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Acoustic Heating Produced in the Boundary Layer

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Instantaneous acoustic heating of a viscous fluid flow in a boundary layer is the subject of investigation. The governing equation of acoustic heating is derived by means of a special linear combination of conservation equations in the differential form, which reduces all acoustic terms in the linear part of the final equation but preserves terms belonging to the thermal mode. The procedure of decomposition is valid in a weakly nonlinear flow, it yields the nonlinear terms responsible for the modes interaction. Nonlinear acoustic terms form a source of acoustic heating in the case of the dominative sound. This acoustic source reflects the thermoviscous and dispersive properties of a fluid flow. The method of deriving the governing equations does not need averaging over the sound period, and the final governing dynamic equation of the thermal mode is instantaneous. Some examples of acoustic heating are illustrated and discussed, and conclusions about efficiency of heating caused by different waveforms of sound are made.

Keywords: acoustic heating, weak dispersion, boundary layer.

1. Introduction

This study focuses on the nonlinear dissipation of the sound energy into the non-wave energy in the boundary layer of the fluid flow. Propagation of the finite amplitude sound is not longer described by the Burgers equation, as it is in the standard thermovisous flows. Fluid flows in a boundary viscous layer reveal a weak dispersion. As an example of the weak dispersion mechanisms, thermodynamic relaxation towards the equilibrium state may be mentioned. It takes place in many biological liquids described by the Maxwell or even more complex dependence of the shear stress on the shear rate. Dispersion always follows attenuation (Alekseev, Rybak, 2002). Temkin (1990) indicated that dispersion and attenuation are connected by the Kramers-Kronig relations. Propagation of the sound in horns at the frequencies near cutoff, in waveguides, and in bubbly liquids represent examples of a strong dispersion (Hamilton *et al.*, 1998).

It is well-known that the standard attenuation of fluids leads to a linear dissipation of the sound. The acoustic heating is an increase of the ambient fluid temperature caused by a *nonlinear* loss in the acoustic energy. This isobaric increase in the temperature is not an acoustic quantity but a value referred to as the entropy, or (the alternative name) the thermal mode. The variations in the ambient temperature should be distinguished from an excess temperature associated with the sound wave, the latter of which is a wave quantity. The periodic sound as the origin of acoustic heating in the standard thermoviscous fluid flows was studied by Rudenko and Soluyan (1977), Makarov and Ochmann (1996), and Rudenko (2007). Interest in acoustic heating has grown over the last few years in connection with biomedical and technical applications. The majority of liquids being of interest are non-newtonian. That imposes unusual dependence of the shear stress on the shear rate, including rheological and elastic behavior (which is specific for solids) (Alekseev, Rybak, 2002; Collyer, 1974; Mewis, 1979). Such applications, among others, require an accurate estimation of heating during the medical therapy involving scanning acoustic microscopy mentioned by Gudra (2008) which uses sound of different kinds, including impulses (Rudenko, 2007; Hartman *et al.*, 1992; Wójcik *et al.*, 2008).

Flows of even newtonian liquids in the viscous boundary layer reveal dispersive properties (Hamilton *et al.*, 1998; Blackstock, 1985; Makarov, 1994). The reason for that is a non-newtonian dependence of components of the shear stress (i.e., the surface forces which appear in the vicinity of the rigid boundary) on the shear rate. This dependence is no longer local, and it is expressed by means of the coefficient which is proportional not only to the total attenuation of a flow in unbounded volumes, but also to the inverse hydraulic diameter of the duct and its cross-sectional area. The monochromatic sound of frequency ω attenuates proportionally to $\sqrt{\omega}$ in contrast with standard newtonian flows with proportion ω^2. There appears a dispersion proportional to $\sqrt{\omega}$ as well. The dispersion and attenuation, as the linear phenomena which influence sound propagation in a boundary layer, were described by Hamilton *et al.* (1998) and Coppens (1971); they are well-understood. The nonlinear dispersion and attenuation are the reasons for an unusual generation of the entropy mode due to boundary-layer effects in ducts. Propagation of sound in biological tissues is also followed by the boundary-layer absorption and dispersion. The general mathematical technique to describe the interaction of wave and non-wave motions has been worked out and applied previously by one of the authors in some problems of a weakly nonlinear flow. It allows to distinguish equations governing the sound, vorticity, and entropy modes, and to account for their interaction (Perelomova, 2003; 2006; 2008). The method and results based on its application are described in Secs. 3, 4. Some illustrations and conclusions concerning the acoustic heating caused by some types of sound are discussed in Sec. 5.

