EFFICIENCY OF ACOUSTIC HEATING IN THE MAXWELL FLUID

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The nonlinear effects of sound in a fluid describing by the Maxwell model of the viscous stress tensor is the subject of investigation. Among other, viscoelastic biological media belong to this non-newtonian type of fluids. Generation of heating of the medium caused by nonlinear transfer of acoustic energy, is discussed in details. The governing equation of acoustic heating is derived by means of the special linear combination of conservation equations of fluid motion in differential form. The method to derive the governing equations does not need averaging over the sound period, and the final governing dynamic equation of the thermal mode is instantaneous. It is valid for both periodic and aperiodic sound. The efficiency of acoustic heating for different shapes of acoustic wave is evaluated.

INTRODUCTION

It is well-known, that the standard attenuation of fluid leads to linear dissipation of sound. The acoustic heating is the increase of ambient temperature, caused by loses in acoustic energy. The acoustic heating is the value related to the entropy or thermal mode. The increase in the ambient temperature should be distinguished from the excess temperature associated with the sound wave. The latter of which is a wave quantity, damped during sound propagation in a fluid with standard attenuation. The role of periodic sound in the origin of acoustic heating in standard thermoviscous fluid flows is well-studied theoretically and experimentally [1, 2]. Interest in acoustic heating has grown over the last few years in connection with biomedical applications. Such applications require accurate estimation of heating during medical therapy, which applies sound of different kinds including impulses.

This study is devoted to nonlinear dissipation of sound energy in a fluid where among standard attenuation, relaxation processes take place. The special mathematic method allows to separate equations governing acoustic, vorticity and entropy modes. The method was used previously by one of the authors to solve some problems of nonlinear flows [3, 4]. Application of this method leads, among other, to the governing equation of acoustic heating. Examples of acoustic heating in three-dimension case are discussed in Sec. (5).

1. DYNAMIC EQUATIONS GOVERNING FLOW OF FLUID

The continuity, momentum and energy equations for a thermoviscous fluid flow without external forces read:

$$\begin{aligned} &\frac{\partial \rho}{\partial t}+\vec{\nabla}(\rho\cdot\vec{v})=0,\\ &\frac{\partial \vec{v}}{\partial t}+(\vec{v}\,\vec{\nabla})\vec{v}=\frac{1}{\rho}\left(-\vec{\nabla}p+Div\,\mathbf{P}\right),\\ &\frac{\partial e}{\partial t}+(\vec{v}\,\vec{\nabla})e=\frac{1}{\rho}\left(-\vec{\nabla}p\,\vec{v}+\chi\Delta T+\mathbf{P}:grad\,\vec{v}\right), \end{aligned} \tag{1}$$

Here, v denotes the velocity of the fluid, ρ, p are density and pressure, e, T mark the energy per unit mass and temperature, correspondingly, χ is the thermal conductivity, and x_i, t spatial coordinates and time. The operators *Div* and *Grad* denote the tensor divergence and dyad gradient respectively. **P** is the tensor of viscous stress. The equation connecting the viscous stress tensor and particle displacements $u_i(r, t)$ in the medium at a given point in space and time, for viscous liquids fits the Maxwell model, in two equivalent forms (τ_R is the relaxation time) [5, 6]:

$$\frac{\partial P_{i,k}}{\partial t}+\frac{1}{\tau_R}P_{i,k}=\mu\frac{\partial}{\partial t}\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right), P_{i,k}=\mu\int\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)\mathrm{e}^{-(t-t')/\tau} \tag{2}$$

Two thermodynamic functions $e(p, \rho)$, $T(p, \rho)$ complete the system (1). Their excess quantities may be written as a series of excess internal energy $e' = e-e_0$ and temperature $T' = T-T_0$ in powers of excess pressure and density $p' = p - p_0$, $\rho' = \rho - \rho_0$ (ambient quantities are marked by the index 0):

