & Sacco [55, 56, 65] who presented a collection of different *Refined* FOSD models, all based on independent approximations of the transverse stress fields introduced in the (partial-)mixed variational principle, but varied in the final number of unknowns. A slight different version of *Refined* FOSD theory, although corresponding to some extent with the model of Tanov & Tabiei [61], was proposed by Qi & Knight [66] and was labeled as a *Consistent* FOSD in the subsequent paper of those authors [67]. In their approach the effective transverse shear strains of the FOSD was treated as the stress-weighted average of the through-the-thickness transverse shear strains based on the equivalent shear strain energy. Evaluation of the effective shear stiffness in that formulation is comparable to the application of the transverse shear correction factor.

For thick panels a significant improvement of results can be obtained by applying the HOSD models, where the conventional displacement equations of FOSD are supplemented with various higher-order terms. In the most popular methodology of the "*derived approach*" a 2-D plate/shell model is constructed by applying a power series expansion of the displacements and strains with respect to the thickness coordinate. Starting with that formulation one can obtain a shear deformation model of an arbitrary order depending on the selected level of truncation. In 1984 Reddy [68] developed a *Third Order Transverse Shear Deformation* (TOSD) theory for laminated plates assuming a cubic representation of the displacement field with respect to the height coordinate (see also Khdeir *et al.* [69], Phan & Reddy [70], Reddy [24, 71] and Reddy & Arciniega [39]). An extension of that model for laminated shells was presented by Reddy & Liu [72] (see also Reddy's handbook [11]). By using the condition of vanishing transverse shear stresses on top and bottom surfaces they reduced a set of unknowns to exactly the same five displacement components as in the FOSD model. Three various TOSD models of multilayered plates were examined in the linear analysis by Bose & Reddy [73]. Reddy extended his TOSD model for the von Kármán-type non-linear plate theory in [74]. Dennis [75] and Simitses [76] used a similar approach to obtain analytical TOSD solutions for circular laminated cylindrical panels modeled within the range of the von Kármán geometric non-linearity by means of the modified Galerkin method. A corresponding formulation for large rotation shell theory was presented by Başar [77] and by Başar *et al.* [78], who considered also a 7-dof model where two extra displacement variables were included due to enriched approximation of the displacement field across the thickness of the panel. A similar TOSD model with 7 dofs was proposed for geometrically non-linear shells by Balah & Al-Ghamedy [79], who additionally applied singularity-free description of finite rotations based on exponential mapping after Simo *et al.* [80]. Another implementation of the Reddy concept of TOSD [68, 72] can be found in the *Simplified Large Rotation* (SLR) formulation proposed for laminated shells by Palazotto and co-workers, Dennis & Palazotto [81, 82], Naboulsi & Palazotto [83], and Tsai *et al.* [84]. The *Second-Order Shear Theory* (SOSD) for laminated shells in the range of moderate rotations was examined by Sacco & Reddy [85] who found that inclusion of second order terms did not significantly improve the linear solution over the FOSD model. Moita *et al.* [86] applied the *Higher-Order Shear Theory* (HOSD) model to investigate the buckling behavior of laminated panels. Piskunov [87] described an iterative analytical theory of composites, where starting from the CLT formulation, one can obtain a HOSD-equivalent model by successive approximations. Quite a general geometrically non-linear HOSD laminated shell theory was presented by Librescu [88].

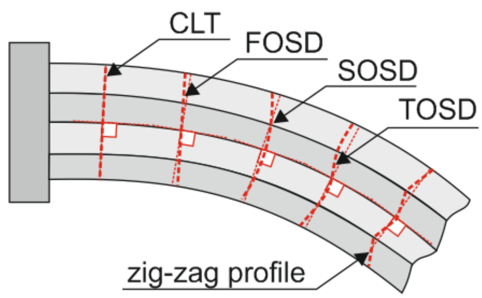

Figure 3. Deformation profiles of a layered panel represented by different shear deformation models.

The 3-D elasticity solutions of laminated plates (see *e.g.* Pagano & Hatfield [89]) exhibited rapid changes of the displacement profile at the interfaces between two contiguous layers. This phenomenon is commonly classified as the *zig-zag effect*. To account for that feature the kinematical model of the layered shell should be enhanced by adding some *warping functions* that are capable to represent the deformed profile with a different slope in each layer. A similar zig-zag effect can be recognized also in displacement fields obtained in some stress based formulations (see *e.g.* Ambartsumyan [34]). The ability of different shear deformation theories to represent the deformation profiles of a layered panel is illustrated in Fig. 3.

By assuming a piecewise linear approximation for the warping function one in fact adopts the FOSD hypothesis for each individual layer of a multilayered shell (see *e.g.* Brank [90], Brank & Carrera [91, 92], Carrera [93, 94], Di Sciuva [95], Toledano & Murakami [96]). However, in the pioneering zig-zag model for multilayered plates presented by Ambartsumyan [34] the resulting through-the-thickness distribution of in-plane displacement field is cubic in each layer; similar piecewise TOSD zig-zag models were considered by Di Sciuva [97], Di & Rothert [98, 99], Lee *et al.* [100], Toledano & Murakami [101]. The warping function can be given explicitly, for example as a zig-zag function connected with two additional unknowns for the whole cross section, [93, 96, 98, 99, 101, 102]. Savithri & Varadan [103] presented a TOSD formulation for composite plates, where the zig-zag effect was included by application of Heaviside step function in the description of the displacements distribution across the plate thickness. Another way is to construct the warping function by invoking interlayer shear stress continuity conditions and zero shear traction boundary conditions on the upper and lower bounding surfaces. Then the resulting model contains exactly the same number of unknowns as the standard FOSD formulation, but requires C^1 type continuity in the FEM implementation (see *e.g.* formulations proposed by Di Sciuva [95, 97], Lee *et al.* [100], Librescu & Schmidt [104, 105], He [106], Shu [107]). Lee & Cao [108] proposed a predictor-corrector method of laminated shells analysis based on zig-zag models of Di Sciuva [95] and Lee *et al.* [100]. An interesting variant of that approach was presented by Soldatos & Watson [109] who proposed to enhance a standard 5-dof model for small displacement analysis of laminated plates by an introduction of special *shape functions*, which could be determined *a posteriori* from the stress equilibrium equations. Working along similar lines, Cho *et al.* [110] proposed the *Efficient Higher-Order Shell Theory* based on an overall cubic distribution of in-plane displacements combined with a piecewise linear profile. Similar model but with cubic function replaced with the sinus function was proposed by Idlbi *et al.* [111] and by Fernandes [112]. Arya *et al.* [113] proposed a zig-zag model enhanced by the application of several trigonometric functions in the displacement field. Hassis [114] in his higher order plate theory introduced a warping function constructed on the basis of deformation modes of the normal fiber treated as a geometrical beam. Such an approach resembles the earlier idea of Sutyrin & Hodges [115], who applied the variational-asymptotical method to split the 3-D analysis of plate deformation into two separate reduced-dimensional problems: a 2-D Reissner–type plate theory analysis and a 1-D through-the-thickness analysis (an extension of that idea for non-linear shell theory was presented by Yu & Hodges [116] and Yu *et al.* [117]). Quite recently, Kim & Cho [118] presented an *Enhanced* FOSD theory for laminated plates constructed as weighed least-square approximation of a 3-D theory. The warping function incorporated in that formulation was obtained with the HOSD plate theory [110] and the resulting effective transfer shear stiffness could be considered as being analogous to that used in the *Consistent* FOSD of Knight & Qi [67].

2.2. Discrete-layer theories

Despite the fact that the performance of ESL models can be significantly improved by inclusion of various warping functions, it is almost impossible to construct a universal ESL model which would be equally efficient for symmetrically and asymmetrically laminated panels. Therefore, the next step on the way to increase the accuracy of the multi-layered shell models has to go beyond the limits of a single layer model, i.e. it is necessary to consider each layer separately within *discrete-layer* (DL) theories named also the *layer-wise* formulations.

At this point, it is worth to notice that some authors used to extend the term "*layer-wise formulation*" also to include those equivalent single layer models which were enhanced by addition of some warping functions (see *e.g.* Rohwer *et al.* [25]). To some extent such an approach can be justified by the common in the both models abandonment of the $C_{z=}1$ requirements, what means that functions describing the displacement distribution in thickness direction can exhibit rapid changes of slopes at the interfaces between two contiguous layers. Nevertheless, the main difference between the ESL model with the warping function and the DL formulation is that the number of unknowns in the ESL model does not depend on the number of layers (usually after taking advantage of some compatibility conditions to eliminate local unknowns).

The laminate in the DL theory is treated as a stack of laminas bounded together by appropriate conditions at ply interfaces. Since each lamina is treated individually, the number of unknowns in DL theories depends on the number of layers, *N*. Kulikov [22] (see also Piskunov [87]) claimed that the DL theory originated from the layer-wise description of sandwich panels introduced by Grigoluk in the 1950s. On the other hand, any DL model was mentioned neither in a review article on layered shells theories by Ambartsumyan [119] from 1962, nor in a survey of developments in the analysis of sandwich structures published three years later by Habip [120]. A layer-wise theory presented for laminated plates by Mau in 1973 [121] used *4N+1* displacement unknowns accompanied by *2(N-1)* additional unknown variables representing the inter-lamina shear stresses. In 1978 Pagano [122] presented an approximate theory for stress analysis in composite laminates, where he assumed in-plane stresses represented within each layer by linear functions of the thickness coordinate. The stress equilibrium equations were expressed in force and moment resultants and formulated separately for each layer, and the set of equations were supplemented with appropriate interface conditions. A final number of unknowns in the Pagano model [122] was equal to *13N*. One can notice that the order of computational complexity of both just mentioned formulations is relatively high, especially that Mau [121] as well as Pagano [122] suggested to model each physical layer with two or three computational sub-layers to provide a satisfactory accuracy of the results. Much more economical DL theories were based on an independent shear deformation of the director associated with each individual layer and involving just *3+2N* displacement unknowns (three global displacements for the whole laminate and two local rotations for each layer). An example of such an approach was a layer-wise laminate plate theory proposed by Mawenya & Davies [123] and described also by Reddy [24, 71]. A corresponding laminated shell model was presented by Chaudhuri & Seide [124] (see also [125]) as the "*layerwise constant shear-angle theory*", and by Noor & Burton [21, 126] as the "*discrete layer shell theory*". A similar formulation but with *3(N+1)* unknowns due to accounting for 3-D effects was presented for laminated shells by Huttelmaier & Epstein [127] as well as by Masud & Panahandeh [128].

