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Navier number and transition to turbulence

P Ziółkowski[1,2,3] **and J Badur**[1,4]

[1] Energy Conversion Department, Institute Fluid-Flow Machinery PAS-ci, Fiszera 14, 80-231 Gdańsk, Poland
[2]Conjoint Doctoral School at the Faculty of Mechanical Engineering, Gdansk University of Technology, Gdańsk, Poland

Abstract. The motivation behind this article is to explain a role of the Navier number (*Na* - dimensionless slip-length) in prediction of closures for laminar to turbulent transition undergoing via eddies detachment from the slip layer in nano-cannals. Additionally the role of the Navier number *Na* in universal modeling of phenomenon of enhanced mass flow rate reported in micro- and nano-channels has been explained. The *Na* number should be treated as a ratio of internal viscous to external viscous momentum transport and therefore this notion cannot be extended onto whole friction resistance phenomena. Our proposal for unique expressing of a critical point in turbulence transition is that on a plane of f_D - *Re* one needs two coordinates. The second critical coordinate has been discovered by Stanton and Pannell and is known to be the Stanton-Pannell number *StPa*. Finally dependence of the Stanton-Pannell number *StPa* on Navier number *Na* and Reynolds number *Re* is presented.

1. Introduction

In the MEMS and NEMS literature, it is assumed that the external friction between solid and liquid surfaces cannot be further neglected [3]. It involves the velocity slip, the slip length l_s and the Navier number (*Na* - dimensionless slip-length), which is important in the surface momentum transfer, coupled with an interfacial transport [2].

Generally, from the point of view of slip flow enhancement, two kinds of the "$\ln f_{\mathrm{D}} - \ln Re$" diagrams should be considered. In the first one a type of fluid is fixed and different material of the wall surface is considered – in the second one, conversely, the material of wall is fixed and the sort of fluid is changed [12]. In both cases we have with a whole manifold of the external viscosity ν, the lengths of slip l_s and the Navier numbers *Na*. Unfortunately, this key distinguish for microflows is not clearly exposed in the literature – even in the recent works [7,19] a problem of different wall frictions for different fluids and wall solids is omitted.

In this article we study a consistency of the Navier-Stokes model of fluid friction on the slip boundary layer form the point of view of a possible prediction of enhanced mass flow rate. Our subject of investigation coming from a fact that in the literature there is no of unified approach to modeling of the slip length since its role is not clarified in classical fluid dynamics with the slip boundary conditions. However it appears, studding more precisely pioneering papers of Navier (1827);

[3] E-mail: pawel.ziolkowski@imp.gda.pl
[4] E-mail: jb@imp.gda.pl

Helmholtz and Piotrowski (1860); Maxwell (1879), one could find that there exist also some additional developments of a slip-boundary concept, which can be overlapped by a one concept of the Navier number.

1.1. Friction and Navier number Na

The external friction between different continua is a physical reason of the breakdown of conventional macroscopic ideas at small scales [2,3,9,15]. Indeed, there exist many physical reasons for the slip over solid surfaces, among which are: molecular slip small dipole moment of polar liquids, and trapping the naturally present gas bubbles on the solid surface [16,24].

The slip velocity cannot be arbitrary – it should undergo of the laws of external friction between three-dimensional fluid continua touching a solid surface. According to early Newton's hypothesis [11], the external friction between bodies depends mainly from three mechanisms: a) the pressure within the contacting layer, b) the relative velocity of the contacting bodies, and 3) square of their relative velocity, nowadays identified with a relative kinetic energy. Leonard Euler was a scientist who proposed two kinds of the external friction – the first, called "static", that dominates for small relative velocity and the second one, called "dynamic", is proper for fully developed motion. This line of reasoning even nowadays is to be observe in the literature and is called "rate dependent external friction" [18,25,27]. The concept of "external friction" between laminas of different bodies and the "internal friction" between lamina of the same body has been introduced by Coulomb [4], who has made a first repetitive experiments having the aim to verify his own laws of friction.