$$\begin{aligned} &e'(p',\rho')=\frac{E_1}{\rho_0}p'+\frac{E_2p_0}{\rho_0^{\,2}}\rho'+\frac{E_3}{p_0\rho_0}p'^2+\frac{E_4p_0}{\rho_0^{\,3}}\rho'^2+\frac{E_5}{\rho_0^{\,2}}\rho'\,p',\\ &T'(p',\rho')=\frac{\Theta_1}{\rho_0C_v}p'+\frac{\Theta_2p_0}{\rho_0^{\,2}C_v}\rho'+\frac{\Theta_3}{p_0\rho_0C_v}p'^2+\frac{\Theta_4p_0}{\rho_0^{\,3}C_v}\rho'^2+\frac{\Theta_5}{\rho_0^{\,2}C_v}\rho'\,p'. \end{aligned} \tag{3}$$

where $E_1, \ldots \Theta_5$ are dimensionless coefficients, and C_v marks the heat capacity per unit mass under constant volume. A small variation in entropy is a total differential, that gives the link of the first coefficient in the series of excess temperature (3):

$$\Theta_2=\frac{C_v\rho_0T_0}{E_1p_0}-\frac{(1-E_2)\Theta_1}{E_1} \tag{4}$$

The expressions for coefficients E_1 and E_2 are as follows

$$E_1=\frac{\rho_0C_v\kappa}{\beta},\quad \mathrm{E}_2=\frac{C_p\rho_0}{\beta p_0}, \tag{5}$$

where Cp denotes the heat capacity per unit mass under constant pressure, κ and β are the compressibility and thermal expansion, correspondingly:

$$\kappa=\frac{-1}{V}\left(\frac{\partial V}{\partial p}\right)_T=\frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T, \beta=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p=\frac{-1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p \tag{6}$$

2. DEFINITION OF MODES IN THE THREE-DIMENSIONAL FLOW OF INNITELY-SMALL MAGNITUDE

Based on the linearised version of Eq. (1), dispersion relations can be obtained for three independent modes: the acoustic (two branches), vorticity (two branches) and entropy (thermal) modes. In general case each of the field variables contains a contribution from all modes, for example: $\rho = \rho_a + \rho_v + \rho_e$. This allows, not only to decompose the main vector for each components, but also to separate the governing equations themselves using special properties of modes. Let y designates the nominal axis of the beam pointing in the propagation direction, and let (x, z) be the coordinates perpendicular to this axis. It is convenient to rearrange formulae in the dimensionless quantities as follows:

$$p' = \frac{p - p_0}{c_0^2 \rho_0}, \rho' = \frac{\rho - \rho_0}{\rho_0}, \vec{v} = \frac{\vec{v}}{c_0}, x' = \frac{\varepsilon x}{\lambda}, y' = \frac{e}{\lambda}, z' = \frac{\varepsilon z}{\lambda}, t' = \frac{c_0}{\lambda} t \tag{7}$$

where p_0 is static pressure and ε is the diffraction parameter describing relations between the characteristic longitudinal and transverse scale. Everywhere below in the text, primes by dimensionless quantities are dropped. In the dimensionless quantities, Eqs (1) read:

$$\frac{\partial v_x}{\partial t} + \sqrt{\varepsilon}\frac{\partial p}{\partial x} - \hat{A}\left(\Delta v_x + \sqrt{\varepsilon}\frac{\partial}{\partial x}\left(\vec{\nabla}\vec{v}\right)\right) = -\left(\vec{v}\vec{\nabla}\right)v_x + \sqrt{\varepsilon}\rho\frac{\partial p}{\partial x} - \hat{A}\rho\left(\Delta v_x + \sqrt{\varepsilon}\frac{\partial}{\partial x}\left(\vec{\nabla}\vec{v}\right)\right)$$

$$\frac{\partial v_y}{\partial t} + \frac{\partial p}{\partial y} - \hat{A}\left(\Delta v_y + \frac{\partial}{\partial y}\left(\vec{\nabla}\vec{v}\right)\right) = -\left(\vec{v}\vec{\nabla}\right)v_y + \rho\frac{\partial p}{\partial y} - \hat{A}\rho\left(\Delta v_y + \frac{\partial}{\partial y}\left(\vec{\nabla}\vec{v}\right)\right)$$

