

# Flow of liquid in flat gaps of the satellite motor working mechanism

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## ABSTRACT

*The article describes the methodology and results of investigations of the flow of oil and HFA-E emulsion in flat gaps of the working mechanism of a satellite motor. The flow of liquid in those gaps is turbulent and not fully developed. The article presents two methods of modelling this flow. Method I makes use of the Darcy-Weisbach formula, while Method II bases on the assumption that in the variable-length gaps the flow is turbulent in the area where the length is the smallest and laminar where the length is the largest. Consequently, the flow in such gaps can be modelled as the sum of laminar and turbulent flows. The results obtained in the experiment have made the basis for calculating relevant coefficients and assessing the proportion of turbulence in the flow modelled using both methods.*

**Keywords:** flow in flat gap; satellite motor; HFA-E emulsion; oil

## INTRODUCTION

For some years, the investigations of hydraulic satellite motors with small working volumes have been performed in the Department of Hydraulics and Pneumatics, Gdansk University of Technology [1,4-12]. The most recent design of the motor, worked out by the author, is shown in Fig. 1.

The working mechanism of the motor is a toothed unit (Figs. 1 and 2) consisting of: the toothed rotor R (4 teeth), the toothed curvature C (6 teeth), and the toothed wheels S (satellites). The number of satellites is equal to the sum of the numbers of rotor teeth and curvature teeth (i.e. 10 satellites in total).

The principle of operation of the satellite mechanism is the following. When the rotor rotates, the volume of the space between the satellites changes. This space is the working

chamber. When its volume increases the filling cycle takes place, while when it decreases we have to deal with the emptying cycle. 24 cycles correspond to one shaft revolution. The chambers in the satellite mechanism are closed by the distribution plates (Fig. 1 - element 7 and 8, and Fig. 2), which also play a role of compensation plates. Thus the satellite motor has an ability to compensate axial clearances of the rotor and satellites.

The satellite motors reveal the smallest mass and overall dimensions as compared to other hydraulic motors. The power [kW] to mass [kg] ratio in those motors exceeds 4. Moreover, these motors can be fed with various liquids, including water.

The satellite motors can be widely used in the shipbuilding industry. For instance, they can be used as rudder drives, anchor

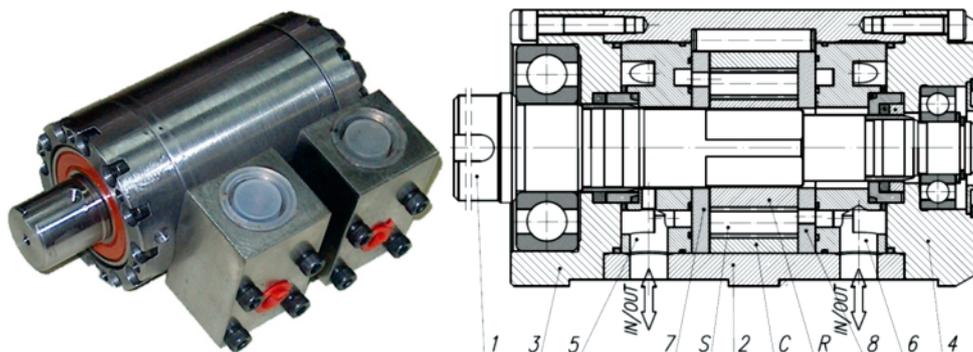


Fig. 1. General view and axial cross section of the motor SM-06: C – curvature, S – satellite, R – rotor; 1 – shaft, 2 – casing, 3 – front casing, 4 – rear casing, 5 and 6 – inlet and exit manifold, 7 and 8 – compensation plates

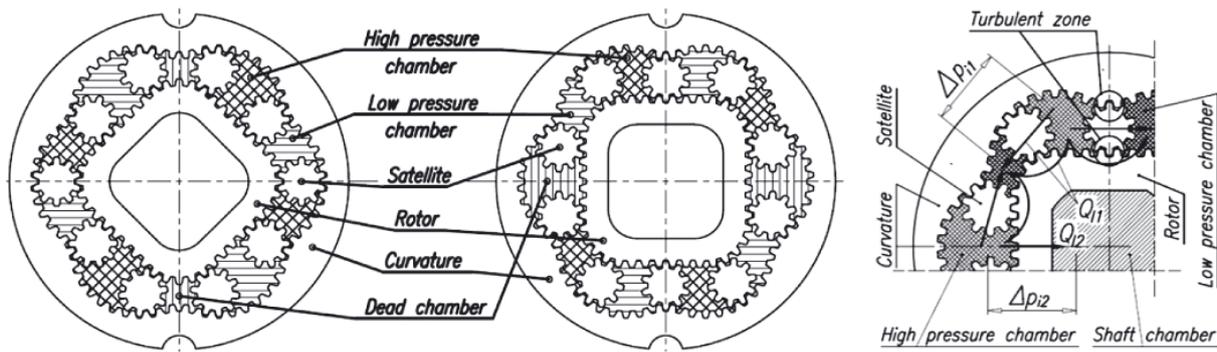


Fig. 2. Working mechanism of satellite motor (left) and the areas of occurrence of gap leakages (right)

hoists, and/or drives of various deck winches and davits. The motors with small working volumes are ideal drives for capstans and capstan winches on yachts (Fig. 3).



