

# Evolutionary Sets of Safe Ship Trajectories: the Method's Development and Selected Research Results

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Received January 2011

## Abstract

The Evolutionary Sets of Safe Ship Trajectories is a method solving ship encounter situations. The method combines evolutionary approach to planning ship trajectory with some of the assumption of game theory. For given positions and motion parameters the method finds a near optimal set of safe trajectories of all ships involved in an encounter. This paper presents framework of the method and its development. Additionally, selected method's results for two multi-ship encounters are provided.

**Keywords:** marine navigation, collision avoidance, evolutionary algorithms

## 1. Introduction

A desired solution to a multi-ship encounter situation would include a set of planned, optimal trajectories for all the ships involved in an encounter, such that no collision or domain violations occur when these ships follow the trajectories. When solving this situation the key difficulty is that even a single course change performed by one ship involved in the encounter may force one or even more the other ships to manoeuvre. Thus the optimization method utilized to find a solution to the problem should be flexible enough to efficiently look through the vast search space and handle even minor changes in the ship's behaviour e.g. in its motion parameters.

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Different approaches exist to solving a multiship encounter situation. Two basic trends are either utilization of differential games [6] or searching for a single trajectory (for the own ship) by evolutionary algorithms [7]. The former method assumes that the process of steering a ship in multi-ship encounter situations can be modelled as a differential game played by all ships involved, each having their strategies. Unfortunately, high computational complexity is its serious drawback. The latter approach is the evolutionary method focused on finding only a single trajectory of the own ship. In short, the evolutionary method uses genetic algorithms, which, for a given set of pre-determined input trajectories find a solution that is optimal according to a given fitness function. However, the method's limitation is that it assumes targets motion parameters not to change and if they do change, the own trajectory has to be recomputed. This limitation becomes a serious one on restricted waters. If a target's current course collides with a landmass or another target of a higher priority, there is no reason to assume that the target would keep such a disastrous course until the crash occurs. Consequently, planning the own trajectory for the unchanged course of a target will be futile in the majority of such cases. Also, the evolutionary method does not offer a full support to VTS operators, who might face the task of synchronizing trajectories of multiple ships with many of these ships manoeuvring.

Therefore, the authors have proposed a new approach, which finds a set of optimal trajectories and combines some of the advantages of both the existing trends: the low computational time, supporting all domain models and handling stationary obstacles (all typical for evolutionary method), with taking into account the changes of motion parameters (changing strategies of the players involved in a game). Instead of finding a single optimal own trajectory for the unchanged courses and speeds of targets, an optimal set of safe trajectories of all ships involved is searched for. The method is called evolutionary sets of safe trajectories and its early version has been presented by one of the authors in [8].

While developing the method, the authors came across some problems, which could not be solved efficiently enough using typical evolutionary mechanisms. Consequently, a number of changes had to be brought to the traditional evolutionary scheme. These modifications are emphasised in the paper.

A software simulation tool implementing the method has been constructed by the authors. Using this tool, comprehensive simulation tests of the method for both open and restricted waters have been conducted. In this paper two scenarios with 6-ship encounters have been presented and the results obtained by the method have been analysed and described.

The rest of the paper is organized as follows. In the next section the task – finding sets of safe trajectories – is presented as an optimisation problem. Then some basics of the evolutionary approach are given in Section 3. This is followed by a detailed description of the proposed method (Section 4), including the modifications of the typical evolutionary mechanisms. Section 5 presents the selected method's

results accompanied with a description of collision avoidance manoeuvres. Finally the method's summary is given in Section 6.

## 2. Optimisation Problem

It is assumed that we are given the following data:

- stationary constraints (such as landmasses and other obstacles),
- positions, courses and speeds of all ships involved,
- ship domains,
- times necessary for accepting and executing the proposed manoeuvres.

Ship positions and ship motion parameters are provided by ARPA (Automatic Radar Plotting Aid) and AIS (Automatic Identification System) systems. A ship domain can be determined mainly based on the ship's length and its motion parameters. Since the shape of a domain is also dependant on the type of water region, the author has decided to use a ship domain model by Davis [3], which updated Goodwin model [5], for open waters and to use a ship domain model by Coldwell [2], which updated Fuji model [4], for restricted waters. The last parameter – the necessary time, it is computed on the basis of navigational decision time and the ship's manoeuvring abilities. By default a 6-minute value is used here.

Knowing all the abovementioned parameters, the goal is to find a set of trajectories, which minimizes the average way loss spent on manoeuvring, while fulfilling the following conditions:

- none of the stationary constraints are violated,
- none of the ship domains are violated,
- the minimal acceptable course alteration is not lesser than 15 degrees,
- the maximal acceptable course alteration is not to be larger than 60 degrees,
- speed alteration are not to be applied unless necessary (collision cannot be avoided by course alteration up to 60 degrees),
- a ship only manoeuvres, when she is obliged to,
- manoeuvres to starboard are favoured over manoeuvres to port board.

