

## DYNWIR S-70 PROGRAM FOR MODAL ANALYSIS OF MULTISUPPORTED AND MULTIMASS ROTORS

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### Summary

The article provides a brief description of the program Dynwir-S-70 (currently being developed in the Rotor Dynamics and Slide Bearings Department in the Institute of Fluid-Flow Machinery Polish Academy of Sciences in Gdansk) which is a program for analysis of multi-supported and multi-mass rotors. Used algorithms and the construction of mass and stiffness matrix were presented. Damping matrix structure based on Rayleigh damping was presented. The article also shows the influence of alpha and beta factors on form and frequency of proper vibrations of rotors. Results of the author's program were compared with the commercial program Madyn 2000.

Keywords: modal analysis, rotor dynamics.

### DYNWIR S-70 PROGRAM DO ANALIZY MODALNEJ WIELOPODPOROWYCH I WIELOMASOWYCH WIRNIKÓW

#### Streszczenie

W artykule zamieszczono krótki opis programu Dynwir-S-70 (rozwijanego w Zakładzie Dynamiki Wirników i Łożysk Ślizgowych IMP PAN w Gdańsku) który jest programem do analizy wielopodporowych i wielomasowych wirników. Przedstawiono stosowane algorytmy oraz budowę macierzy mas i sztywności. Przedstawiono budowę macierzy tłumienia w oparciu o tłumienie Rayleigha. W artykule pokazano również wpływ współczynników alfa i beta na częstotliwości i postaci drgań własnych wirników. Wyniki programu porównano z wynikami komercyjnego programu Madyn 2000.

Słowa kluczowe: analiza modalna, dynamika wirników.

## 1. INTRODUCTION

There are many programs for modal analysis, why is being developer another one? The answer to this question is simple. Modal analysis programs usually do not allow the analysis of power machinery rotors supported on sliding bearings – and such is the purpose of the development program Dynwir-S-70. This article applies to Dynwir-S-70 developed in the Institute of Fluid-Flow Machinery Polish Academy of Sciences in Gdansk in the Department of Rotors Dynamics and Slide Bearings. This program is part of the system MESWIR. Dynwir-S-70 is used for the analysis of free bending, torsional and longitudinal free vibrations of multi-mass and multi-supported rotors [1]. The program calculates all the values of the vibrations and the corresponding values of damping coefficients.

There are commercial programs used for modal analysis of rotating machinery - such as program Madyn 2000. Work is underway to upgrade Dynwir-S-70 for nonlinear elements in the modal analysis. Madyn 2000 was used to verify the Dynwir-S-70 in some cases.

## 2. DESCRIPTION OF THE PROGRAM

One of the main criteria used in the design of mechanical systems are dynamic properties of the structure. They have a direct impact on the vibrations, emitted noise, fatigue, controllability and stability of the structure [2]. One of the basic mechanical properties of the rotating machines are natural frequencies and mode shapes.

Free vibrations are the vibrations of the system, on which - after the initial excitation – do not act any forces. The frequencies with which the system vibrates we call natural frequencies, because they characterize the mechanical properties of the system.

There are many ways to solve natural frequency problem and the literature on this issue is very extensive. It is impossible to provide the best method, because whether it is depended from type of problem and computer capabilities. Types of problems are distinguished by specific cases of proper vibration equations of motion. Thus, we have the following types of equations [3]:

a) Undamped free vibration

$$M\ddot{x} + Kx = 0 \quad (1)$$

b) Damped free vibration

$$M\ddot{x} + C\dot{x} + Kx = 0 \quad (2)$$

where:

M – inertia matrix,

C – damping matrix,

K – stiffness matrix,

x – generalized displacement vector.