Fig. 3. Hydraulic motor driven capstan (up) and capstan winch (down) [13]

Mechanic, pressure and volumetric losses affect the efficiency of energy conversion in the hydraulic motor. It is the leakage flow through flat gaps in the working mechanism which affects the volumetric losses the most (Fig. 1 and Fig. 2). Results of past experimental investigations have made it possible to formulate a hypothesis that the non-fully developed turbulent flow takes place in these gaps. This is especially noticeable when the hydraulic motor is fed with low-viscosity medium, water or HFA-E emulsion for instance. The largest flow disturbance is observed in the areas where the gaps are the shortest, i.e. in the areas of teeth cooperation (Fig. 2).

The literature provides no mathematical models to describe flows in flat gaps with changing length, in particular the flow in the flat gaps of the satellite working mechanism. Publications can only be found which describe the volumetric losses in a global sense, without naming types of gaps [1, 2, 3, 11, 12].

Therefore, a detailed analysis of the flow in flat gaps of the working mechanism of the satellite motor is advisable, as it will provide opportunities for a more detailed simulation of volumetric loss characteristics in the hydraulic motor.

## METHODOLOGY OF EXAMINATION OF THE FLOW IN FLAT GAPS

In order to determine experimentally the flow rate  $Q_{Lfg}$  in flat gaps, we have to measure the absorbing power  $Q$  at small constant velocity  $n$  and at given pressure drop  $\Delta p$  in the motor.

Keeping the rotational speed of the motor low is only possible when it is coupled with the worm gear (Fig. 4). The speed of the examined motor is controlled using the electric motor with a frequency converter. The accumulator damps pressure pulsations in the system.

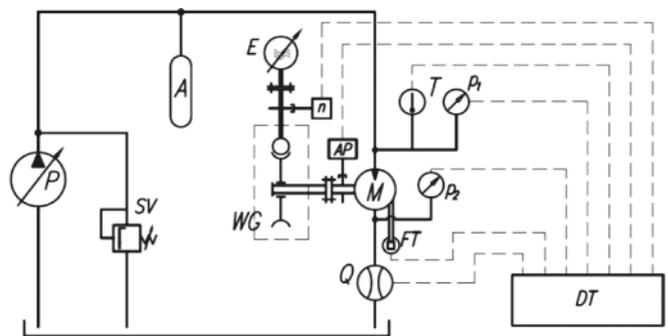


Fig. 4. Scheme of the measuring system:  $M$  – examined motor,  $P$  – pump,  $A$  – accumulator,  $E$  – electric motor with frequency converter,  $SV$  – safety valve,  $WG$  – worm gear,  $DT$  – experimental data recorded,  $Q$  – flow meter,  $FT$  – force sensor (for moment measurement),  $p1$  and  $p2$  – pressure sensors,  $T$  – temperature sensor,  $n1$  – rotational speed sensor,  $AP$  – sensor measuring angular position of the shaft

It is advisable to measure all parameters at very low and constant rotational speed of the shaft. As a result, the characteristic of instantaneous absorbing power  $Q_m$  of the motor as a function of shaft rotation angle is obtained  $\alpha$  (Fig. 5). The absorbing power  $Q_m$  is the sum:

$$Q_m = \frac{q_{th} \cdot n}{Q_{th}} + \frac{Q_{Lfgm} + Q_{Cm}}{Q_{Lm}} \quad (1)$$

where:

- $q_{th}$  – theoretical working volume ;
- $n$  – rotational speed of the shaft;
- $Q_{th}$  – theoretical absorbing power of the motor;
- $Q_{Lm}$  – instantaneous volumetric loss;
- $Q_{Lfgm}$  – instantaneous flow rate in flat gaps;
- $Q_{Cm}$  – instantaneous flow rate in the gaps when inflow and outflow passages are closed by satellites (the flow in short gaps when the dead chamber is created).

We can assume that:

$$Q_{Lfgm} = Q_{Lfg} \quad (2)$$

Where:

- $Q_{Lfg}$  – the average flow rate in flat motor gaps.

Then:

$$Q_{Lfg} + Q_{th} = Q_{fg} \quad (3)$$

The quantity  $Q_{fg}$  can easily be assessed from experimental data (Fig. 5).

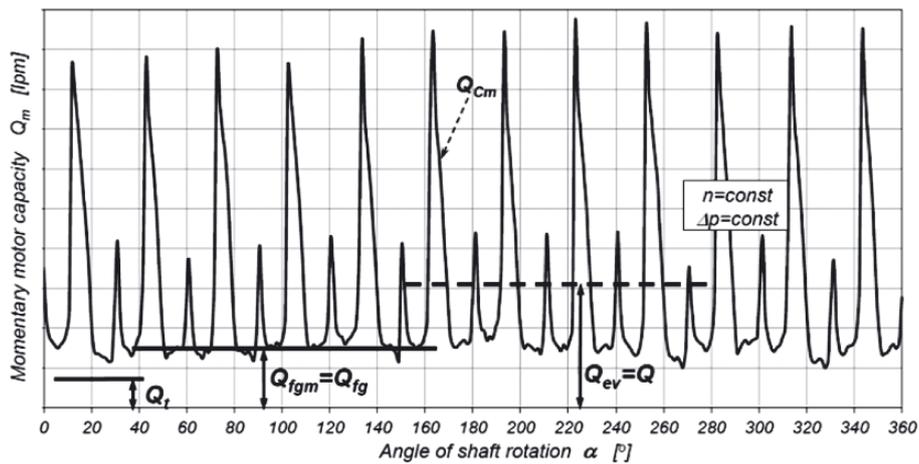


Fig. 5. Characteristics of instantaneous absorbing power  $Q_m$  of the motor vs. shaft rotation angle  $\alpha$

Since the rotational speed of the motor is very small, the pressure drop in inner motor passages can be neglected and:

$$\Delta p_i = \Delta p = p_1 - p_2 \quad (4)$$

The flow rate  $Q_{Lfg}$  in flat gaps is the sum:

$$Q_{Lfg} = Q_{i1} + Q_{i2} \quad (5)$$

where:

- $Q_{i1}$  – flow rate from high pressure chambers to low pressure chambers;
- $Q_{i2}$  – flow rate from high pressure chambers to shaft chamber.

