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MASKING OF FERROMAGNETIC ELLIPTICAL SHELL IN TRANSVERSE MAGNETIC FIELD

A ferromagnetic object, located in the Earth's magnetic field, changes its distribution. Through measuring such disturbances, one can locate the object and destroy it. To conceal the object, a special winding is placed inside its ferromagnetic shell, and its task is to eliminate the disturbances in the distribution of the Earth's magnetic field. A thin walled elliptical shell, made of ferromagnetic material, is examined as the object model. There are coils, placed inside the shell, and their task is to generate a magnetic field, which is eliminating the effect, the shell is making on the distribution of the Earth's magnetic field in the surrounding area. Such a procedure is called magnetic masking and the winding used for this purpose is called the masking winding. The possibility of building the masking windings for the ferromagnetic elliptical shell, situated in a transverse magnetic field respectively to its major axis, is also examined. The solution of Maxwell's equations, which are describing the magnetic field distribution caused by the ferromagnetic shell presence in the Earth's magnetic field, is found. Furthermore, the ability of selecting coils, which are eliminating the perturbations of the magnetic field outside the shell completely, is proven.

1. INTRODUCTION

Usually, ship hulls are constructed of ferromagnetic steel. The ferromagnetic hull causes significant disturbances in the Earth's magnetic field distribution. These disturbances are detected at a considerable distance outside the ship and they allow to recognize it. In order to avoid the influence of the ferromagnetic hull on the Earth's magnetic field distribution, specially shaped windings are placed inside the hull. The magnetic field, generated by these windings, reduces the changes in the magnetic field distribution outside the ship significantly. In this paper, one considers the ship hull model as an elongated ellipsoid shell of a circular cross section in the plane z = const and the shell thickness δ (Fig. 1). Due to the fact that the Earth's magnetic field flux does not exceed a value of about $50\mu T$, it is assumed that the shell material has the linear magnetization characteristics. The adoption of the linear magnetization characteristics allows

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considering the masking winding for each of the three components of the Earth's magnetic flux separately. The calculation of the masking coil for the component in the z axis (Fig. 1) is shown in paper [1]. This paper presents a calculation of masking windings for the field B_0 directed along the x axis (Fig. 1).

2. MATHEMATICAL MODEL

One uses a system of elliptical coordinates η, θ, φ, which are related to the rectangular coordinates x, y, z associated with the ship (Fig. 1) with the following dependencies:

$$\begin{aligned} x &= a \sinh\eta \sin\theta \cos\varphi \\ y &= a \sinh\eta \sin\theta \sin\varphi \\ z &= a \cosh\eta \cos\theta \end{aligned} \tag{1}$$

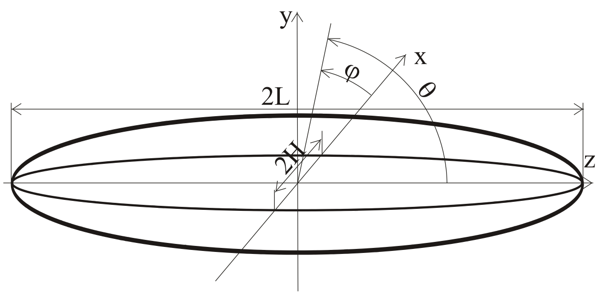

Fig. 1. The ship model and its own rectangular coordinates system x, y, z and the elliptical ones θ, φ

where the constant a is given by the formula:

$$a = \sqrt{L^2 - H^2} \tag{2}$$

and 2L and 2H are the length and width of the ship model respectively.

The outer shell surface is defined by the constant elliptical coordinate η_2:

$$\eta_2 = 0.5 \ln\left(\frac{L+H}{L-H}\right) \tag{3}$$

Maintaining a constant shell thickness δ is not possible in the same elliptic coordinates as are assumed for the outer surface. The shell thickness was assumed as δ in the middle of the ship hull length for the coordinate η_1, which defines the location of the inner shell surface:

$$\eta_1 = \ln\left[\frac{H-\delta}{a} + \sqrt{\left(\frac{H-\delta}{a}\right)^2 + 1}\right] \tag{4}$$