Methods of solving of natural frequency problems are divided into: transformational and iterative. Iterative methods are: the method of Jacobi, Givens, Householder, LR and QR. In Dynwir-S-70 program we use connection of QR and Householder methods, and so the program is based on the iterative methods. In Dynwir S-70 matrix from which they are calculated eigenvalues (3) is a unsymmetric matrix. The exact process is to reduce the computational matrix using Householder transformation to the almost upper triangular matrix (Hessenberg form), after that eigenvalues are calculated by QR algorithm. Transformation time to Hessenberg form takes the most time in the whole process of calculating the eigenvalues

It is worth noting that for a system with n degrees of freedom, the procedure used in Dynwir-S-70, solves the eigenvalue problem for the matrix  $AP = 2n \times 2n$  (3), which gives 2n eigenvalues and 2n eigenvectors.

$$AP = \begin{bmatrix} -MC^{-1} & -M^{-1}K \\ I & 0 \end{bmatrix} \quad (3)$$

where:

I- identity matrix,

0- zero matrix.

The resulting eigenvalues in the considered issue are complex or real numbers. Imaginary parts of the complex numbers represent the natural frequencies of the system expressed in [rad / s], whereas the real parts correspond to damping [ - ] corresponding to the frequency. Eigenvectors corresponding to complex eigenvalues have complex coordinates and are treated as forms of vibrations occurring at particular frequencies. It is assumed that the eigenvalues which are real numbers do not correspond to any natural frequencies. In the output files eigenvalues in which the absolute value of the imaginary part is less than 1 are skipped (when natural frequency is lower than 1 [rad/s]).

### 3. BASE MODEL

Calculations in Dynwir-S-70 was presented at a simple example: rotor supported on two supports at each end of the shaft are bearings (fig. 1).

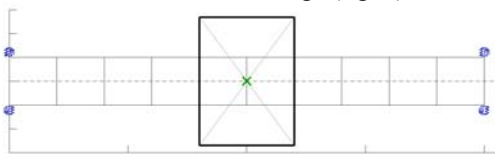


Fig. 1. Construction of the base model. At the ends of the rotor are two slide bearing supports.

The following are the characteristics of the system:

- the length of the rotor: 20 [mm],
- the diameter of the rotor shaft: 20 [mm],
- the diameter of the rotor blade: 50 [mm],
- the thickness of the rotor blade: 40 [mm],
- the bearings stiffness:  $5 \cdot 10^7$  [N/m],
- the bearings damping:  $5 \cdot 10^5$  [N s/m]

### 4. NUMERICAL MODEL BUILT USING BERNOULLIE-EULERA AND TIMOSHENKO BEAMS

The following are the results of a test of the analysis of the base model (fig. 1). This model has been discredited using two types of beams (fig. 2). The first type is a Bernoulli-Euler beam, the second type is a Timoshenko beam. The difference between them lies in the fact that, in the numerical model constructed with the Bernoulli-Euler beam are taken into account the bending, compressing, extension, and torsion stresses, built in the model of Timoshenko beams are also taken into account shear stresses.

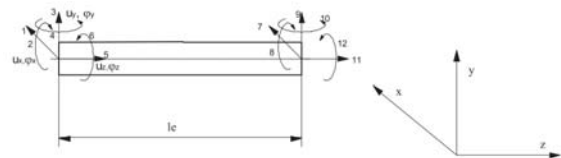


Fig. 2. Beam finite element with two nodes and 12 degrees of freedom [4].

The following indices of directions were determined:

- 1 – x-axis displacement –  $U_x$
- 2 – rotation about the x-axis –  $\varphi_x$
- 3 – y-axis displacement –  $U_y$
- 4 – rotation about the y-axis –  $\varphi_y$
- 5 – z-axis displacement (longitudinal vibrations) –  $U_z$
- 6 – rotation about the z-axis (torsional vibrations) –  $\varphi_z$

Vibration forms described in this article as the first and second, third and fourth, and the next ones are like a pair of similar nature to each other. They are inclined at different angles in space (Fig. 3).