From the shaft chamber the medium is passed to the low-pressure inner outflow chamber. Hence we can assume that:

$$\Delta p_{i1} = \Delta p_{i2} = \Delta p_i \quad (6)$$

where:

- $\Delta p_i$  – pressure drop in working chambers;
- $\Delta p_{i1}$  – pressure drop in satellite and rotor gaps;
- $\Delta p_{i2}$  – pressure drop in rotor gaps between high pressure chambers and the shaft chamber.

The motor examination was carried out at the rotational speed  $n = 1$  rpm.

## PARAMETERS OF EXAMINED MOTORS AND WORKING MEDIA

In the satellite motor the axial clearance  $h_R$  in the rotor is not equal to the axial clearances  $h_S$  of the satellites. In order to simplify the problem description, a definition of the equivalent assembly axial clearance is introduced  $h_o$ :

$$h_o = \frac{h_R + h_S}{2} \quad (7)$$

In the loaded engine the gap height  $h$  is the function of pressure:

$$h = h_o - (D_C \cdot H_C + D_k) \cdot \Delta p_i \quad (8)$$

where:

- $D_k$  – constant dependent on rigidity of the axial clearance compensation unit;
- $D_C$  – constant dependent on curvature rigidity;
- $H_C$  – curvature height.

The constant  $D$  represents the average change of gap height caused by the pressure drop in the motor. The parameters  $D_k$  and  $D_C$  are determined from numerical calculations of

deformation and dislocations of working mechanism and clearance compensation unit elements.

In the satellite working mechanism the shape of the flat gaps is irregular. Moreover, the width  $b$  and the length  $l$  of the gaps change during the operation of the mechanism. All this makes precise description of their dimensions impossible. Therefore an assumption is justified that the gap length  $l$  and width  $b$  are proportional to the module pitch  $m$  of the teeth in the mechanism:

$$l = A_1 \cdot m \quad (9)$$

$$b = A_2 \cdot m \quad (10)$$

Parameters of the examined motors are collected in Tab. 1.

Tab. 1. Parameters of satellite motors

Parameter	Motor			
	SM-0.4/20	SM-0.5/25	SM-0.6/25	SM-0.75/25
$q_t$ [cm <sup>3</sup> /rev]	5.35	14.49	20.86	32.94
$m$ [mm]	0.4	0.5	0.6	0.75
$H_C$ [mm]	20	25	25	25
$h_R$ [μm]	11.25	4.7	4.0	6.38
$h_S$ [μm]	13.6	4.0	3.7	5.35
$D_k$ [1/MPa]	-0.04	-0.1375	-0.13	-0.131
$D_C$ [μm/(MPa·mm)]	0.0064	0.0064	0.0064	0.0064

The motors were examined using:

- the oil Total Azolla 46 ( $\nu = 40$  cSt,  $\rho = 873$  kg/m<sup>3</sup>);
- 1 % emulsion HFA-E made on the basis of the Isosynth VX110BF concentrate (1% concentrate in water) ( $\nu = 0.853$  cSt,  $\rho = 996$  kg/m<sup>3</sup>).

## RESULTS OF EXPERIMENTAL EXAMINATION

The characteristics of oil and emulsion leakage flows in flat motor gaps are shown in Fig. 6, while Fig. 7 presents the characteristics of leakage flow ratios in these gaps.

The results of the experimental examination have revealed that the ratio of the emulsion flow rate to the oil flow rate in these gaps can be described by the equation:

$$\frac{Q_{Lfg,E}}{Q_{Lfg,O}} = E \cdot \Delta p^F \quad (11)$$

where  $E$  is a constant and  $F$  is a coefficient. The values of  $E$  and  $F$  for the examined motors are given in Table 2. In

