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APPLICATION OF THE THEORY OF SEMI-MARKOV PROCESSES TO DETERMINE A LIMITING DISTRIBUTION OF THE PROCESS OF CHANGES OF ABILITY AND INABILITY STATES OF FUEL SUPPLY SYSTEMS IN HEAVY FUEL DIESEL ENGINES

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Abstract

The paper presents applicability of the theory of semi-Markov processes to determine a limiting distribution of the process of changes of technical states of fuel systems for marine engines running on heavy fuel oils. The proposed study of this process includes the components of such fuel systems like: 1 - injectors, 2 - high pressure hoses, 3 - injection pumps, 4 - low pressure hoses, 5 – fine filters, 6 - coarse filters, 7 – fuel-feed pump, 8 – fuel heater and 9 – viscosistat with a viscometer. A semi-Markov state transition model consisting of ten states has been developed for such systems. Application of technical diagnostics has been pointed to be necessary to investigate the state transition process for the systems. The conclusions presented furthermore in the paper provide advantages which (according to the author) are of the most importance in the design and operation phases of fuel systems in marine diesel engines that run on heavy fuel oils.

1. Introduction

During operation of internal combustion piston engines, the fuel combustion process which proceeds in marine diesel engines is particularly important. This process should run in a way that provides as low fuel consumption as possible. It is difficult to reach such a combustion process in internal combustion engines running on heavy fuel oils. This is due to the need of heating the fuel to a high temperature to be able to produce in the engine working spaces a fuel-air mixture having a proper micro- and macro-structure. Heating the fuel is indispensable to reduce the viscosity of heavy fuel oils which should range from 15 ÷ 25cSt (for low speed engines), and 12÷16cSt (for medium speed engines) [11, 12]. Such a structure can be gained when the engine power supply system (engine fuel system) for heavy fuel oil finds itself in a state of full ability (s_0). This means that all components of the system must stay in ability states. Such components include [11, 12, 14]: 1 - injectors, 2 - high pressure hoses 3 - injection pump, 4 - low pressure hoses, 5 - fine filters, 6 - coarse filters, 7 – fuel-feed pump, 8 – fuel heater and 9 – viscosistat with a viscometer [11, 12]. Failure of any of the components in the engine fuel system makes the component disable. This causes a failure and inability state of the entire fuel system. Therefore, the users of marine heavy fuel engines are interested in what the probability is that at any moment in a long time of operation (theoretically $t \to \infty$) the engine fuel system is in the state of ability (s_0) or states of inability ($\bar{s}_0 :\ s_1,\ s_2,\ ...,\ s_9$).

2. General identification of state changes for marine diesel engines

The state of ability (s_0) of each fuel system in marine internal combustion engine is found (as mentioned in the introduction) when all components in the system stay in ability states. If any of the components gets a failure, the fuel system must be considered to be damaged and, therefore, in state of inability ($\overline{s}_0$). A failure of the i-th (any) of the components in this system is fixed by the engine user during performance of a maintenance service. As a result of this action the faulty component is renovated and in consequence stays no longer in the inability state $s_i \in S^*$ (i = 1, 2, 3,…., 9), instead of this, it takes a condition that provides an ability state s_0 to the entire fuel system. Thus, a marine engine fuel system in the phase of operation can take one of the states from the set:

$$S = \{s_0, s_1, s_2, \ldots, s_9\} \tag{1}$$

with the following interpretation:

- s_0 – ability state of a fuel system,
- (s_i, i = 1, 2, 3,…., 9) – inability state of a system due to a failure of, respectively: injectors (s_1), high pressure hoses (s_2), injection pumps (s_3), low pressure hoses (s_4), fine filters (s_6), coarse filter (s_6), fuel-feed pumps (s_7), fuel heater (s_8) and viscosistat including a viscometer (s_9), i.e., that $S^* \subset S$.