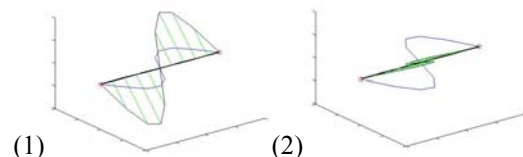


Fig. 3. The difference between the third (1) and fourth (2) form of vibrations in the analyzed cases is the different position in the space.

Table 1 shows the natural frequencies and corresponding damping coefficients for the basic model built using the Timoshenko beam. Six consecutive natural forms of vibrations were

indicated in parentheses. They are shown in Figure 4.

Tab. 1. The first eight natural frequencies and corresponding damping coefficients for the basic model with coefficients  $\alpha=3,0$ ;  $\beta=0,000005$

	Timoshenko beam	
	$\alpha=3,0$ ; $\beta=0,000005$	
	frequency[Hz]	damping [-]
1	568,51 (1)	-38,09
2	568,51 (2)	-38,09
3	2929,75 (3)	-864,74
4	2929,75 (4)	-864,74
5	6259,80 (5)	-3943,53
6	6259,80 (6)	-3943,53
7	7281,64	-5330,29
8	7281,64	-5330,29

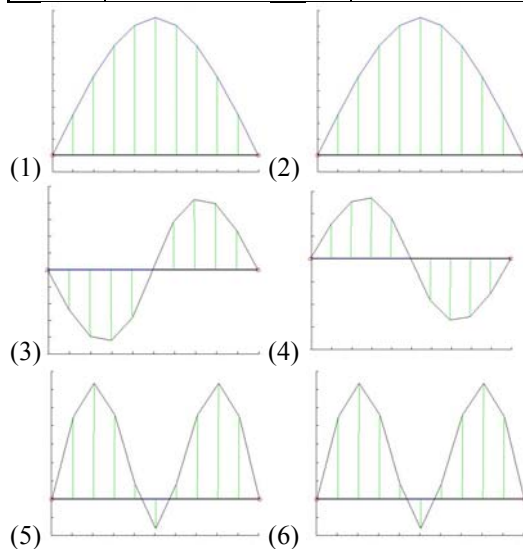


Fig. 4. Form of vibrations for the base model built using Timoshenko beam. The frequencies of these vibrations are given in the Table 1.

Table 2 shows the natural frequencies and corresponding damping coefficients for the base model built using the Euler-Bernoulli beam. Six consecutive natural forms of vibrations were indicated in parentheses. They are shown in Figure 5.

In the model constructed from Euler-Bernoulli beam at some frequencies (tab. 2. position 3 i 4) appear the forms of vibrations, which, should not be disclosed in the real object due to the high damping. The differences between the results of the model built from the Bernoulli-Euler beam and Timoshenko are small. The difference between the frequencies of the first four forms of vibrations is about 1%. The difference between the frequencies of fifth and sixth eigenvector is about 8%. Model adopted for further analysis (after verification of the results with the program Madyn 2000) was a model built with Timoshenko beam (taking into account the shear stresses).

Tab. 2. The first eight natural frequencies and corresponding damping coefficients for the basic model with coefficients  $\alpha=3,0$ ;  $\beta=0,000005$

	Bernoulli-Euler beam	
	$\alpha=3,0$ ; $\beta=0,000005$	
	frequency [Hz]	damping [-]
1	574,14 (1)	-38,84
2	574,14 (2)	-38,84
3	1162,08	-399598,49
4	1162,08	-399598,49
5	2966,76 (3)	-886,51
6	2966,76 (4)	-886,51
7	6808,11 (5)	-4673,91
8	6808,11 (6)	-4673,91