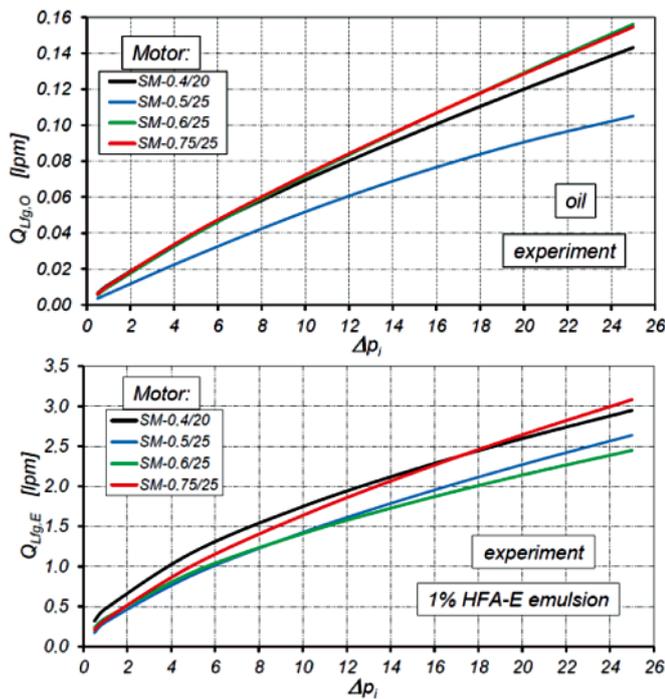


Fig. 6. Characteristics of oil and emulsion leakage flows in flat motor gaps

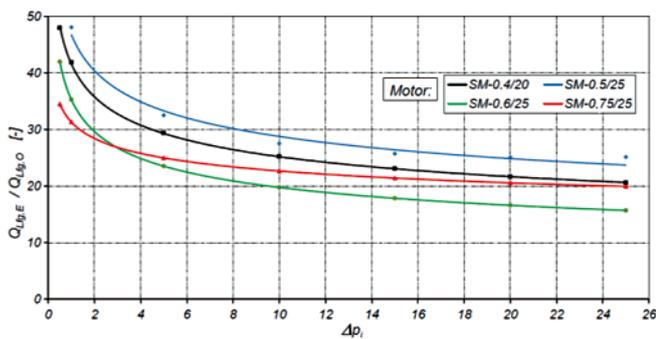


Fig. 7. Ratio of emulsion-to-oil flow rate in motor gaps

order to calculate the constant E it was assumed that the coefficient F should be constant. The value of F assumed for further calculations was calculated as the average value over the experimental data of all motors. On the basis of the experimental data the constant E could be calculated for each motor individually using the least square method (Table 2).

Tab. 2. Values of constants E and F

Parameter		Motor			
		SM-0.4/20	SM-0.5/25	SM-0.6/25	SM-0.75/25
From experiment	E [-]	41.566	46.726	31.5	35.247
	F [-]	-0.218	-0.211	-0.140	-0.252
Calculated	F [-]	-0.20	-0.20	-0.20	-0.20
	E [MPa <sup>0.2</sup> ]	39.941	45.706	32.118	35.0

## MATHEMATICAL MODEL

Below are characterised two methods of modelling the flow in flat gaps of the satellite motor working mechanism. In both methods the following simplifying assumptions were adopted:

- the viscosity of the medium in the gaps is constant. In the real flow, the rise of the medium temperature in the gaps is observed. For instance: at  $\Delta p_i = 25$  MPa the oil flow temperature rises by 12.5 °C, while the water flow

temperature by 6 °C. Consequently, the oil viscosity decreases approximately by 22 % and the emulsion viscosity by about 12 %;

- the gap walls are parallel to each other. In the loaded motor the gap walls can be skewed, especially as a result of action of the axial clearance compensation unit;
- the height of all satellites and the rotor is the same. However, in the real motor the heights of these elements differ from each other and, consequently, the heights of flat gaps are also different;
- the gaps are situated only on one side of a satellite and the rotor. In the real motor in operation, gaps can be observed on both front sides of the rotor and the satellites.

## Method 1

The pressure drop in the gap can be described by the equation [11]:

$$\Delta p_i = \lambda \cdot \rho \cdot \frac{1}{h} \cdot \frac{c^2}{2} \quad (12)$$

where:

- c – velocity of medium flowing in the gap,
- $\rho$  – medium density.

In the literature, for the non-fully developed turbulent flow a number of formulas can be found which describe the loss coefficient  $\lambda$  as a function of the Reynolds number Re. The formula for loss coefficient which is suggested here for further analyses has the form:

$$\lambda = \frac{K}{Re^\beta} \quad (13)$$

The flow velocity in the gap can be described using the following formula:

$$c = \frac{Q_{Lfg}}{A_2 \cdot m \cdot h} \quad (14)$$

while the Reynolds number is [11]:

$$Re = \frac{c \cdot h}{\nu} \quad (15)$$

where:

- $\beta$  – degree of laminarity of the flow (for the laminar flow  $\beta = 1$ , and for the turbulent flow  $\beta = 0$ , i.e.;  $\beta \in <0;1 >$ )
- $\nu$  – kinetic viscosity of the liquid.

After placing Equations (13), (14) and (15) into Equation (12) and further transformations we arrive at:

$$Q_{Lfg} = \left(\frac{1}{\nu}\right)^{\left(\frac{\beta}{2-\beta}\right)} \cdot \left(\frac{1}{K \cdot \rho}\right)^{\left(\frac{1}{2-\beta}\right)} \cdot A_1 \cdot m \cdot \left(\frac{2 \cdot h^3}{A_2 \cdot m}\right)^{\left(\frac{1}{2-\beta}\right)} \cdot \Delta p_i^{\left(\frac{1}{2-\beta}\right)} \quad (16)$$

When analysing the obtained results it was noticed that the pressure increase is accompanied by the increasing level of flow disturbance in the motor gaps, and this disturbance level depends of the viscosity of the liquid. Consequently, the value of the coefficient  $\beta$  will also depend on the viscosity of the liquid and the pressure drop inside the motor. That means that for a given  $\Delta p_i$ :  $\beta_E \neq \beta_O$ . The subscripts O and E refer to the oil and the emulsion, respectively.

The values of the coefficients K,  $A_1$ ,  $A_2$  and  $\beta$  can be calculated from experimental data using the non-linear least



square method with the Levenberg-Marquardt algorithm. In this case we should assume that for oil flow in motor gaps  $\beta_0 \approx 1$ . When the pressure drop in the motor increases, the height  $h$  of the gaps decreases (the effect of action of the compensation unit) and the velocity of the medium flowing in these gaps changes. Therefore we can expect that  $\beta_0 > 1$ .

