POLISH MARITIME RESEARCH 4(84) 2014 Vol. 21; pp. 90-99
10.2478/pomr-2014-0045

Mathematical model defining volumetric losses of hydraulic oil compres-sion in a variable capacity displacement pump

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ABSTRACT

The objective of the work is to develop the capability of evaluating the volumetric losses of hydraulic oil compression in the working chambers of high pressure variable capacity displacement pump. Volumetric losses of oil compression must be determined as functions of the same parameters, which the volumetric losses due to leakage, resulting from the quality of design solution of the pump, are evaluated as dependent on and also as function of the oil aeration coefficient ε. A mathematical model has been developed describing the hydraulic oil compressibility coefficient $k_{lc|\Delta p_{Pi};b_P;\varepsilon;\nu}$ *as a relation to the ratio* $\Delta p_{Pi}/p_n$ *of indicated increase* Δp_{Pi} *of pressure in the working chambers and the nominal pressure pn, to the pump capacity coefficient* b_P*, to the oil aeration coefficient ε and to the ratio* ν/ν_n *of oil viscosity ν and reference viscosity* ν_n*. A mathematical model is presented of volumetric losses* $q_{Pvc|\Delta p_{Pi};b_P;\varepsilon;\nu}$ *of hydraulic oil compression in the pump working chambers in the form allowing to use it in the model of power of losses and energy efficiency*

Keywords: hydrostatic drive, variable capacity displacement pump, volumetric losses of oil compression, mathematical model

Introduction

In references [1–4], this author attempted to evaluate the influence of working liquid compressibility on the picture of volumetric and mechanical losses in a high-pressure variable capacity displacement pump. The considerations were based on the assumptions made by the author in the developed theoretical and mathematical models of torque of mechanical losses in the pump used in the hydrostatic drive [5–7]. The models assume, that **increase** $\Delta M_{Pm|\Delta p_{Pi},q_{Pgv}}$ **of torque of mechanical losses in the pump „working chambers - shaft" assembly**, compared with torque of losses in that assembly in a no-load pump (when indicated increase Δp_{Pi} of pressure in the working chambers equals to zero (Δp_{Pi} = 0)), **is proportional to torque** M_{Pi} **indicated in the pump working chambers**:

$$\Delta M_{Pm|\Delta p_{Pi};q_{Pgv}} \sim M_{Pi|\Delta p_{Pi};q_{Pgv}} . \qquad (1)$$

In references [1– 4], the author introduced also **the working liquid compressibility coefficient** $k_{lc|p_n}$. It determines the degree of decrease of the active volume of working liquid displaced by the pump during one shaft revolution as an effect of increase $\Delta p_{Pi} = p_n$ of pressure in pump working chambers equal to the pump nominal pressure p_n (nominal pressure p_n of the hydrostatic drive system where the pump operates). The pump active working volume $q_{Pt|p_n}$ or $q_{Pgv|p_n}$ is smaller compared with the active volume equal to theoretical working volume q_{Pt} or geometrical working volume q_{Pgv} (determined at the increase Δp_{Pi} of pressure equal to zero – $\Delta p_{Pi} = 0$). It decreases then the indicated torque M_{Pi} and indicated power P_{Pi} in the pump working chambers, which can be generated (and calculated) with the increase Δp_{Pi} of pressure in the chambers, for example with $\Delta p_{Pi} = p_n$. In effect, decreases also torque M_P on the pump drive shaft and power P_{Pc} consumed by the pump on shaft that the pump driving motor can be loaded with.

The $\mathbf{k_{lc|p_n}}$ coefficient is described by the expressions:

$$k_{lc|p_n} = \frac{q_{Pt} - q_{Pt|p_n}}{q_{Pt}} \tag{2}$$

and

$$k_{lc|p_n;b_P} = \frac{q_{Pgv} - q_{Pgv|p_n}}{q_{Pgv}}. \tag{3}$$

Coefficient $\mathbf{k_{lc|p_n}}$ of the working liquid compressibility can also be described by the formulae:

$$k_{lc|p_n} = \frac{q_{Pvc|p_n}}{q_{Pt}} \tag{4}$$

and

$$k_{lc|p_n;b_P} = \frac{q_{Pvc|p_n}}{q_{Pgv}}, \tag{5}$$

where

$q_{Pvc|p_n}$ are losses of pump capacity during one shaft revolution due to compression of non-aerated (or aerated) liquid (volumetric losses of liquid compression), the losses determined at indicated increase Δp_{Pi} of pressure in the pump working chambers equal to the nominal pressure p_n of pump in the hydrostatic drive system.

Volumetric losses q_{Pvc} of liquid compression are an effect of not only the liquid compressibility but also of the variable capacity displacement pump operating principle. The change of geometrical working capacity q_{Pgv} of the pump is accompanied also with the change of the ratio of compressed liquid volume in the pump working chambers and volume q_{Pgv} and, in effect, with the change of ratio of losses q_{Pvc} due to liquid compressibility and the volume q_{Pgv}. Therefore, the compressibility coefficient $\mathbf{k_{lc|p_n}}$ of the same liquid increases in the pump with decreasing capacity q_{Pgv} per one shaft revolution.

In references [1 – 4] the author searched for value of the hydraulic oil compressibility coefficient $\mathbf{k_{lc|p_n}}$, which, with increase Δp_{Pi} of pressure in the working chambers equal to nominal pressure p_n, will give the increase $\Delta M_{Pm|p_n, q_{Pgv}}$ of torque of mechanical losses proportional to q_{Pgv}, i.e to indicated torque $M_{Pi|p_n, q_{Pgv}}$. The author determined, in the tested pump HYDROMATIK A7V.58.1.R.P.F.00 [8], an approximate value of oil compressibility coefficient during the pump test, equal to .$k_{lc|32MPa} = 0{,}030$ Such value of the compressibility coefficient resulted also from aeration ($\varepsilon > 0$) of oil in conditions of the test stand.