Therefore, the process of occurring states $s_i \in S$ (i = 0, 1, 2, 3,…., 9) in sequence can be observed. The operational practice shows that the unconditional duration T_i of any state s_i, and T_{ij} of the state s_i, provided that the state s_j is the next one, are random variables [2, 3, 7, 8]. Furthermore, it is obvious that the system can stay in these states with a determined probability $P_{i,}$, while a transition from the state s_i to s_j proceeds with the probability p_{ij} [3, 8, 10, 13].

It can be assumed that a fuel system in a new engine is in the ability state (s_0). The same state of it can be found after its renovation made during performance of an appropriate maintenance service [3, 7, 12]. Due to the fact that the time of renovation performance is a random variable, the occurrence of the state s_0 is a random event. The system may stay in this state for the time period T_0 with the probability P_0. The time period T_0 is a random variable, because the value of the time results from the occurrence of a failure in the system, which is a random event. A state of inability of the fuel system occurs as a result of its failure. This state is a consequence of occurrence of disability of any component within the system. Thus, for this system there can be distinguished the following inability states: $s_i \in S$ (i = 1, 2, 3,…., 9). When the cause of the inability is a failure of injectors, the state transition from s_0 to s_1 takes place after the time period T_{01} with the probability p_{01}. The occurrence of the state s_1 will cause a need to implement some service in order to renovate the system and regain its state s_0. The fuel system returns to the state $s_{0\text{v}}$ after the time period T_{10} with the probability p_{10}. However, if the cause of the state of inability of the fuel system is a failure of high pressure hoses, the transition proceeds from the state s_0 to s_2. This transition takes place after the time period T_{02} with the probability p_{02}. In this case, the system can return to the state s_0 after the time period T_{20} with the probability p_{20}, provided that a proper service is performed. The same situation applies when the fuel system loses s_0 in the event of a failure of any of its other components. For each of the considered cases the conclusion can be made that the current state of a fuel system and the time period of its duration depend significantly (even exclusively) on the immediate previous state. This is obvious when the consideration includes the fuel system states occurring in sequence: $s_i \in S$ (i = 0, 1, 2, 3,…., 9). For example, if a fuel system is in the state s_0, with the time of its failure this state transits to one of the possible states of $s_i \in S$ (i = 1, 2, 3,…., 9), depending on which of the components of the system is damaged. When the inability state of the fuel system is s_1, then after its renovation and return to the state s_0, just this state (s_0) and the time period of its duration will depend only on the state s_1, not on those states that were earlier, so the states of $s_i \in S$ (i = 2, 3,…., 9). Duration of the state s_0, after implemented service will in turn depend on how well the service restoring this state (s_0) was performed and also on how much the system will be loaded during operation [5, 6, 7, 12, 14]. The above considerations show that a semi-Markov model of a real process of changes of its states can be considered in case of fuel systems for marine diesel engines running on heavy fuel oils. Such a model is possible to be built because the process states can be determined in such a way that the duration of the state existing at the instant τ_n and the state obtainable at the instant τ_{n+1} do not depend stochastically on the states that occurred earlier or duration of these states. The theory of semi-Markov processes can be applied to build a semi-Markov model $\{W(t)\colon t \geq 0\}$ of the real process of changes of technical states for the operation phase of propulsion systems. The characteristics for the model are [7, 8, 10, 12, 13]:

1) the Markov condition is satisfied so that in future the evolution of the process of changes of states of a fuel system during its operation phase, for which the semi-Markov model has been built, would depend only on its state at the given time, not on the system operation in the past, thus so that *the future of this system would not depend on the past but on the present,*
2) random variables T_i (denoting the duration of the state s_i, independently which state it will be followed by) and T_{ij} (denoting the duration of the state „s_i", on the condition that the state "s_j" will be the next one in the process) have other than exponential distributions.

Modeling, which allows to develop a semi-Markov model of the process of changes of fuel system states includes the outputs of the conducted analysis of the changes in these states [2, 5, 8, 12, 14]. The states have been recognized to be the most important in studying the real process of changes of technical states of fuel systems for marine diesel engines on heavy fuel oils.