$$\frac{\partial v_z}{\partial t} + \sqrt{\varepsilon}\frac{\partial p}{\partial z} - \hat{A}\left(\Delta v_z + \sqrt{\varepsilon}\frac{\partial}{\partial z}\left(\vec{\nabla}\vec{v}\right)\right) = -\left(\vec{v}\vec{\nabla}\right)v_z + \sqrt{\varepsilon}\rho\frac{\partial p}{\partial z} - \hat{A}\rho\left(\Delta v_z + \sqrt{\varepsilon}\frac{\partial}{\partial z}\left(\vec{\nabla}\vec{v}\right)\right)$$

$$\frac{1}{E_1}\left(2\varepsilon\frac{\partial v_x}{\partial x}\hat{A}\frac{\partial v_x}{\partial x} + 2\frac{\partial v_y}{\partial y}\hat{A}\frac{\partial v_y}{\partial y} + 2\varepsilon\frac{\partial v_z}{\partial z}\hat{A}\frac{\partial v_z}{\partial z}\right) +$$

$$\frac{1}{E_1}\left(\varepsilon\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)\hat{A}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) + \left(\frac{\partial v_x}{\partial y} + \sqrt{\varepsilon}\frac{\partial v_y}{\partial x}\right)\hat{A}\left(\frac{\partial v_x}{\partial y} + \sqrt{\varepsilon}\frac{\partial v_y}{\partial x}\right)\right) +$$

$$\frac{1}{E_1}\left(\frac{\partial v_z}{\partial y} + \sqrt{\varepsilon}\frac{\partial v_y}{\partial z}\right)\hat{a}\left(\frac{\partial v_z}{\partial y} + \sqrt{\varepsilon}\frac{\partial v_y}{\partial z}\right) + \delta_3\Delta\left(p^2\right) + \delta_4\Delta\left(\rho^2\right) + \delta_5\Delta(\rho p)$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\vec{v} = -\left(\vec{v}\vec{\nabla}\right)\rho - \rho\vec{\nabla}\vec{v} \tag{8}$$

where $\vec{\nabla} = \left(\sqrt{\varepsilon}\partial/\partial x,\ \partial/\partial y,\ \sqrt{\varepsilon}\partial/\partial z\right)$ and $\Delta = \varepsilon\partial^2/x^2 + \partial^2/y^2 + \varepsilon\partial^2/z^2$, $\hat{A}$ denotes a dimensionless operator acting on a scalar function $\varphi(x, y, z, t)$: $\hat{A}\varphi = m\int_{-\infty}^{t}\varphi e^{-(t-t')/\tau}dt'$, $m = \frac{\mu}{\rho_0 c_0{}^2} = \frac{c_\infty{}^2}{c_0{}^2} - 1$ is dimensionless dispersion (c_∞ at infinitively large frequency is a frozen sound speed).

The dynamic equations in the rearranged form involve dimensionless quantities

$$\delta_1 = \frac{\chi\Theta_1}{\rho_0 c_0 \lambda C_v E_1}, \delta_2 = \frac{\chi\Theta_2}{\rho_0 c_0 \lambda C_v (1-E_2)}, \delta_3 = \frac{\chi\Theta_3}{\rho_0 c_0 \lambda C_v E_1}\frac{1-E_2}{E_1}, \delta_4 = \frac{\chi\Theta_4}{\rho_0 c_0 \lambda C_v (1-E_2)},$$

$$\delta_5 = \frac{\chi\Theta_5}{\rho_0 c_0 \lambda C_v E_1}, D_1 = \frac{1}{E_1}(-1+2\frac{1-E_2}{E_1}E_3 + E_5), D_2 = \frac{1}{1-E_2}(1+E_2+2E_4+\frac{1-E_2}{E_1}E_5). \quad (9)$$