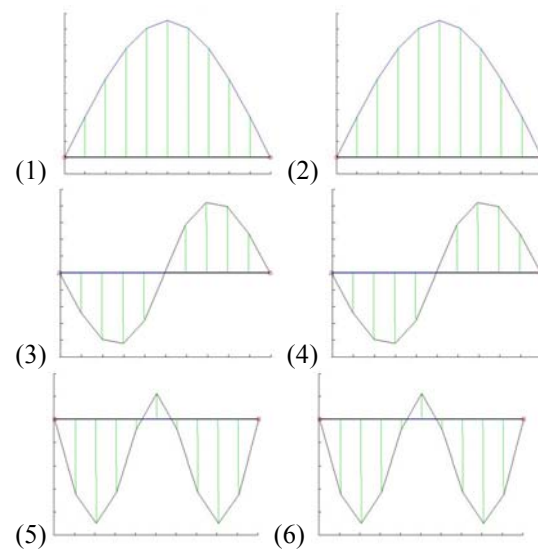


Fig. 5. Form of vibrations for the base model built using Bernoulli-Euler beam. The frequencies of these vibrations are given in the Table 2.

## 5. RAYLEIGH DAMPING

Damping material has been set by the adoption of proportional coefficients  $\alpha$  and  $\beta$  (Rayleigh damping). For such a model of damping the damping matrix (C) of the system is created by the linear combination of a inertia matrix [M] and stiffness matrix [K] that is :

$$[C]=\alpha [M]+\beta [K] \quad (4)$$

Assuming constant damping coefficient  $\zeta$  understood as the ratio between the damping and critical damping in the case of steel  $\zeta=0.01$  and using the relationship (5) [5]:

$$\zeta=\frac{\alpha}{w_{kr}}+\frac{\beta\omega_{kr}}{2} \quad (5)$$

and then analyzing various values (for some natural forms of vibrations) coefficients  $\alpha$  and  $\beta$  were obtained namely  $\alpha=3$  and  $\beta=0,000005$ .

Table 3 shows the process of building mass and stiffness matrix. Based on those matrixes, the

damping matrix is constructed. After the construction of the damping matrix some elements are added into mass, stiffness and damping matrix.

Tab. 3. Construction of the stiffness, inertia and damping matrix and then supplement these with additional elements.

The construction of the inertia of the shaft matrix [M]
The construction of the stiffness of the shaft matrix [K]
The construction of the damping matrix $C = \alpha [M] + \beta [K]$
Supplementing the inertia matrix [M] among others in parts of inertia associated with disks
Supplementing the stiffness matrix [K] including stiffness parts associated with the foundation, external seals and dampers
Supplementing the damping matrix [C] among others in damping parts associated with the torsion vibration dampers, gyroscopic damping, damping foundation, seals and bearings damping.

During the construction of the damping matrix (C) from Rayleigh damping coefficients we need to know  $\alpha$  and  $\beta$ . The following section shows how these coefficients affect the frequency and the forms of rotor vibrations.

### 5.1. Influence of $\beta$ coefficient

Two analyzes were performed. In the first one  $\alpha$  coefficient remained unchanged ( $\alpha=3,0$ ) and  $\beta$  coefficient has been increased twice (from  $\beta=0,000005$  to  $\beta=0,00001$ ). In the second analysis  $\beta$  coefficient has been increased tenfold ( $\beta=0,00005$ )

In the first test, the coefficient  $\beta$  was increased twice in relation to the basic. The results are shown in Tab. 4. A view of the individual forms of vibration was shown in Figure 6. The differences between the first six frequencies in basic model and this are about 1 %.

In a next analysis, the coefficient  $\beta$  was multiplied by ten. The results are shown in tab. 5. A view of the individual mode shapes is shown in Figure 7. The differences between the first and the second frequency in this model and base model is approximately 1 %. The difference between the third and fourth frequency is approximately 11 %. The difference between the fifth and sixth frequency is as high as 84 %.

Tab. 4. The first eight natural frequencies and corresponding damping coefficients for the basic model with coefficients  $\alpha=3,0$ ;  $\beta=0,00001$ .