In references [11, 12] the author presents **the method of determining the value of liquid aeration coefficient ε during pump operation in a hydrostatic drive system or on a test stand**, consisting in finding such value of ε, with which calculated increase $\Delta M_{Pm|p_n, q_{Pgv}}$ of torque of mechanical losses is proportional to indicated torque $M_{Pi|p_n, q_{Pgv}}$ determined (calculated) at constant increase (Δp_{Pi} = cte) of pressure in the pump working chambers. The constant value of Δp_{Pi}, assumed in searching for liquid aeration coefficient ε, equals to pump nominal pressure p_n ($\Delta p_{Pi} = \mathrm{cte} = p_n$).

During the considerations on compressibility of aerated liquid, the values of modulus B of volume elasticity of the hydraulic oils used in hydrostatic drive and control systems , were taken after M. Guillon's reference [10].

Increase $\Delta M_{Pm|p_n, q_{Pgv}}$ of torque of mechanical losses in the pump „working chambers - shaft" assembly, at a constant value of Δp_{Pi} (Δp_{Pi} = cte), is (in this author's opinion) proportional to the pump geometrical working capacity q_{Pgv} ; therefore:

the (calculated) relation :

$$\Delta M_{Pm|p_n, q_{Pgv}} \sim q_{Pgv} \tag{6}$$

can be obtained only with taking into account the actual value of aeration coefficient ε of liquid displaced by the pump,

At the same time, only with accounting for actual value of liquid aeration coefficient ε the calculated increase $\Delta M_{Pm|p_n; q_{Pgv}; \varepsilon}$ of torque of mechanical losses tends to zero at the geometrical working capacity q_{Pgv} per one shaft revolution tending to zero:

$$\Delta M_{Pm|p_n; q_{Pgv} \to 0; \varepsilon} \to 0 \text{ when } q_{Pgv} \to 0. \tag{7}$$

The calculated aeration coefficient ε of hydraulic oil used during tests of A7V.58.1.R.P.F.00 HYDROMATIK pump, corresponding to the situation described by the expressions

(6) and (7) had the value ε = 0,0135 [8, 11, 12].

The method, proposed by the author, of determining (calculating) the working liquid aeration coefficient ε was for the first time used in the research work (carried out by Jan Koralewski) concerning the influence of viscosity and compressibility of aerated hydraulic oil on volumetric and mechanical losses of a A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 9, 13, 14].

The method of determining (calculating) of the aeration coefficient ε of working liquid displaced by displacement pump of variable capacity per one shaft revolution, allows to subdivide the volumetric losses per one shaft revolution in pump working chambers into volumetric losses q_{Pvc} of compression of aerated (or non-aerated) liquid and volumetric losses q_{Pvl} due to leakage.

The method allows also to evaluate the increase $\Delta M_{Pm|\Delta p_{Pi}, q_{Pgv}}$ of torque of mechanical losses in the pump „working chambers - shaft" assembly being a function of torque $M_{Pi|\Delta p_{Pi}, q_{Pgv}}$ indicated in the pump working chambers (by making more precise calculation of indicated torque M_{Pi} , possible).

In this author's opinion, the above mentioned possibilities have not existed so far. They are important for evaluation of volumetric losses due to leakage of liquid in the working chambers and for evaluation of mechanical losses in the pump „working chambers - shaft" assembly, hence they are important for evaluation of a design solution of displacement pump generating those losses, particularly when operating in the conditions of high increase Δp_{Pi} of pressure in the chambers.

Objective of the work is to develop a capability of evaluating the volumetric losses resulting from hydraulic oil compression in the pump working chambers in function of the same parameters on which volumetric losses due leakage, resulting from the quality of design solution of the pump, are dependent and evaluated in function of the oil aeration coefficient ε. Therefore, it is necessary to develop a mathematical model describing the hydraulic oil compressibility coefficient $k_{lc|\Delta p_{Pi}; b_P; \varepsilon; \nu}$ as a relation to :

- the ratio $\Delta p_{Pi}/p_n$ of indicated increase Δp_{Pi} of pressure in the working chambers to the nominal pressure p_n,
- the pump capacity coefficient b_P,
- the oil aeration coefficient ε,
- the ratio ν/ν_n of oil viscosity ν and reference viscosity ν_n.

It is also necessary to present a mathematical model of volumetric losses $q_{Pvc|\Delta p_{Pi}; b_P; \varepsilon; \nu}$ of hydraulic oil compression in the pump working chambers in the form allowing to use it in the model of power of oil compression in the pump and also in the model of power of losses and pump energy efficiency.

2. Mathematical model of hydraulic oil compressibility coefficient $k_{lc|\Delta p_{Pi}; b_P; \varepsilon; \nu}$ compatible with model of energy losses in a displacement pump

The knowledge of oil compressibility coefficient $k_{lc|p_n}$ allows to make a **numerical** evaluation of subdivision of the volumetric losses in pump into losses due to oil leakage in the working chambers and losses of oil compression in chambers.