The first two conditions are obvious: all obstacles have to be avoided and the ship domain is an area that should not be violated by definition. All the other conditions are either imposed by COLREGS [1] and good marine practice or by the economics. In particular, the course alterations lesser than 15 degrees might be misleading for the ARPA systems (and therefore may lead to collisions) and the course alterations larger than 60 degrees are not recommended due to efficiency reasons. Also, ships should only manoeuvre when necessary, since each manoeuvre of a ship makes it harder to track its motion parameters for the other ships ARPA systems.

### 3. Evolutionary Programming in Brief

The general idea of evolutionary programming is shown in Figure 1. First, the initial population of individuals (each being a potential solution to the problem) is generated either randomly or by other methods. This initial population is a subject to subsequent iterations of evolutionary algorithm. Each of these iterations consists of the following steps:

1. **Reproduction:** sets of parents are selected from all of the individuals and they are crossed to produce offspring. The offspring inherits some features from each parent.
2. **Evolutionary operations:** the offspring is modified by means of random mutation operators as well as specialized operators dedicated to the problem.
3. **Evaluation:** each of the individuals is assigned a value of a fitness function, which reflects the quality of the solution represented by this individual.
4. **Succession:** the next generation of individuals is selected. Usually the individuals are chosen randomly, with the probability strictly depending on the fitness function value. The evolutionary algorithm ends when one of the following happens:
  - maximum acceptable time or number of iterations is reached,
  - the satisfactorily high value of fitness function has been reached by one of the individuals,
  - further evolution brings no improvement.

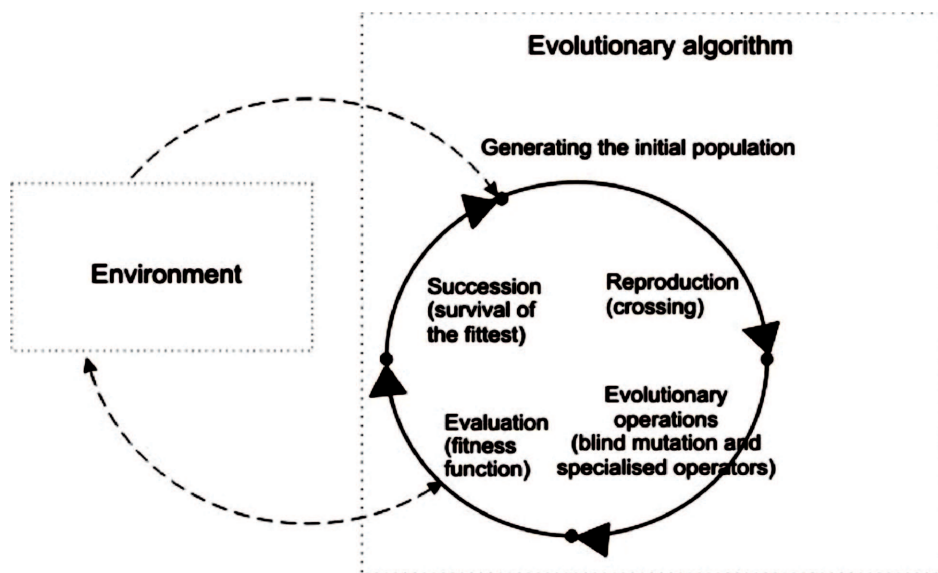


Fig. 1. Evolutionary algorithms – general idea

## 4. Evolutionary Sets of Safe Ship Trajectories and its Amendments to the Evolutionary Scheme

This section presents elements of the original evolutionary scheme customized by the authors to support the Evolutionary Sets of Safe Ship Trajectories method.

### 4.1. Generating initial population

Each individual (a population member) is a set of trajectories, each trajectory corresponding to one of the ships involved in an encounter. A trajectory is a sequence of nodes, each node containing the following data:

- geographical coordinates  $x$  and  $y$ ,
- the speed between the current and the next node.

Typically, the initial population is generated randomly or by some very generic methods, so as not to invest computational time into this phase. Here however, the initial population contains three types of individuals:

- a set of original ship trajectories – segments joining the start and destination points,
- sets of safe trajectories determined by other methods,
- randomly modified versions of the first two types – sets of trajectories with additional nodes, or with some nodes moved from their original geographical positions.

