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Optimization of quadrilateral mesh for ship hull modelling

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Abstract

In this paper a method for improving quadrilateral mesh quality is presented. It assumes the proper topology of a mesh and applies gradient search to optimize the objective function for a mesh. The objective function may be any quality assessment function (metrics) appropriate for the FEM requirements. Quadrilateral meshes are most preferable way of modeling ship hull structures for the FEM analysis. There are many algorithms which produce such meshes. However, apart from providing the proper topology, only some of these algorithms assure quality of mesh elements. To provide a reliable result of FEM calculations a model must consist of elements of appropriate size and quality. Application of the presented method provides a mesh with optimized quality for a given topology.

Keywords: mesh optimization, ship hull modeling, finite element method (FEM), computer aided design (CAD).

Optymalizacja siatek elementów skończonych używanych do modelowania kadłuba statku

Streszczenie

Artykuł przedstawia metodę poprawy jakości czworokątnej siatki elementów skończonych, które najlepiej nadają się do modelowania kadłuba statku dla potrzeb analizy MES. W metodzie zakłada się poprawną topologię siatki i stosuje gradient funkcji celu do poszukiwania optymalnego rozwiązania. Funkcja celu może być dowolną funkcją, opisująca jakość elementu (metryka), odpowiednią dla wymagań metody elementów skończonych. W artykule przedstawiono najczęściej używane metryki wraz z krótkim opisem. Znanych jest wiele algorytmów, tworzących takie siatki. Jednakże oprócz zapewnienia poprawnej topologii tylko nieliczne z nich zapewniają odpowiednią jakość elementów skończonych. Aby uzyskać wiarygodne wyniki obliczeń MES, model musi składać się z elementów o odpowiednich rozmiarach i odpowiedniej jakości. Zastosowanie przedstawionej metody pozwala poprawić jakość elementów siatki dla zadanej topologii. W artykule przedstawiono także przykład poprawy jakości czworokątnej siatki elementów skończonych dla przegrody zbiornika obłowego.

Słowa kluczowe: optymalizacja siatek elementów skończonych, modelowanie kadłuba statku, metoda elementów skończonych (MES), komputerowe wspomaganie projektowania (CAD).

1. Introduction

In shipbuilding the FEM is used for strength and fluid dynamics calculations. Models for strength and fluid dynamics analyses can differ significantly as in the second case only outer shell is considered. In case of strength analysis the internal structure of a hull must be included in a model. Ship hull models of different levels of details are prepared for different phases of the design process. Usually in earlier stages less detailed models conform to calculation requirements.

A ship hull model for strength analysis consists of surfaces. It means that plates and profiles, which are used for an outer shell, frames, girders, decks, bulkheads, tanks, partitions, brackets, stiffeners etc. are modelled with surfaces. The topology of a model is defined by the mutual relations of the hull components. The surface model is a typical model for the FEM analysis. There are many known methods for meshing such a model. These methods are meshing properly large surfaces with relatively small number of constraints. However, the ship hull model is particular because of its topology. In case of a ship hull almost all plates are stiffened with stiffeners and all joints are stiffened with gussets. Such a complex topology results in a great number of constraints. Maintaining constraints leads to meshing a model with poorer quality elements. On the other hand, obtaining reliable results with FEM analysis requires a model meshed with quality elements.

2. Background

Meshing surfaces is a well known problem. The meshing algorithms may be classified according to the kind of elements used in the mesh (triangular, quadrilateral, hexagonal), according to the kind of a meshed surface (planar, curved, explicit, parametric) or according to the methods applied to creating mesh elements (unstructured/structured, direct/indirect).

Triangle meshing algorithms including Delaunay and advancing front methods [2, 3, 4, 7, 11] dominate in the literature. The reason is that they are simple to implement and have provable mathematical properties. There exists a smaller set of literature that describes quadrilateral meshing algorithms. Quadrilateral meshing is not so well mathematically described as triangle meshing and requires more heuristic approach.

When the surface geometry is suitable, quadrilateral mapped meshing, or sub-mapping methods produce very high quality elements. These methods break the surface into a structured set of quadrilaterals, where all interior nodes have exactly four adjacent elements and thus are called structured methods. Where applicable, these methods work faster and produce more reliable results. Unfortunately, because only a limited class of problems may be resolved using mapping methods, more general approach must be sometimes applied.

Unstructured quadrilateral meshing algorithms can be grouped into two main categories: direct and indirect. With an indirect approach, the domain is first meshed with triangles. Various algorithms are then employed to convert the triangles into quadrilaterals [1, 10]. With a direct approach, quadrilaterals are placed on the surface directly, without first meshing with triangles.

3. Quality assessment - metrics

To assess mesh quality many quality metrics have been defined e.g. in [5]. The metrics describe either an element size or its shape. Sometimes complex metric functions are defined to balance both size and shape. The metric function is different for different kinds of elements. Below in Table 1. different metrics for quadrilaterals and their acceptable values are presented.