2. Dynamic equations in a fluid with dispersive properties

The continuity, momentum, and energy conservation equations describing a viscous fluid flow without external mass forces read (Landau, Lifshitz, 1987):

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot(\rho\mathbf{v}) = 0,$$
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v} = \frac{1}{\rho}\left(-\boldsymbol{\nabla}p + \mathrm{Div}\,\mathbf{P}\right), \tag{1}$$
$$\frac{\partial e}{\partial t} + (\mathbf{v}\cdot\boldsymbol{\nabla})e = \frac{1}{\rho}\left(-p(\boldsymbol{\nabla}\cdot\mathbf{v}) + \mathbf{P} : \mathrm{Grad}\,\mathbf{v}\right).$$

Here, $\mathbf{v}$ denotes velocity of the fluid, ρ, p are density and pressure, e marks the internal energy per unit mass, and x_i, t are spatial coordinates and time. The operators Div and Grad denote the tensor divergence and dyad gradient respectively. $\mathbf{P}$ is the tensor of the viscous stress. To complement the system (1), the viscous stress tensor should be expressed in terms of shear rate. The thermodynamic function $e(p,\rho)$ is also required. An excess internal energy $e' = e - e_0$ may be represented as a series in powers of excess pressure and density $p' = p - p_0$, $\rho' = \rho - \rho_0$ (ambient quantities are marked by the index 0):

$$e' = \frac{E_1}{\rho_0}p' + \frac{E_2 p_0}{\rho_0^2}\rho' + \frac{E_3}{p_0\rho_0}p'^2 + \frac{E_4 p_0}{\rho_0^3}\rho'^2 + \frac{E_5}{\rho_0^2}\rho' p', \tag{2}$$

where $E_1,\ldots,E_5$ are dimensionless coefficients. The series (2) allows consideration of a wide variety of fluids in the general form. A variation in the thermodynamic properties of fluids is manifested namely by the coefficients different for different fluids, and various dependence of the shear stress on the shear rate. The expressions for the coefficients E_1 and E_2 are as follows:

$$E_1 = \frac{\rho_0 C_V \kappa}{\beta}, \qquad E_2 = -\frac{C_p\rho_0}{\beta p_0} + 1, \tag{3}$$

where C_V, C_p marks the heat capacity per unit mass under constant volume and under a constant pressure, and κ, β are the compressibility and thermal expansion, correspondingly:

$$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T,$$
$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p. \tag{4}$$

A common practice in nonlinear acoustics is to focus on the equations of the second order of acoustic Mach number $M = v_0/c_0$, where v_0 is the magnitude of a particles' velocity, and

$$c_0 = \sqrt{\frac{(1-E_2)p_0}{E_1\rho_0}}$$

is the propagation speed of infinitely small signals without an account for the viscosity and dispersion. The present study is further constrained by considering nonlinearities of the second order, so that we shall consider weakly nonlinear flows discarding $O(M^3)$ terms in all the expansions. The resulting model will account for the combined effects of the nonlinearity, weak attenuation, and dispersion.

3. Definition of modes in the planar flow of an infinitely small amplitude

We will consider a one-dimensional flow along the axis Ox. In one dimension, there is only one compound of the viscous stress tensor,

$$P_{xx} = \frac{4}{3}\mu\frac{\partial v}{\partial x} + \widehat{A}v, \tag{5}$$

where μ is the shear viscosity, and $\widehat{A}$ is an operator describing dispersive properties of a flow. At this point, we do not specify its form. The only requirement is that the attenuation and dispersion are small, for the sound not to distort considerably at distances compared to its

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wavelength. It is convenient to rearrange the formulae into the corresponding dimensionless quantities in the following way:

$$p^* = \frac{p'}{c_0^2 \cdot \rho_0}, \qquad \rho^* = \frac{\rho'}{\rho_0}, \qquad v^* = \frac{v}{c_0},$$
$$x^* = \frac{\omega x}{c_0}, \qquad \widehat{A}^* = \frac{\widehat{A}}{\rho_0 c_0}, \qquad t^* = \omega t, \tag{6}$$
$$\delta = \frac{4\mu\omega}{3c_0^2\rho_0},$$

where ω is the characteristic frequency of sound (or inverse duration in the case of pulses). All formulae that follow, including the relationships of modes and dynamic equations, will be written in the leading order. We suppose that δ is of order M, as well as the result of applying $\widehat{A}^*$. Starting from Eq. (7), the upper indexes (asterisks) denoting dimensionless quantities will be omitted throughout the text. In the dimensionless quantities, accounting for Eqs. (2), (5), (6), Eqs. (1) take the form:

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + \frac{\partial v}{\partial x} = -v\frac{\partial \rho}{\partial x} - \rho\frac{\partial v}{\partial x},\\
&\frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} - \delta\frac{\partial^2 v}{\partial x^2} - \widehat{A}\frac{\partial v}{\partial x}\\
&\qquad = -v\frac{\partial v}{\partial x} + \rho\frac{\partial p}{\partial x} - \rho\delta\frac{\partial^2 v}{\partial x^2} - \rho\widehat{A}\frac{\partial v}{\partial x},\\
&\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} = -v\frac{\partial p}{\partial x} + (D_1 p + D_2\rho)\frac{\partial v}{\partial x}\\
&\qquad + \frac{\delta}{E_1}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{E_1}\frac{\partial v}{\partial x}\widehat{A}v.
\end{aligned} \tag{7}$$

The quadratic nonlinear terms form the right-hand side of the set (7). The dynamic equations in the rearranged form include the following dimensionless quantities:

$$\begin{aligned}
D_1 &= \frac{1}{E_1}\left(-1 + 2\frac{1-E_2}{E_1}E_3 + E_5\right),\\
D_2 &= \frac{1}{1-E_2}\left(1 + E_2 + 2E_4 + \frac{1-E_2}{E_1}E_5\right).
\end{aligned} \tag{8}$$

The linearized version of Eqs. (7) describes a flow of an infinitely small amplitude, when $M \to 0$:

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + \frac{\partial v}{\partial x} = 0,\\
&\frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} - \delta\frac{\partial^2 v}{\partial x^2} - \widehat{A}\frac{\partial v}{\partial x} = 0,\\
&\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} = 0.
\end{aligned} \tag{9}$$

The linear hydrodynamic field is represented by acoustic modes, propagating in the positive and negative directions of the axis Ox and the entropy mode. Every type of motion is determined by one of the roots of the dispersion relation of the linear flow, $\omega(k)$ (k is the wave number) (Rudenko, Soluyan, 1977; Makarov, Ochmann, 1996; Chu, Kovasznay, 1958) and fixes relationships of perturbations, which are independent of time (Perelomova, 2003; 2006; 2008). The dispersion relations for acoustic modes propagating in the positive direction of the axis Ox (marked by index 1), the negative direction of the axis Ox (marked by index 2), and the entropy modes (marked by index 3), determine relations of an excess pressure and perturbations in density and velocity for every mode. They are independent of time. In the leading order, the relations take the form:

$$\begin{aligned}
\psi_{a,1} &= \begin{pmatrix} \rho_{a,1} \\ v_{a,1} \\ p_{a,1} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 - \frac{1}{2}\left(\delta\frac{\partial}{\partial x} + \widehat{A}\right) \\ 1 \end{pmatrix} \rho_{a,1},\\
\psi_{a,2} &= \begin{pmatrix} 1 \\ -1 - \frac{1}{2}\left(\delta\frac{\partial}{\partial x} + \widehat{A}\right) \\ 1 \end{pmatrix} \rho_{a,2},\\
\psi_e &= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rho_e.
\end{aligned} \tag{10}$$

The relations (10) may be established demanding the equivalence of the dynamic equations for every perturbation (ρ, v and p) specifying every mode. Equations for any perturbation in the frames of every mode should be equivalent in the leading order, i.e. involving terms standing by powers of small viscous and dispersive parameters not higher than the first one. Every equation includes the first-order derivative with respect to time. The linear dynamic equations do not obviously indicate interaction between modes. The equation describing the acoustic excess density in a wave propagating in the positive direction of the axis Ox, is:

$$\frac{\partial \rho_{a,1}}{\partial t} + \frac{\partial \rho_{a,1}}{\partial x} - \frac{\delta}{2}\frac{\partial^2 \rho_{a,1}}{\partial x^2} - \frac{1}{2}\widehat{A}\frac{\partial \rho_{a,1}}{\partial x} = 0. \tag{11}$$

The density perturbation for entropy motion satisfies the equation describing the stationary field:

$$\frac{\partial \rho_e}{\partial t} = 0. \tag{12}$$

The linear dynamic equations for every type of motion may be extracted from the system (9) by means of projecting of the equations into specific sub-spaces (Perelomova, 2003; 2006; 2008). In general, every perturbation of the field variables contains contributions from each of the three modes, for example, $\rho = \rho_{a,1} + \rho_{a,2} + \rho_e$.

