Article citation info:
GIRTLER J, ŚLĘZAK M. Four-state stochastic model of changes in the reliability states of a motor vehicle. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2013; 15 (2): 156–160.

Jerzy GIRTLER
Marek ŚLĘZAK

FOUR-STATE STOCHASTIC MODEL OF CHANGES IN THE RELIABILITY STATES OF A MOTOR VEHICLE

MODEL STOCHASTYCZNY CZTEROSTANOWY ZMIAN STANÓW NIEZAWODNOŚCIOWYCH SAMOCHODU*

The properties of semi-Markov processes have been generally characterized and the applicability of the theory of such processes to the determining of the reliability of motor cars and other road vehicles has been explained. A formal description of the process of changes in the motor vehicle technical states considered as reliability states and a model of this process in the form of a one-dimensional stochastic process have been presented. The values of this process are the technical states of the motor vehicle in question that have significant practical importance. A four-state set of states interpreted as follows has been adopted: full (complete) serviceability, partial (incomplete) serviceability,task-limiting serviceability,and complete (total) unserviceability. Based on the initial distribution adopted and the functional matrix worked out, the boundary distribution of the process of changes in the technical (reliability) states of the motor vehicle has been defined. The probability of the vehicle being fully serviceable has been considered a measure of the vehicle reliability for a long period of vehicle operation. A possibility of defining the vehicle reliability in the form of a probability that a task would also be fulfilled by the vehicle being partially serviceable has also been indicated.

Keywords: *reliability, semi-Markov process, motor vehicle.*

W artykule scharakteryzowano ogólnie własności procesów semimarkowskich i uzasadniono możliwości ich zastosowania do określenia niezawodności samochodów i innych pojazdów drogowych. Przedstawiono formalny opis procesu zmian stanów technicznych samochodów uznanych za stany niezawodnościowe oraz model tego procesu w postaci jednowymiarowego procesu stochastycznego. Wartościami tego procesu są występujące w czasie eksploatacji stany techniczne samochodów, mające istotne znaczenie praktyczne. Przyjęto czterostanowy zbiór stanów o następującej interpretacji: stan zdatności pełnej (całkowitej), stan zdatności częściowej (niepełnej, niecałkowitej), stan niepełnej zdatności zadaniowej i stan niezdatności pełnej (całkowitej). Na podstawie przyjętego rozkładu początkowego i opracowanej macierzy funkcyjnej został określony rozkład graniczny procesu zmian stanów technicznych (niezawodnościowych) samochodu. Prawdopodobieństwo istnienia stanu zdatności pełnej (całkowitej) samochodu zostało uznane za miarę jego niezawodności w długim okresie czasu eksploatacji. Wskazano też na możliwość określenia niezawodności samochodu w formie prawdopodobieństwa, w którym uwzględniony został przypadek wykonania zadania przez samochód także wtedy, gdy znajduje się on w stanie zdatności częściowej.

Słowa kluczowe*: niezawodność, proces semi-Markowa, samochód.*

1. Introduction

In paper [12], a single-state reliability model of a passenger car was presented where one state of serviceability and ten states of unserviceability of the car were singled out, with the latter covering the cases where any of the major functional components of the car would fail. The following major functional components were selected there: 1) engine with fuel, lube oil, and coolant feeding systems; 2) clutch; 3) gearbox; 4) drive shaft; 5) driving axle; 6) steering and suspension system; 7) braking system; 8) electrical system; 9) body with chassis; and 10) measuring and monitoring equipment. An important good point of such a model is the fact that it reflects the serial reliability structure of the vehicle type considered and that it has arisen from the use of alternative classification of the vehicle reliability states into the state of serviceability s_0 and the states of unserviceability s_i (i = 1, 2, 3, …, 10), the latter constituting a set of unserviceability states S_n, i.e. $S_n = \{s_1, s_2, s_3, \ldots, s_{10}\}$. However, in the practice of operation of motor vehicles, understood as both passenger cars and delivery vehicles, the states of partial serviceability, i.e. those intermediate between the full serviceability and unserviceability of the vehicle, may also be important; even the information whether the vehicle is fully serviceable may be of considerable value. In such a case, the approach to the reliability issue may be similar to that presented in paper [11], where the problem of building a model of the process of operation of diesel engines was addressed. The theory of semi-Markov processes was used to develop this model, too, as it was in the case described in [12]. This is important and, simultaneously, worth being emphasized here inasmuch as diesel engines are applied to some motor vehicles, both passenger cars and delivery truck or vans, and to other road transport facilities, e.g. buses etc. The process of occurrence of specific reliability states of motor vehicles is closely connected with the technical condition of the vehicles under consideration and it is one of the most important processes taking place during the vehicle operation stage. This process is composed of the technical states of a specific vehicle, following each other in succession and being causally connected together in time. It is obvious that the course of this process should be reasonable, i.e. it should be dictated by the optimizing criterion adopted, e.g. the expected value of the cost of operation of a vehicle of the specific type (which is important for passenger cars and delivery vehicles) or the any-time startability coefficient (which a top-priority parameter in the case of ambulances, fire-fighting vehicles, or police patrol cars). For the emergency vehicles mentioned here, a task may