But it was Navier [23], who standing on the data of Girard's capillary experiment, has proposed to use both: an idea of dissipative kinetic energy of external friction and the idea of external friction force between fluid and a surface of a fixed solid body, defined by him to be: $\mathbf{f}_f = \nu \mathbf{v}_s$. He called the coefficient of the external friction ν as "la résistance provenant du glissement" – the external viscosity. The tangential component of the fluid velocity $\mathbf{v}$ on surface has been called the "slip velocity". The magnitude of the slip velocity is governed by the value of external friction coefficient – if $\nu \to \infty$ we have the no-slip condition, for $\nu = 0$ we have an ideal slip – the boundary condition which correspond to the Euler model of an internally perfect fluid [5,6].

In contrast, in some special applications, for example, in microfluidic and nanofluidic devices, where the surface-to-volume ratio is huge, the slip velocity behaviour is more typical, and the "slip" hydrodynamic boundary condition is usually used. Regardless the slip physical mechanism, the degree of slip is normally quantified through the "slip length" l_s, or a dimensionless slip length (Navier number) Na [21,28], namely:

$$l_s = \frac{\mu}{\nu} \quad ; \qquad Na = \frac{l_s}{a} \tag{1}$$

being the ratio of the internal viscosity μ and the external viscosity ν. This last coefficient depends form, both, a kind of fluid and a contacting solid. The characteristic dimension of a channel a usually is identified with a radius of tube. The value of Na can be: $0 < Na < \infty$, for instance; $Na = 50$ for water flowing through a carbon nanotube [22]. The slip length l_s can be identified as the distance from the liquid to the surface within the solid phase, where the extrapolated flow velocity vanishes.

In spite of a numerous controversy concerning recognition of the possibility of slippnes, let us recall shortly the main measurements of the external viscosity coefficient, called also sometime the "external friction coefficient". If the linear (Newtonian) bulk shear viscosity of fluid μ is known, then the problem of measurement of ν can be reduced to determination of the slip length $l_s = \mu / \nu$. The first technical closure for ν between water and glass has been proposed yet by [23] in a proportion to the water density ρ:

$$\nu = \rho \times 0{,}0023 \quad [\text{mm}] \tag{2}$$

and was rigorously checked in work by Helmholtz & von Piotrowski [4]. The first closure for the external friction between rarefied air and glass has been prepared by Knudt & Warburg [4] in the following form:

$$l_s = \frac{\mu}{\nu} = 0.7122\, l = 0.7122\, l_0 \frac{760}{p} \qquad \text{in [mm]} \tag{3}$$

where l, l_0 are the mean free paths of molecule in the actual pressure and atmospheric pressure, respectively. At the same time, a systematic experimental study reporting the external viscosity in as well liquids as gases has been apparently reported by Wlademen who was also an inventor for technical devices important for measurements of ν or l_s [17]. Many years after, Schnell [25], has measured the flow rate of water in glass capillaries of radius of the order 100 μm .
Trying to control the external friction by any "interfacial lubrication", Schnell treated the capillaries with dimethyldichlorosilane, making its surface hydrophobic one. Then larger flow rates has been obtained and they were interpreted as a lowering of friction and increasing of the slip length at the wall. Schnell's slip length data was consistent with the analytical solution of Navier that predict the enhancement of the mass flow rate within the capillary tube. Historically, many of the pioneering investigations of non-continuum flows were conducted by researchers in the rarefied gas community who were primarily interested in low-pressure applications. Apart from a numerous experimental data still now there are no benchmarks experiments for identifying of the length-slip – the best candidates are the Couette and Poiseuille flows [7,20].