The relation between  $\beta_E$  and  $\beta_0$  can be calculated from the ratio of the emulsion and oil flows in the motor gaps. Hence, taking into account the relation (16), the ratio of the emulsion to oil flow rates is equal to:

$$\frac{Q_{Lfg,E}}{Q_{Lfg,O}} = \frac{v_O \left(\frac{\beta_O}{2-\beta_O}\right) \cdot (K_O \cdot \rho_O) \left(\frac{1}{2-\beta_O}\right)}{v_E \left(\frac{\beta_E}{2-\beta_E}\right) \cdot (K_E \cdot \rho_E) \left(\frac{1}{2-\beta_E}\right)} \cdot \left(\frac{2 \cdot h^3}{A_2 \cdot m}\right) \left(\frac{1}{2-\beta_E} - \frac{1}{2-\beta_O}\right) \cdot \Delta p_i \left(\frac{1}{2-\beta_E} - \frac{1}{2-\beta_O}\right) \quad (17)$$

$$\cdot \left(\frac{2 \cdot h^3}{A_2 \cdot m}\right) \left(\frac{1}{2-\beta_E} - \frac{1}{2-\beta_O}\right) \cdot \Delta p_i \left(\frac{1}{2-\beta_E} - \frac{1}{2-\beta_O}\right)$$

Comparing the formulas (11) and (17) gives:

$$E = \frac{v_O \left(\frac{\beta_O}{2-\beta_O}\right) \cdot (K_O \cdot \rho_O) \left(\frac{1}{2-\beta_O}\right)}{v_E \left(\frac{\beta_E}{2-\beta_E}\right) \cdot (K_E \cdot \rho_E) \left(\frac{1}{2-\beta_E}\right)} \cdot \left(\frac{2 \cdot h^3}{A_2 \cdot m}\right) \left(\frac{1}{2-\beta_E} - \frac{1}{2-\beta_O}\right) \quad (18)$$

$$F = \frac{1}{2-\beta_E} - \frac{1}{2-\beta_O} \quad (19)$$

Consequently the coefficient  $E$  depends on parameters of the liquid, geometry of the gaps, and the degrees of laminarity of the flows of oil and emulsion. At the same time the constant  $F$  is only dependent on the degrees of laminarity of the emulsion and oil flows in flat gaps of the motor.

From Equation (19) we get:

$$\beta_E = 2 - \frac{2-\beta_O}{1+F \cdot (2-\beta_O)} \quad (20)$$

while after taking into account Equations (19) and (20), Equation (18) takes the form:

$$E = \left(\frac{v_O}{v_E}\right) \left(\frac{\beta_O}{2-\beta_O}\right) \cdot \left(\frac{K_O \cdot \rho_O}{K_E \cdot \rho_E}\right) \left(\frac{1}{2-\beta_O}\right) \cdot \left(\frac{2 \cdot h^3}{K_E \cdot v_E^2 \cdot \rho_E \cdot A_2 \cdot m}\right)^F \quad (21)$$

The solution to this equation is:

$$\beta_O = \frac{2 \cdot \ln\left(E \cdot \left(\frac{1}{2} \cdot \frac{m}{h^3} \cdot K_E \cdot v_E^2 \cdot \rho_E \cdot A_2\right)^F\right) + \ln\left(\frac{K_E \cdot \rho_E}{K_O \cdot \rho_O}\right)}{\ln\left(E \cdot \left(\frac{1}{2} \cdot \frac{m}{h^3} \cdot K_E \cdot v_E^2 \cdot \rho_E \cdot A_2\right)^F\right) + \ln\left(\frac{v_O}{v_E}\right)} \quad (22)$$

Therefore both parameters  $\beta_0$  and  $\beta_E$  are functions of the gap height, which in turn depends on the pressure drop in the motor.

In order to find the relation between coefficients  $K_O$  and  $K_E$ , simplifying assumptions have been adopted that  $\beta_0 \approx 1$  and  $h = h_0$ . In that case from Equation (22) we get:

$$K_O = K_E^{(F+1)} \cdot E \cdot \frac{\rho_E}{\rho_O} \cdot \frac{v_E}{v_O} \cdot \left(\frac{1}{2} \cdot \frac{m}{h_0^3} \cdot v_E^2 \cdot \rho_E \cdot A_2\right)^F \quad (23)$$

The values obtained for the examined motors are given in Table 3, while Fig. 8 shows the characteristics of  $\beta_0$  and  $\beta_E$  as functions of  $\Delta p_i$ .

The calculated results presented in Table 3 indicate that the following simplified relation between the coefficients  $K_O$  and  $K_E$  exists:

$$\frac{K_O}{K_E} = \frac{\rho_E}{\rho_O} \quad (24)$$

Tab. 3. Coefficient of model I at the assumption that  $F = -0,2$

Coefficient/ constant	Motor			
	SM-0.4/20	SM-0.5/25	SM-0.6/25	SM-0.75/25
$A_1 [-]$	---	0.580	0.125	0.145
$A_2 [-]$	---	0.396	0.0393	0.17
$K_O [m/(MPa \cdot s)]$	---	1.141	1.140	1.141
$K_E [\mu m/MPa]$	---	1	1	1

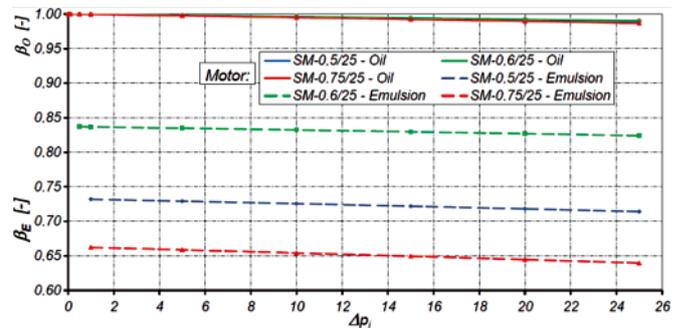


Fig. 8. Degree of laminarity of oil flow  $\beta_0$  and emulsion flow  $\beta_E$  in flat motor gaps

Equation (16), Table 3 and Fig. 8 have made the basis for calculating the characteristics of the oil and emulsion flows in flat gaps of the working mechanisms of the examined motors (Fig. 9).