The presented description of the changes of technical states for fuel systems is a result of the noticed fact that for these systems, just like for other functional systems in diesel engines, prediction of changes in their technical states can be made when the current states of the systems and the conditions under which they are expected to operate in future are known. This also applies to many other technical objects, for example such as gas turbine engines, reciprocating compressors, centrifugal pumps, etc. [2, 4, 7, 12, 14]. This fact which also indicates that changes of technical states of this kind of objects are not closely related to the time of their operation (work), is explained as a scientific hypothesis by the publications [2, 7]. With regard to the fuel systems investigated herein, this can be explained in a form of the following scientific hypothesis (***H***): ***the process of changes of technical states of any fuel system*** *(defined as a random function whose the argument is time and the values are random variables denoting the current technical states),* ***running under rational operation*** *(that is operation based on economic calculation)* ***is a process with asymptotically independent values, because any particular state of the process investigated at any instant*** τ_n (***n*** **= 0, 1,** ***…, m;*** $\tau_0 < \tau_1 < … < \tau_m$)***, depends significantly on the immediate previous state, not on the states that occurred earlier or the periods of their duration.***

It should be noticed that the formulated hypothesis does not contain any such contradictions that might falsify it in the logical sense, still before testing. The consequences of this hypothesis are as follows [2, 7, 8]:

1) K_1 – probabilities (p_{ij}; $i \neq j$; $i,j \in N$) of the process transition, i.e. changes of fuel system states from any state „s_i", which the process (system) is in, to the next (any) state " s_j ", do not depend on the states which the process was earlier in,
2) K_2 – periods of unconditional duration of the particular states "s_i" of the process of changes of fuel system states are stochastically independent random variables (T_i; $i \in N$),
3) K_3 – periods of duration of each of the possible-to-occur states "s_i" of the process of changes of fuel system states, provided that the next state is one of the remaining process states "s_j", are stochastically independent random variables (T_{ij}; $i \neq j$; $i,j \in N$).

The presented consequences reveal the probabilistic law for the changes of technical states of any fuel system. They are not mutually contradictory, and their logic truth raises no doubts. Thus, the non-contradiction condition for the consequences is satisfied. This means that nothing stands in a way to make advantage of the mentioned consequences regarded jointly as one consequence (K) in order to test the hypothesis (H) empirically, so to make its verification if should be accepted or falsified. This verification consists in experimental testing of the truthfulness of the consequences $K_i \in K$, $i = 1, 2, 3$. A non deductive inference method called a reductive inference can be applied to verify this hypothesis.

3. Semi-Markov model of the process of changes of technical states of fuel systems for marine diesel engines

Application of a semi-Markov model of the process of changes of technical states of a fuel system for a heavy fuel engine allows to:

- take into account preventive maintenance required to restore the system which becomes completed upon completion of maintenance of particular system's components and
- investigate more than two reliability states of an engine fuel system as well as its components.

This allows to investigate a semi-Markov model of changes of technical states of fuel systems for marine internal combustion engines with a ten-element set S (1). This means that the limiting distribution of the process of changes of ability and inability states of heavy fuel supply systems for marine diesel engines, can be defined by using the model of the process of changes of the system states, developed in a form of a semi-Markov process $\{\{W(t)$: $t \geq 0\}$ with a set of states $S = s_i$; $i = 0, 1, 2, ... ,9$. Interpretation of the states $s_i \in S(i = 0, 1, 2, ... ,9)$ in accordance with the description in equation (1) is as follows: s_0 – ability state of a fuel system, s_1 – inability state of a fuel system due to a failure of an injector (injectors), s_2 – inability state of a fuel system due a failure of a high pressure hose (hoses), s_3 – inability state of a fuel system due to a failure of an injection pump (injection pumps), s_4 – inability state of a fuel system due to a failure of a low pressure hose (hoses), s_5 – inability state of a fuel system due to a failure of fine fuel filters, s_6 – inability state of a fuel system due to a failure of coarse filters, s_7 – inability state of a fuel system due to a failure of a fuel-feed pump (fuel-feed pumps), s_8 – inability state of a fuel system due to a failure of a fuel heater, s_9 – inability state of a fuel system due to a failure of viscosistat or viscometer.