Approximate quadrilateral quality metrics definitions:

aspect ratio: maximum edge length ratios

skew: 4/(sum of 1/sin(α_k)), where α_k, k = 0, 1, 2, 3 are angles between neighbour quad edges

taper: max. ratio of lengths derived from opposite edges

warpage: cosine of min. dihedral angle formed by planes intersecting in diagonals

stretch: sqrt(2) * min. edge length / max. diagonal length

min. angle: smallest included quad angle (degrees)

max. angle: largest included quad angle (degrees)

condition no.: max. condition number of the Jacobian matrix at 4 corners

scaled jacobian: min. Jacobian divided by the lengths of the 2 edge vectors
shear: 2/condition no. of Jacobian skew matrix
shape: 2/condition no. of weighted Jacobian matrix
relative size: min(J, 1/J), where J is determinant of weighted Jacobian matrix
shear & size: product of *shear* and *relative size*
shape & size: product of *shape* and *relative size*
distortion: {min(|J|)/actual area}*parent area, parent area = 4 for quad

Tab. 1. Quality metrics for quadrilateral elements

metrics	range	acceptable
aspect ratio	1.0 – inf.	1.0 – 4.0
skew	0.0 – 1.0	0.5 – 1.0
taper	1.0 – inf.	0.0 – 0.7
warpage	0.0 – 1.0	0.9 – 1.0
stretch	0.0 – 1.0	0.25 – 1.0
min. angle	0.0 – 90.0	45.0 – 90.0
max. angle	0.0 – 360.0	90.0 – 135.0
condition no.	1.0 – inf.	1.0 – 4.0
scaled Jacobian	-1.0 – 1.0	0.5 – 1.0
shear	0.0 – 1.0	0.3 – 1.0
shape	0.0 – 1.0	0.3 – 1.0
relative size	0.0 – 1.0	0.3 – 1.0
shear & size	0.0 – 1.0	0.2 – 1.0
shape & size	0.0 – 1.0	0.2 – 1.0
distortion	-1.0 – 1.0	0.6 – 1.0

A Jacobian matrix for a quadrilateral is calculated in its node. A quadrilateral has four Jacobian matrices of the form presented by the formula (1).

$$\mathbf{J}_k = \begin{vmatrix} x_{k+1} - x_k & x_{k+3} - x_k \\ y_{k+1} - y_k & y_{k+3} - y_k \end{vmatrix} \qquad (1)$$

where (x_k, y_k) are the coordinates of the k-th quadrilateral node.

In the three dimensional space Jacobian matrix should be 2x3 matrix. However, it is always possible to find a local coordinate system whose *X-Y* plane includes the quadrilateral. Changing to this coordinate system allows calculating Jacobian matrix as in two dimensions.

Not all metrics are used in every FEM software. For example in ASYS v.12 there are the following metrics: aspect ratio, Jacobian ratio, warping factor, parallel deviation, maximum corner angle and skewness.

4. Meshing method

Every ship hull plate is represented with a set of polygons, which describes its contour. This is a set of polygons because there might be holes inside a plate. Nodes of the polygons are placed in the corners of the plate and at every seam or common edge with neighbour element. The polygons are to be meshed with as few as possible elements of maximum quality.

Since it is difficult to control the mesh quality in direct quadrilateral meshing, an indirect method for mesh generation has been chosen. First, a triangular mesh is generated by the Delaunay triangulation divide-and-conquer algorithm described in [11]. Its advantage is ability to produce triangles of large size while maintaining their basic properties like the minimum angle and maximum area. To ensure the minimum angle and maximum area constraint some additional points inside the meshed polygon may be added. These points are called Steiner points.

After completing Delaunay triangulation the resulting triangles must be converted into quadrilaterals. This is achieved by joining the neighbour triangles by removing their common edge. The algorithm responsible for this process is called advanced front because triangles which are closer to the outside of the polygon are joined first. The front of the joined triangles moves from the outside towards inside of the polygon. If the number of triangles in the triangular mesh is odd or if the joined triangles form concave quadrilateral, some triangles are left in the mesh. The quality of so created mostly quadrilateral mesh is dependent on the quality of the background triangular mesh.

The quality of the quadrilateral mesh, which is a result of converting triangles into quadrilaterals by the advancing front algorithm, can be further improved.

5. Quadrilateral mesh optimization

The optimization of a mostly quadrilateral mesh produced by the indirect advancing front algorithm is aimed at improving the shape of the mesh elements. However, it must not change the shape of the meshed surface. The optimization is performed by moving internal nodes on the meshed surface. The boundary nodes are excluded because it would violate the constraint and change the shape of the meshed surface. Thus, it is assumed that the coordinates of the internal mesh nodes are the decision variables in the optimization process.

Any quality metrics presented in Section 3 may be used to define the objective function for the optimization. The choice of metrics determines what modifications in the element shape are most preferable. However, it is assumed that neither number of quadrilateral and triangular elements of the mesh nor the number of mesh nodes can be changed. Thus, only the shape metrics are applicable since they influence the angles and edge proportions of the mesh elements. The objective function for the optimization process is a sum of the metrics for all quadrilateral mesh elements.