	$\alpha=3,0$ ; $\beta=0,00001$	
	frequency [Hz]	damping [-]
1	568,41 (1)	-69,95
2	568,41 (2)	-69,95
3	2920,04 (3)	-1712,63
4	2920,04 (4)	-1712,63
5	6164,74 (5)	-7842,80
6	6164,74 (6)	-7842,80
7	7131,85	-10625,86
8	7131,85	-10625,86

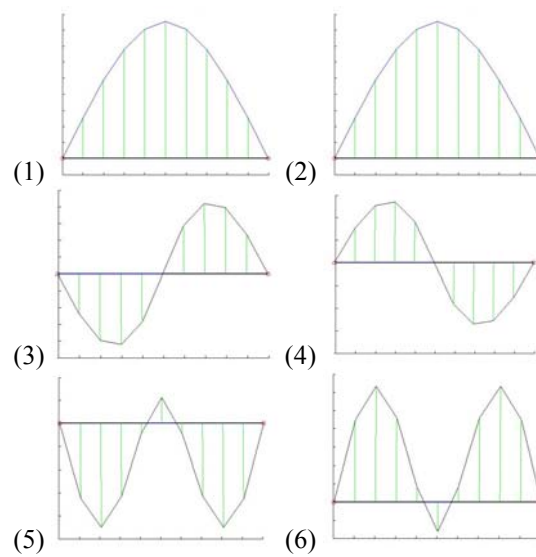


Fig. 6. Forms of vibration for the base model  $\alpha=3,0$ ;  $\beta=0,00001$ .

Tab. 5. The first eight natural frequencies and corresponding damping coefficients for the basic model with coefficients  $\alpha=3,0$ ;  $\beta=0,00005$ .

	$\alpha=3,0$ ; $\beta=0,00005$	
	frequency [Hz]	damping [-]
1	566,03 (1)	-324,75
2	566,03 (2)	-324,75
3	1009,69 (3)	-38924,78
4	1009,69 (4)	-38924,78
5	2600,80 (5)	-8486,10
6	2600,80 (6)	-8486,10
7	2249352,99	-11126,08
8	2249352,99	-11126,08



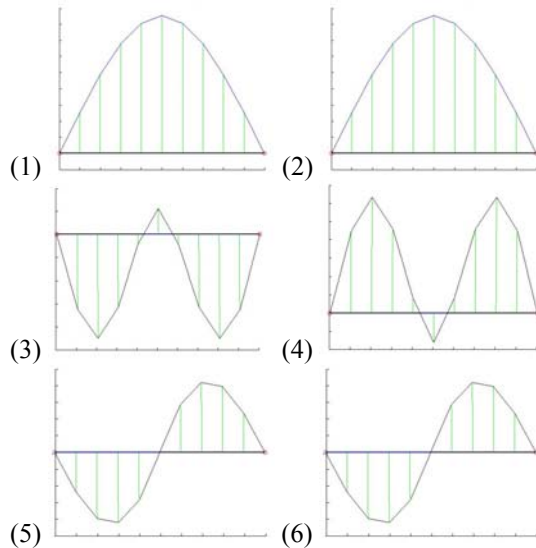


Fig. 7. Mode shapes for the base model,  $\alpha = 3,0$ ;  $\beta = 0,000005$ .

### 5.2. Influence of $\alpha$ coefficient

Two tests were performed during which  $\beta$  coefficient was constant ( $\beta = 0,000005$ ) while the coefficient  $\alpha$  was decreased tenfold ( $\alpha = 0,3$ ), and increased it tenfold ( $\alpha = 30,0$ ). Below are the results. In the first test, the coefficient  $\alpha$  has been reduced in relation to the base of ten. The results are shown in Tab. 6. In the second test coefficient  $\alpha$  was increased tenfold ( $\alpha = 30,0$ ). The results are shown in Tab. 7. Mode shapes at all frequencies are the same as in the base case. The difference between the frequencies of the first six mode shapes in this model and base model are negligible, and thus it can be concluded that the coefficient  $\alpha$  slightly affects the natural frequency of the rotor in this case.