The first type of individuals results in an immediate solution in case of no collisions, or in faster convergence in case of minor constraint violations. The second type provides sets of safe (though usually not optimal) trajectories. Depending on the type of water region, they are mostly generated by the method of planning a trajectory on raster grids [9], which enables avoiding collisions with other ships as well as with stationary obstacles (for restricted waters) and by the method of planning a sequence of necessary manoeuvres on open waters [10]. Both methods return more useful results than plain randomly-generated trajectories, at the cost of consuming more computational time. The third type of individuals (randomly modified individuals of the previous two types) is used to generate the majority of a diverse initial population and thus to ensure a vast searching space.

### 4.2. Reproduction

In this phase pairs of individuals (parents) are crossed to generate new individuals (offspring). Two types of crossing have been used:

1. An offspring inherits whole trajectories from both parents.
2. Each of the trajectories of the offspring is a crossing of the appropriate trajectories of the parents.



### 4.3. Random mutation

Evolutionary operations that have been used include random mutation and three groups of specialized operators. Four types of random mutation operators have been used, all operating on single trajectories. These random operators are:

1. Node insertion: a node is inserted randomly into the trajectory.
2. Node joining: two neighbouring nodes are joined, the new node being the middle point of the segment joining them.
3. Node shift: a randomly selected node is moved (its polar coordinates are altered).
4. Node deletion: a randomly selected node is deleted.

A modification applied here is that trajectory mutation probability decreases with the increase of the trajectory fitness value, so as to mutate the worst trajectories of each individual first, without spoiling its best trajectories. In the early phase of the evolution all random operators: the node insertion, deletion, joining and shift are equally probable. In the later phase node shift dominates with its course alteration changes and distance changes decreasing with the number of generations. For node insertion and node shift instead of Cartesian coordinates  $x$  and  $y$ , the polar coordinates (course alteration and distance) are mutated in such a way that the new manoeuvres are between 15 and 60 degrees. As a result, fruitless mutations (the ones leaving to invalid trajectories) are avoided for these two operators.

### 4.4. Specialised operators

Specialised operators, responsible for more conscious improving of trajectories (as opposed to random mutation) result in a faster convergence to a solution. The evolutionary operators, which have been used here, can be divided into following groups, with group 2 only applied for restricted waters.

1. Operators avoiding collisions with prioritised ships. Five types of these operators have been used, all operating on single trajectories. If a collision with a prioritised ship has been registered, an operator is selected depending on the values of a time remaining to a collision and a time remaining to reaching the next node:
  - a) Segment insertion – if only there is enough time for three course alterations, a new segment is inserted.
  - b) Node insertion – if there is not enough time for a whole new segment (additional three course alterations), a single node is inserted.
  - c) First node shift – if there is not enough time for a node insertion (additional two course alterations) and the collision point is much closer to the first node of a segment, the first node is moved away from the collision point.
  - d) Second node shift – if there is not enough time for a node insertion (additional two course alterations) and the collision point is much closer to the second node of a segment, the second node is moved away from the collision point.

- e) Segment shift – if there is not enough time for a node insertion (additional two course alterations) and the collision point is close to the middle of a segment, the whole segment is moved away from the collision point.

None of these operations guarantees avoiding the collision with a given target but they are likely to do so and therefore highly effective statistically, which is enough for evolutionary purposes.

2. Operators avoiding collisions with stationary obstacles (restricted waters only). If a segment of a trajectory crosses a landmass or other stationary obstacle, similarly as in a case of a collision with a target, depending on the values of a time remaining to collision and a time remaining to reaching the next node, one of the abovementioned five operators is chosen, based on similar rules as in point 1).
3. Validations and fixing. This group includes three operators:
  - a) Node reduction – its purpose is to eliminate all the unnecessary nodes. If a segment, which bypasses a given node by joining its neighbours, is safe, the node is deleted.
  - b) Smoothing – if a course alteration is larger than 30 degrees, a node is replaced with a segment to smoothen the trajectory.
  - c) Adjusting manoeuvres – each trajectory of an individual is analysed and in case of unacceptable manoeuvres (such as slight course alterations), the nodes being responsible are moved so as to round a manoeuvre up or down to an acceptable value.

#### 4.5. Evaluation

The following basic fitness function is used first:

$$fitness = \sum_{i=1}^n [tr\_fit_i], \quad (1)$$

where:

$$tr\_fit_i = \left( \frac{tr\_length_i - way\_loss_i}{tr\_length_i} \right) * sf_i * of_i, \quad (2)$$

$sf_i$  – ship collision factor [1] of the  $i$ -th ship computed over all prioritised targets:

$$sf_i = \prod_{j=1, j \neq i}^n (\min(fmin_{i,j}, 1)) \quad (3)$$

$of_i$  – obstacle collision factor [1] of the  $i$ -th ship computed over all stationary constraints:

$$of_i = \left( \frac{trajectory\_length_i - trajectory\_cross\_length_i}{trajectory\_length_i} \right)^2 \quad (4)$$

$n$  – the number of ships [1],  
 $m$  – the number of stationary constraints [1],  
 $i$  – the index of the current ship [1],  
 $j$  – the index of a target ship [1],  
 $k$  – the index of a stationary constraint [1],  
 $f_{min_{i,j}}$  – the approach factor value for an encounter of ships  $i$  and  $j$  [1],  
 $trajectory\_length_i$  – the total length of the  $i$ -th ship's trajectory [nautical miles],  
 $trajectory\_cross\_length_i$  – the total length of the parts of the  $i$ -th ship's trajectory, which violate stationary constraints [nautical miles].