In a pump of variable capacity per one shaft revolution, operating with variable working capacity q_{Pgv} (at the pump capacity coefficient $b_P = q_{Pgv}/q_{Pt}$ changing in the $0 < b_P \le 1$ range), the $k_{lc|p_n}$ liquid (hydraulic oil) compressibility coefficient is described (in reference to test conditions presented in [8,9]) by the formulae:

$$k_{lc|p_n; b_P} = \frac{q_{Pvc|\Delta p_{Pi} = p_n}}{q_{Pgv}} = \frac{0{,}5 q_{Pt} + 0{,}5 q_{Pgv}}{q_{Pgv}} \cdot \left[\frac{1}{B|_{p_{P1ia} = 0{,}15\text{MPa},\, \vartheta = 20^\circ C} \left(1 + a_p p_n + a_\vartheta \Delta\vartheta\right)} + \frac{\varepsilon}{p_{P1ia} + p_n} \right] p_n \tag{8}$$

or

$$k_{lc|p_n; b_P} = \frac{1 + b_P}{2 b_P} \cdot \left[\frac{1}{B|_{p_{P1ia} = 0{,}15\text{MPa},\, \vartheta = 20^\circ C} \left(1 + a_p p_n + a_\vartheta \Delta\vartheta\right)} + \frac{\varepsilon}{p_{P1ia} + p_n} \right] p_n , \tag{9}$$

and with geometrical variable working capacity q_{Pgv} equal to the theoretical capacity q_{Pt} per one shaft revolution ($q_{Pgv} = q_{Pt}$) (at $b_P = 1$), by the formula:

$$k_{lc|p_n} = \frac{q_{Pvc|\Delta p_{Pi} = p_n}}{q_{Pt}} = \left[\frac{1}{B|_{p_{P1ia} = 0{,}15\text{MPa},\, \vartheta = 20^\circ C} \left(1 + a_p p_n + a_\vartheta \Delta\vartheta\right)} + \frac{\varepsilon}{p_{P1ia} + p_n} \right] p_n . \tag{10}$$

Therefore, in a variable capacity displacement pump operating at the theoretical working capacity q_{Pt} per one shaft revolution, the oil compressibility coefficient $k_{lc|p_n}$ (formula (10)) results from:

- the oil volume elasticity modulus B (value B = 1500MPa [10] assumed at absolute pressure p_{P1ia} = 0,15MPa and temperature ϑ = 20°C),
- coefficient a_p = 0,005/1MPa of the increase of modulus

B along with increase of pressure p,
- coefficient a_ϑ = – 0,005/1°C of the decrease of modulus B along with increase of temperature ϑ,
- oil temperature ϑ (temperature increase $\Delta\vartheta$ in relation to the reference temperature ϑ = 20°C),
- oil aeration coefficient ε determined at absolute pressure p_{P1ia},
- absolute pressure p_{P1ia} in the working chambers during their connection with the inlet channel (during tests presented in [8, 9], pressure in the working chambers was $p_{1ia} \approx$ 0,15MPa),
- the system nominal pressure p_n.

In the same displacement pump operating with the variable geometrical working capacity q_{Pgv} per one shaft revolution, smaller than q_{Pt}, the value of oil compressibility coefficient $k_{lc|p_n;b_P}$ (formulae (8) and (9)) increases in comparison with the value $k_{lc|p_n}$ during pump operation at theoretical working capacity q_{Pt}. This is an effect of an increase of initial oil volume subjected to compression, i.e. volume ($0,5q_{Pt} + 0,5q_{Pgv}$) to the set working capacity q_{Pgv}.

Therefore, the decreasing of the pump capacity setting q_{Pgv} (b_P coefficient in the $0 < b_P \le 1$ range) causes, in a displacement pump with variable capacity per one shaft revolution , an increase of the working liquid compressibility coefficient $k_{lc|p_n;b_P}$ (formulae (8) and (9)).

The value $k_{lc|p_n}$ of liquid (oil) compressibility coefficient must be determined in the same conditions as the coefficient k_l of volumetric losses due to leakage in the pump working chambers , used in the model of losses and energy efficiency of the pump, i.e.:
- system nominal pressure p_n,
- theoretical capacity q_{Pt} per one shaft revolution (b_P = 1),
- oil temperature ϑ_n corresponding to the oil viscosity coefficient ν/ν_n = 1, i.e. at the viscosity ν_n = 35mm²s⁻¹ (during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], (the temperature corresponding to the oil reference viscosity ν_n = 35mm²s⁻¹ was ϑ_n = 43°C), - and also with the actual oil aeration coefficient ε determined at oil temperature ϑ_n corresponding to the oil viscosity coefficient ν/ν_n = 1.

Fig. 1 presents an example of relations of the hydraulic oil compressibility coefficient $k_{lc|p_n;b_P}$ (determined by formulae (8) and (9)) to the oil aeration coefficient ε at different values of the pump capacity coefficient b_p, determined at the nominal pressure p_n = 32MPa, at temperature ϑ_n = 43°C of oil used during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], the temperature corresponding to oil viscosity ratio ν/ν_n = 1 (reference viscosity ν_n = 35mm²s⁻¹).

Fig. 2 presents relations of hydraulic oil compressibility coefficient $k_{lc|p_n;b_P}$ (determined by formulae (8) and (9)) to the pump capacity coefficient b_P at different values of oil aeration coefficient ε determined at the nominal pressure p_n =32MPa, hydraulic oil temperature ϑ_n = 43°C during tests of A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12] , i.e. the temperature corresponding to oil viscosity coefficient ν/ν_n = 1 (at the reference viscosity ν_n = 35mm²s⁻¹).

Relation of hydraulic oil compressibility coefficient $k_{lc|p_n;b_P}$ to the pump capacity coefficient b_P is based on the displacement pump operation principle and described (in the $0 < b_P \le 1$ range) by the formula:

$$k_{lc|p_n;b_P} = \frac{1+b_P}{2b_P} k_{lc|p_n} , \qquad (11)$$

where $k_{lc|p_n}$ is a liquid compressibility coefficient during pump operation at the theoretical working capacity q_{Pt} per one shaft revolution (at b_P = 1) and at liquid (oil) temperature ϑ_n corresponding to the oil viscosity ratio ν/ν_n = 1, i.e. at the reference viscosity ν_n = 35mm²s⁻¹.