It is crucial to distinguish between two quite different concepts of shell kinematics that are quite often identified in the literature with the common label of the "*multi-director model*". On the one hand, Pinsky & Kim [129], Braun *et al.* [130] and Wagner & Gruttmann [131] used this term to describe DL models with independent director vectors in each layer, on the other hand, Krätzig [132] presented a *multi-director* single-layer shell model that can be considered as being analogous to the general HOSD shell theory of Librescu [88], but in contrast to the latter, belonging to the category of the "*direct approach*" methodologies (see also [133–135]). A prototype of that formulation was given earlier by Naghdi (see *e.g.* equation (2.22) in [136]) but without introducing the name: "*multi-director model*". To avoid any possible confusion, in the present report the expression "*multi-director model*" stands only for the DL formulations being equivalent to that presented by Pinsky & Kim [129].

A "*multi-director formulation*" of Pinsky & Kim [129] accounted for visco-elastic material behavior and large deformations including the thickness stretching; therefore the number of unknowns in that model was extended to *3+4N*. Cho & Averill [137] combined the zig-zag model of Di Sciuva [95] with a layer-wise formulation obtaining a "*First order zig-zag sublaminate plate theory*" with *5(N+1)* unknowns. The generalized laminated plate theory of Reddy [71] (see also [138]) offers a quite universal description of the layer-wise model with an arbitrary order of displacement interpolation within each layer assuming kinematical variables located at the interfaces. A similar concept was considered by Gaudenzi *et al.* [139] who, however, imposed the continuity of interlaminar stresses only at selected interfaces in order to limit the total number of unknowns. Carrera [140] presented a mixed variational formulation of a layer-wise multilayered plate theory with variable fields of displacements and transverse stresses interpolated by Legendre polynomials of a chosen order (see also [27–29]). A displacement formulation of that model was considered in [140] as a special reduced variant of the layer-wise plate theory with limited number of unknowns but also without continuity of transverse shear and normal stresses. Başar [77] and Başar *et al.* [78] introduced DL models with inextensible multi-director (*3+2N* unknowns) for finite rotation analysis of composite shells. A corresponding model based on the *geometrically exact* shell formulation of Simo *et al.* [80] was presented by Vu-Quoc *et al.* [141] with the main assumption that "*the transverse fiber across the whole multilayer shell deforms as a chain of rigid links that are connected to each other by universal joints*". Başar *et al.* [133] and Braun *et al.* [130] applied the 7-parameter FOSD shell theory for each single layer, what resulted in DL formulations with *3+4N* unknowns. Başar & Ding [26] considered the transverse normal strains in their DL models with *3+3N*, *3+4N*, and *3+6N* unknowns. Gruttmann & Wagner [142] presented a DL multilayered shell model based on the HOSD theory with *3+9N* unknowns. Williams & Addessio [143] constructed a DL model for analysis of plate delamination problem; in their model the layer displacement variables were supplemented with interfacial traction terms and appropriate evolution laws describing the damage growth.

It is quite interesting that traces of DL formulations can also be found in some ESL zig-zag models. As a typical example one can consider the ESL zig-zag theory of Di Sciuva [95], who started his derivations assuming a multi-director description of the displacement field with independent rotational parameters in each layer. In the next step of the formulation those parameters were eliminated by invoking the shear stress constraints. As it was mentioned earlier in this chapter, the final number of unknowns in the ESL model of Di Sciuva [95] (and also in other models of that kind [104–107]) was equal to five. A quite similar concept can be recognized in the laminate theories proposed by Li & Liu [144], where the displacement field was expressed with global components of TOSD theory and local components of DL model combined within so called *global-local superposition technique*. After taking advantage of continuity conditions the model constructed by Li & Liu [144] used 13 layer independent variables, what makes six more unknowns than in a standard TOSD model.

2.3. Three-dimensional and combined models

It should be emphasized that either the ESL or DL formulations described above are in fact 2-D models; one can quite easily imagine that multilayered shells can be analyzed also with the use of 3-D models based on elastic three-dimensional continuum. However, practical application of such models is very limited due to the dramatic increase of the number of unknowns. Analytical 3-D solutions presented by Pagano in 1969 [145] and

1970 [146] for selected examples of simply supported laminated composite plates under distributed transverse loads serve as classical benchmark problems. They are still used by many researchers for the purpose of validating their diverse theories of laminated plates [89, 122, 147]. Bogdanovich & Yushanov [148] presented a 3-D displacement-assumed variational analysis of laminated composite plates. Williams & Addessio [143] extended Pagano 3-D model to include the problem of delamination. Desai *et al.* [149] and Feng & Hoa [150] modeled layered composites as a stack of 3-D solid sub-elements along the thickness of composite laminates.

It seems quite obvious that any attempt to use a detailed 3-D model for the whole analyzed panel very soon would finish in exceeding reasonable limits of practical application. Therefore a concept of *global-local* analysis (Feng & Hoa [150], Cho & Averill [137], Kong & Cheung [151]) appears to represent a rational compromise between the requirements of precise analysis and realistic costs of calculations. The primary step is to select those regions of the analyzed structure, named *local zones*, where a more detailed analysis (3-D or at least DL model) is required due to vicinity of holes, constrained boundaries or other discontinuities. One can assume that the remaining parts of the structure, the *global zones* are more regular and therefore they can be treated with accepted accuracy as regions of a single layer panel with equivalent laminate properties (ESL model). Depending on specifics of the applied approach a special treatment of transition zones may be necessary, see *e.g.* [150–152]. By combining local and global zones in one computational model one can significantly improve the accuracy of the analysis staying within a moderate range of computational costs.

2.4. Recovery of transverse stress

Using a FOSD model with a proper estimation of a transverse shear stiffness one can attain a quite satisfying accuracy of a global response of moderately thick laminated panels. However, whereas the FOSD results for deflections and rotations or even for in-plane stresses are acceptable, a direct application of constitutive relations must result in a wrong profile of the transverse shear stresses (being constant across the thickness of each layer). A simple correction of the FOSD model resulting in a much more realistic distribution of the transverse shear stress can be obtained when the transverse stresses are calculated from the stress equilibrium condition of the in-plane stress components (see *e.g.* [54, 58, 153–155]). Carrera [156], Das et al [157] and Rohwer *et al.* [25] showed that using the stress equilibrium for the estimation of transverse stresses can also be very effective for HOSD or DL models. The same strategy was applied also in the CLT models; see *e.g.* Ambartsumyan [34] or Jones [1]. A slightly different post-processing method was constructed by Cho & Kim [158], who applied a displacement approximation of the HOSD theory to reinterpret the results from the FOSD analysis by matching rotational variables of both kinematical models (similar approach can be found also in [159]). The improved displacement field predicted by that procedure provided a satisfactory accuracy of transverse shear stress calculated directly from the constitutive relations. Wisniewski & Schrefler [160] introduced a post-processing procedure for a stress recovery in multilayered beams based on the sub-discretization of material layers of the beam into quadrilateral 2D elements, followed by an application of the smoothing technique. Later on this approach was extended for 3D problems by Galvanetto *et al.* [161]. It is also worthy to notice that in mixed formulations transverse stresses can be calculated directly as primary variables (see *e.g.* [65]) or at least the shear stress profiles can be substantially improved by direct calculation of stress resultants being primary variables, [55, 56]. Post-processing method of stress recovery can also be applied in geometrically non-linear analysis of laminated shells, [162, 163]. Lee & Lee [162] developed an equilibrium-based stress recovery method that utilizes the in-plane stresses and shear forces obtained by a shell element analysis and one-dimensional FEM approximation introduced along the thickness in the post-processing phase. Park *et al.* [163] calculated transverse stress in finite rotation analysis by piecewise integration of the three-dimensional stress equilibrium equations in the thickness direction. A review on *a priori* and *a posteriori* methods of transverse stress evaluation in multilayered plates was presented by Carrera [156] and Kant & Swaminathan [164].

2.5. Sandwich panels as a special case of multilayered structures

It is worth to notice that sandwich panels can be in general considered as multilayered structures and as such they can be analyzed with most of the theoretical models surveyed above. However, there is also quite a broad category of computational models that are specially adjusted to deal with sandwich panels by a compliance with the specific features of those structures. A typical sandwich panel is constructed from two thin face-sheets separated by a thick but usually lightweight core. As a rule, the core is made of a low strength material: the most popular are foam or honeycomb cores but one can also find a core constructed as a 3-D truss-structure. All those materials are characterized by low in-plane and flexural rigidities, for that reason usually only shear stiffness is considered for the core, whereas the thin plate/shell model is applied for the faces. Such assumptions were introduced in one of the earliest sandwich plate models proposed by Hoff [165] in 1950, similar approach was also adopted in one of the first non-linear finite element formulations for sandwich plates presented by Schmit Jr. & Monforton [166] in 1970. On the other hand, the thickness of outer face-sheets is significantly smaller than the height of the core; hence already in 1947 Reissner [41] suggested that the bending stiffness of facings about their own midsurface can be neglected. In 1991 Lewiński [167] proposed a more general formulation of a sandwich plate model, which by introducing some simplifying assumptions can be reduced to that of Hoff [165] or Reissner [41]. Malcolm & Glockner [168] and Glockner & Malcolm [169] constructed a computational model based on the Cosserat surface theory, where they treated the face-sheets as membranes of negligible thickness. Das *et al.* [157] proposed a HOSD sandwich shell model with seven weighted-average displacement variables. Vu-Quoc *et al.* [141] presented a DL formulation for sandwich shells based on *geometrically exact* shell formulation of Simo *et al.* [80]. Borsellino *et al.* [170] performed experimental tests and numerical simulations of sandwich structures with composite facings.

Developments in the analysis and modeling of sandwich structures were reviewed in 1965 by Habip [120], and more recently by Burton & Noor [171] (1995), Librescu & Hause [7] (2000) and by Hohe & Librescu [9] (2004).

2.6. Material modeling in analysis of multilayered shells

The most popular, linear elastic material models of multilayered shells are very well established in the technical literature. Probably the most frequently cited resources in that context are handbooks of Jones [1] and of Vinson & Chou [2], both published in 1975; more recent texts were prepared by Nettles [172], Stockton [173], and Vasiliev & Morozov [3]. Phenomenological observations of most high-strength fiber reinforced composites seem to justify assumption of their linearly elastic response; however, one can easily indicate also many cases where material non-linearity should be taken into consideration. Nevertheless, the label of "composite materials" covers so large range of materials that it is just impossible to get a general approach of the mechanical behavior; the following short survey is limited to some selected examples only.

