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The analytical design method of railway route's main directions intersection area

Abstract: The paper presents a design method related to railway track sections situated in a bend. In the method advantage is taken of an analytical form of description by using appropriate mathematical formulae. This may become particularly useful when the investigations are based on the data obtained from mobile satellite measurements. In the paper attention is concentrated on an universal approach creating an opportunity to diversify the type and length of the transition curves in use. The design procedure of the geometric lay-out is allocated to a particular local system of coordinates to be followed by a special transfer of the obtained solution to the national frame of space references 2000. The adopted course of procedure together with adequate theoretical relations has been provided. All the study has been illustrated with calculation examples based on data obtained from the railway line in operation.

Keywords: Railway route; Geometric lay-out; Design method

1 Introduction

The issue of designing geometrical layouts of civil communication routes (roads and railways engineering) is being still developed. Undoubtedly, progress in realization techniques and obtained accuracy are the reasons of development in the field of designing. Satellite measurements play essential role in this process [5]. Within the scope of creating a theoretical basis for the new designing method works such as [2–12] may be mentioned.

In Poland in the middle of 2008 there was established an institution, Active Geodetic Network ASG-EUPOS, which is a national network of the permanent stations GNSS [5, 13]. The results of several measuring cam-

paignes carried out on the railway track by an investigation team of the Gdansk University of Technology, Naval Academy in Gdynia, the Department of Railways PKP PLK S.A. in Gdynia, and the Leica Geosystems GA firm [14] qualify for making a statement on the application of the measuring technique. Even at its present stage of development it opens entirely new perspectives. Its application makes it possible to determine very precisely the basic data for designing the railway line modernization (main directions if the route and its intersection angle), and additionally, with relatively small error, the coordinates of the existing track axis. The technique under consideration creates an opportunity to reproduce the area of the route direction change in compliance with the requirements of the modernization project [15, 16].

Papers [17] and [18] deal with the presentation of the design procedure related to the area of the railway route direction change adapted to the mobile satellite measurements technique. The procedure concerns a model case, that is, such a situation when the geometric system indicates full symmetry, which means, that use is made of the same type of transition curves of equal length. However, the situation is not always the same, and in general one should take into consideration a possibility of variation in the type and length of the transition curves. The design procedure should then be significantly modified.

2 Determination of basic data for design

As a result of the satellite measurements it is possible to obtain a set of coordinates Y_i, X_i of the points situated along the track axis, indicated in the national space reference system 2000. Its presentation in graphic form can be used for a general assessment of the geometric conditions. Figure 1 illustrates a chosen fragment of a geometric lay-out consisting of two straight segments and an arc between them tending to form a circle (evidently deformed).

The measured coordinates of straight 1 on the left side of the geometric lay-out under consideration and straight 2 on the right of the system can take advantage from the

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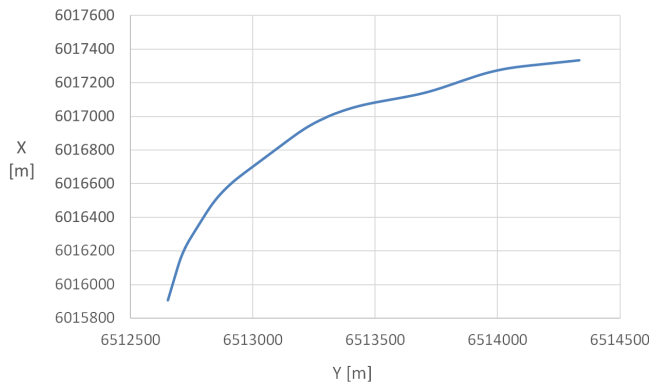
Figure 1: Illustrative segment of the railway route in the national space reference system 2000.

determination of the equations of both the straights in the Y, X system, by the use of the least squares method, in the form of $X = A + BY$. A knowledge of the equations of both the straights makes it possible to determine the coordinates of the point cutting the main directions of the route in terms of the system 2000 by using the equations:

$$Y_W = \frac{A_2 - A_1}{B_1 - B_2} \tag{1}$$

$$X_W = A_1 + \frac{A_2 - A_1}{B_1 - B_2} B_1 \tag{2}$$

From the point of view of the real route direction search the major value lies in the slope coefficient of each straight $B = \tan\phi$. The determination of the slope angles ϕ_1 and ϕ_2 of both the straights relating to axis Y enables us to find the intersection angle α of the route. A general formula for the intersection angle α is,

$$\alpha = |\varphi_1 - \varphi_2| + \gamma \tag{3}$$

where: $\gamma = \pi$ for the case, when the route turns right and $B_1 > 0$, $B_2 < 0$ and for the situation when the route turns left and $B_1 < 0$, $B_2 > 0$,

$\gamma = 0$ for other cases.

The determination of new ordinates of the track axis will relate to the following output data:

- the intersection angle α,
- radius of the circular arc R,
- superelevation on arc h_0,
- lengths l_1 and l_2 of the assumed transition curves.

In order to make use of the obtained measuring data to design a route's main directions intersection area it is necessary to isolate its fragment, being of interest to us, from the whole geometric lay-out, and to carry out an appropriate transformation (a shift and a turn) of the coordinate

system. In fact, every design of geometric layout can be presented in terms of its own local system of coordinates as shown in Figure 2. The outset of the system is situated on one of the major directions (on the straight *1*). To compare the ordinates it is possible to transfer the coordinates of the existing route determined by satellite measurements to the local coordinate system. Equations for the new route coordinates with the local coordinate system x, y shifted to point $O(Y_0, X_0)$ and turned through angle β, are described by relations [19]:

$$x = (Y - Y_0)\cos\beta + (X - X_0)\sin\beta \tag{4}$$

$$y = -(Y - Y_0)\sin\beta + (X - X_0)\cos\beta \tag{5}$$

The value of angle β is described by the following equation:

$$\beta = \frac{1}{2}(\varphi_1 + \varphi_2) + \delta \tag{6}$$

where: $\delta = -\frac{\pi}{2}$ in case the route turns right and $B_1 > 0$, $B_2 < 0$,

$\delta = \frac{\pi}{2}$ in case the route turns left and $B_1 < 0$, $B_2 > 0$,

$\delta = 0$ in other cases.