Fig. 3 presents examples of the relations of hydraulic oil compressibility coefficient $k_{lc|\Delta p_{Pi};b_P}$ and $k_{lc|\Delta p_{Pi}}$ (described by formulae (8), (9) and (10)) to ratio $\Delta p_{Pi}/p_n$ of indicated increase Δp_{Pi} of pressure in the pump working chambers and nominal pressure p_n = 32MPa, at pump capacity coefficient b_P = 0,229 and b_P = 1, at different values of aeration coefficient ε, at temperature ϑ_n = 43°C of hydraulic oil used in the tests of A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], the temperature corresponding to oil viscosity ratio ν/ν_n = 1 (reference viscosity ν_n = 35mm²s⁻¹).

The proposed relations (in the $0 < b_P \le 1$ range):

$$k_{lc|\Delta p_{Pi};b_P;\varepsilon} = \frac{1+b_P}{2b_P}\left[k_{lc|p_n;\varepsilon=0}\left(\Delta p_{Pi}/p_n\right)^{a_{pc}} + \varepsilon\right] \qquad (12)$$

and

$$k_{lc|\Delta p_{Pi};\varepsilon} = k_{lc|p_n;\varepsilon=0}\left(\Delta p_{Pi}/p_n\right)^{a_{pc}} + \varepsilon \qquad (13)$$

allow to evaluate, with satisfying precision, the liquid compressibility coefficient $k_{lc|\Delta p_{Pi};b_P}$ and $k_{lc|\Delta p_{Pi}}$ in a range of indicated increase Δp_{Pi} of pressure in the pump working chambers:

$$\Delta p_{Pi} > 3{,}2\text{MPa}. \qquad (14)$$

Exponent a_{pc} in Eq. (12) and (13), describing the relation of coefficient $k_{lc|\Delta p_{Pi};b_P;\varepsilon}$ and $k_{lc|\Delta p_{Pi};\varepsilon}$ of oil compressibility to the expression $(\Delta p_{Pi}/p_n)^{a_{pc}}$ is independent of the pump capacity coefficient b_P and the oil aeration coefficient ε.

The value of exponent a_{pc} is:

$$a_{pc} = 0{,}89.$$

Expressions (12) and (13) (Fig. 3) **have a form allowing to use them in equations describing energy losses due to liquid compressibility in pump**.

Fig. 4 shows examples of relations of hydraulic oil compressibility coefficient $\mathbf{k_{lc|p_n;b_P}}$ and $\mathbf{k_{lc|p_n}}$ (determined by formulae (8), (9) and (10)) to oil temperature ϑ, at pump capacity coefficient $b_p = 0{,}229$ and $b_p = 1$, with different values of aeration coefficient ε used in the tests of A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12]. Temperature ϑ_n, corresponding to the recommended reference viscosity $\nu_n = 35 mm^2s^{-1}$ (oil viscosity ratio $\nu/\nu_n = 1$) was equal to $\vartheta_n = 43^oC$. The tests were carried out in the temperature range $20^oC \le \vartheta \le 68^oC$ (at $b_p = 1$ – in the $24^oC \le \vartheta \le 68^oC$ range)

In the description of volumetric losses in the pump due to oil compression in the pump working chambers, it was decided to replace the relations of oil compressibility coefficient $\mathbf{k_{lc|p_n;b_P}}$ and $\mathbf{k_{lc|p_n}}$ to oil temperature ϑ by relation to oil viscosity ratio ν/ν_n. This is justified on one hand by dependence of oil viscosity ν on temperature ϑ and on the other hand on the fact, that the energy losses: volumetric losses due to leakage in the working chambers, pressure losses in the pump channels, and mechanical losses in the pump „working chambers - shaft" assembly are determined in function of oil viscosity ν, and exactly in function of oil viscosity ratio ν/ν_n.

Fig. 5 shows examples of relations of the hydraulic oil compressibility coefficient $\mathbf{k_{lc|p_n;b_P}}$ and $\mathbf{k_{lc|p_n}}$ to oil viscosity ν described by the expressions:

$$k_{lc|p_n;b_P;\nu} = k_{lc|p_n;b_P}\left(\nu/\nu_n\right)^{a\nu c} \qquad (15)$$

and

$$k_{lc|p_n;\nu} = k_{lc|p_n}\left(\nu/\nu_n\right)^{a\nu c} \qquad (16)$$

allowing to evaluate the dependence of oil compressibility coefficient $k_{lc|p_n;b_P;\nu}$ and $k_{lc|p_n;\nu}$ on viscosity ν in the temperature $20^oC \le \vartheta \le 68^oC$ range, at pump capacity coefficient $b_p = 0{,}229$ and $b_p = 1$, at different values of aeration coefficient ε of oil during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12].

Exponent a_{vc} in Eq. (15) and (16), describing the relations of coefficient $k_{lc|p_n;b_P}$` and $\mathbf{k_{lc|p_n}}$ of oil compressibility to oil viscosity ν by means of expression $\left(\nu/\nu_n\right)^{a\,\nu c}$ replacing the relation to oil temperature ϑ by relation to oil viscosity ν, is independent (in the $0 < b_p \le 1$ range) of pump capacity coefficient b_p.

However, exponent a_{vc} changes with the change of oil aeration coefficient ε, i.e.:

with:

ε = 0	–	a_{vc} = - 0,12,
ε = 0,004	–	a_{vc} - 0,1,
ε = 0,008	–	a_{vc} = - 0,086,
ε = 0,012	–	a_{vc} = - 0,076,
ε = 0,0135	–	a_{vc} = - 0,072,
ε = 0,016	–	a_{vc} = - 0,067.