A constitutive model applied by Hu [174] in buckling analysis of fiber-composite plates accounted for elastic material non-linearity restricted only to in-plane shear terms. This issue corresponds to the important question of an effective experimental determination of the in-plane shear stiffness of a composite plate. It seems quite obvious that the non-linearity of the relation between in-plane shear stress and in-plane shear strain was treated by Hu [174] as a material non-linearity. However, another option was chosen by Pai & Palazotto [175], who handled this problem exclusively within geometrical non-linearity. Pai & Palazotto [175] presented a geometrically non-linear co-rotational formulation for laminated shells accounting for large strains and a change of fiber directions during deformation of a laminate. They used the right-stretch (Biot) strain tensor and the work-conjugate Jaumann-Biot stress tensor, declaring that only those strain and stress measures allow for using "*experimentally obtained material constants in the constitutive equations*" [175]. The effect of fiber rotation was also investigated by Wisnom [176], who examined how much the change of fiber directions during deformation of a glass-epoxy laminate in $\pm 45°$ tension tests can influence the measured in-plane shear stiffness. Wisnom [176] concluded that the effect of fiber rotation is small at shear strains below 7%, what means that this factor can be neglected in small strain analysis. Abu-Farsakh *et al.* [177] examined inelastic static response of laminated composite beams using the secant modulus model. Woo *et al.* [178] analyzed laminated orthotropic plates using anisotropic elastic-plastic material model, based on Prandtl–Reuss flow rule with strain hardening and Huber–Mises yield criterion modified by introducing the parameters of anisotropy. A rheological behavior of laminated plates and shells was considered *e.g.* by Kennedy [179], Kłosowski & Woźnica [180] and Wagner & Gruttmann [131]. Kennedy [179] analyzed the time-dependent response of composites assuming viscoelastic constitutive relations. Visco-plastic material models of Perzyna, Chaboche and Bodner-Partom were used in the creep analysis of composite panels by Kłosowski & Woźnica [180]. Wagner & Gruttmann [131] employed visco-plastic material model in an examination of delamination problems in layered panels. Başar *et al.* [133] applied a hyper-elastic Mooney-Rivlin type constitutive model in a large strain analysis of a sandwich shell with a rubber core.

An interesting example of micro-macro modeling of composite materials based on the homogenization theory of periodic media was described recently by Takano *et al.* [15]. In this formulation the composite is treated as the assembly of periodic microscopic structures. Assuming that microscopic periodicity remains in the local region also under large deformation, the local region is replaced by the homogenized model. However, in general, due to large deformations the change of microstructures in one local region is different from that in other region; therefore during deformation the microstructures have to be updated for local regions. An application of that procedure in an analysis of knitted fabric composite materials seems to be especially promising. More on a multi-scale modeling of composite materials can be found in a recent book by Böhm [19] (see also [14, 18]).

A review of recent developments in theoretical modeling of composite materials including inelastic behavior and damage was given by Dvorak [181], who among other things indicated also growing "*ability to design physical properties of composite material systems and structures for different specific purposes*", [181]. Hohe & Becker [8] presented a survey on material representation for cellular sandwich cores, including plastic and non-linearly elastic models.

2.7. More reading on multilayered plates and shells modeling

This chapter does not pretend to present a complete survey of all theoretical models proposed for layered thin-walled structures, but rather to show the most relevant ideas in that field. A comprehensive discussion on different modeling aspects of multilayered panels can be found in the fundamental handbook of Reddy [11]. Extensive reviews on analysis of multilayered plates and shells can be also found in [21–29, 39, 40, 71, 73, 76, 87, 94, 126, 164, 182–189]. See also collection of papers edited by Guz [13] and Hult & Rammerstorfer [190].

3. Finite element analysis of layered thin-walled structures

3.1. Equivalent Single Layer FE models of laminated composites and sandwich panels

It seems that the most straightforward construction of a 2D FE model for a laminated composite or a sandwich panel can be obtained within the ESL approach. One can simply employ one of existing finite elements prepared for homogeneous plates or shells and all necessary modifications in that case consist in the introduction of an anisotropic material model with material parameters estimated according to selected lamination theory. Rao [191] performed a linear FE analysis of shallow laminated shells using 48 dof finite elements based on the CLT (*Classical Lamination Theory*); a similar element was used in geometrically non-linear analysis by Saigal *et al.* [36].

For the reasons described in the previous chapter, it seems quite obvious that shear deformation theories are more suitable as the basis for a construction of finite elements to model laminated composites. There is a big number of finite elements for laminated shells that are formulated within the FOSD (*First Order Shear Deformation*) theory. A basic FE formulation of the classical linear FOSD theory of layered shells can be found in the book of Reddy [192]. Large displacement FOSD FE analysis of layered plates (in the range of *von Kármán* non-linearity) was presented in a review paper of Reddy [182]; a corresponding FE model for laminated composite shells was examined by Reddy & Chandrashekhara [45] (see also Reddy's handbook [11]). Palmerio *et al.* [193] described a 9-node shell element for moderate rotation FOSD analysis of laminated shells. Different aspects of the FE implementation of the FOSD moderate rotation shell theory were examined by Kreja *et al.* [48] (see also [194]). In 1979 Panda & Natarajan [195] constructed their FE FOSD laminated plate model as a displacement based degenerated isoparametric element with quadratic in-plane interpolation and reduced integration; the corresponding FE formulation for laminated shells was presented by Chang & Sawamiphakdi [196]. A very similar element was used by Jun & Hong [197] (see also [198]) in a non-linear UL analysis of cylindrical composite panels performed with the arc-length control method. Wagner [199] analyzed large deformations and buckling of cylindrical composite laminated shells using a 4-node finite element with reduced integration and hour-glass stabilization. Ferreira & Barbosa [200] presented a 9-node element based on the Marguerre shallow shell theory (in the range of *von Kármán* non-linearity) and ANS approach. Laschet & Jeusette [201] performed post-buckling analysis of laminated composites applying an under-integrated solid-shell element possessing only translational degrees of freedom. Rikards *et al.* [202] analyzed buckling and vibration of composite stiffened shells using triangular FOSD shell elements with selective integration. The concept of FOSD finite elements based on mixed interpolation of tensor components (*MITC* elements) was extended for laminated plates by Alfano *et al.* [155], and for laminated shells by Haas & Lee [203] and Hossain *et al.* [154]. Somashekar *et al.* [204] examined a 4-node field-consistent shell element for a linear analysis of laminated composite panels. Groenwold & Stander [205] developed a 4-node 24 dof shells element for layered composite panels. Dorninger [206] (see also [207]) extended the non-linear formulation of the degenerated shell element of Ramm [208] to include the anisotropic layered material behavior of laminated composites. Brank *et al.* [209] presented an ANS formulation of a 4-node FOSD shell element for large-rotation analysis of laminated elastic shells; however, only one out of eight numerical examples was devoted to multi-layered shell problem, and the magnitude of rotations in that particular example stayed within the range of moderate rotations (compare [210]). More examples of the large rotation FEA of laminated shells within the FOSD theory can be found in the recent papers by Kreja & Schmidt [211] and Kreja [210]. Kim [212] (see also Kim & Voyiadjis [213]) developed under-integrated 8-node non-linear composite FOSD shell element based on corotational formulation. A modified version of that element based on the ANS formulation was examined by Kim & Park [214] (see also Kim *et al.* [215]). Kim *et al.* [216] presented large rotation analysis of laminated composite structures using a co-rotational 4-node shell element with enhanced strains. A co-rotational formulation was applied also by Barut *et al.* [217], who analyzed large displacements of shallow laminated shells applying triangular finite elements. Han *et al.* [218] performed a large deformation analysis of laminated shells using an element-based 9-node stress-resultant ANS shell element with 54 dofs. A new version of that element modified according to the *Corrected* FOSD model of Tanov & Tabiei [61] was recently presented by Han *et al.* [219]. Pai [220] developed a 4-node laminated shell element using the co-rotational formulation of Pai & Palazotto [175] and the energy-consistent FOSD theory of Pai [57]; with 14 dofs per node including the derivatives of deflections that element was declared by the author to be locking-free. Arciniega & Reddy [221] analyzed large deformation of composite panels using high order interpolation shell elements based of 7-parameter FOSD shell theory. Hashagen *et al.* [222] adopted the solid-like shell element introduced by Parisch [223] for homogeneous structures to perform materially and geometrically non-linear analysis of fiber reinforced metal laminates. Kulikov & Plotnikova [224?] presented an extended mixed field formulation of a multi-layered shell element with fundamental unknowns consisted of six displacement parameters, eleven strains and eleven stress resultants; however, the non-displacement unknowns were eliminated on the element level resulting in the FE model with displacement dofs only. Cen *et al.* [153] proposed a 4-node laminated FOSD plate element based on utilization of Timoshenko beam theory (a mixed field formulation corresponding to the ANS) combined with a hybrid stress approach for improving the accuracy of stress recovery; a very similar approach was also used by Zhang & Kim [226] in a construction of their 20 dof and 24 dof quadrilateral laminated plate elements. An eighteen-node hybrid-stress solid-shell element for laminated structures was presented by Sze *et al.* [227] and by Sze & Zheng [228].

Looking for a possible improvement of the FOSD results Tanov & Tabiei [61] proposed a simply correction to a standard FE FOSD shell model by enforcing a parabolic shear strain distribution across the shell thickness. Fares & Youssif [63], Fares *et al.* [64], and Auricchio & Sacco [65? ?] presented a collection of different finite shell elements based on the *refined* FOSD theory and mixed variational principle. In 1985 Phan & Reddy [70] constructed a 4-node finite element based on the Reddy TOSD theory of laminated plates [68] assuming Hermite interpolation of the transverse deflection and Lagrange interpolation of the other displacement unknowns. An extension of that FE model for inclusion of the *von Kármán* non-linearity was presented by Reddy [24]. Finite elements constructed according to various TOSD plate theories were examined also by Bose & Reddy [229]. High order interpolation shell elements based on TOSD small displacement theory were described recently by Reddy & Arciniega [39]. Dennis & Palazotto [81, 82] developed finite shell elements based on their own *Simplified Large Rotation* (SLR) TOSD theory of cylindrical laminated shells (see also [83, 84, 230]); a combination of Hermite and Lagrange interpolation schemes was applied, similarly as used earlier by Phan & Reddy [70], however, a quadratic shape functions were used for "in-plane" displacement components, u and v. Das *et al.* [157] presented a rather complex formulation of a triangular finite element based on HOSD model with seven weighted-average displacement variables; a special procedure based on the hybrid energy functional was applied to satisfy the C^1 inter-element continuity requirements what resulted in the FE with 13 dofs per node. Moita *et al.* [86] used 80 dof finite shell elements based on the HOSD theory in the buckling analysis of laminated panels. Başar *et al.* [78] developed 4-node ANS shell elements based on a large-rotation TOSD theory of laminated shells; variants with 7 and 5 dofs per node were considered. Balah & Al-Ghamedy [79] presented a similar 4-node ANS shell element for the TOSD formulation with seven degrees of freedom but they applied exponential mapping of finite rotations instead of Euler angles used by Başar *et al.* [78].