The sum of two first coefficients in series of temperature perturbation is a linear attenuation due to thermal conductivity, as follows from (9):

$$\delta = \delta_1 + \delta_2 \quad (10)$$

The linear hydrodynamic field is represented by two acoustic modes, two vorticity modes and the entropy, or thermal mode. Every type of motion is determined in fact by one of the root of dispersion relation of the linear flow, $\omega(k)$ [1, 2, 6] and fixes links of perturbations, which are independent on time [3, 4]. The dispersion relations for both acoustic modes (marked by index 1 and index 2), and entropy mode (marked by index 3) are as follows:

$$\omega_1 = \sqrt{\Delta} + i\Delta\left(\hat{A} + \frac{\delta}{2}\right),\ \omega_2 = -\sqrt{\Delta} + i\Delta\left(\hat{A} + \frac{\delta}{2}\right),\ \omega_3 = -i\Delta\,\delta_2 \quad (11)$$

where

$$\Delta = -k_x^2 - k_y^2 - k_z^2,\ \sqrt{\Delta} = -i\sqrt{k_x^2 + k_y^2 + k_z^2} \quad (12)$$

In the linear flow ($M \to 0$), the equations for every type of motion may be extracted from the system in accordance to specific links inside every mode. That may be formally proceeded bymeans of projecting the equations into specific sub-spaces [3, 4, 5]. The dynamic equations are obviously independent. The equation for the first acoustic branch is following:

$$\frac{\partial \rho_a}{\partial t} + \sqrt{\Delta}\rho_a - \left(\hat{A} + \frac{\delta}{2}\right)\Delta\rho_a = 0 \quad (13)$$

The density perturbation for entropy motion satisfy the diffusion equation:

$$\frac{\partial \rho_e}{\partial t} + \delta_2 \Delta \rho_e = 0$$

3. EQUATION GOVERNING SOUND

The nonlinear terms in every conservation equation from the right-hand side of system (8) include, in general, inputs of every mode in the weakly nonlinear flow. We fix links determining every mode in the linear flow and consider every excess quantity as a sum of the specific excess quantities of every mode. The consequent decomposing of the governing equations for sound and thermal modes may still be achieved by means of linear projection, for details refer to [5]. In simple terms, projecting is the linear combination of equations in such a way as to keep the terms of the chosen mode in the linear part, and reduce all other terms. Corresponding projectors are linear, application of them at the set (8) yield correct terms of order M^2. Keeping only the terms corresponding to the acoustic rightwards progressive wave, one can easily obtain the equation analogous to the Khokhlov-Zabolotskaya-Kuznetsov one:

$$\frac{\partial \rho_a}{\partial t} + \frac{\partial \rho_a}{\partial y} + \frac{\varepsilon}{2}\int \Delta_\perp \rho_a dy - \left(\hat{A} + \frac{\delta}{2}\right)\frac{\partial^2 \rho_a}{\partial y^2} + \frac{1-D_1-D_2}{2}\rho_a \frac{\partial}{\partial y}\rho_a = 0 \quad (15)$$

In Eq. (15), the series of square root of Laplacian $\Delta = \partial^2/\partial y^2 + \varepsilon\Delta$ is used: $\sqrt{\Delta} \approx \partial/\partial y + 1/2\varepsilon\Delta_\perp \int dy$, that is a common practise in wave theory. The nonlinear term in Eq. (15) may be considered as a result of the self-action of sound, which corrects the dynamic equations by nonlinear terms.

4. INTERACTION BETWEEN MODES. ACOUSTIC HEATING

The important property of projection is not only to decompose the specific perturbations in the linear part of equations, but to distribute nonlinear terms correctly between different dynamic equations. In the context of acoustic heating, the magnitude of excess density specific for the entropy mode is small compared to that of the sound. Multiplying the first, second, third equation form the system (8), by:

$$-\delta\frac{\partial}{\partial x}, -\delta\frac{\partial}{\partial y}, -\delta\frac{\partial}{\partial z}$$

respectively and forth and fifth equation by -1 and 1, respectively and taking a sum of all equations reduces all terms belonging to the acoustic, and vorticity modes in the linear part of the final equation. As for the nonlinear part of the final equation, only acoustic quadratic terms are considered there. That yields the dynamic equation for acoustic heating:

$$\frac{\partial}{\partial t}\rho_{ent} + \delta_2\Delta\rho_{ent} - \frac{\delta}{4}(3 + D_1 + D_2)\frac{\partial^2 \rho_a{}^2}{\partial y^2} + (1 + D_1 + D_2)\left(-\rho_a\left(\vec{\nabla}\vec{v}_a\right) + \frac{\delta_2}{2}\frac{\partial^2 \rho_a{}^2}{\partial y^2}\right) =$$
$$-(1 + D_1 + D_2)\rho_a\left(\vec{\nabla}\vec{v}_a\right) - \frac{2}{E_1}\frac{\partial}{\partial y}\rho_a\hat{A}\frac{\partial}{\partial y}\rho_a + \delta\left(D_1\left(\frac{\partial\rho_a}{\partial y}\right)^2 - \rho_a\frac{\partial^2\rho_a}{\partial y^2}\right) - \tag{16}$$
$$-(\delta_3 + \delta_4 + \delta_5)\left(\Delta\rho_a{}^2\right),$$

which becomes simpler after ordering:

$$\frac{\partial}{\partial t}\rho_{ent} + \delta_2\Delta\rho_{ent} = \frac{-2}{E_1}\frac{\partial}{\partial y}\rho_a\hat{A}\frac{\partial}{\partial y}\rho_a + \left(\left(\frac{\delta}{2} - \delta_2\right)(1 + D_1 + D_2) - 2(\delta_3 + \delta_4 + \delta_5)\right)\rho_a\frac{\partial^2\rho_a}{\partial y^2} +$$
$$\left(\frac{\delta}{2}(3D_1 + D_2 + 3) - \delta_2(1 + D_1 + D_2) - 2(\delta_3 + \delta_4 + \delta_5)\right)\left(\frac{\partial\rho_a}{\partial y}\right)^2 \tag{17}$$

5. NUMERICAL EXAMPLES

The solution of Eq. (17) governing the decrease in the ambient density ρ_{ent}, is a fairly complex problem considering that the excess acoustic density itself should satisfy Eq. (15), which itself is nonlinear and accounts for attenuation due to thermal conduction and dispersion. It should be noted that ρ_{ent} is not an acoustic quantity. The acoustic terms in right hand side of Eq. (17) play the role of nonlinear source of heating. The acoustic source in Eq. (17) is instantaneous and describes dynamic of the thermal mode in any time. Let us consider only terms originating from relaxation, both in the governing equations for sound and entropy excess density (Eqs. (17, 15)). In terms of dimensional temperature T_e, and accounting for Eqs. (3, 17), the governing equation of acoustic heating becomes:

$$\frac{\partial T_e}{\partial t}=\frac{1}{\phi}\frac{\partial \rho_a}{\partial y}\int \frac{\partial \rho_a}{\partial y}e^{-(t-t')/\tau}dt' \tag{18}$$

where

$$\phi=\frac{\rho_0 C_v}{\Theta_2 p_0}=\frac{-\rho_0 C_v E_1}{2\Theta_2 p_0 m} \tag{19}$$

Assuming that

$$\Theta_2=\frac{\rho_0^2 C_v}{p_0}\left(\frac{\partial T}{\partial \rho}\right)_p=-\frac{\rho_0 C_v}{p_0 \beta}. \tag{20}$$

and using (5) we obtain the following relation for ϕ:

$$\phi=\frac{\rho_0 C_v \kappa}{2m} \tag{21}$$

The coefficient ϕ depends exclusively on the molecular properties of a fluid and for example for water equals: $\phi = 4,\ 38 \cdot 10^{-6}\,1/K$ [8, 9]. The excess acoustic density in Eq. (18) should satisfy the Eq. (15), but the solution of Eq. (15) is a fairly complex problem itself. However, it is reasonable as the first approximation to solve the simplified version of linear Eq. (15) $\left(\hat{A}\rightarrow\ 0,\delta\rightarrow\ 0\right)$. Expanding operator $\sqrt{\Delta}$ in series with respect to power of ε and keeping only $O(\varepsilon)$ terms, we obtain

$$\frac{\partial}{\partial y}\left(\frac{\partial \rho_a}{\partial t}+\frac{\partial \rho_a}{\partial y}\right)+\frac{1}{2}\varepsilon\Delta_\perp \rho_a=0 \tag{22}$$