Tab. 6. The first eight natural frequencies and corresponding damping coefficients for the reference model, the coefficients  $\alpha = 0,3$ ;  $\beta = 0,000005$ .

	$\alpha = 0,3$ ; $\beta = 0,000005$	
	frequency [Hz]	damping [-]
1	568,51 (1)	-37,66
2	568,51 (2)	-37,66
3	2929,75 (3)	-864,06
4	2929,75 (4)	-864,06
5	6259,82 (5)	-3942,38
6	6259,82 (6)	-3942,38
7	7281,65 (7)	-5329,52
8	7281,65 (8)	-5329,52

Tab. 7. The first eight natural frequencies and corresponding damping coefficients for the reference model, the coefficients  $\alpha = 30,0$ ;  $\beta = 0,000005$ .

	$\alpha = 30,0$ ; $\beta = 0,000005$	
	frequency [Hz]	damping [-]
1	568,50 (1)	-42,39
2	568,50 (2)	-42,39
3	2929,70 (3)	-871,48
4	2929,70 (4)	-871,48
5	6259,62 (5)	-3955,11
6	6259,62 (6)	-3955,11
7	7281,50 (7)	-5337,97
8	7281,50 (8)	-5337,97

### 6. COMPARISON WITH COMMERCIAL SOFTWARE

The results of the Dynwir-S-70 were compared with a commercial program for analysis of rotors - Madyn 2000. In both programs, models have been constructed with regard to the same geometrical parameters and the same conditions in the bearing support. Natural frequencies received using Madyn 2000 software are presented in Table 8. The corresponding mode shapes are shown in Figure 8.

Tab. 8. The first eight natural frequencies and corresponding damping in Madyn 2000 software.

	$\zeta = 0,1$	
	frequency [Hz]	damping [%]
1	568,40 (1)	0,2
2	568,59 (2)	0,2
3	2918,95 (3)	0,1
4	2948,99(4)	0,1
5	6320,71(5)	0,1
6	6323,07(6)	0,1
7	7344,37	0,1
8	7370,84	0,1

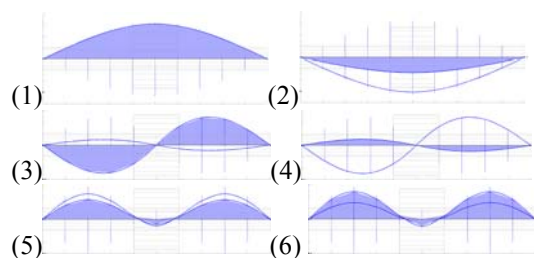


Fig. 8. Mode shapes – calculated in Madyn 2000.

The differences in frequencies between particular mode shapes in Madyn 2000 and Dynwir-S-70 for the base case are approximately 1 %. These differences may be due to a different algorithm of calculating the eigenvalues, or a different discretization of the model.

## 7. SUMMARY AND CONCLUSIONS

The paper presents basic information about the Dynwir-S-70 software. Influence of discretization model (using Bernoulli-Euler beam and Timoshenko beam) on natural mode shapes and natural frequencies of the system was presented. This paper briefly discusses the Rayleigh proportional damping model and the impact of the coefficients ( $\alpha$  and  $\beta$ ) for the natural frequencies and. By appropriate selection of  $\alpha$  and  $\beta$  coefficient some forms of vibration can "reveal" or can "disappear" in modal analysis. As it turns out, in this case much greater impact on the results of the modal analysis is the value of the coefficient  $\beta$  than  $\alpha$ .

Developing the Dynwir-S-70 software allows the modal analysis of the rotor supported on slide bearings. In the nearest future it is planned to expand the program Dynwir-S-70 with non-linear elements in the modal analysis.

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