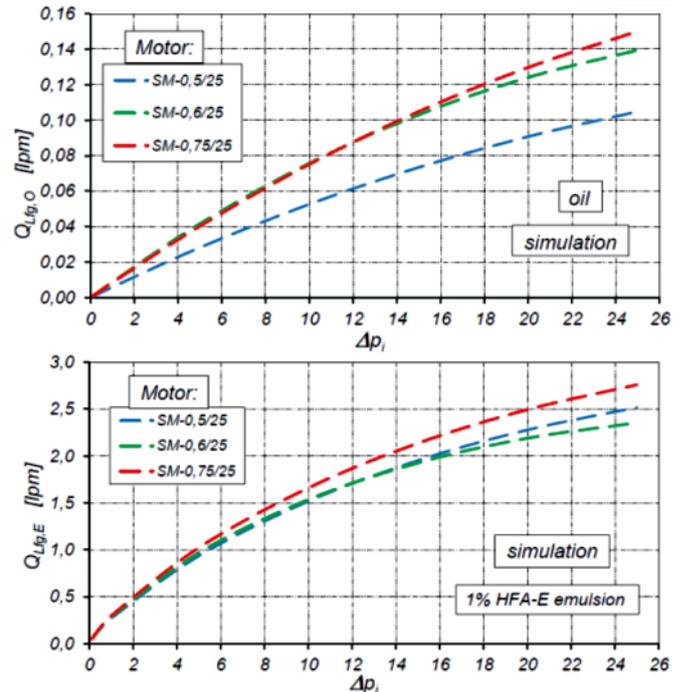


Fig. 9. Characteristics of oil and emulsion leakage flows in flat motor gaps (result of calculations)

For the laminar flow of both the emulsion and the oil we have:  $\beta_0 = \beta_E = 1$ , while for the turbulent flow:  $\beta_0 = \beta_E = 0.5$ . After assuming  $\beta_0 = \beta_E = \beta$  and comparing Equations (11) and (17) we can calculate  $\Delta p_i$  for the laminar and turbulent flows of the both liquids:

$$E \cdot \Delta p_i^F = \left(\frac{v_O}{v_E}\right) \left(\frac{\beta}{2-\beta}\right) \cdot \left(\frac{K_O \cdot \rho_O}{K_E \cdot \rho_E}\right) \left(\frac{1}{2-\beta}\right) \quad (25)$$

For  $\beta = 0$  the limiting pressure above which the turbulent flow of the both liquids takes place is:

$$\Delta p_{i,t} = \sqrt[5]{\frac{1}{E} \cdot \left( \frac{K_E}{K_O} \cdot \frac{\rho_E}{\rho_O} \right)^{0,5}} \quad (26)$$

The quantity  $\Delta p_{i,t}$  takes values which are not reachable in technique. That means that the fully developed turbulent flow of oil will never occur.

For  $\beta = 1$  the limiting pressure below which the laminar flow of the both liquids takes place is:

$$\Delta p_{i,l} = \sqrt[5]{\frac{1}{E} \cdot \frac{v_O}{v_E} \cdot \frac{K_O}{K_E} \cdot \frac{\rho_O}{\rho_E}} \quad (27)$$

The values for  $\Delta p_{i,L}$  are given in Table 4.

Tab. 4.  $\Delta p_{i,L}$  values for the examined motors

Motor:	SM-0.4/20	SM-0.5/25	SM-0.6/25	SM-0.75/25
for F – acc. to experiment	---	0.98	0.058	0.32
for F – 0.20	---	0.87	0.15	0.23

### Method II

This method assumes that:

- in the region of cooperation of working mechanism teeth, where the gap length reaches the minimum, the flow is turbulent;
- in the gaps with maximal length (satellite centre and rotor front) the flow is laminar;
- the region of turbulent flow increases and the region of laminar flow decreases with the increasing pressure drop in the working mechanism:

$$b = \underbrace{X \cdot b}_b + \underbrace{(1 - X) \cdot b}_b \quad (28)$$

where:

$b_l, b_t$  – gap widths corresponding to the laminar and turbulent flow, respectively;

$X$  – gap width weight  $b$ .