The influence, in the $3{,}5 \ge \nu/\nu_n \ge 0{,}4$ range, **of ratio** ν/ν_n **of hydraulic oil viscosity** (influence of change of oil temperature ϑ in the $20^oC \le \vartheta \le 68^oC$ range) **on the change of oil compressibility coefficient $\mathbf{k_{lc|\Delta p_{Pi};b_P;\varepsilon;\nu}}$ is small** in comparison with the influence of ratio $\Delta p_{Pi}/p_n$ of indicated increase Δp_{Pi} of pressure in the working chambers and nominal pressure p_n, with the influence of pump capacity coefficient b_p and with the influence of oil aeration coefficient ε. Therefore, **in order to simplify the expression describing the mathematical model of oil compressibility coefficient $\mathbf{k_{lc|\Delta p_{Pi};b_P;\varepsilon;\nu}}$** (and also model of volumetric losses q_{Pvc} of oil compression and model of power ΔP_{Pvc} of volumetric losses of oil compression), **it was decided to adopt a single value of a_{vc} exponent** equal:

$$a_{vc} = - 0{,}12,$$

corresponding to non-aerated (ε = 0) oil condition.

Mathematical model of relation of the hydraulic oil compressibility coefficient $k_{lc|\Delta p_{Pi};b_P;\varepsilon;\nu}$ to the non-aerated oil compressibility coefficient $k_{lc|p_n;\varepsilon=0}$ (formula (10) at ε = 0) in a variable capacity displacement pump and:

- to ratio $\Delta p_{Pi}/p_n$ of indicated increase Δp_{Pi} of pressure in the pump working chambers and nominal pressure p_n (in the $\Delta p_{Pi} > 3{,}2 MPa$ range),
- to pump capacity coefficient b_p (in the $0 < b_p \le 1$ range),
- to oil aeration coefficient ε,
- to oil viscosity relation ν/ν_n

takes, in reference to formulae (11) ÷ (16), the form:

$$k_{lc|\Delta p_{Pi};b_P;\varepsilon;\nu} = \frac{1+b_P}{2b_P}\left[k_{lc|p_n;\varepsilon=0}\left(\Delta p_{Pi}/p_n\right)^{apc} + \varepsilon\right]\left(\nu/\nu_n\right)^{a\nu c}, \qquad (17)$$

with exponent $a_{pc} = 0{,}89$

and with exponent $a_{vc} = - 0{,}12$.

The form of expression (17) allows to determine the impact of hydraulic oil compression in mathematical model of the power of oil compression in pump operating in the conditions produced by change of Δp_{Pi}, b_p, ε and ν.

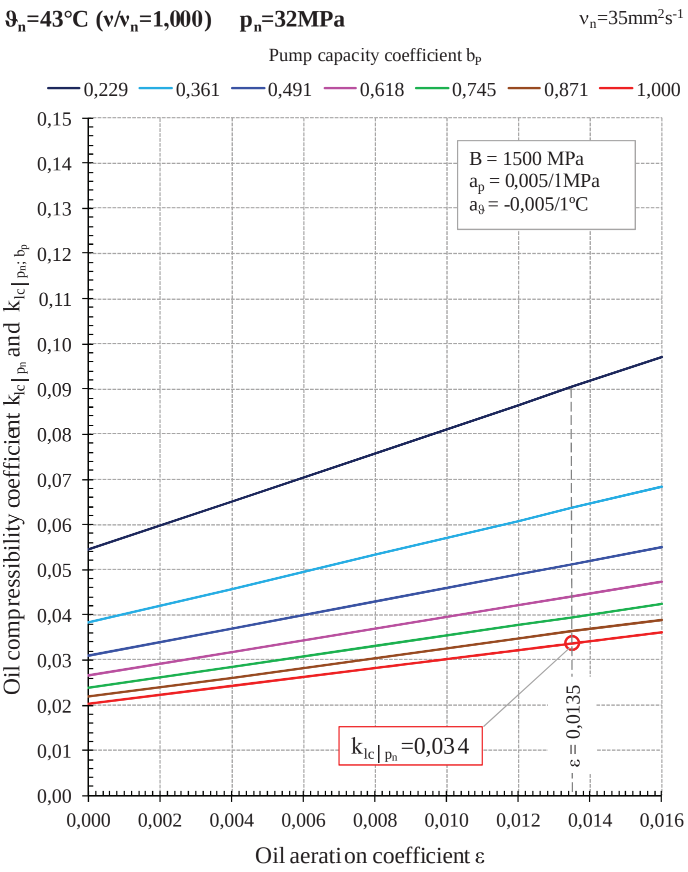


Fig.1 Example of relation of the oil compressibility coefficient $k_{lc|p_n;b_P}$ to the oil aeration coefficient ε at different values of the pump capacity coefficient b_P, determined at nominal pressure p_n = 32MPa, at temperature ϑ_n = 43°C of hydraulic oil used in tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], temperature corresponding to the oil viscosity ratio ν/ν_n = 1 (reference viscosity ν_n = 35mm^{2}s^{-1}); the value of oil compressibility coefficient $k_{lc|p_n}$ used in the model of losses and energy efficiency is determined in the same conditions as coefficient k_1 of volumetric losses due to leakage in the pump working chambers, i.e. at the theoretical capacity q_{Pt} per one shaft revolution (at the pump capacity coefficient b_P = 1); the actual oil aeration coefficient determined during the pump tests was ε = 0,0135 and the corresponding value of oil compressibility coefficient $k_{lc|p_n}$ = 0,034