A separate group among the FE implementations of the ESL models consists of finite elements constructed according to the zig-zag deformation theory with interlaminar stress continuity (compare a review article of Carrera [96]). Various FE realizations of the theoretical zig-zag model of Toledano & Murakami [95, 101] were presented by Carrera and co-workers; their FE formulations with seven displacement unknowns at each node were characterized by the C^0 type continuity. Carrera [231] described 4-, 8- and 9-node plate elements following his own theoretical model [93]; selectively and uniformly reduced integration schemes were considered. A corresponding multilayer 4-node shell element was presented by Carrera & Parisch [232], who started from the existing finite-rotation assumed strain shell element proposed for homogeneous shells by Parisch [233]. Another FE implementation of the zig-zag model of Carrera [93] was prepared by Brank & Carrera [91, 92]; their formulation based on the ANS shell element by Brank *et al.* [209]. As it was mentioned earlier the zig-zag model proposed by Di Sciuva [94] required a C^1 type continuity in the FEM implementation, therefore the triangular fully conforming multilayered plate element presented by Di Sciuva [97] had 10 dofs at each node, with first and second derivatives of the transverse deflection in the list of displacement unknowns.

3.2. Discrete Layer FE models of laminated composites

Mawenya & Davies [123] presented a linear bending analysis of laminated plates employing a DL finite element with quadratic interpolation and *3+2N* dofs at each node (three global translations for the whole laminate with two local rotations for each layer). A similar FE formulation of a DL model for laminated plates by accounting for the *von Kármán* non-linearity was developed by Reddy [24]. An analogous DL shell element was constructed within the degenerated formulation by Rammerstorfer *et al.* [234]. Chaudhuri & Seide [124] described a corresponding triangular multi-layered shell element based on their "*layer-wise constant shear-angle theory*" with quadratic in-plane interpolation; more recently, Chaudhuri [125] applied that element in the analysis of angle-ply composite plates. The DL shell elements of Pinsky & Kim [129] accounted for large deformations including the thickness stretching with the number of nodal unknowns extended to *3+4N*. Similar discrete layer FE models of composite shells based on the 7-parameter FOSD large rotation shell theory were developed by Braun *et al.* [130].

Başar & Ding [26] and Başar *et al.* [133] examined 4-node ANS/EAS shell elements for various DL models accounting for the thickness stretching using *3+3N*, *3+4N* and *3+6N* dofs per node. A DL multi-layered shell model with *3+9N* unknowns based on the HOSD theory was implemented into FEM by Gruttmann & Wagner [142]. Naboulsi & Palazotto [83] examined a discrete layer formulation FE model of cylindrical composite shells based on the co-rotational concept of Pai & Palazotto [175]. Vu-Quoc & Tan [235] presented a DL formulation based on a solid-shell element without any rotational degrees of freedom what made it especially well suited for modeling multilayer shells with geometrical thickness discontinuities like ply drop-offs or (piezoelectric) patches. A comparable multi-layered DL shell element based on the mixed field formulation was analyzed by Kulikov & Plotnikova [236, 237]. Dakshina Moorthy & Reddy [238] considered a related finite element formulation of DL model of laminated panels; however, their analysis was performed for a simplified 2D geometry of a vertical cross-section. They applied 6-node plane element with quadratic interpolation for the in-plane approximation and a linear function for the thickness approximation; the EAS formulation was employed to prevent locking. A similar concept was applied by Krätzig & Jun [134, 135], who considered a quite general formulation of the *discrete-layer* models with two different layer-wise refinement concepts, *internal* for improved modeling of complicated stress states and *external* for better kinematic approximation properties. In the two examples presented by Krätzig & Jun [134], an automatic 3-D refinement procedure based on the error estimation was applied; the obtained final h-refined FE meshes directly corresponded to some extend with the global-local solution concept. Desai *et al.* [149] applied 3-D finite elements in a layer-wise discretization of layered composites (i.e. using one element per each layer across the thickness). The set of nodal parameters in their mixed formulation based 3-D finite elements contained displacements and transverse stress components therefore the through-the-thickness continuity requirements of displacements and transverse stress fields were satisfied automatically. A similar hybrid approach was used by Feng & Hoa [150] in their multi-layered 3-D model build as a stack of 8-node solid sub-elements along the thickness of composite laminates. Two dimensional shell elements were combined with 3-D solid elements in materially and geometrically non-linear analysis of composites by Rammerstorfer *et al.* [239]. A slight different variant of a global-local FEA can be obtained by a combination of DL and ESL composite shell elements as it was done by Kong & Cheung [151] for linear analysis and by Gruttmann & Wagner [142] in non-linear applications. Yu *et al.* [152] presented a detailed FEA of composites plates using a discrete layer approach where a single layer is modeled with the mixed-field eight-node plate element with the total number of 104 (stress-displacement) unknowns. Within a global-local concept proposed by Yu *et al.* [152] some physical layers can be modeled as a single *global* (ESL) region; nevertheless, the numerical size of the problem remained enormous. Crisfield *et al.* [240] introduced special interface elements to analyze delamination problem in composites.

Aitharaju & Averill [241] (see also Cho & Averill [137]) proposed an interesting tactic to circumvent problems of the C^1 continuity requirements related to the application of the zig-zag model of Di Sciuva [94] in the FEM. Their shell element had a form of an 8-noded cube with 5 dofs (three translations and two rotations) at each node; therefore they could increase the refinement of the model by using more than one element in the thickness direction.

3.3. Special FE models for sandwich structures

Generally, the most of FE models of multi-layered plates and shells reviewed above in the context of composite panels can be directly applied also in the analysis of sandwich structures; however, there is also a group of finite elements exclusively dedicated to sandwich structures. In 1970 Schmit Jr. & Monforton [166] presented one of the first non-linear finite element models of sandwich plates, accounting only for membrane and bending deflections of the outer faces and transverse shear deformations of the core. Marcinowski [242] constructed his 8-node shell element for a geometrically non-linear analysis of sandwich shells assuming after Reissner [41] that outer facings worked as thin membranes without bending stiffness. Das *et al.* [157] used a hybrid-stress formulation to develop a new 3-node triangular HOSD sandwich finite element with 39 degrees of freedom. Vu-Quoc *et al.* [141] analyzed sandwich shells applying a geometrically exact 4-node shell element with selective reduced integration based on a general DL multi-layered shell formulation. An interesting way of putting the idea of ESL modeling of sandwich panels into practice was presented by Tanov & Tabiei [33] who suggested performing a FEA of any sandwich shell with an existing FE FOSD model of homogeneous shells, simply entering equivalent material parameters provided by their *sandwich homogenization procedure.*

3.4. Analysis of composites and sandwich panels with commercial FEA codes

The growing contribution of laminated composite applications in engineering structures initiated an increasing interest in appropriate analysis tools; as a result some professionals reached for ready computational models offered by the commercial Finite Element Analysis systems. Ali [243] applied the MSC Nastran [59] to perform a linear analysis of a petrol engine oil sump pan made of fiberglass composite. Rolfes & Rohwer [58] analyzed laminated composite plates using the MSC Nastran with their self-written preprocessor and postprocessor to implement the "*improved*" transverse shear stiffness for the FOSD model together with the special procedure to evaluate the transverse shear stresses. Sze *et al.* [244] performed geometrically non-linear analysis of selected benchmark problems of laminated shells using SR4 element of the FEA system ABAQUS. Hu [174] applied ABAQUS with user defined composite material model where non-linearity of strain-stress relation was associated with in-plane shear. Eason & Ochoa [245] presented the modeling progressive damage in composites with the ABAQUS. Manet [246] investigated an application of ANSYS 5.2 in the analysis of sandwich structures behavior examining various finite elements available in that FEA system. Quite recently, Borsellino *et al.* [170] applied ANSYS 5.6 to perform a 2-D (plane) computer simulation of static mechanical tests for sandwich panels.

3.5. More reading on FEA of multi-layered panels

Noteworthy review articles on FEM modeling of multi-layered shells were published *e.g.* by Ferreira & Fernandes [185], Noor & Burton [21], Qatu [189], Toorani & Lakis [186]; FEA of sandwich structures was surveyed *e.g.* by Librescu & Hause [7], Burton & Noor [171].

4. Concluding Remarks

With the increasing application of laminated composites and sandwich panels in various fields of structural engineering, there is a great concern about their appropriate computational representation.

The main conclusions one can draw from the presented survey on computational modeling of laminated composite and sandwich structures are as follows:

1. A single numerical model capable for a universal representation of all layered composite and sandwich panels does not exist; depending on the particular problem different formulations can be the most effective choice. Frequently, the best results can be obtained with a combined global-local analysis where various numerical models are used for separate parts of the structure.

2. A detailed through-the-thickness representation of deformation profiles and/or distribution of stresses usually accompany small or moderate displacement formulations, on the other hand, most of large rotation analyses are performed for a simplified FOSD type models.

On the base of the literature survey reported in the paper, one can perform a general classification of computational models for undamaged multilayered plates and shells. While, the micro-mechanical scale analysis, and even, multi-scale modeling calculations are still too costly

Table 1. General classification of macro-mechanical models for multi-layered panels.

Models			References
2-D models	ESL	CLT	1, 2, 11,25, 34, 36, 42-43, 191
		FOSD	1, 2, 11, 25, 37-48, 182, 192-218
		mFOSD*	38, 49 , 50-67, 118, 153-159, 219-221
		HOSD	11, 24, 39, 68-79,81-88, 157, 229, 230
		Zig-zag	34, 90-118, 231, 232, 241
	DL		11, 21, 24, 71, 77, 78, 121-131, 133-135, 137-144, 234-238
3-D models			89, 143, 145, 146, 148-150
Global-local strategies			115, 127, 134, 137, 150-152, 239, 240

* mFOSD stands for “modified”, “consistent”, “improved”, “refined” or “corrected” FOSD formulations

for practical applications, the dominating group consists of computational models associated with the macro-mechanical scale. A general classification of computational models of multilayered panels in this group is shown in Table 1, together with an indication of the appropriate references.

Acknowledgements

The financial support of the Polish Ministry of Science and Higher Education under Grant No. N N506 254237 is gratefully acknowledged.