2. Dimensionless coefficients at a bulk and surface

Traditionally, the question of dimensionless numbers in fluid dynamics appears when the set of governing equations (mass, momentum, angular momentum, entropy and total energy) is undergoing of a sort of purely mathematical analysis in a co-called dimensionless form. The fundamental dimensionless number related with the integral form of equation of mass balance is the Reynolds number:

$$\mathrm{Re} = \dot{m}/(\pi\, d\, \mu) = \dot{m} d_n /(A\mu) \tag{4}$$

which is understood as a dimensionless mass flow rate $\dot{m}$. Here, in a case of rectangular cross section we use d_n - hydraulic diameter. Since for many flows viscosity μ weakly depends on the temperature and pressure, and since $\dot{m}_{in} = \dot{m}_{out}$, the Reynolds number can be interpreted as an integral (total) flow parameter. One should remember that in above definition the mass flow rate is defined via normal component of velocity $v_n = \mathbf{v} \cdot \mathbf{n}$, which ordinary is less than the velocity length $c = |\mathbf{v}| > v_n$. For open canals, where $\dot{m}$ cannot be determined, Re usually means dimensionless inflow velocity u_∞ [6].

2.1. Navier flow enhancement

For much longer tubes, for a slow (laminar) flow, yet another type of integral characteristics for different length and diameter of glass tubes has been experimentally found by Poiseuille [16] as a linear formula:

$$\dot{m}_{no-slip} = \dot{m}_{Poiseuille} = \frac{\pi \rho^2 d^4}{128 \mu L}(p_{in} - p_{out}) \sim \Delta p \tag{5}$$

It is a celebrated referential formula for no-slip resistance of flow in a capillary tube. Since in this formula only one coefficient μ is responsible for fluid resistance, then during the next century (refer

with equation (5)) and Poiseuille-like experiments was a base for experimental executing the value of the internal viscous friction. In hydraulics it is known as a $\Delta pd^4/L$ law.
The original slip Navier's solution [23] up to year 2000, was treated, in the literature, as wrong law $\Delta pd^3/L$. However, it is also a linear in Δp, but only with a little bit grater coefficient of discharge such that:

$$\dot{m} = \dot{m}_{Navier} = \frac{\pi a^2 \rho}{\nu} \frac{\Delta p}{L} \frac{a}{2} \left(1 + \frac{\nu}{\mu} \frac{a}{2}\right)^{-1} = \frac{\pi a^3 \rho^2}{2\mu} \frac{\Delta p}{L} \left(1 + \frac{1}{2} \frac{a}{l_s}\right)^{-1} \tag{6}$$

which leads to the following flow enhancement (where $a = d$):

$$\eta_{\mathrm{enh}} = \frac{\dot{m}_{Navier}}{\dot{m}_{Poiseuille}} = (1 + \frac{l_s}{8a}) \left(1 + \frac{a}{2l_s}\right)^{-1} = (1 + Na/8)(1 + Na^{-1}/2)^{-1} . \tag{7}$$

It practically means that a notion of flow enhancement can't be discovered by Navier solely, since the referential no-slip rate of mass flow has been finding latter by Poiseuille. Nevertheless, the expression for $\dot{m}_{Navier}$ is a first case of integral characteristics where the slip length explicitly appears. In order to best appreciate the effect of a wall friction Hagen preformed experiments in long capillary tubes with pressure-driven flows of water of different temperature. He fined the following integral characteristics [11]:

$$\alpha_1 \dot{m}^2 + \alpha_2 L \dot{m} = a^4 \Delta p \tag{8}$$

where α_1 is a temperature-dependent constant related with some *vis-viva* flow resistance and α_2 is a temperature-independent constant. Perhaps because the concept of "*vis viva* wall friction" was treated to be mistaken, the credit to the discovery of a linear part has been often being given to Poiseuille and Hagen and is known as the "Hagen-Poiseuille law".
Quite similar linear characteristics for pressure driven flow in a long tube have been find by Helmholtz & von Piotrowski [4]:

$$\eta_{\mathrm{enh}} = \frac{\dot{m}_{slip}}{\dot{m}_{Poiseuille}} = \left(1 + 4\frac{l_s}{a}\right) = 1 + 4Na \quad . \tag{9}$$