Graph of state transitions for the process $\{W(t)$: $t \geq 0\}$ is shown in Fig. 1

Transitions of the states s_i ($i = 0, 1, 2, ..., 9$) occur in succeeding times t_n ($n \in N$),, where at the time $t_0 = 0$, the engine fuel system is in state s_0. The state s_0 lasts until a failure of any of the components of the engine fuel system. However, the states ($s_i(i = 1, 2, ..., 9$) last until the damaged engine fuel system is restored.

The above considerations show that it can be assumed that the state of an engine fuel system at time t_{n+1} and the time period of duration of the state at a time t_n do not depend on states that occurred at times t_0, t_1, ..., t_{n-1} or time periods of their durations. Therefore, the process $\{W(t): t \geq 0\}$ is a discrete-state continuous-time semi-Markov process [2, 3, 4, 8, 10, 13].

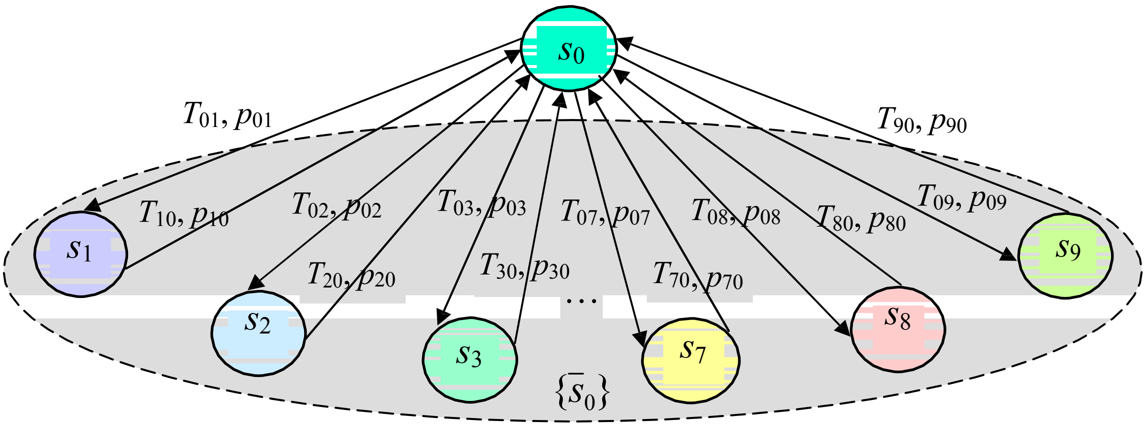

Fig. 1. Graph of state transitions for the process $\{W(t): t \geq 0\}$: s_0 – ability state of an engine fuel system, $\{\bar{s}_0\}$ – set of inability states of an engine fuel system: $\{\bar{s}_0\} = \{s_1, s_2, s_3, ..., s_9\}$, $s_i \in S(i = 1, 2, ..., 9)$ – states with the interpretations respectively: s_1 – inability state of an engine fuel system due to a failure of an injector (injectors), s_2 – inability state of an engine fuel system due a failure of a high pressure hose (hoses), s_3 – inability state of an engine fuel system due to a failure of an injection pump (injection pumps), s_4 – inability state of an engine fuel system due to a failure of a low pressure hose (hoses), s_5 – inability state of an engine fuel system due to a failure of fine fuel filters, s_6 – inability state of an engine fuel system due to a failure of coarse filters, s_7 – inability state of an engine fuel system due to a failure of a fuel-feed pump (fuel-feed pumps), s_8 – inability state of an engine fuel system due to a failure of a fuel heater, s_9 – inability state of an engine fuel system due to a failure of viscosistat or viscometer, T_{ij} - time period of the process duration in the state s_i provided that the state s_j is the next one, p_{ij} – probability of staying the process in the state s_i, provided that the state s_j ($i, j = 0, 1, 2, ..., 9$; $i \neq j$) is the next one.

An exemplary realization of the process $\{W(t): t \geq 0\}$, depicting occurrence of technical states of a fuel system in any heavy fuel engine when operated, is shown in Figure 2.