The proposed definition of the coefficient  $X$  is:

$$X = e^{-\left(\frac{Re}{C_1}\right)} \quad (29)$$

$$Q_{Lfg} = \underbrace{A \cdot e^{-C \cdot \left(\frac{h_R + h_S}{2} - D \cdot \Delta p_i\right) \frac{\Delta p_i}{v}} \cdot \frac{1}{v} \cdot \frac{1}{\rho} \cdot \left(\frac{h_R + h_S}{2} - D \cdot \Delta p_i\right)^3 \cdot \Delta p_i}_{Q_l} + \underbrace{B \cdot m \cdot \left(1 - e^{-C \cdot \left(\frac{h_R + h_S}{2} - D \cdot \Delta p_i\right) \frac{\Delta p_i}{v}}\right) \cdot \left(\frac{h_R + h_S}{2} - D \cdot \Delta p_i\right) \cdot \sqrt{\frac{2}{\rho}} \cdot \sqrt{\Delta p_i}}_{Q_t} \quad (39)$$

In the above formula the coefficients  $A$  and  $B$  and the constant  $C$  are unknown. Their values for the examined motors were calculated from the experimental data  $Q_{Lfg} = f(\Delta p_i)$  using the nonlinear least square method and the Levenberg–Marquardt algorithm. The values of the coefficients  $A$  and  $B$ , and the constant  $C$  are given in Table 5.

where  $C_1$  is the weight coefficient and the Reynolds number is [11]:

$$Re = c \cdot h / v \quad (30)$$

The average flow velocity  $c$  in the flat gaps of the working mechanism affects the flow rate  $Q_{Lfg}$  and is not known. Therefore the following simplifying assumption is proposed:

$$c = C_2 \cdot \Delta p_i \quad (31)$$

where  $C_2$  is the proportionality constant. Then:

$$X = e^{-\left(C_2 \frac{h}{v} \Delta p_i\right)} \quad (32)$$

where:

$$C = C_2 / C_1 \quad (33)$$

The values of the constant  $C$  should be different for the oil and the emulsion.

The laminar flow  $Q_l$  in the gap can be modelled by the formula:

$$Q_l = X \cdot A \cdot \frac{1}{v} \cdot \frac{1}{\rho} \cdot h^3 \cdot \Delta p_i \quad (34)$$

in which:

$$A = \frac{1}{12} \cdot \frac{b}{l} \quad (35)$$

and  $b_l$  is the gap width in the motor area where the laminar flow is observed.

The turbulent flow  $Q_t$  can be modelled by the formula:

$$Q_t = (1 - X) \cdot B \cdot m \cdot h \cdot \sqrt{\frac{2}{\rho}} \cdot \sqrt{\Delta p_i} \quad (36)$$

In the above formula  $B$  is the proportionality coefficient meeting the condition:

$$B \cdot m = B_1 \cdot b = B_1 \cdot A_2 \cdot m \quad (37)$$

In the discussed case the flow rate  $Q_{Lfg}$  in the flat motor gaps can be expressed as the sum of the laminar component  $Q_l$  and the turbulent component  $Q_t$ :

$$Q_{Lfg} = Q_l + Q_t \quad (38)$$

After relevant replacements, Equation (38) takes the form:

The characteristics of the oil and emulsion flow rates in flat gaps of working mechanisms of the examined motors, worked out using Equation (38) and the data from Table 2, are shown in Fig. 10.

Tab. 5. Values of A, B, C and D

Coefficient/ constant	Motor			
	SM-0.4/20	SM-0.5/25	SM-0.6/25	SM-0.75/25
A <sub>O</sub> [-]	0.17	2.6	5.8	1.5
A <sub>E</sub> [-]	0.082	1.6	2.5	0.73
B <sub>O</sub> [-]	1.6	1.0	0.8	0.6
B <sub>E</sub> [-]	10.0	7.0	6.0	4.1
C <sub>O</sub> [m/(MPa·s)]	0.0035	0.003	0.004	0.004
C <sub>E</sub> [m/(MPa·s)]	0.00048	0.002	0.0026	0.002
D [μm/MPa]	0.088	0.0225	0.03	0.029

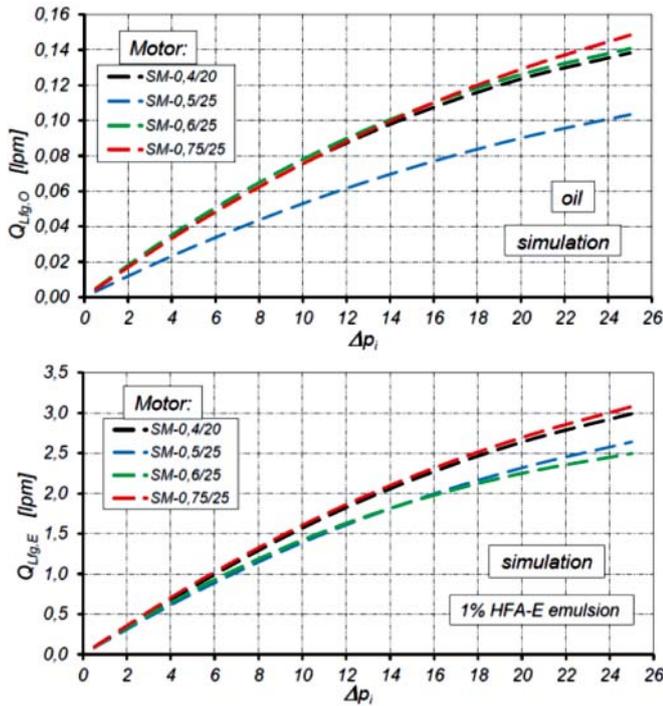


Fig. 10. Characteristics of oil and emulsion flow rates in flat gaps of the working mechanism (results of calculations)

## DISCUSSION

The results of examination of the selected motors have proved that for a given pressure drop in flat gaps of the working mechanism of the motor, the flow rate of the 1 % HFA-E emulsion is at least from ten to twenty times larger than the oil flow rate. The dynamic viscosity of the oil is more than forty times larger than that of the 1 % HFA-E emulsion, and

in this context it would be reasonable to expect that the flow rate of the emulsion is also more than forty times larger than the oil flow rate. However, this is true only for a narrow range of motor pressure drop. The present analysis has revealed that the pressure limit  $\Delta p_{i,L}$ , above which disturbances begin to appear in the emulsion flow is not the same for all motors. In the motors with typical clearances (up to 5 μm) this pressure can even reach 1 MPa, while in motors with very small working mechanism module pitches and large axial clearances (see the SM-0.4/20 motor) the flow disturbance is already observed as early as for  $\Delta p_{i,L} > 0$ .