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** be ordered at any time where immediate engine start and correct (reliable) functioning of all other equipment of such vehicles would be essential [8, 10]. Information about the reliability of motor vehicles of various types is critical for operating specific vehicles in a reasonable way. One of the most important indicators describing the reliability of motor vehicles is the probability that the vehicles will function correctly. For the solving of many problems related to the reliability of various devices, the theory of semi-Markov processes is successfully used more and more often. This theory may also be applicable when similar motor vehicle reliability problems are to be solved [2, 12]. Therefore, a semi-Markov reliability model of any motor vehicle with a four-state set of reliability states has been proposed here, with the interpretation of the reliability states having been presented in item 2 of this paper.

The semi-Markov processes are stochastic processes having special properties. Different definitions of such processes, with different ranges of generalization and exactitude, can be found in relevant publications. For the purposes of modelling the occurrence of specific technical states as reliability states of motor vehicles, the semi-Markov process (a family of random variables $\{W(t): t \geq 0\}$) may be defined with the use of the so-called homogenous Markov renewal process [1, 3, 13, 14, 17].

According to its definition, the semi-Markov process is a stochastic process with a discrete set of states and its realizations are constant functions in individual intervals (the functions having constant values within specific intervals of the operation time are random variables), continuous on the right. This definition also implies that this process is definite when its initial distribution $P_i = P\{Y(0) = s_i\}$ and functional matrix $\mathbf{Q(t)} = [Q_{ij}]$ are known, with the matrix being so defined that its elements are probabilities of transition from state s_i to state s_j within time not exceeding t ($i \neq j$; $i, j = 1, 2, \ldots, k$) and the probabilities are non-decreasing functions of variable t, which are denoted by $Q_{ij}(t)$ [4, 10].

A semi-Markov model of any real process may only be built when the states of this process can be so defined that the time of duration of the state existing at instant τ_n and the state obtainable at instant τ_{n+1} would not stochastically depend on the states that took place previously and on the time intervals of duration of those states.

The construction of a semi-Markov model $\{W(t): t \geq 0\}$ of a real process of changes in the technical states as reliability states of any motor vehicle is prerequisite for the applicability of the theory of semi-Markov processes. Such models are characterized by the following [5, 7, 8, 13, 14]:

1) The Markov condition is met, according to which the future evolution of the state of any motor vehicle as a research specimen (the process of changes in the reliability states during the vehicle operation stage) for which the semi-Markov model has been built should only depend on the vehicle state at the specific instant instead of the functioning of the same vehicle in the past, i.e. the *future* of the vehicle should exclusively depend on its *present* instead of its *past*.
2) The random variables T_i (representing the time of duration of state s_i regardless of the state to follow) and T_{ij} (representing the time of duration of state s_i providing that the next state of the process is s_j) have distributions other than exponential.

This means that the modelling aimed at designing a semi-Markov model of the process of changes in the technical states considered reliability states of motor vehicles should be done with taking into account an analysis of changes in the states of the real process, i.e. changes in the reliability states that occur during the stage of operation of the vehicles involved.