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4. Dynamic equations in a weakly nonlinear dispersive flow

4.1. Weakly nonlinear dynamic equation governing the sound

The nonlinear terms in every conservation equation from the right-hand side of the system (7) include, in general, a contribution of every mode. In the studies of a weakly nonlinear flow, we fix linear links determining every mode and consider every excess quantity as a sum of the specific excess quantities of every mode. The consequent decomposing of the governing equations for the sound and thermal modes may still be achieved by means of the linear projection. In simple terms, projecting is a linear combination of equations in a way that allows to keep the terms of the chosen mode in the linear part and reduce all other terms. One can readily derive the equation governing the sound multiplying the second equation from (7) by 1/2, applying $1/2 + \widehat{A}/4$ at the third equation and taking their sum. That reduces all linear terms belonging to the second acoustic and the entropy modes. Keeping only the terms corresponding to the acoustic rightwards progressive wave in the nonlinear part, and expressing all acoustic quantities in terms of excess acoustic density by use of links ($\psi_{a,1}$ from Eqs. (10)), one can readily obtain an equation supplementing the well-know Burgers' equation (accounting for the standard attenuation exclusively) by the terms responsible for dispersion (Blackstock, 1985):

$$\frac{\partial\rho_{a,1}}{\partial x}+\frac{\partial\rho_{a,1}}{\partial t}-\frac{\delta}{2}\frac{\partial^2\rho_{a,1}}{\partial x^2}-\frac{1}{2}\widehat{A}\frac{\partial\rho_{a,1}}{\partial x} =-\frac{1-D_1-D_2}{4}\frac{\partial\rho_{a,1}^2}{\partial x}. \tag{13}$$

The nonlinear term in the right-hand side of Eq. (13) may be considered as a result of the self-action of the sound, which corrects the dynamic equation (11) by nonlinear terms.

4.2. Interaction of the thermal mode with the dominant sound. Acoustic heating

An important property of projection is not only to decompose the specific perturbations in the linear part of equations but to distribute nonlinear terms correctly between different dynamic equations. In the context of acoustic heating, the magnitude of excess density specific to the entropy mode is small compared to that of sound. It may be easily verified that the modes with relationships (10) satisfy, in the leading order (up to the terms proportional to the squared parameters responsible for dispersion and attenuation), the equality below:

$$\begin{pmatrix}1 & 0 & -1\end{pmatrix}\begin{pmatrix}\rho_{a,1}+\rho_{a,2}+\rho_e\\ v_{a,1}+v_{a,2}+v_e\\ p_{a,1}+p_{a,2}+p_e\end{pmatrix}=\rho_e, \tag{14}$$

which suggests a way of combining of the set of Eqs. (7). The relationships which determine the sound should be completed by nonlinear quadratic terms making it isentropic in the leading order. These corrections are similar to those specific to the Riemann wave in the ideal gas (Riemann, 1953). In the case of the wave progressive in the positive direction of the axis OX, they are

$$\begin{aligned}v_{a,1}&=\rho_{a,1}-\frac{1}{2}\widehat{A}\rho_{a,1}-\frac{\delta}{2}\frac{\partial\rho_{a,1}}{\partial x}\\&\quad-\frac{1}{4}(3+D_1+D_2)\rho_{a,1}^2,\\ p_{a,1}&=\rho_{a,1}-\frac{1}{2}(1+D_1+D_2)\rho_{a,1}^2.\end{aligned} \tag{15}$$

The nonlinear corrections of the second and higher order terms depend on the equation of state, and in the case of an ideal gas they coincide with the well-known links, originally derived by Riemann (1953), in the Riemann wave with $D_1=-\gamma$, $D_2=0$, $\widehat{A}=0$, and $\delta=0$.