References

[1] Jones R. M., Mechanics of composite materials, Second Editions, Taylor & Francis, Inc., Philadelphia, PA, 1999

[2] Vinson J. R., Chou T.-W., Composite Materials and Their Use in Structures, Applied Science Publishers Ltd, London, 1975

[3] Vasiliev V. V., Morozov E. V., Mechanics and Analysis of Composite Materials, Elsevier Science Ltd, Oxford 2001

[4] Halle D. K., Kelly A., Strength of fibrous composite materials, Annual Review of Materials Science 2, 1972, 405-462

[5] Chou T.-W., Kelly A., Mechanical properties of composites, Annual Review of Materials Science 10, 1980, 22-59

[6] Tong L., Mouritz A. P., Bannister M. K., 3D Fibre Reinforced Polymer Composites, Elsevier Science Ltd., Oxford 2002

[7] Librescu L., Hause T., Recent developments in the modeling and behavior of advanced sandwich constructions: a survey, Composite Structures 48, 2000, 1-17

[8] Hohe J., Becker W., Effective stress-strain relations for two-dimensional cellular sandwich cores: Homogenization, material models, and properties, Applied Mechanics Review 55, 2002, 61-87

[9] Hohe J., Librescu L., Advances in the Structural Modeling of Elastic Sandwich Panels, Mechanics of Advanced Materials & Structures 11, 2004, 395-424

[10] Reddy J.N., On laminated composite plates with integrated sensors and actuators, Engineering Structures 21, 1999, 568-593

[11] Reddy J.N., Mechanics of Laminated Composite Plates and Shells: Theory and Analysis, CRC Press, Boca Raton, FL, Second Edition, 2004

[12] Hachenberg D., The Role of Advanced Numerical Methods in the Design and Certification of Future Composite Aircraft Structures, WCCM V: Proceedings of the Fifth World Congress on Computational Mechanics, July 7-12, 2002, Vienna, Austria, H.A. Mang, F.G. Rammerstorfer & J. Eberhardsteiner (eds.), CD-ROM, Vienna 2002

[13] Guz A. N. (ed.), Micromechanics of composite materials: Focus on Ukrainian Research, Applied Mechanics Review 45, 1992, 13-101

[14] Pagano N. J., Yuan, F. G., The significance of effective modulus theory (homogenization) in composite laminate mechanics, Composites Science and Technology 60, 2000, 2471-2488

[15] Takano N., Ohnishi Y., Zako M., Nishiyabu K., The formulation of homogenization method applied to large deformation problem for composite materials, Int. Journal of Solids & Structures 37, 2000, 6517-6535

[16] Schmauder S., Computational Mechanics, Annual Review of Materials Research 32, 2002, 437-465

[17] Pahr D. H., Rammerstorfer F. G., A fast multi-scale analyzing tool for the investigation of perforated

laminates, Computers & Structures 82, 2004, 227-239

[18] Ladevéze P., Multiscale Modeling and Computational Strategies for Composites, WCCM V: Proceedings of the Fifth World Congress on Computational Mechanics, July 7-12, 2002, Vienna, Austria, H.A. Mang, F.G. Rammerstorfer & J. Eberhardsteiner (eds.), CD-ROM, Vienna 2002

[19] Böhm H. J. (Ed.), Mechanics of Microstructured Materials, CISM Courses and Lectures No. 464, Springer-Verlag, Vienna 2004

[20] Zhang W. C., Evans K. E., Numerical prediction of the mechanical properties of anisotropic composite materials, Computers & Structures 29, 1988, 413-422

[21] Noor A. K. & Burton W. S., Assessment of computational models for multilayered composite shells, Applied Mechanics Reviews 43, 1990, 67-97

[22] Kulikov G. M., Non-linear analysis of multilayered shells under initial stress, Int. Journal of Non-Linear Mechanics 36, 2001, 323-334

[23] Carrera E., Developments, ideas, and evaluations based upon Reissner's Mixed Variational Theorem in the modeling of multilayered plates and shells, Applied Mechanics Reviews 54, 2001, 301-329

[24] Reddy J. N., On refined computational models of composite laminates, Int. Journal for Numerical Methods in Engineering 27, 1989, 361-382

[25] Rohwer K., Friedrichs S., Wehmeyer C., Analyzing laminated structures from fibre-reinforced composite material - an assessment, Technische Mechanik 25, 2005, 59-79

[26] Başar Y., Ding Y., Interlaminar stress analysis of composites: layer-wise shell finite elements including transverse strains, Composites Engineering 5, 1995, 485-499

[27] Carrera E., An assessment of mixed and classical theories on global and local response of multilayered orthotropic plates, Composite Structures 50, 2000, 183-198

[28] Carrera E., Demasi L., Classical and advanced multilayered plate elements based upon PVD and RMVT. Part 1: Derivation of finite element matrices, Int. Journal for Numerical Methods in Engineering 55, 2002, 191-231

[29] Carrera E., Demasi, L., Classical and advanced multilayered plate elements based upon PVD and RMVT. Part 2: Numerical implementations, Int. Journal for Numerical Methods in Engineering 55, 2002, 253-291

[30] Chen C.-M., Kikuchi N., Farzad R.-A., An enhanced asymptotic homogenization method of the static and dynamics of elastic composite laminates, Computers & Structures 82, 2004, 373-382

[31] Lewińsk, T., On recent developments in the homogenization theory of elastic plates and their application to optimal design: Part I, Structural Optimization 6, 1993, 59-64

[32] Rabczuk T., Kim J. Y., Samaniego E., Belytschko T., Homogenization of sandwich structures, Int. Journal for Numerical Methods in Engineering 61, 2004, 1009-1027

[33] Tanov R. and Tabiei A., A note on finite element implementation of sandwich shell homogenization, Int. Journal for Numerical Methods in Engineering 48, 2000, 467-473

[34] Ambartsumyan S. A., Theory of Anisotropic Plates, Technomic Publishing Co., Inc., Stamford, 1970

[35] Sun C.T., Chin, H., Analysis of asymmetric composite laminates, AIAA Journal 26, 1988, 714-718

[36] Saigal S., Kapania R. K., Yang T. Y., Geometrically nonlinear finite element analysis of imperfect laminated shells, Journal Composite Materials 20, 1986, 197-214

[37] Whitney J. M., Pagano, N. J., Shear deformation in heterogeneous anisotropic plates, Journal of Applied Mechanics, Trans. ASME 37, 1970, 1031-1036

[38] Dong S. B., Tso F. K. W., On a laminated orthotropic shell theory including transverse shear deformation, Journal of Applied Mechanics, Trans. ASME 39, 1972, 1091-1096

[39] Reddy J. N., Arciniega R. A., Shear deformation plate and shell theories: From Stavsky to Present, Mechanics of Advanced Materials and Structures 11, 2004, 535-582

[40] Ghugal Y. M., Shimpi R. P., A review of refined shear deformation theories of isotropic and anisotropic laminated plates, Journal of Reinforced Plastics and Composites 21, 2002, 775-813

[41] Reissner E., Small bending and stretching of sandwich-type shells, NACA Report No. 975, 1950, 483-508

[42] Chandrashekhara K., Pavan Kumar D. V. T. G., Assessment of shell theories for the static analysis of cross-ply laminated circular cylindrical shells, Thin-Walled Structures 22, 1995, 291-318

[43] Chandrashekhara K., Pavan Kumar D. V. T. G., Static response of composite circular cylindrical shells studied by different theories, Meccanica 33, 1998, 11-27

[44] Tarn J.-Q., Wang Y.-B., A refined asymptotic theory and computational model for multilayered composite plates, Computer Methods in Applied Mechanics & Engineering 145, 1997, 167-184

[45] Reddy J. N., Chandrashekhara K., Nonlinear analysis of laminated shells including transverse shear strains, AIAA Journal 23, 1985, 440-441

[46] Schmidt R., Reddy J. N., A refined small strain and moderate rotation theory of elastic anisotropic shells, Journal of Applied Mechanics, Trans. ASME 55, 1988, 611-617

[47] Palmerio A. F., Reddy J. N., Schmidt R., On a moderate rotation theory of elastic anisotropic shells - Part 1. Theory, Int. Journal of Non-Linear Mechanics 25, 1990, 687-700

[48] Kreja I., Schmidt R., Reddy J. N., Finite elements based on a first-order shear deformation moderate rotation shell theory with applications to the analysis of composite structures, Int. Journal Non-Linear Mechanics 32, 1997, 1123-1142

[49] Whitney J. M., Shear correction factors for orthotropic laminates under static load, Journal of Applied Mechanics, Trans. ASME 40, 1973, 302-303

[50] Wittrick W. H., Analytical, three-dimensional elasticity solutions to some plate problems, and some observations on Mindlin's plate theory, Int. Journal of Solids & Structures 23, 1987, 441-464

[51] Vlachoutsis S., Shear correction factors for plates and shells, Int. Journal for Numerical Methods in Engineering 33, 1992, 1537-1552

[52] Jemielita G., Coefficients of shear correction in transversely nonhomogeneous moderately thick plates, Journal of Theoretical and Applied Mechanics (Polish Society of Theoretical and Applied Mechanics) 40, 2002, 73-84

[53] Noor A. K., Peters J. M., A posteriori estimates for shear correction factors in multilayered composite cylinders, Journal of Engineering Mechanics ASCE 115, 1988, 1225-1244

[54] Sze K. Y., He L.-W., Cheung Y. K., Predictor-corrector procedures for analysis of laminated plates using standard Mindlin finite element models, Composite Structures 50, 2000, 171-182

[55] Auricchio F., Sacco E., Partial-mixed formulation and refined models for the analysis of composite laminates within an FSDT, Composite Structures 46, 1999, 103-113

[56] Auricchio F., Sacco E., A mixed-enhanced finite-element for the analysis of laminated composite plates, Int. Journal for Numerical Methods in Engineering 44, 1999, 1481-1504

[57] Pai P. F., A new look at shear correction factors and warping functions of anisotropic laminates, Int. Journal of Solids & Structures 32, 1995, 2295-2313

[58] Rolfes R., Rohwer K., Improved transverse shear stresses in composite finite elements based on First Order Shear Deformation Theory, Int. Journal for Numerical Methods in Engineering 40, 1997, 51-60

[59] MSC/NASTRAN Encyclopedia for Version 69, Mac Neal-Schwendler Corp., 1996

[60] Altenbach H., An alternative determination of transverse shear stiffnesses for sandwich and laminated plates, Int. Journal of Solids & Structures 37, 2000, 3503-3520

[61] Tanov R., Tabiei A., A simple correction to the first-order shear deformation shell finite element formulations, Finite Elements in Analysis and Design 35, 2000, 189-197

[62] Reissner E., A consistent treatment of transverse shear deformations in laminated anisotropic plates, AIAA Journal 10, 1972, 716-718

[63] Fares M. E., Youssif Y. G., A refined equivalent single-layer model of geometrically non-linear doubly curved layered shells using mixed variational approach, Int. Journal Non-Linear Mechanics 36, 2001, 117-124

[64] Fares M. E., Zenkour A. M., El-Marghany M. Kh., Non-linear thermal effects on the bending response of cross-ply laminated plates using refined first-order theory, Composite Structures 49, 2000, 257-267

[65] Auricchio F., Sacco E., Refined First-Order Shear Deformation Theory models for composite laminates, Journal of Applied Mechanics, Trans. ASME 70, 2003, 381-390

[66] Qi Y., Knight Jr. N. F., A refined first-order shear deformation theory and its justification by plane-strain bending problem of laminated plates, Int. Journal of Solids & Structures 33, 1996, 49-64

[67] Knight Jr. N. F., Qi, Y., On a consistent first-order shear deformation theory for laminated plates, Composites, Part B 28B, 1997, 397-405.