This basic fitness function focuses on way loss and safe distances between ships, with COLREGS only being applied via ship domain models [2, 3] used to compute the approach factor value [11]. The impact of ship domain model on COLREGS compliance is as follows. Domain shape affects the size of necessary course alteration manoeuvres to starboard and port board, thus affecting way loss and indirectly – fitness function values assigned to different trajectories. Therefore applying asymmetrical ship domain, whose port board area is larger than starboard area, favours manoeuvres to starboard over manoeuvres to port board. Also, larger bow area makes it less likely to cross ahead of stand-on targets. Apart from ship domains, two other means of reaching compliance with COLREGS have been applied here:

- a) Only collisions with prioritised ships were taken into account so as not to encourage unnecessary or unlawful manoeuvres from so-called “stand-on” vessels.
- b) Manoeuvres to starboard are encouraged by a larger probability of course alteration to starboard than port board in mutation and specialised operators:
  - node shift,
  - node insert,
  - segment shift,
  - segment insert in and mutation.

After computing the basic fitness function value, additional penalties are applied for collision avoidance actions not recommended by COLREGS. The rules of applying these penalties are different for restricted and open waters due to the fact that on restricted waters manoeuvres may result from avoiding collisions with land and other stationary obstacles as well as with targets. These rules are as follows:

1. On open waters:
  - a) if a ship is not obliged to give way, any manoeuvre it performs is penalized,
  - b) if a ship is obliged to give way, and does not perform a manoeuvre it is penalized,
  - c) all manoeuvres to port board are penalized.
2. On restricted waters: every trajectory node, which is a part of a manoeuvre, contains special information on the reason why this particular node has been inserted or shifted: land or other stationary obstacle avoidance, target avoidance or accidental manoeuvre generated by evolutionary mechanisms. Based on this penalties are applied as follows:



- a) if a ship does not initially have to give way to any target and its first manoeuvre has reason other than stationary obstacle avoidance, it is penalized,
- b) any manoeuvre to port board of reason other than stationary obstacle avoidance is penalized.

For normalized basic fitness function values, the penalties resulting from the unlawful manoeuvres have been set to 0.05. The penalties are additive: that is a manoeuvre might be penalized twice. For example a manoeuvre to port board from a stand-on ship would be first penalized for performing any manoeuvre at all (rule 1a) and then, additionally for altering its course to port board (rule 1c).

#### 4.6. Succession

A number of selection methods have been tried by the authors with the most successful being the truncation method (with the truncation threshold of 50%). In this method the random factor is eliminated and the highest-ranked individuals constitute the next generation. Although this kind of selection means a loss of diversity (and thus the risk of stopping at local optimums), it has the benefit of a fast convergence to a solution. This fast convergence is essential for a method designed to operate in real time. Instead of finding the globally optimal solution in a longer time, finding an acceptable sub-optimal solution in a given time is needed. However, when combined with specialised operators described in Section 4.4, the solution, which the process converges to, is usually close to the optimal one.

### 5. Method's Results for Selected Scenarios

In this section exemplary method's results for scenarios covering open waters and restricted waters cases are presented and discussed.

#### 5.1. Open water scenario

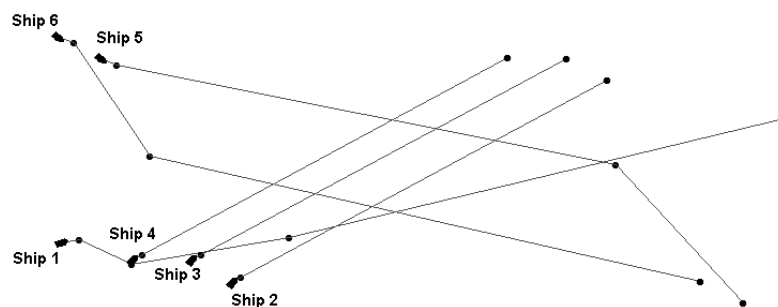


Fig. 2. Open water scenario (initial ship positions)