2. Formulation of the problem of reliability of a motor vehicle

The passenger car, like any other present-day road vehicle, is a complex technical system (Fig. 1), consisting of many elements having specific durability and reliability, which have been grouped into the functional components mentioned in the introduction [18].

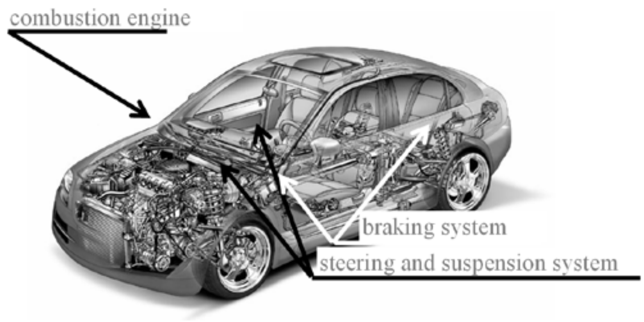

Fig. 1. Overall view of a passenger car, with indicated examples of the major functional components that are critical for the safety of car movement

The vehicle can function correctly (reliably) if all its elements, i.e. actually all its functional components, operate reliably when the vehicle is driven. However, such a situation during the vehicle operation stage is a probable rather than certain event. The probability of correct functioning of the vehicle decreases with time. The knowledge of this probability is important, especially after a longer time of the vehicle operation. When this probability is known, the preventive maintenance of the vehicle may be reasonably planned, because the knowledge of this probability and of the costs that might arise from a vehicle failure makes it possible to determine the risk of a failure to carry out a transport task. The probability of correct (reliable) functioning of a motor vehicle for a longer time of vehicle operation (theoretically at $t \to \infty$) may be relatively easily determined when a semi-Markov reliability model of the vehicle has been developed [5, 6, 9, 12, 13, 14]. Such a model represents changes in the technical states of the vehicle, which simultaneously are the reliability states of the vehicle.

3. The semi-Markov model of changes in the technical states considered reliability states of a motor vehicle

For every motor vehicle, as it is in the case of any diesel engine, the process of changes in its technical states is a process in which the time intervals of duration of every state are random variables. Individual realizations of these random variables depend on multiple factors, including the degree of wear of parts of the functional components of the said vehicles. For all the vehicle types, the wear of vehicle parts is weakly correlated with time [4, 7, 11, 16, 19]. This finding made it possible to forecast the technical state of the vehicles under consideration with ignoring the states that took place previously. This means that the semi-Markov process theory may be used to develop the reliability model of motor vehicles and thus, a more appropriate probabilistic mathematical model necessary for determining motor vehicle reliability indicators, especially the probability of correct functioning of the vehicles, may be obtained.

According to the deliberations presented in the introduction to this paper, the process of changes in motor vehicle reliability states $\{W(t): t \geq 0\}$ may be modelled by stochastic processes with a discrete set of states and with continuous time of duration of specific technical states of the vehicles. In mathematical terms, the models considered here as representing the process of changes in the technical state of motor vehicles, like in other technical objects, are functions that map the set of instants T into a set of technical states S. Therefore, for a model like this to be developed, a finite set of changes in the reliability (technical) states of the vehicles must be set up. Having adopted the usability of a motor vehicle for the carrying out of specific tasks as a criterion for the defining of separate states, we may differentiate the following set of classes (subsets) of the technical states, the classes being simply named "states" (that have significant importance in the vehicle operation practice), which simultaneously are vehicle reliability states [5, 7, 14]:

$$S = \{s_i;\ i = 1, 2, 3, 4\}. \tag{1}$$

Individual states s_i ($i = 1, 2, 3, 4$) being elements of set S should be interpreted as follows:

- s_1 – state of full (complete) serviceability, i.e. the technical state of any motor vehicle in which the vehicle may be used in the whole range of its capacity for which it was prepared at the designing and manufacturing stage;
- s_2 – state of partial (incomplete) serviceability, i.e. the technical state of a motor vehicle in which the vehicle may be used in the whole range of its capacity as it is in state s_1, but with significantly higher fuel consumption due to excessive engine wear or with increased braking distance due to wear of the brake mechanism (braking system) (Fig. 1);
- s_3 – state of task-limiting serviceability, i.e. the technical state of a motor vehicle in which the vehicle makes it possible to carry out only selected tasks, e.g. the state that has arisen from such a degree of wear of the combustion engine that higher vehicle speeds cannot be achieved (Fig. 1);
- s_4 – state of complete (total) unserviceability, i.e. the technical state of a motor vehicle in which the vehicle cannot be operated at all due to failure of the engine, brake mechanism (braking system), steering mechanism (steering system), suspension system, etc. (Fig. 1).