[68] Reddy J. N., A simple higher-order theory for laminated composite plates, Journal of Applied Mechanics, Trans. ASME 51, 1984, 745-752

[69] Khdeir A. A., Reddy J. N., Librescu L., Analytical solution of a refined shear deformation theory for rectangular composite plates, Int. Journal of Solids & Structures 23, 1987, 1447-1463

[70] Phan N. D., Reddy J. N., Analysis of laminated composite plates using a higher-order shear deformation theory, Int. Journal for Numerical Methods in Engineering 21, 1985, 2201-2219.

[71] Reddy J. N., On the generalization of displacement-based laminate theories, Applied Mechanics Reviews 42, 1989, S213-S222

[72] Reddy J. N., Liu, C. F., A higher-order shear deformation theory of laminated elastic shells, Int. Journal

of Engineering Sciences 23, 1985, 669-683
[73] Bose P., Reddy J. N., Analysis of composite plates using various plate theories Part 1: Formulation and Analytical Solutions, Structural Engineering & Mechanics 6, 1998, 583-612
[74] Reddy J. N., A general non-linear third-order theory of plates with moderate thickness, Int. Journal of Non-Linear Mechanics 25, 1990, 677-686
[75] Dennis S. T., A Galerkin solution to geometrically nonlinear laminated shallow shell equations, Computers & Structures 63, 1997, 859-874
[76] Simitses G. J., Buckling of moderately thick laminated cylindrical shells: A review, Composites Part B 27B, 1996, 581-587
[77] Başar Y., Finite-rotation theories for composite laminates, Acta Mechanica 98, 1993, 159-176
[78] Başar Y., Ding Y., Schultz R., Refined shear-deformation models for composite laminates with finite rotations, Int. Journal of Solids & Structures 30, 1993, 2611-2638
[79] Balah M., Al-Ghamedy H. N., Finite element formulation of a third order laminated finite rotation shell element, Computers & Structures 80, 2002, 1975-1990
[80] Simo J. C., Fox D. D., Rifai M. S., On a stress resultant geometrically exact shell model. Part III: Computational aspects of the nonlinear theory, Computer Methods in Applied Mechanics & Engineering 79, 1990, 21-70
[81] Dennis S. T., Palazotto A. N., Transverse shear deformation in orthotropic cylindrical pressure vessels using a higher-order shear theory, AIAA Journal 27, 1989, 1441-1447
[82] Dennis S. T., Palazotto A. N., Large displacement and rotational formulation for laminated shells including parabolic transverse shear, Int. Journal of Non-Linear Mechanics 25, 1990, 67-85
[83] Naboulsi S. K., Palazotto A. N., Non-linear static-dynamic finite element formulation for composite shells, Int. Journal of Non-Linear Mechanics 38, 2003, 87-110
[84] Tsai C. T., Palazotto A. N., Dennis S. T., Large-rotation snap-through buckling in laminated cylindrical panels, Finite Elements in Analysis and Design 9, 1991, 65-75
[85] Sacco E., Reddy J.N., On first- and second-order moderate rotation theories of laminated plates, Int. Journal for Numerical Methods in Engineering 33, 1992, 1-17
[86] Moita J. S., Mota Soares C. M., Mota Soares C. A., Buckling behaviour of laminated composite structures using a discrete higher-order displacement model, Composite Structures 35, 1996, 75-92
[87] Piskunov V. G., An iterative analytical theory in the mechanics of layered composite systems, Mechanics of Composite Materials 39, 2003, 1-16
[88] Librescu L., Refined geometrically non-linear theories of anisotropic laminated shells, Quarterly of Applied Mathematics 45, 1987, 1-22
[89] Pagano N. J., Hatfield S.J., Elastic behavior of multilayered bidirectional composites, AIAA Journal 10, 1972, 931-933
[90] Brank B., On composite shell models with piecewise linear warping function, Composite Structures 59, 2003, 163-171
[91] Brank B., Carrera E., A family of shear-deformable shell finite elements for composite structures, Computers & Structures 76, 2000, 287-297
[92] Brank B., Carrera E., Multilayered shell finite element with interlaminar continuous stresses: a refinement of the Reissner-Mindlin formulation, Int. Journal for Numerical Methods in Engineering 48, 2000, 843-874
[93] Carrera E., On the use of the Murakami's zig-zag function in the modeling of layered plates and shells, Computers & Structures 82, 2004, 541-554
[94] Carrera E., Historical review of zig-zag theories for multilayered plates and shells, Applied Mechanics Reviews 56, 2003, 287-308
[95] Di Sciuva M., An improved shear deformation theory of moderately thick multilayered anisotropic shells and plates, Journal of Applied Mechanics, Trans. ASME 54, 1987, 589-596
[96] Toledano A., Murakami H., A composite plate theory for arbitrary laminate configurations, Journal of Applied Mechanics, Trans. ASME 54, 1987, 181-189
[97] Di Sciuva M., A third-order triangular multilayered plate finite element with continuous interlaminar stresses, Int. Journal for Numerical Methods in Engineering 38, 1995, 1-26
[98] Di S., Rothert H., A solution of laminated cylindrical shell using an unconstrained third-order theory, Composite Structures 32, 1995, 667-680
[99] Di S., Rothert H., Solution of laminated cylindrical shell using an unconstrained third-order theory, Computers & Structures 69, 1998, 291-303
[100] Lee K. H., Senthilnathan N. R., Lim S. P., Chow T., An improved zig-zag model for the bending of laminated composite plates, Composite Structures 15, 1990, 137-148
[101] Toledano A., Murakami H., A High-Order Laminated Plate Theory with Improved In-Plane Responses, Int. Journal of Solids & Structures 23, 1987, 111-131

[102] Carrera E., C^0 Reissner-Mindlin multilayered plate elements including zig-zag and interlaminar stress continuity, Int. Journal for Numerical Methods in Engineering 39, 1996, 1797-1820
[103] Savithri S., Varadan T. K., Large deflection analysis of laminated composite plates, Int. Journal Non-Linear Mechanics 28, 1993, 1-12
[104] Librescu L., Schmidt R., Substantiation of a shear-deformable theory of anisotropic composite laminated shells accounting for the interlaminar continuity conditions, Int. Journal of Engineering Science 29, 1991, 669-683
[105] Schmidt R., Librescu L., Further results concerning the refined theory of anisotropic laminated composite plates, Journal of Engineering Mathematics 28, 1994, 407-425
[106] He L.-H., Non-linear theory of laminated shells accounting for continuity conditions of displacements and tractions at layer interfaces, Int. Journal of Mechanical Sciences 37, 1995, 161-173
[107] Shu X.-P., A refined theory of laminated shells with higher-order transverse shear deformation, Int. Journal of Solids & Structures 34, 1997, 673-683
[108] Lee K. H., Cao L., A predictor-corrector zig-zag model for the bending of laminated composite plates, Int. Journal of Solids & Structures 33, 1996, 879-897
[109] Soldatos K. P., Watson P., A method for improving the stress analysis performance of one- and two-dimensional theories for laminated composites, Acta Mechanica 123, 1997, 163-186
[110] Cho M., Kim K.-O., Kim M.-H., Efficient higher-order shell theory for laminated composites, Composite Structures 34, 1996, 197-212
[111] Idibi A., Karama M., Touratier, M., Comparison of various laminated plate theories, Composite Structures 37, 1997, 173-184
[112] Fernande, A., A mixed formulation for elastic multilayer plates, Computes Rendus Mecanique 331, 2003, 337-342
[113] Arya H., Shimpi R. P., Naik N. K., A zigzag model for laminated composite beams, Composite Structures 56, 2002, 21-24
[114] Hassis H., A high-order theory for static-dynamic analysis of laminated plates using a special warping model, European Journal of Mechanics A / Solids 21, 2002, 323-332
[115] Sutyrin V. G., Hodges D. H., On asymptotically correct linear laminated plate theory, Int. Journal of Solids & Structures 33, 1996, 3649-3671
[116] Yu W., Hodges D. H., A geometrically nonlinear shear deformation theory for composite shells, Journal of Applied Mechanics, Trans. ASME 71, 2004, 1-9
[117] Yu W., Hodges D. H., Volovoi V. V., Asymptotic generalization of Reissner-Mindlin theory: accurate three-dimensional recovery for composite shells, Computer Methods in Applied Mechanics & Engineering 191, 2002, 5087-5109
[118] Kim J.-S., Cho, M., Enhanced First-Order Shear Deformation Theory for laminated and sandwich plates, Journal of Applied Mechanics, Trans. ASME 72, 2005, 809-817
[119] Ambartsumyan S. A., Contributions to the theory of anisotropic layered shells, Applied Mechanics Reviews 15, 1962, 245-249
[120] Habip L. M., A survey of modern developments in the analysis of sandwich structures, Applied Mechanics Reviews 18, 1965, 93-98
[121] Mau S. T., A refined laminated plate theory, Journal of Applied Mechanics, Trans. ASME 40, 1973, 606-607
[122] Pagano N. J., Stress fields in composite laminates, Int. Journal of Solids & Structures 14, 1978, 385-400
[123] Mawenya A. S., Davies J. D., Finite element bending analysis of multilayer plates, Int. Journal for Numerical Methods in Engineering 8, 1974, 215-225
[124] Chaudhuri R. A., Seide P., An approximate method for prediction of transverse shear stresses in a laminated shell, Int. Journal of Solids & Structures 23, 1987, 1145-1161
[125] Chaudhuri R. A., Analysis of laminated shear-flexible angle-ply plates, Composite Structures 67, 2005, 71-84
[126] Noor A. K., Burton W. S., Peters J. M., Assessment of computational models for multilayered composite cylinders, Int. Journal of Solids & Structures 27, 1991, 1269-1286
[127] Huttelmaier H. P., Epstein M., A finite element formulation for multilayered and thick shells, Computers & Structures 21, 1985, 1181-1185
[128] Masud A., Panahandeh M., Finite-Element Formulation for Analysis of Laminated Composites, Journal of Engineering Mechanics ASCE 125, 1999, 1115-1124
[129] Pinsky P. M., Kim K. O., A multi-director formulation for nonlinear elastic-viscoelastic layered shells, Computers & Structures 24, 1986, 901-913
[130] Braun M., Bischoff M., Ramm E., Nonlinear 3-dimensional analysis of composite and laminate plate and shell structures, Recent Developments in Finite Element Analysis, T. J. R. Hughes, E. Oñate & O. C. Zienkiewicz (eds.), CIMNE, Barcelona, 1994, 215-224
[131] Wagner W., Gruttmann F., FE–Modeling of Fiber