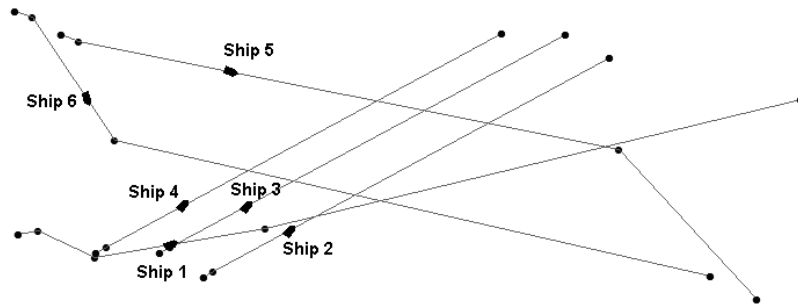


Fig. 3. Open water scenario (positions after 20% of total simulation time)

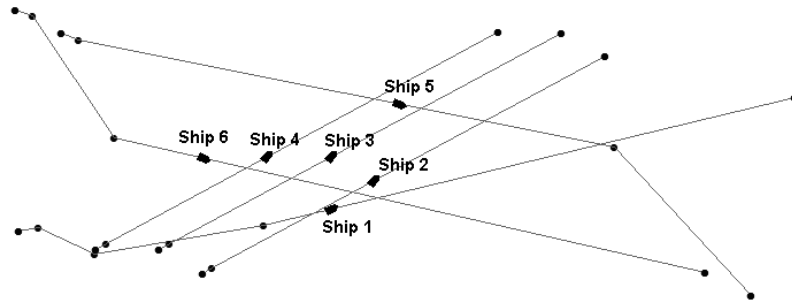


Fig. 4. Open water scenario (positions after 40% of total simulation time)

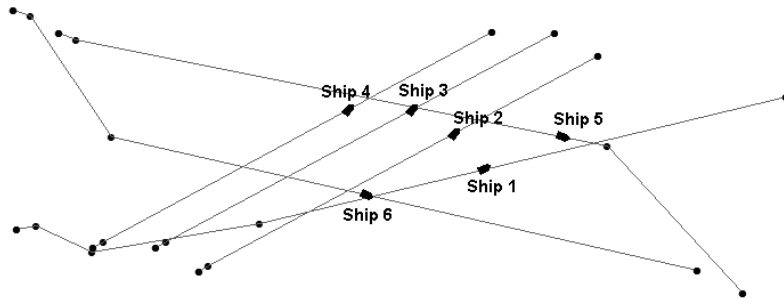


Fig. 5. Open water scenario (positions after 60% of total simulation time)

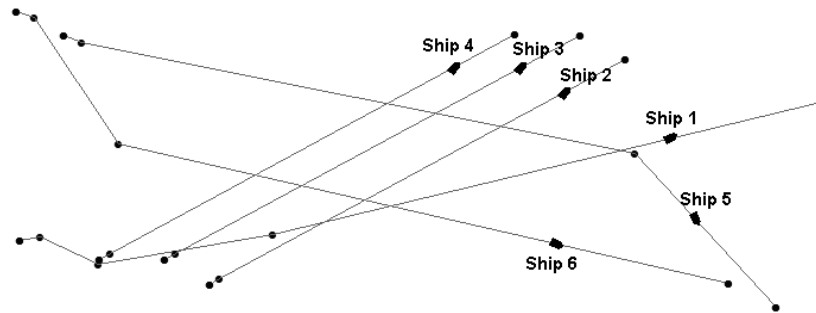


Fig. 6. Open water scenario (positions after 80% of total simulation time)

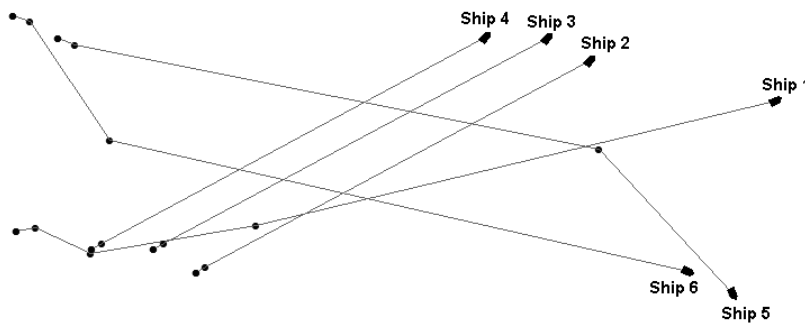


Fig. 7. Open water scenario (destinations reached)

In the scenario presented in Figures 2-7 (with ship positions given in Table 1) there is a single ship (ship 1) crossing with two group of ships, namely:

- first group formed by ship 2, ship 3 and ship 4,
- second group formed by ship 5 and ship 6.