2.2. Navier and Knudsen number at gases
Extending this line of reasoning to taking into account an acceleration of fluid within a tube and the exact form of continuity equation, in work [1] have been find, a integral form of solution of one-dimensional Navier-Stokes equation with the appropriate Navier slip boundary condition. This characteristic written down in terms of "mass flow rate – drop of pressure" is as follows:

$$\dot{m} = \frac{H^3 W}{24 \mu L R T_{out}} \left[p_{in}^2 - p_{out}^2 + 12 Na_{out}\, p_{out} (p_{in} - p_{out})\right] \sim (\Delta p)^2 \tag{10}$$

where H, W, L are height, width and length of the channel, respectively, R is the specific gas constant, T_{out} is the outlet gas temperature, p_{in} and p_{out} are the pressures at the inlet and outlet of the channel, and the Navier number calculated on the outlet is expressed by Maxwell's clousure $Na_{out} = (2 - f)/f\, Kn_{out} = l_s / H$.
Ewart et al [14] have obtained the mass flow rate through a tube of diameter d by solving the Navier-Stokes equations with the second order slip condition in the following form:

$$\dot{m} = \frac{\pi d^4 p_m}{128 \mu R T L}(p_{in} - p_{out})\left(1 + 8 Na_m + 32 A_2 \frac{p_m}{p_{in} - p_{out}} \ln \frac{p_{in}}{p_{out}} Kn_m^2\right) \quad (11)$$

where Kn_m and $Na_m = A_1 Kn_m$ are the mean Knudsen and Navier number based on the mean pressure $p_m = 0.5(p_{in} + p_{out})$. Accordingly to the previous remarks, the second order part described by second slip coefficient A_2 cannot be expressed as a "square of Navier number" - it is rather described by a second Navier number. We should remember that the Navier number always is connected with a ratio of internal and external viscosities (refer with equetion 1). [13] have expressed the flow enhancement as:

$$\eta_{\text{enh}} = \frac{\dot{m}_{slip-2order}}{\dot{m}_{Poiseuille}} = 1 + 8 Na_m + 16 A_2 \frac{\mathscr{P} + 1}{\mathscr{P} - 1} \ln \mathscr{P} \, Kn_m^2 \quad (12)$$

where $\mathscr{P} = p_{in} / p_{out}$ is the inlet-outlet pressure ratio. This compact form of the flow enhancement is a proper for comparisons with the appropriate measured values [8,14]. For another integral flow characteristics that explicitly involve the slip length (or the dimensionless slip length) see: [2,10, 16,17].

3. Transition to turbulence

Stanton and Pannell, [26], making measurements of water flow within a capillary pipe, have proposed a change of the paradigm in approach to description of flow characteristics. They have courage to describe a results of measurements in a quite new way – they resigned with the celebrated pressure-discharge characteristics "$\Delta p - \dot{m}$", deciding to presents their own results as a diagram "dimensionless wall stress – dimensionless mass flow rate". Dimensionless wall stress, called the Stanton-Pannell friction factor, has been defined as a wall stress divided by the *vis viva* of a flow: $f_{\text{SP}} = \tau_w / (\rho U^2)$. Dimensionless mass flow rate they described as the Reynolds number to be: $Re = 4\dot{m} / (\pi d \mu) = \rho U d / \mu$. In this "$f_{\text{SP}} - Re$" chart the analytical solutions of one-dimensional Navier Stokes equation can easily be presented:

$$f_{\text{SP}} = \frac{\tau_w}{\rho U^2} = \begin{cases} \dfrac{4\mu}{a\rho U} = \dfrac{8}{Re} & - \text{no-slip solution} \\ \dfrac{4\mu}{a\rho U(1 + 4 l_s / a)} = \dfrac{8}{Re(1 + 4 Na)} & - \text{slip solution} \end{cases} \quad (13)$$

In contemporary literature instant of *vis viva* we use the kinetic energy which is two-time grater quantity, and such obtained dimensionless wall stress is called the Fanning friction factor : $f_{\text{F}} = 2 f_{\text{SP}}$. Frequently, according to Darcy, we apply four time grater coefficient, called the Darcy friction factor $f_{\text{D}} = 4 f_{\text{F}} = 8 f_{\text{SP}}$ [20], it leads to celebrated in the literature: $f_{\text{D}} = 64 / Re$.