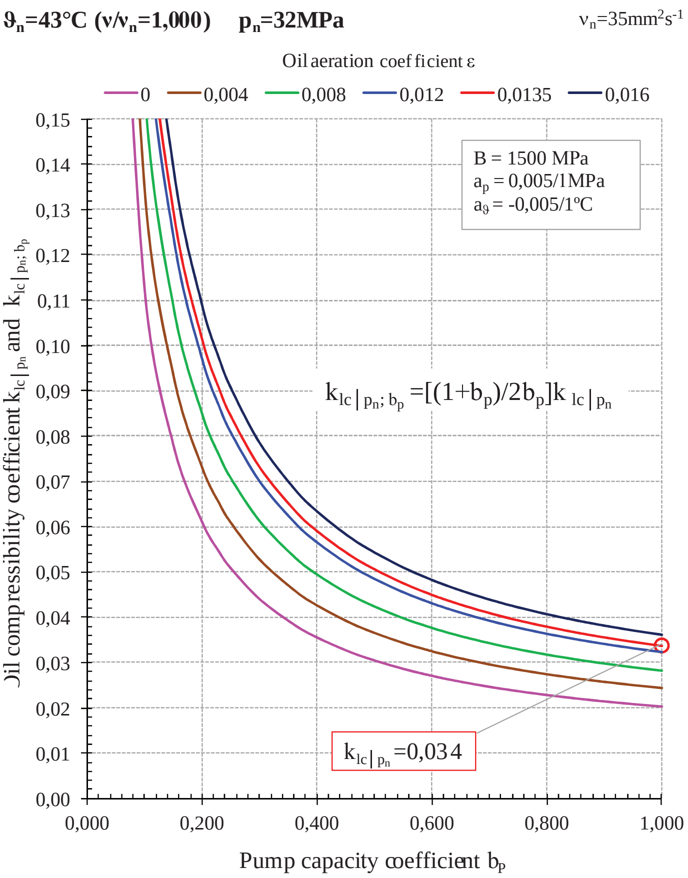


Fig.2 Example of relation of the oil compressibility coefficient $k_{lc|p_n;b_P}$ to pump capacity coefficient b_P at different values of oil aeration coefficient ε, determined at nominal pressure p_n = 32MPa, at temperature ϑ_n = 43°C of hydraulic oil used in tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], temperature corresponding to the oil viscosity ratio ν/ν_n = 1 (reference viscosity ν_n = 35mm^{2}s^{-1}); the value of oil compressibility coefficient $k_{lc|p_n}$ used in the model of losses and energy efficiency is determined in the same conditions as coefficient k_1 of volumetric losses due to leakage in the pump working chambers, i.e. at the theoretical capacity q_{Pt} per one shaft revolution (at the pump capacity coefficient b_P = 1); the actual oil aeration coefficient determined during the pump tests was ε = 0,0135 and the corresponding value of oil compressibility coefficient $k_{lc|p_n}$ = 0,034; the value of liquid compressibility coefficient $k_{lc|p_n;b_P}$ increases with the decreasing value of pump capacity coefficient b_P (in the $0 < b_P \le 1$ range) in accordance with the formula $k_{lc|p_n;b_P} = \left[(1+b_P)/2b_P\right]k_{lc|p_n}$

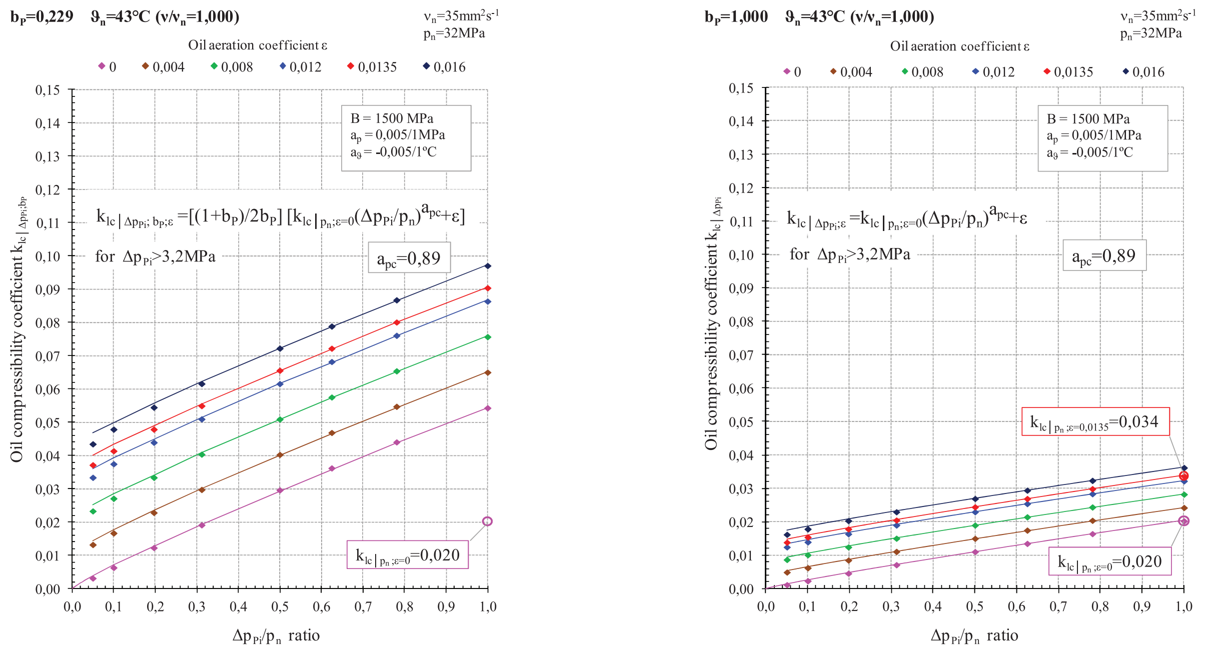


Fig.3 Examples of relation of the hydraulic oil compressibility coefficient $k_{lc|\Delta p_{Pi};b_P}$ and $k_{lc|\Delta p_{Pi}}$ to the ratio $\Delta p_{Pi}/p_n$ of indicated increase Δp_{Pi} of pressure in the pump working chambers to nominal pressure p_n = 32MPa at the pump capacity coefficient b_P = 0,229 and b_P = 1, at different values of oil aeration coefficient ε, at temperature ϑ_n = 43°C of hydraulic oil used during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], temperature corresponding to oil viscosity ratio ν/ν_n = 1 (reference viscosity ν_n = 35mm²s⁻¹); value of the oil compressibility coefficient $k_{lc|p_n}$ used in the model of losses and energy efficiency is determined in the same conditions as coefficient k_1 of volumetric losses due to leakage in the pump working chambers, i.e. at the theoretical capacity q_{Pt} per one shaft revolution (at pump capacity coefficient b_P = 1); the actual oil aeration coefficient determined during the pump tests was ε = 0,0135 and the corresponding value of oil compressibility coefficient $k_{lc|p_n}$ = 0,034