For simplicity, let sound be associated exclusively with a wave propagating in the positive direction of the axis Ox: $p_a=p_{a,1}$, $\rho_a=\rho_{a,1}$, $v_a=v_{a,1}$. The linear combination of the left-hand and the right-hand sides of the Eqs. (7) in accordance to (14) results in:

$$\frac{\partial}{\partial t}(\rho-p)=\frac{\partial}{\partial t}\rho_e=-\frac{1}{E_1}\frac{\partial\rho_a}{\partial x}\widehat{A}\rho_a-\frac{\delta}{E_1}\left(\frac{\partial\rho_a}{\partial x}\right)^2. \tag{16}$$

Only acoustic quadratic terms are kept in the right-hand side of Eq. (16). The acoustic terms of the leftwards propagating sound are also completely reduced in the linear part of the final equation. It is convenient to rearrange Eq. (13) in the dimensionless variables, coordinate x, and the retarded time τ, $\tau=t-x$, assuming its weak dependence on x (Mx) (Rudenko, Soluyan, 1977). It takes the form:

$$\frac{\partial\rho_{a,1}}{\partial x}-\frac{\delta}{2}\frac{\partial^2\rho_{a,1}}{\partial\tau^2}+\frac{1}{2}\widehat{A}\frac{\partial\rho_{a,1}}{\partial\tau}-\frac{1-D_1-D_2}{4}\frac{\partial\rho_{a,1}^2}{\partial\tau}=0. \tag{17}$$

Right up to this point, we did not make any assumptions about a specific form of the operator $\widehat{A}$. Provided that the boundary layer is thin in comparison with the transverse dimension of the duct, the dimensional dispersion operator, which applies on any scalar function φ, is as follows (Blackstock, 1985):

$$\widehat{A}\varphi(x,\tau)=\frac{C}{4B}\sqrt{\frac{4\mu}{\pi\rho_0\omega}}\int\limits_0^\infty\frac{\varphi(x,\tau-\tau')}{\sqrt{\tau'}}\,\mathrm{d}\tau', \tag{18}$$

where C is the perimeter of the duct and B is its cross-sectional area.

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This section is restricted to the acoustic field represented by the rightwards propagating sound, although it may be easily expanded to include leftwards propagating waves or any superposition of two acoustic branches.

5. Numerical examples

The solution of Eq. (16) governing the decrease in the ambient density ρ_e, is a fairly complex problem considering that the excess acoustic density should satisfy Eq. (17), which itself is nonlinear and accounts for attenuation (standard and due to the operator $\widehat{A}$ and dispersion). It should be underlined that ρ_e is not an acoustic quantity. The equation governing its dynamics, Eq. (16), includes nonlinear acoustic terms proportional to coefficients responsible for dissipation and dispersion in the right-hand side. They play the role of a nonlinear source of heating and reflect the fact that the origins of the phenomenon are nonlinearity, viscosity, and dispersion. Equation (16) is instantaneous, it describes the dynamics of the entropy mode at any time, and does not require periodicity of the sound.

Let us consider only the terms originating from dispersion related to the viscous boundary layer along the rigid walls bounding flow. As for the governing equations for the sound, Eq. (17), there is no general analytical solution of it, even without the free-stream losses (Hamilton *et al.*, 1998). The distortions of the initially sinusoidal waveform were computed numerically (Coppens, 1971). They agree with the general conclusions that the wall dispersion produces phase speeds that are smaller than the free-space phase speed 1. The phase speed increases with the frequency, and the classical thermoviscous attenuation coefficient is recovered at high frequencies. Boundary-layer dispersion causes rounding of the positive portions and cusping of the negative portions of the waveforms. In terms of the dimensional temperature $\widetilde{T}_e$, accounting for Eq. (16), the governing equation of acoustic heating takes the form:

$$\left(\frac{\partial \widetilde{T}_e}{\partial t}\right)_{\text{disp}} \equiv Q_{\text{disp}} = -\frac{1}{\beta}\frac{\partial}{\partial t}\rho_e = D\frac{\partial \rho_a}{\partial x}\int_0^\infty \rho_a(x,\tau-\tau')\frac{\mathrm{d}\tau'}{\sqrt{\tau'}}, \qquad (19)$$

where $D = \dfrac{C}{4BE_1\beta}\sqrt{\dfrac{4\mu}{\pi\rho_0\omega}}$. Equation (19), along with Eq. (16), is the main result of this study. The standard attenuation would give the increase in the temperature as follows:

$$\left(\frac{\partial \widetilde{T}_e}{\partial t}\right)_{\text{stand}} \equiv Q_{\text{stand}} = S\left(\frac{\partial \rho_a}{\partial x}\right)^2, \qquad (20)$$

where $S = \dfrac{\delta}{E_1\beta}$.