Ship 1 is a give-way vessel only to the first group of ships, thus it performs a substantial starboard course alteration to avoid ahead crossing. Ships 2, 3 & 4 are stand-on vessels (having no other vessels to their starboard) and due to that their courses remain unchanged until reaching their destination positions (maximum possible trajectory fitness value of 1.0). Unlike group 1, ships 5 & 6 from group 2 must give way to both ship 1 and group 1 ships. Due to mutual relation between origin and destination positions of ship 5 and ship 6 the former alters her course to port board, while the latter – to starboard. This way ship 5 reaches her destination safely bypassing ships 1, 2, 3 & 4 ahead with substantial distance to the ships. On the other hand, ship 6 avoids ahead crossing by her starboard manoeuvre. If both ship 5 & ship 6 changed courses to starboard, ship 6 would be forced to perform a larger alteration and the resulting way loss of the ships would be greater.

Table 1

Open water scenario – ship positions &amp; resulting fitness values

|               | Origin position                | Destination position           | V [kn] | Resulting trajectory fitness value [/] | Resulting general fitness value [/] |
|---------------|--------------------------------|--------------------------------|--------|--|-------------------------------------|
| <b>Ship 1</b> | 20° 18' 29" E<br>58° 28' 08" N | 20° 47' 17" E<br>58° 33' 06" N | 14.73  | 0.9275                                 | 0.9872                              |
| <b>Ship 2</b> | 20° 25' 17" E<br>58° 26' 34" N | 20° 40' 14" E<br>58° 34' 37" N | 10.41  | 1.0000                                 |                                     |
| <b>Ship 3</b> | 20° 23' 42" E<br>58° 27' 27" N | 20° 38' 39" E<br>58° 35' 30" N | 10.41  | 1.0000                                 |                                     |
| <b>Ship 4</b> | 20° 21' 20" E<br>58° 27' 28" N | 20° 36' 16" E<br>58° 35' 31" N | 10.41  | 1.0000                                 |                                     |
| <b>Ship 5</b> | 20° 20' 04" E<br>58° 35' 30" N | 20° 45' 41" E<br>58° 25' 44" N | 15.39  | 0.9575                                 |                                     |
| <b>Ship 6</b> | 20° 18' 21" E<br>58° 36' 21" N | 20° 43' 59" E<br>58° 26' 36" N | 15.39  | 0.8984                                 |                                     |

## 5.2. Restricted water scenario

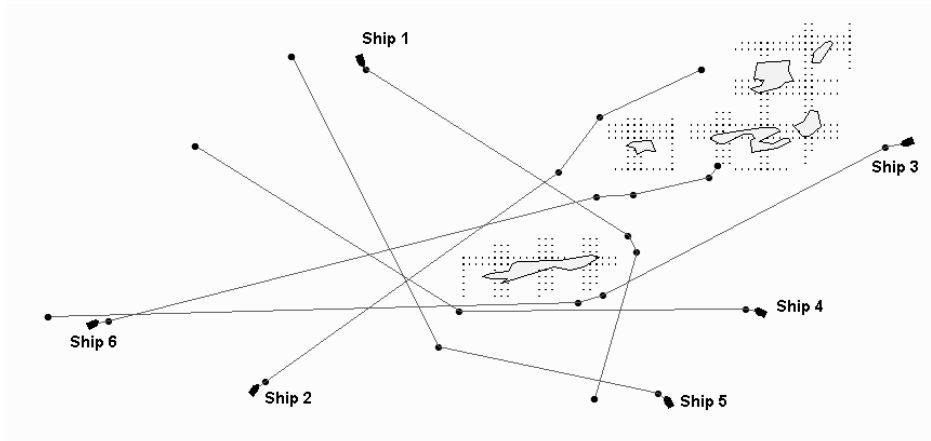


Fig. 8. Restricted water scenario – dotted areas depict shallow waters (initial positions)

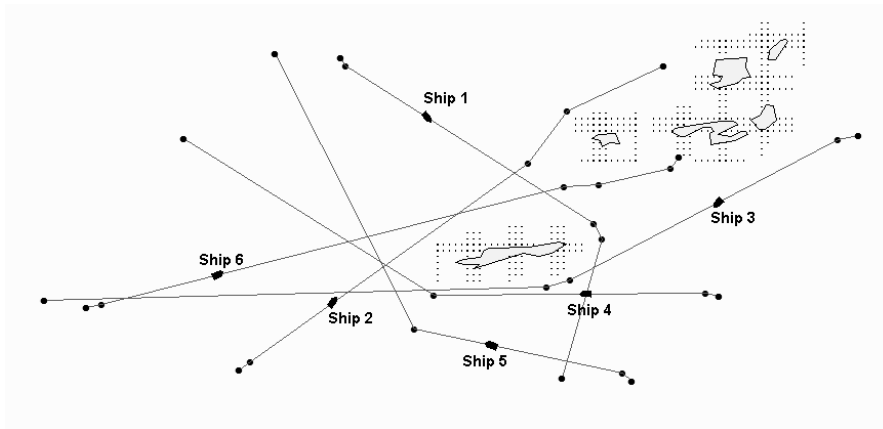


Fig. 9. Restricted water scenario – dotted areas depict shallow waters (positions after 20% of total simulation time)