At large motor pressure drops, the ratio of the emulsion flow rate to the oil flow rate ranges within 15 ÷ 25.

Analysing the results obtained using method I makes it possible to formulate a conclusion that the decreasing height of the working mechanism gaps, being the result of the action of the clearance compensation unit, leads to the decrease of the coefficients  $\beta_o$  and  $\beta_E$  (Fig. 8). That means that increasing  $\Delta p_i$  in the motor results in the increase of the flow velocity and intensification of the level of flow disturbances. Moreover, the value of the coefficient  $K_o$  for oil is larger than the value of  $K_E$  for emulsion, and their ratio is equal to the density ratio between emulsion and oil. Therefore the flow resistance coefficients for oil and emulsion can be defined in the following way:

$$\lambda_o = \frac{1,14}{Re\beta_o} \text{ and } \lambda_E = \frac{1}{Re\beta_E}$$

Having known the value of the coefficient  $\beta$  we can calculate the proportion  $S_{t-I}$  of the turbulent component in the flow rate  $Q_{Lfg}$  in the motor gaps. This component, expressed in %, is equal to:

$$S_{t-I} = (1 - \beta) \cdot 100 \%$$

The percent proportion  $S_{t-II}$  of the turbulent component of the flow rate in the gaps calculated using method II (formula (38)) is equal to:

$$S_{t-II} = (Q_t/Q_{Lfg}) \cdot 100 \%$$

The values of  $S_{t-I}$  and  $S_{t-II}$  are given in Table 6.

It is noticeable that smallest differences between the values of  $S_{t-I}$  and  $S_{t-II}$  are recorded for large pressure drops in the gaps and for low-viscosity liquids.

Based on the values of the coefficients A and B given in Table 5 we can conclude that:

- the largest turbulence area in the gaps is observed in motors with the highest gap height-to-length ratio – here it is the motor SM-0.4/20. That is why model I coefficients could not be calculated for this motor;
- for all motors we can assume:

$$\frac{A_o}{A_E} \approx 2, \frac{B_E}{B_o} \approx 7$$

Tab. 6. Percent proportions  $S_{t-I}$  and  $S_{t-II}$  of the flow  $Q_t$  in total flow  $Q_{Lfg}$  in the gaps

Liquid	$\Delta p_i$ [MPa]	Method	Motor			
			SM-0.4/20	SM-0.5/25	SM-0.6/25	SM-0.75/25
Oil	5	I	---	0.2	0.3	0.2
		II	9.8	1.2	0.8	2.3
	25	I	---	1.1	1.4	1.0
		II	22.4	2.9	2.1	2.7
Emulsion	5	I	---	25.3	25.4	25.3
		II	17.2	9.5	9.4	13.2
	25	I	---	26.7	27.2	26.6
		II	36.7	22.4	22.7	29.9

Tab. 7. Difference between  $Q_{Lfg}$ , in %, obtained experimentally and calculated using method I and II

Liquid	$\Delta p_i$ [MPa]	Method	Motor			
			SM-0.4/20	SM-0.5/25	SM-0.6/25	SM-0.75/25
Oil	5	I	---	2.7	0.2	0.7
		II	2.7	3.9	9.1	0.6
	25	I	---	0.1	0.6	2.9
		II	3.4	1.6	9.8	4.1
Emulsion	5	I	---	4.8	4.4	0.9
		II	28.8	15.7	14.7	14.7
	25	I	---	4.5	3.8	10.4
		II	1.4	0.1	2.0	0.1

In order to compare the accuracy of the method I and II with the experiment, the percent deviations of the results of  $Q_{Lfg}$  calculations were shown in Table 7.

Comparing the result collected in Table 7, we can see that the smallest  $Q_{Lfg}$  deviations from the experiment have been obtained using method I.

## SUMMARY

The article presents the methodology and results of experimental investigations of the flow rate in flat gaps of satellite motor working mechanisms. These gaps are characteristic for variable length and width. The results of the experiments have made it possible to assume that the flow in these gaps is turbulent but not fully developed. In this context two methods were proposed using which the flow can be calculated for liquids revealing large viscosity differences. The results of these calculations have proved that for the nominal pressure drop in the motor with nominal gap heights the turbulent flow component in the gap can amount to 3 % for oil, and as much as to 30 % for the emulsion.

In the future, the here presented methods to describe the liquid flow in flat gaps of the motor working mechanism are planned to be verified using the CFD method. What is also planned is the numerical calculation of the flow of liquid in elementary gaps (of rectangular or toroidal shape, etc.) a, for different-viscosity liquids, in particular for water.

Methods I and II can be used for working out models of volumetric losses in the motor. For this purpose the flow in the gaps of motor distribution system, as well as the flow convected in the inter-teeth spaces of the working mechanism, the flow caused by cyclic deformation of working chambers, and the flow caused by compressibility of the liquid, are to be modelled mathematically. These issues will be the subject of the next publications.

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