5.1. Acoustic heating caused by the periodic sound

As an approximate excess acoustic density, satisfying the linear wave equation without an account for attenuation and dispersion, let us consider the periodic waveform:

$$\rho_a(\tau) = M\sin\tau. \qquad (21)$$

Equations (19), (20) yield:

$$\begin{aligned} Q_{\text{disp}} &= DM^2\sqrt{\frac{\pi}{2}}\cos(\tau)\left(\cos(\tau)-\sin(\tau)\right),\\ Q_{\text{stand}} &= SM^2\cos(\tau)^2, \end{aligned} \qquad (22)$$

and in the quantities averaged over the sound period

$$\langle Q_{\text{disp}}\rangle = \frac{DM^2}{2}\sqrt{\frac{\pi}{2}}, \qquad \langle Q_{\text{stand}}\rangle = \frac{SM^2}{2}.$$

The relative efficiency of heating depends on the ratio of D and S, that is, on the geometry of a duct, the characteristic frequency of the sound and the thermodynamic and viscous properties of a fluid.

5.2. Acoustic heating caused by pulses

Let us consider an excess dimensionless acoustic density in the form of traveling pulses:

$$\begin{aligned} &\mathbf{A}. \quad \rho_a(\tau) = M\exp\left(-\tau^2\right),\\ &\mathbf{B}. \quad \rho_a(\tau) = -2M\tau\exp\left(-\tau^2\right). \end{aligned} \qquad (23)$$

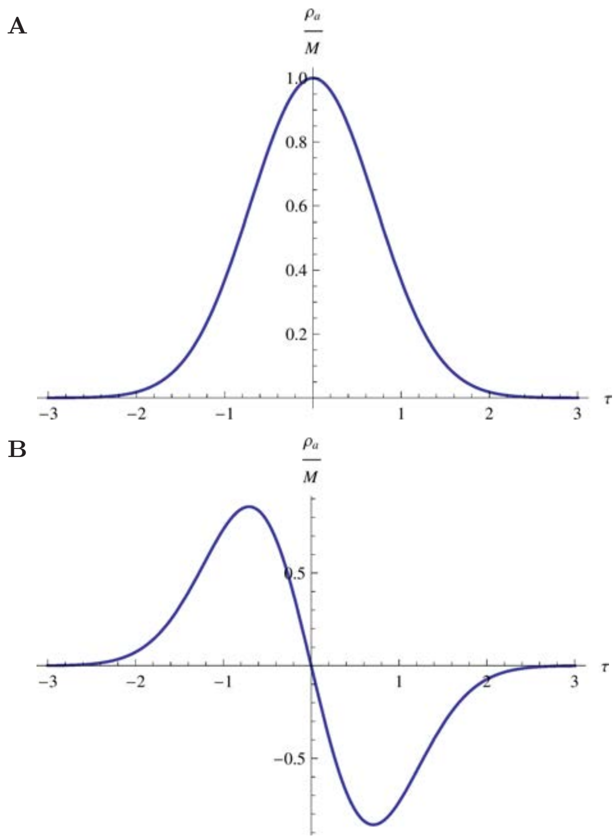

Fig. 1. Acoustic pulses (23) **A**, **B**.

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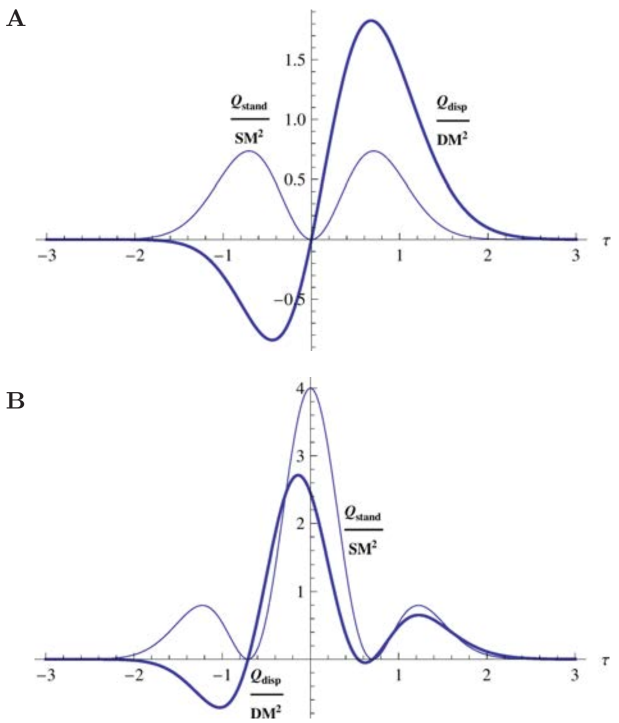