The optimization algorithm uses the objective function gradient and it works iteratively. The vector of decision variables $\mathbf{X_n}$ is increased by the scaled gradient vector $\mathbf{G_n}$ to give a better value of the objective function. Then a new gradient vector is calculated. The process repeats until the gradient is of zero length or increase of the objective function is small enough. One iteration step is presented by the formula (2).

$$\mathbf{X_{n+1}} = \mathbf{X_n} + \mathrm{a} \cdot \mathbf{G_n} \qquad (2)$$

where a is a gradient vector scale coefficient.

The values of the gradient vector are numerically calculated. As there is no guarantee that the optimization process converges to some value, there is a limited number of iteration to be taken.

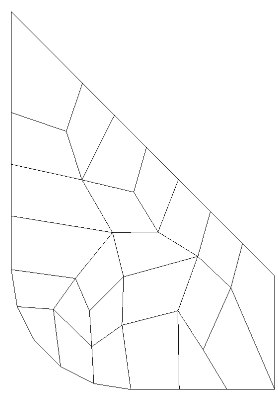

Fig. 1. Not optimized mesh for a partition plate of a hopper tank
Rys. 1. Niezoptymalizowana siatka elementów skończonych dla przegrody zbiornika obłowego

Fig. 1 shows a hopper tank partition plate meshed with the indirect advancing front algorithm without optimization. Because the number of triangles after Delaunay triangulation was even, no triangles were left after joining the triangles with the advancing front algorithm.

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Fig 2. shows the same hopper tank partition plate meshed with the indirect advancing front algorithm with optimization. The stretch metrics was used for the objective function. The differences are most significant in the lower right area of the plate. The degenerate quadrilateral with almost collinear edges resembling a triangle is improved and changed into trapezoid. In this example the optimization process improved the objective function from value of 16.65 to 17.94.

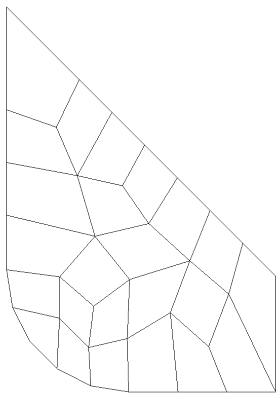

Fig. 2. Optimized mesh for a partition plate of a hopper tank
Rys. 2. Zoptymalizowana siatka elementów skończonych dla przegrody zbiornika obłowego

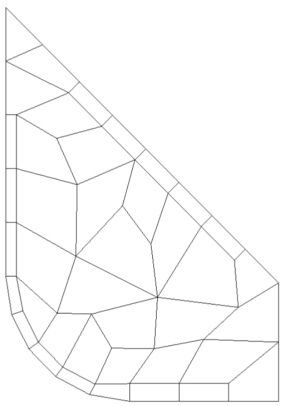

Fig. 3. Not optimized mesh for a partition plate of a hopper tank with stiffeners
Rys. 3. Niezoptymalizowana siatka elementów skończonych dla przegrody zbiornika obłowego z usztywnieniami

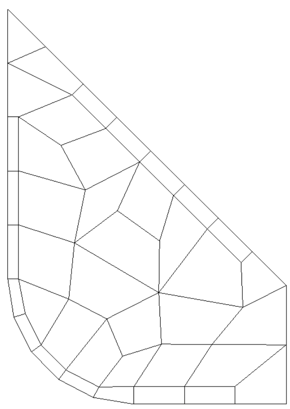

Fig. 4. Optimized mesh for a partition plate of a hopper tank with stiffeners
Rys. 4. Zoptymalizowana siatka elementów skończonych dla przegrody zbiornika obłowego z usztywnieniami

Figs. 3 and 4 also present the meshed hopper tank partition plate, but in this case the tank walls are stiffened. On the boundary of the plate there are rectangular elements placed between stiffeners. In Fig. 3 the mesh was not optimized. In Fig. 4 the optimization changed the shape of internal elements, they are more like trapezoids, compared to Fig. 3. The overall value of the objective function changed from 15.43 to 16.23.

6. Summary

The presented method for mostly quadrilateral mesh generation is a multi-stage indirect method. It generates as few elements as possible and tries to keep comparable dimensions of elements. The last stage in mesh generation is optimization. It is performed to correct the results of triangulation and conversion of triangles into quadrilaterals.

The presented method has the following advantages; it:
- maximizes the size and quality of mesh elements,
- accepts many quality metrics used in the FEM analysis,
- works on very constrained topologies.

The disadvantages of the presented method are as follows; it:
- produces a mostly quadrilateral mesh which may include several triangles,
- the optimization is time consuming on a big mesh and not always converges to the local optimum.

The described above meshing method was implemented as a program and is used to generate a coarse mesh for the strength analysis of a ship hull.

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