The solution of Eq. (22) with the following initial condition: $\rho_a(t = 0, y) = \sin(y)\sin(ny)$ takes form

$$\begin{aligned}\rho_a=\frac{M}{2}\exp\left(\frac{-(n+1)^2 r^2}{n^2+2n+4t^2\varepsilon^2+1}\right)\left(\frac{(n+1)^2\cos(\alpha)}{n^2+2n+4t^2\varepsilon^2+1}+2t\varepsilon\frac{(n+1)\sin(\alpha)}{n^2+2n+4t^2\varepsilon^2+1}\right)+\\ \frac{M}{2}\exp\left(\frac{-(n-1)^2 r^2}{n^2-2n+4t^2\varepsilon^2+1}\right)\left(\frac{(n-1)^2\cos(\eta)}{n^2-2n+4t^2\varepsilon^2+1}-2t\varepsilon\frac{(n-1)\sin(\eta)}{n^2-2n+4t^2\varepsilon^2+1}\right)\end{aligned} \tag{23}$$

where $r=\sqrt{x^2+z^2}$ and α, η are:

$$\begin{aligned}\alpha&=\frac{(n+1)\left(4\varepsilon^2 t^3-4y\varepsilon^2 t^2+\left(n^2+2n-2r^2\varepsilon+1\right)t-(n+1)^2 y\right)}{n^2+2n+4t^2\varepsilon^2+1}\\ \eta&=\frac{(n-1)\left(4\varepsilon^2 t^3-4y\varepsilon^2 t^2+\left(n^2-2n-2r^2\varepsilon+1\right)t-(n-1)^2 y\right)}{n^2-2n+4t^2\varepsilon^2+1}\end{aligned} \tag{24}$$

The numerical examples based on Eq. (18) with the solution (23) are presented at the Fig. (1). The variable ε is equal: $\varepsilon = \lambda / R$ (λ, R are length of sound wave and radius of the transmitter, respectively) and was chosen according to the experimental data: $n = 0.01$. The value n used in the calculations is equal 0.1, $\tau = 10^{-5}$.

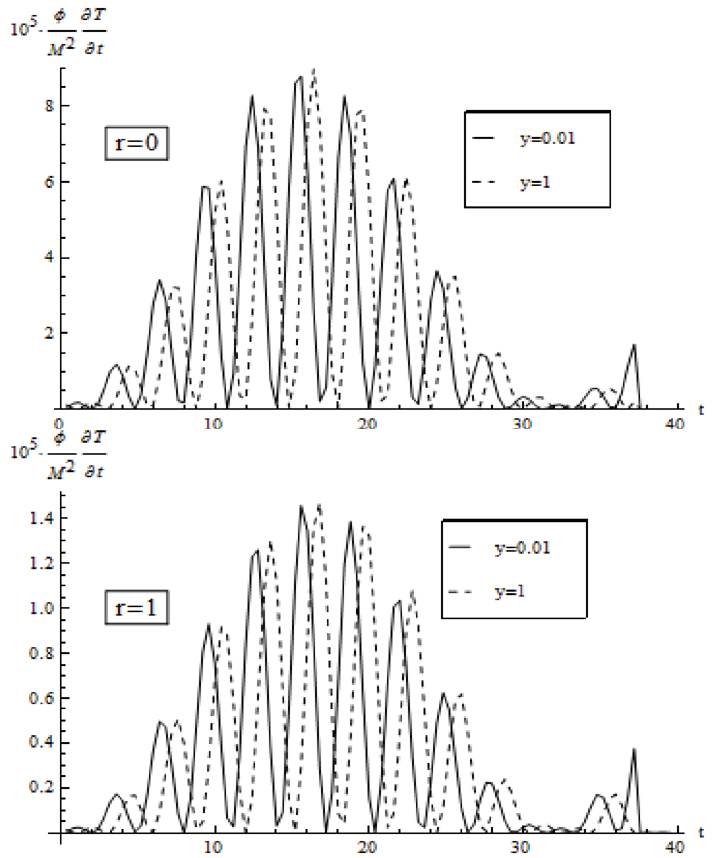

Fig. 1. Dimensionless variations in partial derivative of temperature with respect to time calculated for two values of r: $r = 0$ and $r = 1$, respectively