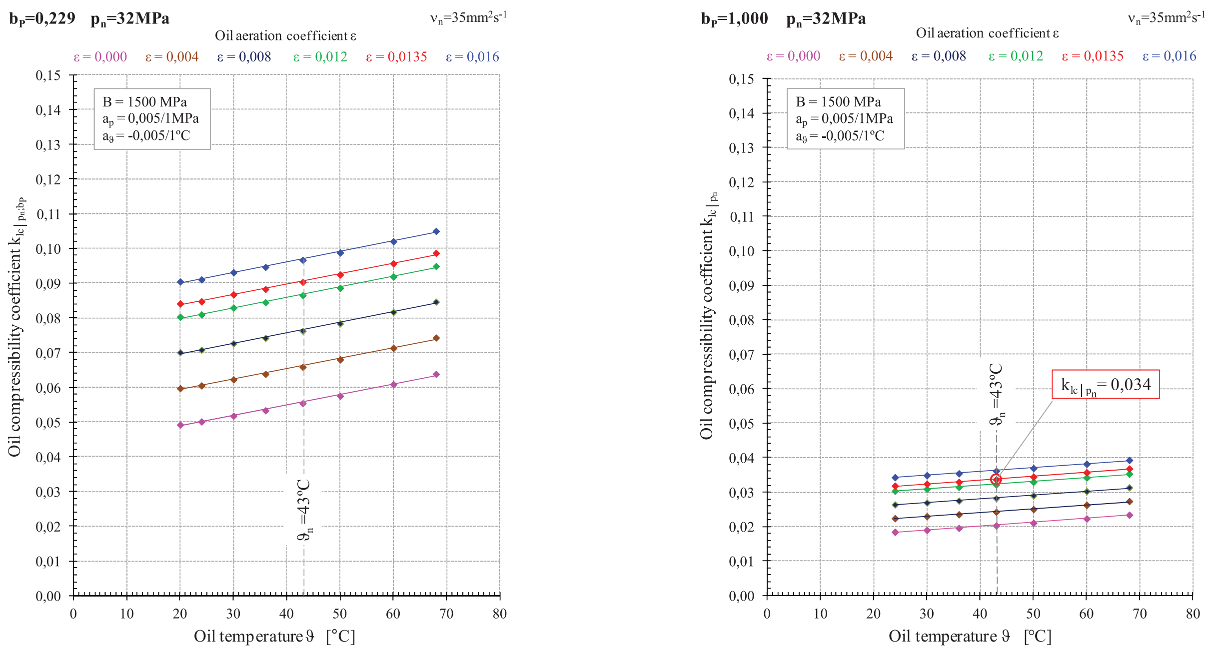
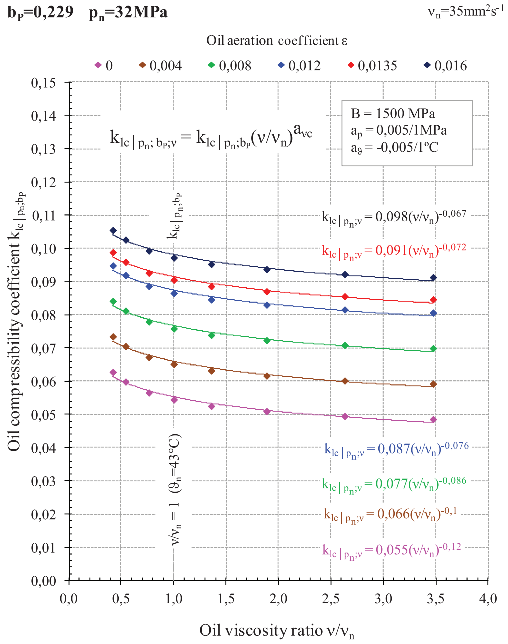


Fig.4 Examples of relation of hydraulic oil compressibility coefficient $k_{lc|p_n;b_P}$ and $k_{lc|p_n}$ to oil temperature ϑ at pump capacity coefficient b_P = 0,229 and b_P = 1, at different values of aeration coefficient ε of oil used during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12]; value of oil compressibility coefficient $k_{lc|p_n}$ used in the model of losses and energy efficiency is determined in the same conditions as coefficient k_1 of volumetric losses due to leakage in the pump working chambers, i.e. at the theoretical capacity q_{Pt} per one shaft revolution (at pump capacity coefficient b_P = 1) and at oil temperature ϑ_n corresponding to oil viscosity ratio ν/ν_n = 1 (reference viscosity ν_n = 35mm²s⁻¹); temperature corresponding to oil viscosity ratio ν/ν_n = 1 was ϑ_n = 43°C; actual value of oil aeration coefficient was ε = 0,0135 and the corresponding value of oil compressibility coefficient $k_{lc|p_n}$ = 0,034