The state of complete or total unserviceability (s_4) is a result of a random event that is referred to as a total failure having occurred. Examples of such events may be [10, 18]: seizure of pistons in cylinders of a combustion engine, causing the pistons and crankshaft to be immobilized; burst of a brake hose, causing the brake fluid to leak out; deformation of a stub axle or a suspension arm, due to which the vehicle drive direction cannot be maintained; shearing failure of the key of an axle shaft spindle, due to which the road wheels cannot be driven, etc. The states of partial serviceability (s_2) or task-limiting serviceability (s_3) result from random events referred to as partial failures (or, often, minor defects or shortcomings), such as e.g. significant wear of injection equipment, pistons with piston rings, camshaft cams, thermostat failure, perforation of an exhaust silencer, fracture of a spring in the torsional vibration damper causing noisy operation of the clutch when disengaged, significant wear of the steering mechanism causing excessive steering play, etc.

Elements of set $S = \{s_i;\ i = 1, 2, 3, 4\}$ are values of process $\{W(t): t \geq 0\}$, which is composed of states $s_i \in S$ following one another in succession and being connected with each other by causality. A realization of such a process taken as an example has been presented in Fig. 2.

For motor vehicles, the differentiation between states $s_i \in S$ ($i = 1, 2, 3, 4$) is important inasmuch as it is essential for the vehicles to be only used when they are in state s_1 or, exceptionally, s_2. In the latter case, however, the vehicles should only be used for a time as short as possible and immediately after thatbe subjected to renovation.

This process is fully definite if its functional matrix is known [6, 9, 13]:

$$Q(t) = [Q_{ij}(t)], \tag{2}$$

with the non-zero elements of the matrix being interpreted as follows:

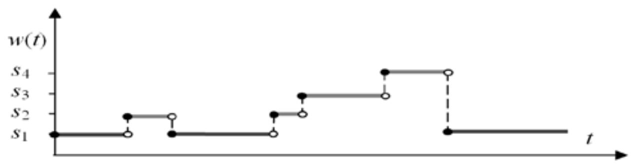

Fig. 2. An example of realizations of process {W(t): t ≥ 0} representing changes in the reliability states of a motor vehicle

$$Q_{ij}(t) = P\{W(\tau_{n+1}) = s_j,\ \tau_{n+1} - \tau_n < t \mid W(\tau_n) = s_i\};\ s_i, s_j \in S;\ i, j = 1, 2, 3, 4;\ i \neq j,$$

and when the initial distribution of this process is given:

$$p_i = P\{W(0) = s_i\},\ s_i \in S;\ i = 1, 2, 3, 4. \tag{3}$$

Depending on the strategy of maintaining the vehicles in a state that would enable them to carry out the tasks for which they were prepared at the designing and manufacturing stage, different variants of realizations of process $\{W(t): t \geq 0\}$ may be taken into account [11]. In the case of, especially, passenger cars and other vehicles prepared for the transportation of people, in consideration of the safety of motion of such vehicles, the most important variant is the one where the initial distribution of process $\{W(t): t \geq 0\}$ is as shown below:

$$p_1 = P\{W(0) = s_1\} = 1,\quad p_i = P\{W(0) = s_i\} = 0 \quad \text{for } i = 2, 3, 4 \tag{4}$$

and its functional matrix has the following form:

$$\mathbf{Q(t)} = \begin{bmatrix} 0 & Q_{12}(t) & 0 & 0 \\ Q_{21}(t) & 0 & Q_{23}(t) & 0 \\ Q_{31}(t) & 0 & 0 & Q_{34}(t) \\ Q_{41}(t) & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