6. VARIATIONS IN TEMPERATURE IN UID WITH STANDARD ATTENUATION. COMPARISON WITH THE UID WITH THE MAXWELL STRESS TENSOR

The equation of acoustic heating for the fluid with the standard attenuation is as follows:

$$\frac{\partial T}{\partial t} = \frac{1}{\Gamma}\left(\frac{\partial \rho_a}{\partial y}\right)^2 \tag{25}$$

where

$$\Gamma = \frac{\rho_0 C_v \kappa}{\dfrac{4/3\mu + \xi}{\rho_0 c_0 \lambda} + \delta_1 + \delta_2} \tag{26}$$

μ, ξ are bulk and shear viscosity, respectively. The solution of Eq. (22) with the following initial condition: $\rho_a(t=0,y) = \sin(y)$, takes form

$$\rho_a = -M\frac{e^{\frac{-r^2}{4t^2\varepsilon^2+1}}\left(2t\varepsilon\cos\left(\dfrac{4t^3\varepsilon^2 - 4y\varepsilon^2 t^2 - 2r^2\varepsilon t + t - y}{4t^2\varepsilon^2+1}\right) + \sin\left(\dfrac{4t^3\varepsilon^2 - 4y\varepsilon^2 t^2 - 2r^2\varepsilon t + t - y}{4t^2\varepsilon^2+1}\right)\right)}{4t^2\varepsilon^2+1} \tag{27}$$

The numerical examples based on Eq. (18) and Eq. (26) with the solution (28) are presented at the Fig. (2).

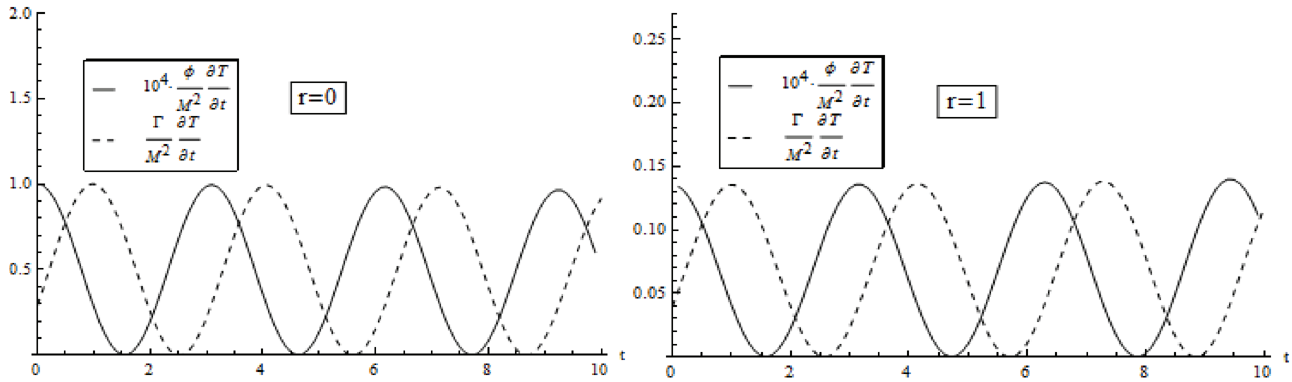

Fig. 2. Dimensionless variations in partial derivative of temperature with respect to time for Maxwell fluid (based on Eq. (18)), marked by solid line and for the fluid with standard attenuation (based on Eq. (26)), marked by dashed line, calculated for values: $y = 0.01$ and for two values of r: $r = 0$ and $r = 1$

7. CONCLUSIONS

The equation governing acoustic heating, Eq. (17) is a result of consequent decomposing of weakly nonlinear equations for sound and non-acoustic motions. It is the main result of this study. The method applies in a wide variety of flows with different mechanisms of dissipation and dispersion, it leads to instantaneous equations and does not need temporal averaging of the conservative equations with respect to sound period.

Illustrations (Fig. 1, 2) reveal, that increase in temperature, associating with the entropy motion, $\int \partial T/\partial t dt$, is always positive. It is simply a square under a curve. The method provides us to equations, dependent on coefficients of thermodynamic state and allows to compare acoustic heating caused by sound in the Maxwell fluid and in the fluid with standard attenuation.

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