Obtaining (obviously in approximation) the values of the probabilities Pj (j = 0, 1, 2, 3) that make a limiting distribution of the process of changes of ability and inability states of a heavy fuel supply system for a marine diesel engine requires determination of the initial distribution of the process $\{W(t): t \geq 0\}$and its matrix function.

The initial distribution of the process is:

$$P\{W(0) = s_i\} = \begin{cases} 1 & dla \quad i = 0 \\ 0 & dla \quad i = 1,\ 2,\ ...,\ 9 \end{cases} \tag{2}$$

and the matrix function has the form:

$$
\mathbf{Q(t)} = \begin{bmatrix}
0 & Q_{01}(t) & Q_{02}(t) & Q_{03}(t) & Q_{04}(t) & Q_{05}(t) & Q_{06}(t) & Q_{07}(t) & Q_{08}(t) & Q_{09}(t) \\
Q_{10}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{20}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{30}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{40}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{50}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{60}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{70}(t) & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{80}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Q_{90}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{3}
$$

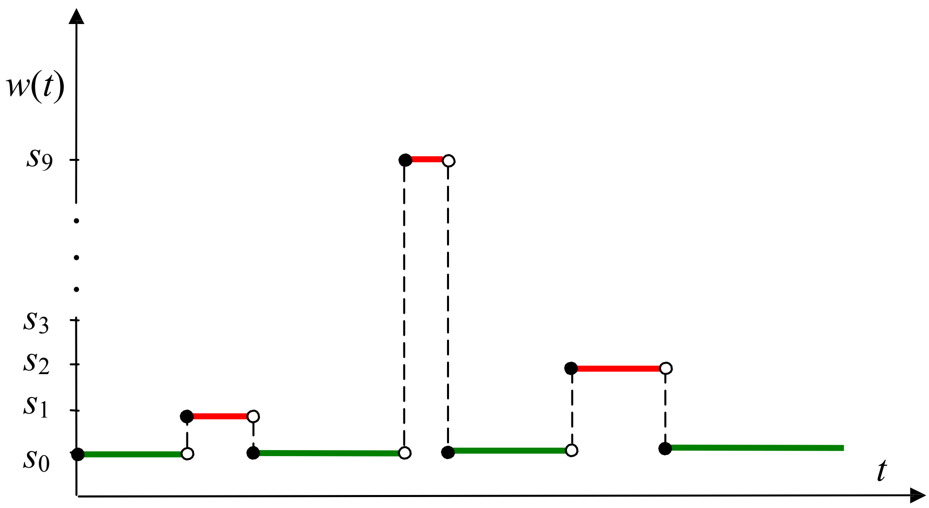

Fig. 2. An exemplary realization of the process $\{W(t): t \geq 0\}$ for the investigated fuel system components: s_0 – ability state of the components and the system at the same time, $s_i(i = 1, 2, 3, \ldots, 9)$ – states as described in Figure 1.

The matrix function $\mathbf{Q}(\boldsymbol{t})$ is a model of changes of technical states of a engine fuel system. Nonzero elements $Q_{ij}(t)$ of the matrix $\mathbf{Q(t)}$ depend on the distributions of random variables which are the time intervals of staying the process $\{W(t): t \geq 0\}$ in the states $s_i \in S(i = 0, 1, \ldots, 9)$. The elements of the matrix function $\mathbf{Q}(\boldsymbol{t})$ are the probabilities of the process transitions from state s_i, to state s_j (s_i, $s_j \in S$) in time no longer than t, which are defined as follows [3, 8]:

$$
\mathrm{Q}_{ij}(t) = P\{W(\tau_{n+1}) = s_j,\ \tau_{n+1} - \tau_n < t | W(\tau_n) = s_i\} = p_{ij}F_{ij}(t) \tag{4}
$$

where:

$p_{(ij)}$ – one-step transition probability of the homogeneous Markov chain;

whereas $p_{ij} = P\{Y(\tau_{n+1}) = s_j | Y(\tau_n) = s_i\} = \lim\limits_{t \to \infty} Q_{ij}(t)$;

$F_{(ij)}(t)$ – distribution of the random variable $T_{(ij)}$ denoting the duration of the state s_i of the process $\{W(t): t \geq 0\}$, provided that the state s_j is the next one in the process.