In this variant, an assumption is made that the vehicle operation is started when the vehicle is in the state of full serviceability (s_1). When the vehicle state changes into the state of partial serviceability (s_2), the vehicle is still used for the time sufficient to complete the transport task that has already been undertaken. The vehicle transition from state s_1 to state s_2 is a random event, which occurs with a probability of $p_{12}$when time T_{12}, which is a random variable, has elapsed. State s_2 lasts for time T_2, which is a random variable, too. When the current task is completed, the vehicle should be renovated by being subjected to an appropriate servicing procedure. Otherwise, when the vehicle use is continued, its technical state will change into the state of task-limiting serviceability (s_3), which may prevent the completion of the next task. The vehicle being in state s_3 should be renovated so that it is brought back to state s_1. The above means that a principle should be observed at the operation of motor vehicles to renovate the vehicles fully rather than partially. For this reason, the transition probabilities p_{32}, p_{42}, and p_{43} are zero (i.e. $p_{32} = 0$, $p_{42} = 0$, and $p_{43} = 0$), which has been taken into account in functional matrix (5) [11].

Functional matrix (5) represents changes in states $s_i \in S$ ($i = 1, 2, 3, 4$) of process $\{W(t): t \geq 0\}$. The matrix shows that these states can change as illustrated in the transition graph in Fig. 3.

According to the theory of semi-Markov processes [10, 13, 14], the probabilities of changes in the states of any technical object, i.e. of a motor vehicle as well, are defined by probabilities p_{ij} of transitions in the Markov chain $\{W(\tau_n): n = 0, 1, 2, ...\}$ inserted in process $\{W(t): t \geq 0\}$. These probabilities may be arranged in the following matrix of the probabilities of transitions:

$$P = [p_{ij};\ i,j = 1, 2, 3, 4] \tag{6}$$

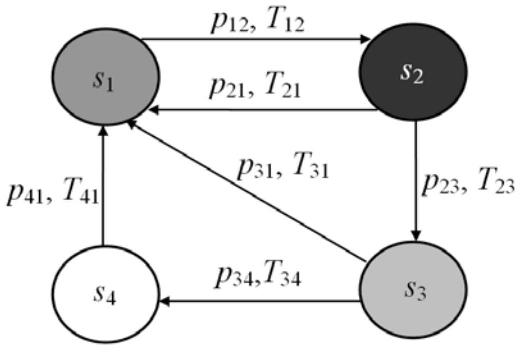

Fig. 3. A graph of changes in states $s_i \in S$ (i = 1, 2, 3, 4) of process $\{W(t): t \geq 0\}$

where: $p_{ij} = P\{W(\tau_{n+1}) = s_j \mid W(\tau_n) = s_i\} = \lim_{t\to\infty} Q_{ij}(t)$.

Matrix (6) makes it possible to determine the boundary distribution of process $\{W(t): t \geq 0\}$, the general interpretation of which is as given below:

$$P_j = \lim_{t\to\infty} P\{W(t) = s_j\} = \lim_{t\to\infty} P\{W(t) = s_j / W(0) = s_i\} \tag{7}$$

It follows from matrix (5) that this matrix has the form:

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ p_{21} & 0 & p_{23} & 0 \\ p_{31} & 0 & 0 & p_{34} \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

From the theorem given in publication [14] on page 40, it is evident that a boundary distribution (7) of the process under consideration exists, which is defined by the formula [10, 12, 13]:

$$P_j = \frac{\pi_j E(T_j)}{\sum_{k=1}^{4} \pi_k E(T_k)}; \quad i = 1,\ 2,\ 3,\ 4 \tag{9}$$

with the boundary distribution π_j (j = 1, 2, 3, 4) of the Markov chain $\{W(\tau_n): n = 0, 1, 2, ...\}$,inserted in process $\{W(t): t \geq 0\}$, meeting the following equations:

$$\left.\begin{aligned} &[\pi1, \pi2, \pi3, \pi4]\cdot\begin{bmatrix} 0 & 1 & 0 & 0 \\ p_{21} & 0 & p_{23} & 0 \\ p_{31} & 0 & 0 & p_{34} \\ 1 & 0 & 0 & 0 \end{bmatrix} = [\pi1, \pi2, \pi3, \pi4] \\ &\pi1+\pi2+\pi3+\pi4 = 1 \end{aligned}\right\} \tag{10}$$