Reinforced Polymer Structures, WCCM V: Proceedings of the Fifth World Congress on Computational Mechanics, July 7-12, 2002, Vienna, Austria, H.A. Mang, F.G. Rammerstorfer & J. Eberhardsteiner (eds.), CD-ROM, Vienna 2002

[132] Krätzig W. B., 'Best' transverse shearing and stretching shell theory for nonlinear finite element simulations, Computer Methods in Applied Mechanics & Engineering 103, 1993, 135-160

[133] Başar Y., Itskov M., Eckstein A., Composite laminates: nonlinear interlaminar stress analysis by multi-layer shell elements, Computer Methods in Applied Mechanics & Engineering 185, 2000, 367-397

[134] Krätzig W. B., Jun D., Multi-layer multi-director concepts for D-adaptivity in shell theory, Computers & Structures 80, 2002, 719-734

[135] Krätzig W. B., Jun D., On 'best' shell models – From classical shells, degenerated and multi-layered concepts to 3D, Archive of Applied Mechanics 73, 2003, 1-25

[136] Naghdi P. M., Finite deformations of elastic rods and shells, Finite Elasticity, Proceedings of the IUTAM Symposium, Lehigh University, Bethlehem, PA, USA, August 10-15, 1980, D.E. Carlson & R. T. Shield (eds.), Martinus Nijhoff Publishers, The Hague/Boston/London, 1982, 47-103

[137] Cho Y.B., Averill R.C., First-order zig-zag sublaminate plate theory and finite element model for laminated composite and sandwich panels, Composite Structures 50, 2000, 1-15

[138] Barbero E. J., Reddy J. N., Teply J., An accurate determination of stresses in thick laminates using a generalized plate theory, Int. Journal for Numerical Methods in Engineering 29, 1990, 1-14

[139] Gaudenzi P., Barboni R., Mannini A., A finite element evaluation of single-layer and multi-layer theories for the analysis of laminated plates, Composite Structures 30, 1995, 427-440

[140] Carrera E., Mixed layer-wise models for multilayered plates analysis, Composite Structures 43, 1998, 57-70

[141] Vu-Quoc L., Deng H., Tan X. G., Geometrically-exact sandwich shells: The static case, Computer Methods in Applied Mechanics & Engineering 189, 2000, 167-203

[142] Gruttmann F., Wagner W., Coupling of 2D- and 3D-composite shell elements in linear and nonlinear applications, Computer Methods in Applied Mechanics & Engineering 129, 1996, 271-287

[143] Williams T. O., Addessio F. L., A general theory for laminated plates with delaminations, Int. Journal of Solids & Structures 34, 1997, 2003-2024

[144] Li X. , Liu D., Generalized laminate theories based on double superposition hypothesis, Int. Journal for Numerical Methods in Engineering 40, 1997, 1197-1212

[145] Pagano N.J., Exact Solutions for Composite Laminates in Cylindrical Bending, Journal of Composite Materials 3, 1969, 389-411

[146] Pagano, N.J., Exact Solutions for Rectangular Bidirectional Composites and Sandwich Plates, Journal of Composite Materials 4, 1970, 20-34

[147] Pagano N. J., Free edge stress fields in composite laminates, Int. Journal of Solids & Structures 14, 1978, 401-406

[148] Bogdanovich A.E., Yushanov S.P., Three-dimensional variational analysis of Pagano's problems for laminated composite plates, Composites Science & Technology 60, 2000, 2407-2425

[149] Desai Y. M., Ramtekkar G. S., Shah A. H., A novel 3D mixed finite-element model for statics of angle-ply laminates, Int. Journal for Numerical Methods in Engineering 57, 2003, 1695-1716

[150] Feng W., Hoa S. V., Partial hybrid finite elements for composite laminates, Finite Elements in Analysis and Design 30, 1998, 365-382

[151] Kong J., Cheung Y.K., Three-dimensional finite element analysis of thick laminated plates, Computers & Structures 57, 1995, 1051-1062

[152] Yu G., Guang-Yau T., Chaturvedi S., Adeli H., *et al.*, A finite element approach to global-local modeling in composite laminate analysis, Computers & Structures 57, 1995, 1035-1044

[153] Cen S., Yuqiu Long Y., Yao Z., A new hybrid-enhanced displacement-based element for the analysis of laminated composite plates, Computers & Structures 80, 2002, 819-833

[154] Hossain S. J., Sinha P. K., Sheikh A. H., A finite element formulation for the analysis of laminated composite shells, Computers & Structures 82, 2004, 1623-1638

[155] Alfano G., Auricchio F., Rosati L., Sacco E., MITC finite elements for laminated composite plates, Int. Journal for Numerical Methods in Engineering 50, 2001, 707-738

[156] Carrera E., A priori vs. a posteriori evaluation of transverse stresses in multilayered orthotropic plates, Composite Structures 48, 2000, 245-260

[157] Das M., Barut A., Madenci E., Ambur D.R., Complete stress ?eld in sandwich panels with a new triangular ?nite element of single-layer theory, Computer Methods in Applied Mechanics & Engineering 194, 2005, 2969-3005

[158] Cho M., Kim M.-H., A postprocess method using a displacement field of higher-order shell theory, Composite Structures 34, 1996, 185-196
[159] Cho M., Kim J.-S., A postprocess method for laminated shells with a doubly curved nine-noded finite element, Composites, Part B 31, 2000, 65-74
[160] Wisniewski K., Schrefler B.A., On recovery of stresses for a multi-layered beam element, Engineering Computations 10, 1993, 563-569
[161] Galvanetto U., Pellegrino C., Schrefler B. A., A three-dimensional stress recovery procedure for composite materials, Computers & Structures 69, 1998, 567-575
[162] Lee K., Lee S. W., A post-processing approach to determine transverse stresses in geometrically nonlinear composite and sandwich structures, Journal of Composite Materials 37, 2003, 2207-2224
[163] Park B. C., Park J. W., Kim Y. H., Stress recovery in laminated composite and sandwich panels undergoing finite rotation, Composite Structures 59, 2003, 227-235.
[164] Kant T., Swaminathan K., Estimation of transverse/interlaminar stresses in laminated composites – a selective review and survey of current developments, Composite Structures 49, 2000, 65-75
[165] Hoff N. J., Bending and Buckling of Rectangular Sandwich Plates, NACA TN No. 2225, Washington 1950
[166] Schmit Jr. L.A., Monforton G.R., Finite deflection Discrete Element Analysis of sandwich plates and cylindrical shells with laminated faces, AIAA Journal 8, 1970, 1454-1461
[167] Lewiński T., On displacement-based theories of sandwich plates with soft core, Journal of Engineering Mathematics 25, 1991, 223-241
[168] Malcolm D. J., Glockner P. G., Nonlinear sandwich shell and Cosserat surface theory, Journal of Engineering Mechanics ASCE 98, 1972, 1183-1203
[169] Glockner P. G., Malcolm D. J., Lagrangian formulation of sandwich shell theory, Journal of Engineering Mechanics ASCE 99, 1973, 445-456
[170] Borsellino C., Calabrese L., Valenza A.: Experimental and numerical evaluation of sandwich composite structures, Composites Science & Technology 64, 2004, 1709-1715
[171] Burton W. S., Noor A. K., Assessment of computational models for sandwich panels and shells, Computer Methods in Applied Mechanics & Engineering 124, 1995, 125-151
[172] Nettles A.T., Basic Mechanics of Laminated Composite Plates, NASA RP-1351, MSFC, Alabama 1994
[173] Stockton S. L., Engineering and Design: Composite Materials for Civil Engineering Structures, Technical Letter No. 1110-2-548, U.S. Army Corps of Engineers Washington, DC 1997
[174] Hu H.-T., Buckling analyses of fiber-composite laminate plates with material nonlinearity, Finite Elements in Analysis & Design 19, 1995, 169-179
[175] Pai P. F., Palazotto A. N., Nonlinear displacement based finite element analysis of composite shells - A new total Lagrangian formulation, Int. Journal of Solids & Structures 32, 1995, 3047-3073
[176] Wisnom M. R., The effect of fibre rotation in ± 45° tension tests on measured shear properties, Composites 26, 1995, 25-32
[177] Abu-Farsakh G.A., Barakat S.A., Al-Zoubi N.R., Effect of material nonlinearity in unidirectional composites on the behavior of beam structures, Int. Journal of Solids & Structures 37, 2000, 2673-2694
[178] Woo K. S., Hong C. H., Basu P. K., Materially and geometrically nonlinear analysis of laminated anisotropic plates by p-version of FEM, Computers & Structures 81, 2003, 1653-1662
[179] Kennedy T.C., Nonlinear viscoelastic analysis of composite plates and shells, Composite Structures 41, 1998, 265-272
[180] Kłosowski P., Woźnica K., Numerical treatment of elasto viscoplastic shells in the range of moderate and large rotations, Computational Mechanics 34, 2004, 194-212
[181] Dvorak G. J., Composite materials: Inelastic behavior, damage, fatigue and fracture, Int. Journal of Solids & Structures 37, 2000, 155-170
[182] Reddy J. N., Analysis of layered composite plates accounting for large deflections and transverse shear strains, Recent Advances in Nonlinear Computational Mechanics, E. Hinton, D. R. J. Owen & C. Taylor (eds.), Pineridge Press Ltd, Swansea 1982, 155-202
[183] Carvelli V., Savoia M., Assessment of plate theories for multilayered angle-ply plates, Composite Structures 39, 1997, 197-207
[184] Jaunky N., Knight Jr. N.F., An assessment of shell theories for buckling of circular cylindrical laminated composite panels loaded in axial compression, Int. Journal of Solids and Structures 36, 1999, 3799-3820
[185] Ferreira A. J. M., Fernandes A. A., A review of numerical methods for the analysis of composite and sandwich structures, Multimaterials Technologies – Solutions and Opportunities, DOGMA Conference in Utrecht, 24-25 October 2000, 71-76
[186] Toorani M. H., Lakis A. A., General equations of