The matrix **P** of transition probabilities of the embedded Markov chain to the process, as this results from the matrix function **Q(t)** (2), is composed of the elements which are [2, 7, 8]: $Q_{i0}(t) = p_{i0} = 1 (i = 1, 2, \ldots, 9)$ and $Q_{0j}(t) = p_{0j}$, because **Q(t)** is a stochastic matrix.

The process $\{W(t)\colon t \geq 0\}$ is irreducible [1, 3, 4], and the random variables $T_{(ij)}$ take finite positive expected values. Therefore, its limiting distribution [7, 8]

$$P_j = \lim_{t\to\infty} P_{ij}(t) = \lim_{t\to\infty} P\{W(t) = s_j\},\ s_j \in S (j = 0, 1, \ldots, 4) \tag{5}$$

is of the following form [8]:

$$P_j = \frac{\pi_j \mathrm{E}(T_j)}{\sum_{l=0}^{9} \pi_l \mathrm{E}(T_l)} \tag{6}$$

The probabilities $\pi_j (j = 0, 1, 2, \ldots, 9)$ in the formula (6) are limiting probabilities of the embedded Markov chain to the process $\{W(t)\colon t \geq 0\}$. However, $\mathrm{E}(T_j)$ and $\mathrm{E}(T_k)$ are expected values of the random variables of T_j and T_k respectively, which are the time periods of staying the system in states: s_j and s_k respectively, regardless of which state will be next.
To determine the limiting distribution (6) it is required to solve the system of equations that comprise the said limiting probabilities $\pi_j (j = 0, 1, \ldots, 9)$ of the embedded Markov chain and the matrix **P** of the probabilities of transition from the state s_i to s_j,. Such a system takes the following form:

$$\left.\begin{array}{l} [\pi_0,\pi_1,\pi_2,\pi_3,\pi_4,\pi_5,\pi_6,\pi_7,\pi_8,\pi_9] = [\pi_0\ \ \pi_1,\pi_2,\pi_3,\pi_4,\pi_5,\pi_6,\pi_7,\pi_8,\pi_9]\cdot\mathbf{P} \\ \sum_{k=1}^{4} \pi_k = 1 \end{array}\right\} \tag{7}$$

As a result of solving the system of equations (7), by using the formula (6), the following relationships can be obtained:

$$\left.\begin{array}{l}
P_0 = \dfrac{\mathrm{E}(T_0)}{\mathrm{E}(T_0) + \sum_{l=0}^{9} p_{(0l)} \mathrm{E}(T_l)},\ P_1 = \dfrac{p_{(01)}\mathrm{E}(T_1)}{\mathrm{E}(T_0) + \sum_{l=0}^{9} p_{(0k)} \mathrm{E}(T_l)},\ P_2 = \dfrac{p_{(02)}\mathrm{E}(T_2)}{\mathrm{E}(T_0) + \sum_{l=0}^{9} p_{(0l)} \mathrm{E}(T_l)}, \\
P_3 = \dfrac{p_{(03)}\mathrm{E}(T_3)}{\mathrm{E}(T_0) + \sum_{l=0}^{9} p_{(0l)} \mathrm{E}(T_l)},\ P_4 = \dfrac{p_{(04)}\mathrm{E}(T_4)}{\mathrm{E}(T_0) + \sum_{l=0}^{9} p_{(0l)} \mathrm{E}(T_l)},\ P_5 = \dfrac{p_{(05)}\mathrm{E}(T_5)}{\mathrm{E}(T_0) + \sum_{l=0}^{9} p_{(0l)} \mathrm{E}(T_l)}, \\
P_6 = \dfrac{p_{(06)}\mathrm{E}(T_6)}{\mathrm{E}(T_0) + \sum_{l=0}^{9} p_{(0l)} \mathrm{E}(T_l)},\ P_7 = \dfrac{p_{(07)}\mathrm{E}(T_7)}{\mathrm{E}(T_0) + \sum_{l=0}^{9} p_{(0l)} \mathrm{E}(T_l)},\ P_8 = \dfrac{p_{08}\mathrm{E}(T_8)}{\mathrm{E}(T_0) + \sum_{l=0}^{9} p_{(0l)} \mathrm{E}(T_l)}, \\
P_9 = \dfrac{p_{09}\mathrm{E}(T_9)}{\mathrm{E}(T_0) + \sum_{l=0}^{9} p_{(0k)} \mathrm{E}(T_l)}
\end{array}\right\} \tag{8}$$