Having solved the system of equations (10) and with making use of relation (9), we will obtain the following formulas:

$$P_1 = E(T_1)M^{-1},\ P_2 = E(T_2)M^{-1},\ P_3 = p_{23}E(T_3)M^{-1},\ P_4 = p_{23}p_{34}E(T_4)M^{-1} \tag{11}$$

with: $M = E(T_1) + E(T_2) + p_{23}E(T_3) + p_{23}p_{34}E(T_4)$,

where:

$E(T_j)$ – expected value of the time of duration of state $s_j \in$ S (j = 1, 2, 3, 4);

p_{ij} – probability of the transition of process $\{W(t): t \geq 0\}$ from state s_i to state s_j (s_i, $s_j \in S$; i,j = 1, 2, 3, 4; $i \neq j$);

P_j – probability of process $\{W(t): t \geq 0\}$ being in state s_j (j = 1, 2, 3, 4).

Individual probabilities P_j (j = 1, 2, 3, 4) defined by formulas (11) should be interpreted as follows:

$$P_1 = \lim_{t\to\infty} P\{W(t) = s_1\},\ P_2 = \lim_{t\to\infty} P\{W(t) = s_2\},\ P_3 = \lim_{t\to\infty} P\{W(t) = s_3\},$$
$$P_4 = \lim_{t\to\infty} P\{W(t) = s_4\}$$

In the presented variant of changes in the differentiated reliability states of motor vehicles of any kind, the situations have also been taken into account where the user may risk thecarrying outof a task when the specific vehicle is in state s_2 (state of partial serviceability) or even to risk the completion of a task having been undertaken when the vehicle already is in state s_3 (task-limiting serviceability). The vehicle reliability may be measured by the probability of the vehicle being in state s_1, i.e. in the state in which it may be used in the whole range of its capacity, for a longer period of vehicle operation. In the situation where the transport task may be carried out by the vehicle being in state s_2, the vehicle reliability may be defined by probability $P = P_1 + P_2$. Probabilities P_3 and P_4 may, and should be, interpreted as the probabilities of an event that the vehicle would fail to carry out a task if it were previously in a state of long-time operation.

4. Recapitulation

The deliberations presented have shown that the process of changes in technical (reliability) states of motor vehicles is a stochastic process,which is discrete relative to the states and continuous relative to time,with four states interpreted as follows to be taken into account: s_1 – state of full (complete) serviceability, s_2 – state of partial (incomplete) serviceability, s_3 – state of task-limiting serviceability, and s_4 – state of complete (total) unserviceability. Actually, the technical state of every motor vehicle changes continuously; therefore, countable sets of vehicle states, i.e. sets consisting of an infinite number of elementary technical states, may be considered. The recognition of all the technical states of motor vehicles is neither possible nor advisable, for both technical and economic reasons. Hence, a need exists to divide the set of the technical states into a small number of classes (subsets) of the states. Having adopted the usability of a motor vehicle for the carrying out of specific tasks as a criterion to define separate states, we may differentiate, as mentioned before,certain classes (subsets) of the elementary technical states,with the classes being grouped into a set of states $S = \{s_1, s_2, s_3, s_4\}$, which may be considered a set of values of a stochastic process $\{W(t): t \in T\}$ whose realizations are constant in intervals and continuous on the right. Thus, this process is, in mathematical terms, a function mapping the set of instants T into a set of technical states S.

The semi-Markov processes used as models of real processes of operation of various technical objects are convenient tools in the research practice. They may also be suitable for the analysing of reliability of motor vehicles. This leads to a conclusion that the designing of a semi-Markov model of the process of operation of a technical object of any kind will provide a possibility of easy (thanks to the existing theory of semi-Markov processes) determining of probabilistic characteristics of motor vehicles.