anisotropic plates and shells including transverse shear deformations, rotary inertia and initial curvature effects, Journal of Sound and Vibration 237, 2000, 561-615
[187] Altenbach J., Altenbach H., Trends in engineering plate theories, Maintenance & Reliability, Polish Academy of Sciences Quarterly 4/2001, 2001, 21-30
[188] Piskunov V. G., Rasskazov A. O., Evolution of the theory of laminated plates and shells, Int. Applied Mechanics (Plenum Publ. Corp.) 38, 2002, 135-166
[189] Qatu M. S., Recent research advances in the dynamic behavior of shells: 1989-2000, Part 1: Laminated composite shells, Applied Mechanics Reviews 55, 2002, 325-350
[190] Hult J., Rammerstorfer F. G. (eds.), Engineering Mechanics of Fibre Reinforced Polymers and Composite Structures, CISM Courses and Lectures No. 348, Springer-Verlag, Vienna - New York 1994
[191] Rao K. P., A rectangular laminated anisotropic shallow thin shell finite element, Computer Methods in Applied Mechanics & Engineering 194, 2005, 2285-2707
[192] Reddy, J. N., Energy Principles and Variational Methods in Applied Mechanics, John Wiley & Sons, Ltd., New York 2002
[193] Palmerio A. F., Reddy J. N., Schmidt R., On a moderate rotation theory of elastic anisotropic shells - Part 2. FE analysis, Int. Journal of Non-Linear Mechanics 25, 1990, 701-714
[194] Kreja I., Schmidt R., Moderate Rotation Shell Theory in FEM application, Proceedings of Gdansk University of Technology Civil Eng series LI.1995, 229-249
[195] Panda S. C.. Natarajan R., Finite element analysis of laminated composite plates, Int. Journal of Non-Linear Mechanics 14, 1979, 69-79
[196] Chang T. Y., Sawamiphakdi K., Large deformation analysis of laminated shells by finite element method, Computers & Structures 13, 1981, 331-340
[197] Jun S. M., Hong C. S., Buckling behavior of laminated composite cylindrical panels under axial compression, Computers & Structures 29, 1988, 479-490
[198] Kweon J. H., Hong C. S., An improved arc-length method for postbuckling analysis of composite cylindrical panels, Computers & Structures 53, 1994, 541-549
[199] Wagner W., On formulation of geometrically nonlinear finite elements for fiber reinforced cylindrical shells, Statik und Dynamik in Konstruktiven Ingenieurbau, Festschrift Wilfried B. Krätzig, SFB 151 – Berichte nr 23, 1992, B3-B10 (in German)
[200] Ferreira A. J. M., Barbosa J. T., Buckling behaviour of composite shells, Composite Structures 50, 2000, 93-98.
[201] Laschet G., Jeusette J.-P., Postbuckling finite element analysis of composite panels, Composite Structures 14, 1990, 35-48.
[202] Rikards R., Chate A., Ozolinsh O., Analysis for buckling and vibrations of composite stiffened shells and plates, Composite Structures 51, 2001, 361-370.
[203] Haas D. J., Lee W., A nine-node assumed-strain finite element for composite plates and shells, Computers & Structures 26, 1987, 445-452
[204] Somashekar B.R., Prathap G., Ramesh Babu C., A field-consistent four nodded laminated anisotropic plate/shell element, Computers & Structures 25, 1987, 345-353
[205] Groenwold A., Stander N., A 24 d.o.f. four-node flat shell finite element for general unsymmetric orthotropic layered composites, Engineering Computations 15, 1998, 518-543
[206] Dorninger K., Computational analysis of composite shells at large deformations, Computer Aided Design in Composite Material Technology, Proceedings of the Int. Conf. Southampton, 1988, C. A. Brebbia, W. P. de Wilde & W. R. Blain (eds.), Springer-Verlag, Berlin 1988, 7/66-7/75
[207] Dorninger K., Rammerstorfer F. G., A layered composite shell element for elastic and thermoelastic stress and stability analysis at large deformations; Int. Journal for Numerical Methods in Engineering 30, 1990, 833-858
[208] Ramm E., A plate/shell element for large deflections and rotations, Proceedings of US-Germany Symp., MIT Boston, August 1976, 264-293
[209] Brank B., Perić D., Damjanić F. B., On implementation of a nonlinear four node shell finite element for thin multilayered elastic shells, Computational Mechanics 16, 1995, 341-359
[210] Kreja I., Critical examination of benchmark problems for large rotation analysis of laminated shells, Shell Structures: Theory and Applications, Proc. the 8th Conf. SSTA, Gdańsk – Jurata, 12-14 October 2005, W. Pietraszkiewicz & C. Szymczak (eds.), Taylor & Francis Group, London 2006, 481-485
[211] Kreja I., Schmidt R., Large rotations in First Order Shear Deformation FE analysis of laminated shells, Int. Journal Non-Linear Mechanics 41, 2006, 101-123
[212] Kim K. D., Buckling behaviour of composite panels using the finite element method, Composite Structures 36, 1996, 33-43
[213] Kim K. D., Voyiadjis G.Z., Non-linear finite element analysis of composite panels, Composites, Part B

30, 1999, 365-381

[214] Kim K. D., Park T. H., An 8-node assumed strain element with explicit integration for isotropic and laminated composite shells, Structural Engineering & Mechanics 13, 2002, 387-410

[215] Kim K. D., Lomboy G. R., Han S. C., A co-rotational 8-node assumed strain shell element for postbuckling analysis of laminated composite plates and shells, Computational Mechanics 30, 2003, 330-342

[216] Kim K.-D., Han S.-D., Suthasupradit S., Geometrically non-linear analysis of laminated composite structures using a 4-node co-rotational shell element with enhanced strains, International Journal of Non-Linear Mechanics 42, 2007, 864-881

[217] Barut A., Madenci E., Tessler A., Nonlinear analysis of laminates through a Mindlin-type shear deformable shallow shell element, Computer Methods in Applied Mechanics & Engineering 143, 1997, 155-173

[218] Han S. C., Kim K. D., Kanok-Nukulchai W., An element-based 9-node resultant shell element for large deformation analysis of laminated composite plates and shells, Structural Engineering and Mechanics 18, 2004, 807-829

[219] Han S.-D., Tabiei A., Park W.-T., Geometrically nonlinear analysis of laminated composite thin shells using a modied First-order shear deformable element-based Lagrangian shell element, Composite Structures 82, 2008, 465-474

[220] Pai P. F., Total-Lagrangian formulation and finite-element analysis of highly flexible plates and shells, Mathematics and Mechanics of Solids 12, 2007, 213-250

[221] Arciniega R.A., Reddy J.N., Tensor-based finite element formulation for geometrically nonlinear analysis of shell structures, Computer Methods in Applied Mechanics and Engineering 196, 2007, 1048-1073

[222] Hashagen E., Schellekens J. C. J., de Borst R., Parisch H., Finite element procedure for modelling fibre metal laminates, Composite Structures 32, 1995, 255-264

[223] Parisch H., A continuum-based shell theory for non-linear applications, Int. Journal for Numerical Methods in Engineering 38, 1995, 1855-1883

[224] Kulikov G. M., Plotnikova S. V., Simple and effective elements based upon Timoshenko–Mindlin shell theory, Computer Methods in Applied Mechanics & Engineering 191, 2002, 1173-1187

[225] Kulikov G. M., Plotnikova S. V., Non-linear strain-displacement equations exactly representing large rigid-body motions. Part I: Timoshenko-Mindlin shell theory, Computer Methods in Applied Mechanics & Engineering 192, 2003, 851-875

[226] Zhang Y.X., Kim K.S., Two simple and efficient displacement-based quadrilateral plate elements for the analysis of composite laminated plates, Int. Journal for Numerical Methods in Engineering 61, 2004, 1771-1796

[227] Sze K. Y., placeCityYao L.-Q., Pian T.H.H., An eighteen-node hybrid-stress solid-shell element for homogenous and laminated structures, Finite Elements in Analysis & Design 38, 2002, 353-374

[228] Sze K. Y., Zheng S.-J., A stabilized hybrid-stress solid element for geometrically nonlinear homogeneous and laminated shell analyses, Computer Methods in Applied Mechanics & Engineering 191, 2002, 1945-1966

[229] Bose P., Reddy J. N., Analysis of composite plates using various plate theories Part 2: Finite element model and numerical results, Structural Engineering & Mechanics 6, 1998, 727-746

[230] Chaplin C. P., Palazotto A. N., The collapse of composite cylindrical panels with variousthickness using Finite Element Analysis, Computers & Structures 60, 1996, 797-815

[231] Carrera E., C_z^0 requirements—models for the two dimensional analysis of multilayered structures, Composite Structures 37, 1997, 373-383.

[232] Carrera E., Parisch H., An evaluation of geometrical nonlinear effects of thin and moderately thick multilayered composite shells, Composite Structures 40, 1998, 11-24

[233] Parisch H., An investigation of a finite rotation four node assumed strain shell element, Int. Journal for Numerical Methods in Engineering 31, 1991, 127-150

[234] Rammerstorfer F. G., Dorninger K., Starlinger A., Composite and sandwich shells, in Nonlinear Analysis of Shells by Finite Elements, Rammerstorfer, F. G. (ed.), CISM Courses and Lectures No. 328, Springer-Verlag, placeCityVienna - placeStateNew York 1992, 131-194

[235] Vu-Quoc L., Tan X.G., Optimal solid shells for non-linear analyses of multilayer composites. I. Statics, Computer Methods in Applied Mechanics & Engineering 192, 2003, 975-1016

[236] Kulikov G. M., Plotnikova S. V., Equivalent Single-Layer and Layerwise Shell Theories and Rigid-Body Motions - Part I: Foundations, Mechanics of Advanced Materials & Structures 12, 2005, 275-283

[237] Kulikov G. M., Plotnikova S. V., Equivalent Single-Layer and Layerwise Shell Theories and Rigid-Body Motions -Part II: Computational Aspects, Mechanics of Advanced Materials & Structures 12,

2005, 331-340
[238] Dakshina Moorthy C. M., Reddy J. N., Modeling of laminates using a layerwise element with enhanced strains, Int. Journal for Numerical Methods in Engineering 43, 1998, 755-779
[239] Rammerstorfer F. G., Dorninger K., Starlinger A., Skrna-Jakl I. C., Computational methods in composite analysis and design, in [190], 1994, 209-231
[240] Crisfield M.A., Jelenic G., Mi Y., Zhong H.-G., Fan Z., Some aspects of the non-linear finite element method, Finite Elements in Analysis & Design 27, 1997, 19-40
[241] Aitharaju V. R., Averill R. C., C^0 zigzag kinematic displacement models for the analysis of laminated composites, Mechanics of Composite Materials & Structures 6, 1999, 31-56
[242] Marcinowski J., Geometrically nonlinear static analysis of sandwich plates and shells, Journal of Theoretical and Applied Mechanics (Polish Society of Theoretical and Applied Mechanics) 41, 2003, 561-574
[243] Ali R., Use of finite element technique for the analysis of composite structures, Computers & Structures 58, 1996, 1015-1023
[244] Sze K. Y., Liu X. H., Lo S. H., Popular benchmark problems for geometric nonlinear analysis of shells, Finite Elements in Analysis and Design 40, 2004, 1551-1569
[245] Eason T. G., Ochoa O. O., Modeling progressive damage in composites: a shear deformable element for ABAQUS, Composite Structures 34, 1996, 119-128
[246] Manet V., The use of ANSYS to calculate the behaviour of sandwich structures, Composites Science & Technology 58, 1998, 1899-1905