In the laminar regime, the classical no-slip solution Hagen-Poiseuille $f_{\text{D}} = 64 / Re$ and the slip Helmholtz-Piotrowski solution $f_{\text{D}} = 64 / Re(1 + 4 Na)$ are plotted. In the turbulent regime for smooth pipes: and the no-slip Blasius solution $f_{\text{D}} = 0.316 Re^{-0.25}$ is plotted as continuous lines. Some experimental results [19] for flow of gas within peek-coated, fused-silica microchannal is shown also. The friction factor is reduced due to slip at the walls as $f_{\text{SP-slip}} / f_{\text{SP-no-slip}} = 1/(1 + 4 Na)$ [10]. For comparisons, slip analytical solution with $Na = l_s / a = Kn$ within the laminar regime is drowned by a dotted line. Unfortunately, up to now there is no analytical solution (like the Blasius one) for turbulent flow with slip.

Due to unusual and rapid development and improvement of specific measurements techniques for the microfluids field, frequently we are ask about any refinement of analytical methodology in expressing the new results. Especially we are asking; do the classical “ $f_D - Re$ ” diagram is sufficient for universal expressing of the specify phenomena of microflows [20].

One of a key element of $f_D - Re$ universality is appearance of a one critical point exactly in this same point on the diagram for every flow. Let recall, that this point appears also on “ $\dot{m} - \Delta p$ diagram, thus in the classical literature by Du Buat, Prony, Darcy, Eytelwein there are exist some qualitative information about “transition” from a quiet form of flow into a loud one. Probably the first quantitative information about this point has been given and measured by Hagen [11] who used some kind of dimensionless critical number $Ha = (2gh)^{0,5}$ where h being the pressure head. Later Reynolds has considered another candidates for a universal number – finally expressing a criterion for laminar-turbulence transmission as: $Re_{\mathrm{cr}} = 2250$.

For unique expressing of a critical point on a plane of $f_D - Re$ we need two coordinates. The second critical coordinate has been discovered by Stanton and Pannell [26], which have been written the following celebrated condition:

$$Re = \frac{\rho U_{\mathrm{cr}} d}{\mu} \quad \text{is constant} \sim 2300 \qquad StPa = \frac{\rho d^3}{\mu^2} \frac{dp}{dx}_{|\mathrm{cr}} \quad \text{is constant} \sim 0.004 \; . \tag{14}$$

The Stanton-Pannell number depends only on a drop of pressure and it can give a comfortable information about of type of flow yet in a moment of an experiment design. This number is connected with another dimensionless factor called the Poiseuille number $Po = f_D \cdot Re$ to be:

$$StPa = \frac{1}{2} Po\, Re = \frac{1}{2} f_D\, (Re)^2 \, . \tag{15}$$

From equation (13) follows that the critical Darcy friction factor always is : $f_D = 0.32$.

Enhancement of the Poiseuille number due to slip is $Po_{slip} / Po_{no-slip} = 1/(1+4Na)$. This result is consistent with [19] where for square channel they have obtained: $f_D \cdot Re = 56.9/(1+7.88Kn)$. Notice that, due to the fact that phenomenon of the laminas-to-turbulent transition do not occurs simultaneously in the whole channel, in practice, we are speaking on a “transition zone” or of the “transition regime” [7,9].

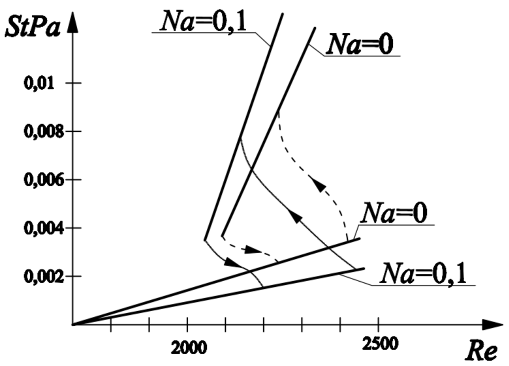

Figure 1. Dependence of the Stanton-Pannell velocity profile (the Stanton-Pannell number $StPa$) on a dimensionless mass flow rate (Reynolds number Re).