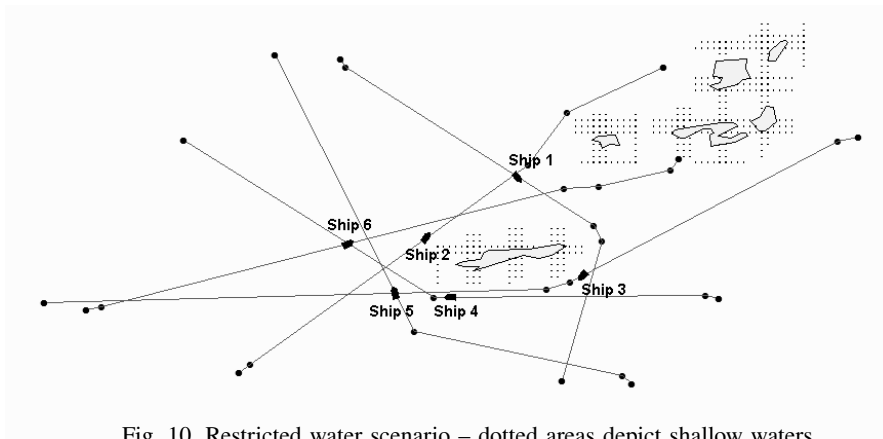


Fig. 10. Restricted water scenario – dotted areas depict shallow waters (positions after 40% of total simulation time)

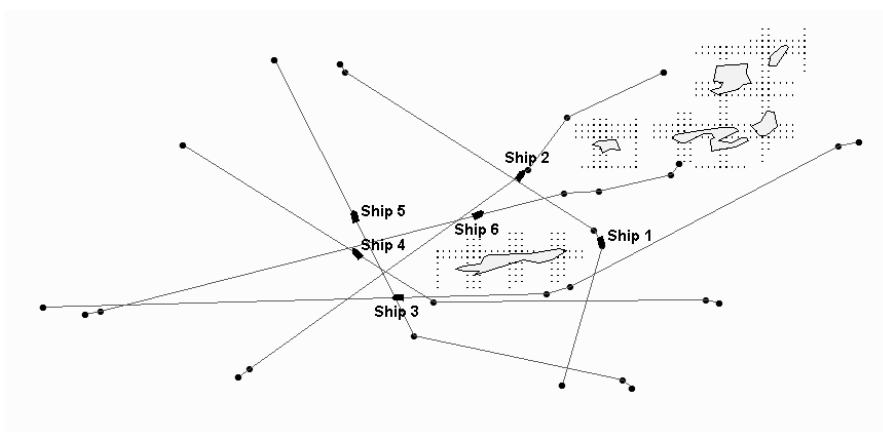


Fig. 11. Restricted water scenario – dotted areas depict shallow waters (positions after 60% of total simulation time)

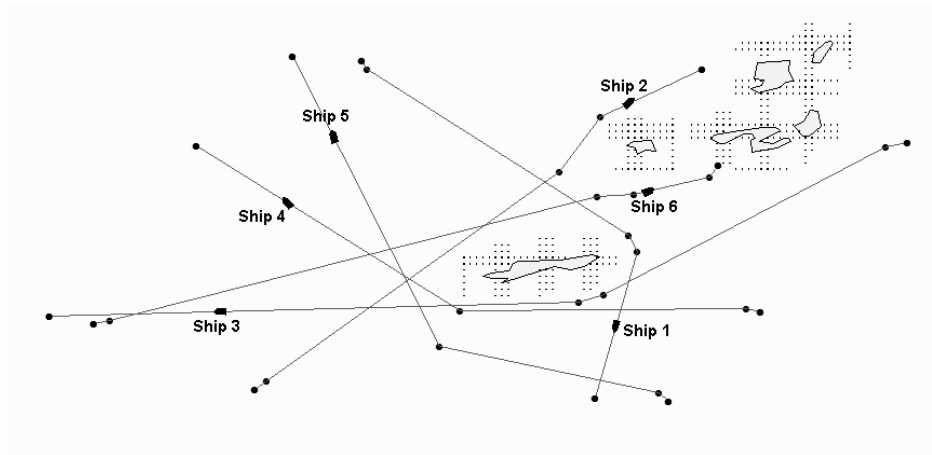


Fig. 12. Restricted water scenario – dotted areas depict shallow waters (positions after 80% of total simulation time)