Assuming, that the Earth's magnetic flux in the space surrounding the ship is a uniform field B_0 directed along the x axis of the ship's own rectangular coordinate, the Earth's flux has following components in the elliptical coordinate system:

$$\begin{aligned}
B_{0\eta} &= \frac{B_0 \cos\varphi \cosh\eta \sin\theta}{\sqrt{\sinh^2\eta + \sin^2\theta}} \\
B_{0\theta} &= \frac{B_0 \cos\varphi \sinh\eta \cos\theta}{\sqrt{\sinh^2\eta + \sin^2\theta}} \\
B_{0\varphi} &= -B_0 \sin\varphi
\end{aligned} \tag{5}$$

The magnetic flux $\mathbf{B_n}$ in each of the areas is presented as the sum of the Earth's magnetic flux $\mathbf{B_0}$ and $\mathbf{B_{fn}}$, hereinafter referred as the perturbation magnetic flux, which is caused by the presence of the ferromagnetic shell:

$$\mathbf{B}_n = \mathbf{B}_0 + \mathbf{B}_{fn} \tag{6}$$

The components of the magnetic flux $\mathbf{B_0}$ in the elliptical coordinates are determined by the relationships (5). The magnetic perturbation flux, while assuming linear magnetization of the elliptical ferromagnetic shell, fulfils the Maxwell's equations in each of the three subareas:

$$\nabla \times \mathbf{B}_{fn} = 0 \qquad \nabla \cdot \mathbf{B}_{fn} = 0 \tag{7}$$

where n=1 – the area inside the ellipsoid, n = 2 – the inside of the ferromagnetic wall of the ellipsoid and n=3 – the area outside the ellipsoid.

The continuity of the normal magnetic flux components occurs on the borders between particular areas:

$$B_{f1\eta} = B_{f2\eta}\Big|_{\eta=\eta_1} \qquad B_{f2\eta} = B_{f3\eta}\Big|_{\eta=\eta_2} \tag{8}$$

Whereas, in the case of the condition for the continuity of the tangential component of the magnetic field, it is assumed that inside the ellipsoid is a thin layer of two masking windings represented as linear currents I_θ and I_φ.

$$\begin{aligned}
B_{f2\theta} - \mu_w B_{f1\theta} &= \mu_0 \mu_w I_\varphi + (\mu_w - 1)\frac{B_0 \cos\varphi \sinh\eta \cos\theta}{\sqrt{\sinh^2\eta + \sin^2\theta}}\Bigg|_{\eta=\eta_1} \\
B_{f2\varphi} - \mu_w B_{f1\varphi} &= \mu_0 \mu_w I_\theta - (\mu_w - 1) B_0 \sin\varphi\Big|_{\eta=\eta_1} \\
B_{f2\theta} - \mu_w B_{f3\theta} &= (\mu_w - 1)\frac{B_0 \cos\varphi \sinh\eta \cos\theta}{\sqrt{\sinh^2\eta + \sin^2\theta}}\Bigg|_{\eta=\eta_2} \\
B_{f2\varphi} - \mu_w B_{f3\varphi} &= -(\mu_w - 1) B_0 \sin\varphi\Big|_{\eta=\eta_2}
\end{aligned} \tag{9}$$

where μ_w – the relative ferromagnetic shell permeability, and $\mu_0 = 4\pi 10^{-7}$ H/m – the magnetic permeability of vacuum.

The winding represented by the linear current $I_\theta(\theta,\varphi)$ is directed along the axis θ, and the second one, with the linear current $I_\varphi(\theta,\varphi)$, is directed along the axis φ. The presence of the masking windings causes a discontinuity of the magnetic field tangential components. Taking into account the linear magnetization characteristics, one obtains the following boundary conditions for the tangential components (9). Taking into account conditions (9), one assumes the linear currents distributions in the form of:

$$I_{\varphi} = \frac{I_{\varphi x}\cos\varphi\cos\theta}{\sqrt{\sinh^2\eta_1 + \sin^2\theta}} \qquad I_{\theta} = I_{\theta x}\sin\varphi \tag{10}$$