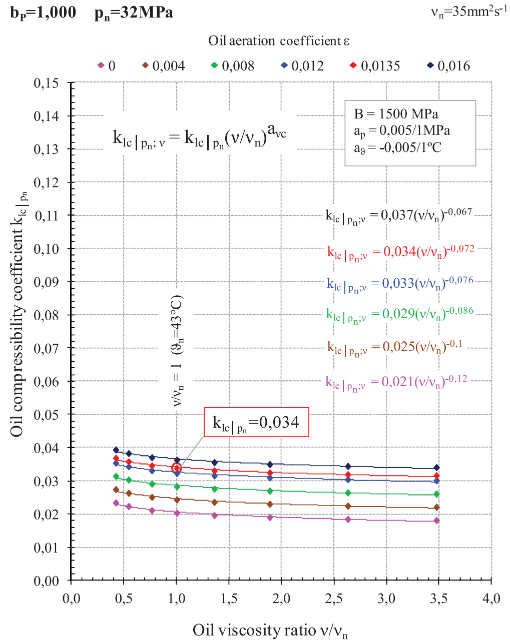


Fig.5 Examples of relation of hydraulic oil compressibility coefficient $k_{lc|p_n;b_P}$ and $k_{lc|p_n}$ to oil viscosity ratio ν/ν_n corresponding to oil temperature ϑ at pump capacity coefficient $b_P = 0{,}229$ and $b_P = 1$, at different values of aeration coefficient ε of oil used during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12]; value of oil compressibility coefficient $k_{lc|p_n}$ used in the model of losses and energy efficiency is determined in the same conditions as coefficient k_1 of volumetric losses due to leakage in the pump working chambers, i.e. at the theoretical capacity q_{Pt} per one shaft revolution (at pump capacity coefficient $b_P = 1$) and at oil temperature ϑ_n corresponding to oil viscosity ratio $\nu/\nu_n = 1$ (reference viscosity $\nu_n = 35 mm^2 s^{-1}$); temperature corresponding to oil viscosity ratio $\nu/\nu_n = 1$ was $\vartheta_n = 43°C$; actual value of oil aeration coefficient was $\varepsilon = 0{,}0135$ and the corresponding value of oil compressibility coefficient $k_{lc|p_n} = 0{,}034$

3. Mathematical model of volumetric losses q_{Pvc} of hydraulic oil compression in the pump working chambers

Volumetric losses q_{Pvc} determined per one shaft revolution of compression of liquid pressed in the pump working chambers are not attributable to the pump design solution. They result from the liquid compressibility itself and from aeration of the liquid. Main reason of working liquid aeration is air dissolved in the liquid (not having in such form any influence on its compressibility) and getting out of the liquid (in the form of bubbles) in the conditions of local (in the system conduits or in pump working chambers during their connection with pump inlet channel) drop of pressure below the atmospheric pressure. One of the reasons may be admitting by the hydrostatic system designer or user too low pressure in the pump inlet conduit, which may cause cavitation in the pump working chambers during their connection with the inlet channel.

Mathematical model of volumetric losses $q_{Pvc|\Delta p_{Pi};b_P;\varepsilon;\nu}$ per one shaft revolution resulting from compression of non-aerated ($\varepsilon = 0$) or aerated ($\varepsilon > 0$) hydraulic oil, determined at indicated increase Δp_{Pi} of pressure in the pump working chambers, at pump capacity coefficient b_P (in the $0 < b_P \le 1$ range) and at the ratio ν/ν_n of oil viscosity ν and reference viscosity ν_n is described (with $\Delta p_{Pi} > 3{,}2MPa$) by the formula:

$$q_{Pvc|\Delta p_{Pi};b_P;\varepsilon;\nu} = k_{lc|\Delta p_{Pi};b_P;\varepsilon;\nu}\, q_{Pgv} = k_{lc|\Delta p_{Pi};b_P;\varepsilon;\nu}\, b_P q_{Pt} =$$
$$= \frac{1+b_P}{2b_P}\left[k_{lc|p_n;\varepsilon=0}\left(\Delta p_{Pi}/p_n\right)^{a_{pc}} + \varepsilon\right]\left(\nu/\nu_n\right)^{a_{vc}} b_P\, q_{Pt} =$$
$$= \frac{1+b_P}{2}\left[k_{lc|p_n;\varepsilon=0}\left(\Delta p_{Pi}/p_n\right)^{a_{pc}} + \varepsilon\right]\left(\nu/\nu_n\right)^{a_{vc}} q_{Pt}\,. \tag{18}$$

with exponent $a_{pc} = 0{,}89$
and with exponent $a_{vc} = -0{,}12$.

Coefficient $k_{lc|p_n;\varepsilon=0}$ of hydraulic oil compressibility **is a coefficient of non-aerated oil compressibility. At pressure p_n = 32MP, this coefficient is of an order $k_{lc|32MPa;\varepsilon=0} = 0{,}020$.**

In reference to formulae (13), (17) and (18), a conclusion may be made that coefficient $k_{lc|p_n;\varepsilon>0}$ of aerated ($\varepsilon > 0$) oil compressibility may be replaced, with sufficient accuracy, (in the $\Delta p_{Pi} > 3{,}2MPa$ range), by the expression:

$$k_{lc|p_n;\varepsilon\rangle 0} = k_{lc|p_n;\varepsilon=0} + \varepsilon \,. \tag{19}$$

Formula (18) describing volumetric losses $q_{Pvc|\Delta p_{Pi};b_P;\varepsilon;\nu}$ of hydraulic oil compression in the working chambers should be used in the model of power of oil compression in the pump.

4. Conclusions

1. The objective of this work was to develop the capability of evaluating the volumetric losses of hydraulic oil compression in the working chambers of high pressure variable capacity displacement pump. Volumetric losses of oil compression must be determined in function of the same parameters, on which the volumetric losses due to leakage, resulting from the quality of design solution of the pump, are dependent and evaluated also in function of the oil aeration coefficient ε.
2. A mathematical model has been developed for describing the coefficient $k_{lc|\Delta p_{Pi};b_P;\varepsilon;\nu}$ of hydraulic oil compressibility in function of:

- the ratio $\Delta p_{Pi}/p_n$ of indicated increase Δp_{Pi} of pressure in the working chambers and nominal pressure p_n,
- the pump capacity coefficient b_P,
- the oil aeration coefficient ε,
- the ratio ν/ν_n of oil viscosity ν and reference viscosity ν_n.

3. A mathematical model of volumetric losses $q_{Pvc|\Delta p_{Pi};b_P;\varepsilon;\nu}$ of hydraulic oil compression has been presented in the form allowing to use it in the model of power of oil compression in the pump.

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