The semi-Markov processes are more useful in practice as models of the real processes of changes in the states of technical objects than the Markov processes are. This is because the semi-Markov processes, where time is a continuous parameter and a finite set of states is considered, are characterized by the fact that the time intervals during which the processes remain in specific states are random variables with any distributions concentrated in the set $R_+ = [0, \infty)$. This feature distinguishes them from the Markov processes, where the intervals are random variables with exponential distributions.

An additional benefit of the use of semi-Markov processes (as it is when the Markov processes are used) is the fact that professional computer tools are available that make it possible to solve various systems of equations of states for such models of real processes.

References

1. Benjamin J.R, Cornell C.A. Rachunek prawdopodobieństwa, statystyka matematyczna i teoria decyzji dla inżynierów. Warszawa: WNT, 1977. Dane o oryginale: Probability, Statistics, and Decision for Civil Engineers. Copyright by McGraw-Hill, Inc., 1970
2. Cempel C, Natke H. G, Yao J.P.T. Symptom reliability and hazard for systems conditio motoring. Mechanical Systems and Signal Processing 2000; 14(3): 495–505.
3. Cinlar E. Markov reneval theory: a survey. Mag. Sciences 1975; 7: 727–752.
4. Gercbach I.B, Kordonski Ch.B. Modele niezawodnościowe obiektów technicznych. Warszawa: WNT, 1968.
5. Girtler J. Stochastyczny model procesu eksploatacji okrętowego silnika spalinowego. Zagadnienia Eksploatacji Maszyn 1989; 2: 79–88.
6. Girtler J. Możliwości zastosowania i przydatność procesów semimarkowskich jako modeli procesów eksploatacji maszyn. Zagadnienia Eksploatacji Maszyn 1996; 3: 419–428.
7. Girtler J. Diagnostyka jako warunek sterowania eksploatacją okrętowych silników spalinowych. Szczecin: WSM, 1997.
8. Girtler J.: Sterowanie procesem eksploatacji okrętowych silników spalinowych na podstawie diagnostycznego modelu decyzyjnego. Gdynia: AMW, 1989.
9. Girtler J. Physical aspects of application and usefulness of semi-Markovian processes for modeling the processes occurring in operational phase of technical objects. Polish Maritime Research 2004; 3: 7–12.
10. Girtler J. Operation of diesel engines as the index of their reliability and safety. Journal of KONES. International Combustion Engines 2003; 10(1-2): 17–24.
11. Girtler J. Semi-Markov model of diesel engines' operating process. The Archives of Automotive Engineering (Archiwum Motoryzacji) 2044; 3:165–196.
12. Girtler J, Ślęzak M. Application of the theory of semi-Markov processes to the development of a reliability model of an automotive vehicle. The Archives of Automotive Engineering (Archiwum Motoryzacji) 2012; 56(2): 15–27.
13. Grabski F. Teoria semi-markowskich procesów eksploatacji obiektów technicznych. Gdynia: AMW, 1982.
14. Grabski F. Semi-markowskie modele niezawodności i eksploatacji. Warszawa: IBS PAN, 2002.
15. Korczewski Z. Entropy function application In the selection process of diagnostic parameter of marine diesel and gas turbine engines. Polish Maritime Research 2010; 17(2): 29–35.
16. Niewczas A. Podstawy stochastycznego modelu zużywania poprzez tarcie w zagadnieniach trwałości elementów maszyn. Radom: Politechnika Radomska, 1989.
17. Pyke R. Markov reneval processes with finitely many states. Ann. Math. Statist. 1961; 32(4): 1243–1259.
18. Trzeciak K. Diagnostyka samochodów osobowych. Warszawa: WKiŁ, 1991.
19. Zwierzycki W. Prognozowanie niezawodności zużywających się elementów maszyn. Radom: ITE, 2000.

Prof. Jerzy GIRTLER, Ph.D. (Eng.)
Department of Ship Power Plants
The Gdansk University of Technology
ul. G. Narutowicza nr 11/12, 80-233 Gdańsk, Poland
e-mail: jgirtl@pg.gda.pl

Marek ŚLĘZAK, M.Sc. (Eng.)
Automotive Industry Institute
ul. Jagiellońska nr 55, 03-301 Warszawa, Poland
e-mail: m.slezak@pimot.org.pl