Do the $f_D - Re$ curve is enough for explaining universality of microflow phenomena ? Probably not. For instance, note that, a real non-measurable wall stress τ_w [treated as surface friction force per unit area] is replacement by means an analytical solution to the measurable fall of pressure dp/dx. Thus,

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practically, the Darcy friction factor f_D, has the same information what keep the Stanton-Pannell's number: $StPa$. Dependence of the Stanton-Pannell number $StPa$ on Navier number Na and Reynolds number Re is presented on figure 1. This figure is obtained on the basis Morini and Celata measurements [7,19,20].

4. Conclusions

Reassuming, the special role of Reynolds number coming from Reynolds' discovery that for geometrically similar glass tubes there was a critical mass velocity U_{cr} when stream-line or laminar in character flow at low values of the velocity, suddenly change in eddying or turbulent flow at high velocities. The special role of the Stanton-Pannell number follows from the discovery of the "critical drop of pressure". Both Reynolds and Stanton and Pannell have discovered, on making a series of observations, that for different geometries in the moment of changing of flow character from the laminar [stream-line] to the turbulent [eddying] that two numbers have still the value $Re_{cr} = 2300$ and $StPa_{cr} = 0.004$ (on Figure 1). This fact that there exists two dimensionless numbers proper for description of arbitrary flows within a frame of a one integral law of "flow resistance" nowadays is fundamental and is a base of paradigmatic view on mathematical modeling.

Unfortunately, an obstacle for a further progress of turbulence understanding is a logical mistake, which lays to identification the "wall stress" with a "surface friction force". Therefore, a reader can be astonished finding the main aim of many experimental works to be: "The object is to furnish evidence conforming the existence the laws of the surface friction of fluids." It is a mysterious "similarity of motion" anticipated by a numerous researchers. After discovery of Stanton and Pannell, the scientific word was absolutely astonish – because instant of numerous laws of pipe flows, separately proposed by Du Buat, Prony, Darcy, and other hydraulics engineers, there is only one for every sort of fluid and type and geometry of pipe. And, against the numerous 3D models – the notion of "wall friction" plays in this diagram the fundamental role. From this moment, the enigma of turbulence was identified with external eddies produced during a wall friction phenomena. In other words, if our turbulence model is based on a notion of turbulent wall stress then we must to consider an additional internal transport of momentum by a flux called the "Reynolds stress tensor". However, if our turbulence model is based on turbulent surface friction force concept when we should take into account the "*vis viva*" .

In turn, Stanton and Pannell were the first experimentalists who have believed in a theoretical anticipation of different profiles of fluid velocity at a pipe cross section, therefore they try to measure two kinds of velocities – the first one is the classical da Vinci mass velocity, measured by weight of the total discharge of fluid passing through the pipe in a given time, and the second one is the maximum velocity at the axis of the pipe, estimated by measuring the difference of pressure between that in a small Pitot tube facing the current and placement in the axis of the pipe and that in a small hole in the wall of the pipe. As we know from a basic consideration, this pressure difference is $½ \rho u_{max}^2$ and from this relation the maximum velocity [speed] can be calculated. Everyone were strongly astonish looking on a plot of a velocity ratio $\varphi_{SP} = U / u_{max}$ has been made for different falls of pressure, expressed by a logarithm of dimensionless [Reynolds] number. Prior studies available in the literature, have no pay attention on the question of the Stanton-Pannell critical number during laminar to turbulent transition [12,7], but should be stated that nanochanels notion of Reynolds number and Stanton-Pannell number is meaningless and should be replaced by the Navier number. Additionally, a role of Navier number leas on expressing flow enhancement.

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