Fig. 2. Increase in the temperature in time unit caused by pulses (23) **A**, **B**. The part associated with the dispersion is plotted in the bold line, and the part relating to the standard attenuation is plotted in the normal line.

For these pulses, Q_{disp} is expressed in terms of special functions.

The increase in the background temperature after passing a pulse, is determined by the integral $\int_{-\infty}^{\infty} Q \, \mathrm{d}\tau$. Numerical evaluations result in the ratio of temperatures caused by dispersion and the standard attenuation, $\int_{-\infty}^{\infty} Q_{\text{disp}} \, \mathrm{d}\tau \Big/ \int_{-\infty}^{\infty} Q_{\text{stand}} \, \mathrm{d}\tau$, which equals $1.03D/S$ (**A**) and $0.52D/S$ (**B**). The "dispersive" part of heating in the case of asymmetric pulse is considerably smaller.

6. Conclusions

The equation governing acoustic heating, Eq. (16), is the result of a decomposition of the weakly nonlinear equations governing the acoustic and non-acoustic motions of a fluid. The method may be applied to a wide variety of flows with different mechanisms of dissipation and dispersion, which are described by an operator $\widehat{A}$. The Maxwell model of liquid is described by a term proportional to $\int_{-\infty}^{t} \frac{\partial v}{\partial x} \exp(-(t-t')/t_R) \, \mathrm{d}t'$, where t_R is the characteristic relaxation time to the thermodynamically equilibrium state (Alekseev, Rybak, 2002). Some evaluations of a nonlinear generation of the entropy mode caused by this type of relaxation may be found in (Perelomova, 2008; Perelomova, Pelc–Garska, 2010). For both tube wall and relaxation dispersions, the phase speed increases with frequency, although tube wall dispersion produces phase speeds that are smaller than the free-space phase speed. The curves of the energy released in both cases distort in a different way as compared to the standard attenuation. Some other linear relaxation mechanisms have been mentioned in the review by Makarov and Ochmann (1997), among them, the frequency independent thermodynamic relaxation, specific for a high-frequency sound. They may be easily considered, accounting that generally $\widehat{A}$ is a sum of all operators responsible for dispersion.

The applied method results in the instantaneous equations and does not need temporal averaging of the conservative equations with respect to the period of sound. This distinguishes it from the traditional decomposition of equations for acoustic and non-acoustic motions which is based on averaging of conservation equations over the sound period (Rudenko, Soluyan, 1977; Makarov, Ochamnn, 1996). The main result of this study, besides Eq. (16), is Eq. (19), describing the excess temperature of the entropy mode of a flow in a duct with rigid walls. The numerical examples reveal that the heating caused by the standard attenuation is much less effective with respect to the "dispersive" one if caused by a bipolar pulse, as compared with a mono-polar. As for the periodic sound, the efficiency takes intermediate place between monopolar and bipolar pulses and equals $0.89D/S$. Some general peculiarities may be concluded a priori. The acoustic heating grows with an increase of the acoustic Mach number M and dispersive parameter D. The efficiency of "dispersive" heating increases in the domain of a small characteristic inverse duration of an impulse ω. For detail evaluations, one requires knowledge about thermodynamic properties of a liquid. In an ideal gas, $\beta E_1 = 1/(T_0(\gamma-1))$, where γ is the ratio of specific heats and T_0 denotes unperturbed temperature of a gas. For the liquid water in normal conditions, $\beta E_1 = 2 \cdot 10^{-9}$ K^{-1}. The dimensionless standard attenuation $\delta = 3.75 \cdot 10^{-15}\omega$, where ω is measured in s^{-1}. The acoustic Mach number M belongs typically to the domain between 10^{-4} and 10^{-2}.

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