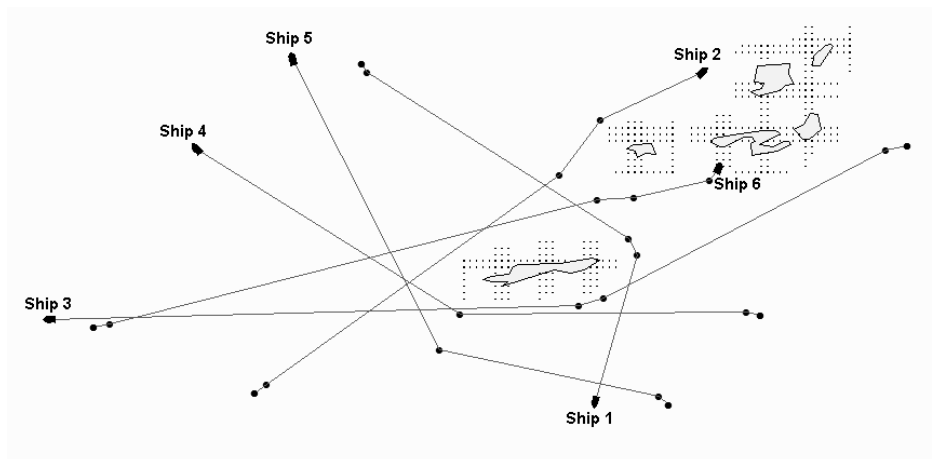


Fig. 13. Restricted water scenario – dotted areas depict shallow waters (destinations reached)

To facilitate analysis of a scenario presented in Figures 8–13 (with ship positions given in Table 2) let's divide the ships as follows:

- ship 3, ship 4 & ship 5, forming group 1, heading westbound,
- ship 2 and ship 6, forming group 2, heading eastbound,
- ship 1 heading southbound.

All group 1 ships must bypass an obstacle and perform this action by port board manoeuvres assuring safe astern crossings. In the group 2 alone there a slight crossing threat and ship 2 & ship 6 are forced to minor course amendments. However, still group 2 ships have impact on ship 1 and ship 5 manoeuvrings. Ship 1

Table 2

Restricted water complex scenario – ship positions &amp; resulting fitness values

|        | Origin position                | Destination position           | V [kn] | Resulting trajectory fitness value [/] | Resulting general fitness value [/] |
|--------|--------------------------------|--------------------------------|--------|--|-------------------------------------|
| Ship 1 | 21° 29' 58" E<br>59° 58' 05" N | 21° 39' 13" E<br>59° 44' 44" N | 13.18  | 0.9137                                 | 0.9565                              |
| Ship 2 | 21° 25' 45" E<br>59° 45' 05" N | 21° 43' 24" E<br>59° 57' 44" N | 14.54  | 0.9909                                 |                                     |
| Ship 3 | 21° 51' 33" E<br>59° 54' 51" N | 21° 17' 38" E<br>59° 47' 58" N | 17.67  | 0.9139                                 |                                     |
| Ship 4 | 21° 45' 43" E<br>59° 48' 07" N | 21° 23' 26" E<br>59° 54' 42" N | 12.43  | 0.9004                                 |                                     |
| Ship 5 | 21° 42' 05" E<br>59° 44' 35" N | 21° 27' 15" E<br>59° 58' 17" N | 14.61  | 0.9374                                 |                                     |
| Ship 6 | 21° 19' 24" E<br>59° 47' 39" N | 21° 44' 04" E<br>59° 53' 56" N | 13.32  | 0.9893                                 |                                     |

is in the worst situation here: she has to bypass a large obstacle (the same as group 1 & 2 but larger north-southbound than west-eastbound), give way to group 2 ships and make sure her manoeuvring won't disturb group 1. Successfully ship 1 makes her so by severe port board course change and astern bypassing trajectories of ship 3, ship 4 and ship 5.

## 6. Summary and Conclusions

In the paper a method of solving encounter situations – Evolutionary Sets of Safe Ship Trajectories – has been presented. The method is a generalization of evolutionary trajectory determining. A set of trajectories of all ships involved, instead of just the own trajectory, is determined. The method avoids violating ship domains and stationary constraints, while obeying the international collision avoidance rules (COLREGS) and minimizing total way loss computed over all trajectories. While developing the method, it turned out, that evolutionary mechanisms had to be adjusted to the particular problem. Some of the adjustments are natural (specialized operators dedicated to particular situations) while others are non-typical or even counter-intuitive. The latter include: strong pre-processing, additional penalties for breaking COLREGS and choosing threshold selection.

The method proves, even in quite complex cases, to return safe trajectories which are compliant with COLREGS. The trajectories have low way loss, another advantage of the method is that the trajectories are relatively simple and do not contain unnecessary manoeuvres.

*The authors thank the Polish Ministry of Science and Higher Education for supporting this research by grant no. N N516 186737.*

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