After taking into account equations (10), the boundary conditions for the tangential components take the form of:

$$\begin{aligned} &B_{f2\theta} - \mu_w B_{f1\theta} = \left.\frac{\left[\mu_0\mu_w I_{\varphi x} + (\mu_w - 1)B_0\sinh\eta_1\right]\cos\varphi\cos\theta}{\sqrt{\sinh^2\eta_1 + \sin^2\theta}}\right|_{\eta=\eta_1} \\ &B_{f2\varphi} - \mu_w B_{f1\varphi} = \left[\mu_0\mu_w I_{\theta x} - (\mu_w - 1)B_0\right]\sin\varphi\Big|_{\eta=\eta_1} \\ &B_{f2\theta} - \mu_w B_{f3\theta} = (\mu_w - 1)\left.\frac{B_0\cos\varphi\sinh\eta\cos\theta}{\sqrt{\sinh^2\eta + \sin^2\theta}}\right|_{\eta=\eta_2} \\ &B_{f2\varphi} - \mu_w B_{f3\varphi} = (\mu_w - 1)B_0\sin\varphi\Big|_{\eta=\eta_2} \end{aligned} \tag{11}$$

Due to the boundary conditions (11), the magnetic flux components are assumed as follows:

$$B_{f\eta k} = b_{\eta k}(\eta,\theta)\cos\varphi \quad B_{f\theta k} = b_{\theta k}(\eta,\theta)\cos\varphi \quad B_{f\varphi} = b_{\varphi k}(\eta,\theta)\sin\varphi \tag{12}$$

where k=1,2,3.

Substituting the magnetic flux given by the formulas (12) to the rotation equations (7) recorded for the axis η, θ elliptical coordinate system respectively, we obtain:

$$b_{\eta k} = -\frac{\sin\theta}{\sqrt{\sinh^2\eta + \sin^2\theta}}\frac{\partial}{\partial\eta}\left(b_{\varphi k}\sinh\eta\right) \tag{13}$$

$$b_{\theta k} = -\frac{\sinh\eta}{\sqrt{\sinh^2\eta + \sin^2\theta}}\frac{\partial}{\partial\theta}\left(b_{\varphi k}\sin\theta\right) \tag{14}$$

The magnetic flux components expressed by (13), (14) satisfy the third rotation equation for φ axis. Substituting (13) and (14) into the divergence equation (7), we obtain the equation for the flux component $b_{\varphi k}$ in the form:

$$\left(\sinh^2\eta + \sin^2\theta\right)b_{\varphi k} - \sin^2\theta\frac{\partial}{\partial\eta}\left[\sinh\eta\frac{\partial}{\partial\eta}\left(b_{\varphi k}\sinh\eta\right)\right] - \sinh^2\eta\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\left(b_{\varphi k}\sin\theta\right)\right] = 0 \tag{15}$$

Assuming the linear current $I_{\theta x}$ in the form:

$$I_{\theta x} = -\frac{I_{\varphi x}}{\sinh\eta_1} \tag{16}$$

and after taking into account (14), it can be proven that the boundary conditions (11) for the tangential components of θ and φ axes are identical and they can be replaced by the boundary conditions for the surface η_1 and η_2 respectively:

$$
\begin{aligned}
&-b_{2\varphi}+\mu_w b_{1\varphi}=\left.\frac{\mu_0\mu_w I_{\varphi x}+(\mu_w-1)B_0\sinh\eta_1}{\sinh\eta_1}\right|_{\eta=\eta_1}\\
&b_{2\varphi}-\mu_w b_{3\varphi}=-(\mu_w-1)B_0\big|_{\eta=\eta_2}
\end{aligned}
\tag{17}
$$

Taking into account the boundary conditions (17) and the limited values of magnetic flux for $\eta = 0$ i $\eta\to\infty$, the solution is adopted in the following form:

$$
b_\varphi=\begin{cases}C_1 & \text{for } \eta<\eta_1\\ C_2+C_3\dfrac{Q_1^1(\cosh\eta)}{\sinh\eta} & \text{for } \eta_1<\eta<\eta_2\\ C_4\dfrac{Q_1^1(\cosh\eta)}{\sinh\eta} & \text{for } \eta>\eta_2\end{cases}
\tag{18}
$$

where: $Q_1^1(\cosh\eta)=\dfrac{\sinh\eta}{2}\ln\left(\dfrac{\cosh\eta+1}{\cosh\eta-1}\right)-\mathrm{ctgh}\eta$ - the spherical function of the general type of the second kind [2, 3]. Constants C_1, C_2, C_3 and C_4, by the boundary conditions (8) and (17) and taking into account (13), is determined from the system of equations:

$$
\begin{aligned}
C_2+C_3a_1-\mu_w C_1&=-\frac{\mu_0\mu_w I_{\varphi x}}{\sinh\eta_1}-(\mu_w-1)B_0\\
C_2+C_3a_2-\mu_w C_4a_2&=-(\mu_w-1)B_0\\
C_1&=C_2+a_3C_3\\
C_4&=a_4C_2+C_3
\end{aligned}
\tag{19}
$$

where:

$$
a_1=\frac{Q_1^1(\cosh\eta_1)}{\sinh\eta_1},\ a_2=\frac{Q_1^1(\cosh\eta_2)}{\sinh\eta_2},\ a_3=\frac{1}{\cosh\eta_1}\left[\frac{dQ_1^1(\cosh\eta)}{d\eta}\right]_{\eta=\eta_1},\ a_4=\frac{\cosh\eta_2}{\left[\dfrac{dQ_1^1(\cosh\eta)}{d\eta}\right]_{\eta=\eta_2}}.
$$

Solving the system of equation (19), we obtain:

$$
C_2=\frac{\mu_w-1}{\Delta}\left\{\frac{\mu_0\mu_w a_2 I_{\varphi_x}}{\sinh\eta_1}+B_0\left[a_1+(\mu_w-1)a_2-\mu_w a_3\right]\right\}
\tag{20}
$$

$$
C_3=\frac{\mu_w-1}{\Delta}\left[(1-a_2a_4)\mu_w B_0-(\mu_w a_2a_4-1)\frac{\mu_0\mu_w I_{\varphi x}}{(\mu_w-1)\sinh\eta_1}\right]
\tag{21}
$$

where: $\Delta=(\mu_w-1)^2a_2-(\mu_w a_3-a_1)(\mu_w a_2a_4-1)$.

The condition, that perturbation flux equals zero outside the ellipsoid, means that $C_4 = 0$ and, hence, the linear current $I_{\varphi x}$ is:

$$
I_{\varphi x}=\frac{\mu_w-1}{\mu_w}\frac{B_0}{\mu_0}\sinh\eta_1\frac{a_4(a_1-a_2)+\mu_w(1-a_3a_4)}{a_2a_4-1}
\tag{22}
$$

and the linear current $I_{\theta x}$ from (16) takes the value:

$$I_{\theta x} = -\frac{\mu_w - 1}{\mu_w}\frac{B_0}{\mu_0}\frac{a_4(a_1 - a_2) + \mu_w(1 - a_3 a_4)}{a_2 a_4 - 1} \tag{23}$$

3. RESULTS ANALYSIS

In order to investigate the practical feasibility of making the winding masking a ship, the size of the necessary linear currents defined by the relations (20) and (21) is defined. The value of Earth's magnetic flux is assumed as B_0 = 50 μT and the object of moderate size, with the length of 2L = 30m and the width of 2H = 6 m, is examined. In Figure 2a one shows the calculated dependence of the line current on the ferromagnetic ship wall thickness, and in Fig. 2b - the line current dependence on the relative permeability of the steel hull. The presented calculations show that the magnitude of the linear current is, practically, a linear function of both: the shell thickness δ and the relative magnetic permeability μ_w. The amplitude of the current $I_{\theta x}$, determined by the relation (16), will be greater because sinh η_1 is less than one. However, the required line current densities are in the range of 2 kA/m for $I_{\varphi x}$ and 10 kA/m for $I_{\theta x}$. It is worth mentioning, that the increase of the object's geometric dimensions does not result in a significant manner in the increase of the masking currents density.

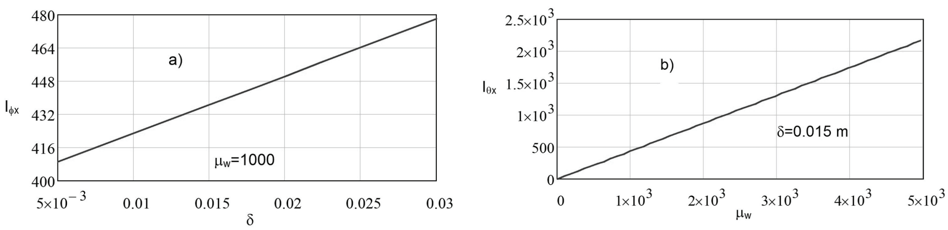

Fig. 2. The line current $I_{\varphi x}$ in A/m as a function of the shell thickness δ in miw (a) and as a function of the relative magnetic permeability μ (b)

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