P_0 is a limiting probability that the engine fuel system stays in the state s_0 (ability state) in a longer period (time interval) of operation (in theory at $t \to \infty$). Thus, this probability defines a coefficient of technical availability of the system for operation. However, $P_j (j = 1, 2,..., 9)$ are limiting probabilities of existence of the states $s_j \in$ S of the systems at $t \to \infty$, thus they are the probabilities that its components (and simultaneously the entire system, due to its serial reliability construction) are in states of inability.

Obtaining the probabilities $P_j (j = 0, 1, 2, \ldots, 9)$ which make a limiting distribution of the process of changes of ability and inability states of fuel supply systems for marine heavy fuel diesel engines requires, as it follows from (6) and (8) formulas, an estimation of the values of the probabilities p_{ij} and expected values $E(T_j)$. Obtainment of the values (obviously, in approximation) is expensive and labor consuming. This is due to the fact that collection of necessary information to estimate p_{ij} and $E(T_j)$ requires long-term empirical research using appropriate diagnosing systems (*SDG*) for engine fuel systems [1, 12, 15, 16, 17].

Estimation of the probabilities p_{ij} and expected values $E(T_j)$ is possible after getting realizations $w(t)$ of the process $\{W(t): t \geq 0\}$ in sufficiently long time period of investigation, thus for $t \in [0, t_b]$, whereas the time of the process investigation is: $t_b >> 0$. Then, it is possible to determine the numbers of: $n_{ij}(i,j = 0, 1, 2, 3;\ i \neq j)$, transitions of the process $\{W(t): t \geq 0\}$ from state s_i to s_j in sufficiently long time and to define the values of the estimator P_{ij} of the unknown probability p_{ij}. The estimator of the transition probability p_{ij} with the highest reliability is the statistics [7, 8]:

$$\hat{\imath}_{ij} = \frac{N_{ij}}{\sum_j N_{ij}}, \quad i \neq j; \quad i, j = 0,\ 1,\ 2,\ 3, \tag{9}$$

whose the value $\hat{p}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$ is an estimation of the unknown transition probability p_{ij}.

The $w(t)$ course of the process $W(t)$ can also give the realizations $t_j^{(m)}$, $m = 1, 2, ..., n_{ij}$ of the random variables T_j. Application of a point estimation allows for ease calculation of $E(T_j)$ as an arithmetic mean value of $t_j^{(m)}$. Acquisition of the necessary information in order to calculate the probabilities requires employing adequate diagnosing systems (*SDG*) for the mentioned fuel systems which in this case become diagnosed systems (*SDN*) [5, 7].

The presented description of changes of states of a fuel system in a heavy fuel diesel engine can be obviously developed when taking into account the states of partial ability of such an engine system.

4. Final remarks and conclusions

Application of the aforesaid processes in the practice requires to satisfy the two following conditions:

- to collect the mathematical statistics that are necessary to determine the probabilities p_{ij} and the expected values T_{ij},
- to develop a semi-Markov model of changes of technical states of an engine fuel system with a small number of its states and simple (in the mathematical sense) matrix function **Q(t)**.

Application of a semi-Markov process as a model of changes of the specified states of an engine fuel system at a defined time (given moment), instead of a Markov process, results from that it should not be expected that the random variable $T_{(ij)}$ denoting the time period of duration of the state s_i on the condition that the state s_j is the next one and the random variable T_i denoting the time period of duration of the state $s_i(i = 1, 2, 3, \ldots, 9)$ of an engine fuel system, independently of which state will be next, have arbitrary distributions included in the set $R_+ = [0, +\infty)$. Employing a Markov process for this case would be justified if the assumption could be made that the random variables $T_{(ij)}$ and T_i have exponential distributions.

The presented model can be of considerable practical importance because of ease in determining the estimators of the transition probabilities p_{ij} which are the elements of the matrix **P** (9) and ease in estimating the expected values $E(T_j)$. However, the considerations should take into account the fact that a point estimation of the expected value $E(T_j)$ does not enable to define the estimation accuracy. Such accuracy is possible by the interval estimation, where a confidence interval $[t_{dj}, t_{gj}]$ with random endpoints is determined that contains with a certain probability (confidence level) β, the unknown expected value $E(T_j)$.

References

1. Cempel C., Natke H., G., Yao J.P.T.: Symptom reliability and hazard for systems conditio motoring. Mechanical Systems and Signal Processing. Vol. 14, No 3, 2000, pp.495-505.
2. Girtler J.: Girtler J.: Physical aspect of application and usefulness of semi-Markovian processes for modeling the processes occurring in operational phase of technical objects. Polish Maritime Research. No 3(41)/2004, vol. 11, pp. 25-30.
3. Girtler J.: Semi-Markovian models of the process of technical state changes of technical objects. Polish Maritime Research No 4(42)/ 2004, vol. 11, pp. 3-7.
4. Girtler J.: Reliability model of two-shaft turbine combustion engine with heat regenerator. Journal of KONES Powertrain and Transport, Vol. 132, No. 4, 2006, pp.15-22.
5. Girtler J.: Problemy oszacowania trwałości i niezawodności silników o zapłonie samoczynnym z zastosowaniem teorii procesów semimarkowskich i diagnostyki. Combustion Engines (Silniki Spalinowe), nr 3, 2013, s. 1-9[pdf].
6. Girtler J.: A method for evaluating the performance of a marine piston internal combustion engine used as the main engine on a ship during its voyage in different sailing conditions. Polish Maritime Research, Vol. 17 No. 4, 2010, s. 31-38
7. Girtler J.: *Diagnostyka jako warunek sterowania eksploatacją okrętowych silników spalinowych*. Studia Nr 28, WSM, Szczeci 1997.
8. Grabski F.: Teoria semi-markowskich procesów eksploatacji obiektów technicznych. Zeszyty Naukowe AMW, nr 75A, Gdynia 1982.
9. Korczewski Z.: Entropy function application In the selection process of diagnostic parameter of marine diesel and gas turbine engines. Polish Maritime Research. Vol. 17, No 2(65), 2010, pp.29-35.
10. Limnios N., Oprisan G.: Semi-Markov Processes and Reliability. Boston, Birkhauser 2001.
11. Piotrowski I. Witkowski K.: Okrętowe silniki spalinowe. Wyd. TRADEMAR, Gdynia 1996.
12. Piotrkowski I., Witkowski K.: Eksploatacja okrętowych silników spalinowych. AM, Gdynia 2002.

13. Сильвестров Д. С.: Полумарковские процессы с дискретным множеством состояний. Издательство „Советское Радио”. Москва, 1980.
14. Wojnowski W.: Okrętowe siłownie spalinowe. Cz. I. Wyd. AMW, Gdynia 1998.
15. *Inżynieria diagnostyki maszyn*. Praca zbiorowa po redakcją B. Żółtowskiego i C. Cempla. PTDT. Wyd. ITE, Warszawa, Bydgoszcz, Radom 2004.
16. MAN B&W Diesel A/S: CoCoS Maintenance, Designed for Maintenance Excellence, Kopenhaga 2005.
17. Wartsila Corporation: Service News from Wartsila Corporation 2 2002/1 2003, CBM for two stroke engines, Kaidara Software, Wartsila Corporation Helsinki, marzec 2003.