When the route turns right and the rotation angle β conforms with the movement of the hands of the clock, after turning the system, the ordinate y obtains positive values. In case the route turns left, the values of ordinate y are negative. However, for practical purposes, it is possible to mirror them in relation to axis x. In this condition use is made of coordinate system x, $\bar{y}$, with $\bar{y} = -y$.

Unfortunately at this stage of procedure it is still impossible to determine the position of initial point $O(Y_0, X_0)$ for the new design of the geometric lay-out. It is obvious that the point should determine the outset of the transition curve and lie on the straight corresponding to the arterial direction of the route.

3 Design procedure

3.1 Local coordinates system

Since the coordinates of point $O(Y_0, X_0)$ in the system 2000 are not known it is necessary to assume first a local coordinates system x, y, not connected with the global system whose outset at point $O(0,0)$ will create the beginning of the first transition curve (*TC1*) which deviates the course of the route from the arterial direction (Fig. 2). It will also create the starting point of the second, auxiliary system of
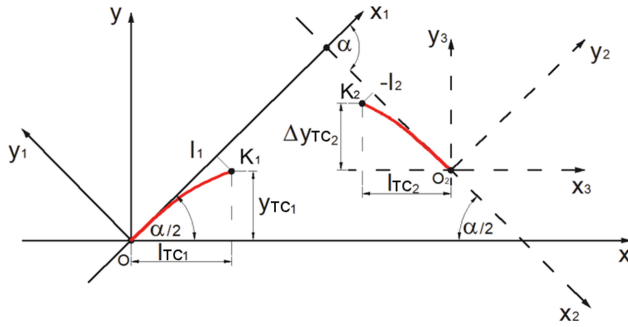
Figure 2: Assumed local coordinates system (with inscribed transition curves).

coordinates Ox_1y_1 used for the determination of points on the transition curve *TC1*.

The position of the auxiliary set of coordinates $O_2x_2y_2$ of the second transition curve (*TC2*) is at this stage unknown. It is assumed that its axis of abscissa is adequate to the straight parallel to the second main direction cutting across the straight line *1* at angle α. The analysis aims at the determination of the position of point O_2 for the Oxy coordinates system (Fig. 2). Apart from the determination of ordinates $y(x)$, the crucial question will pose the length of the total system resulting from the values of projections of transition curves l_{TC1} and l_{TC2} and the circular arc l_{CA} on the axis of abscissae x.

In the design procedure relating to the route's main directions intersection area the most complex operation is the determination of the right coordinates Y_0 and X_0 at the outset of the local coordinates system, for the reason, that they should be adequate to the newly designed geometric lay-out. To find the most optimal coordinates it is necessary to determine the position of the basic points in the system that are responsible for the localization of the transition curves and the circular arc. This necessitates a detailed analysis of the respective geometric elements used. Of course primary attention should be given to both the transition curves. Later the circular arc will be inscribed between them.

3.2 Transition curve *TC1*

The procedure starts with the determination of transition curve *TC1* ordinates, located in the auxiliary coordinates system Ox_1y_1 (Fig. 2). The choice of the type of the curve characterizes the function of its curvature $k_1(l)$ on the basis of which it is possible to determine the transition curve equation, whether it is accurate and written in a parametric form with $x_1(l)$ and $y_1(l)$ (where parameter l is the position of a given point along the curve length), or approx-

imated with $y_1(x_1)$, obtained after adoption of some common simplifying assumptions used in railway routes [19].

From the viewpoint of undertaking further steps in the investigation procedure the value of the tangent at the end of curve *TC1* is of great significance. In order to determine it in an accurate way it is necessary to start the calculations with the curvature equation $k_1(l)$ and the determination of the tangent inclination angle

$$\Theta_1(l) = \int k_1(l)\, dl$$

The tangent value at the end of the transition curve in the working system of coordinates Oxy will, thus, amount to

$$s_1 = \tan\left[\Theta_1(l_1) + \frac{\alpha}{2}\right] \tag{7}$$

The next stage of operation is the transformation of the transition curve *TC1* to the adopted working set of coordinates by making a turn of its reference system through angle $\alpha/2$. Consequently the calculations lead to parametric equations of the transition curve in the Oxy system [20]:

- in the case of *TC1* determined in an accurate way

$$x(l) = x_1(l)\cos\frac{\alpha}{2} - y_1(l)\sin\frac{\alpha}{2} \tag{8}$$

$$y(l) = x_1(l)\sin\frac{\alpha}{2} + y_1(l)\cos\frac{\alpha}{2} \tag{9}$$

- in the case of *TC1* determined approximately

$$x(x_1) = x_1\cos\frac{\alpha}{2} - y_1(x_1)\sin\frac{\alpha}{2} \tag{10}$$

$$y(x_1) = x_1\sin\frac{\alpha}{2} + y_1(x_1)\cos\frac{\alpha}{2} \tag{11}$$

In the above equations both the parameter $l \in \langle 0,\ l_1 \rangle$, and $x_1 \in \langle 0,\ l_1 \rangle$. The abscissa of the transition curve $x \in \langle 0,\ l_{TC1} \rangle$, where l_{TC1} is determined by Equation (8) or (10). The final ordinate y_{TC1} follows from Equation (9) or (11).

3.3 Transition curve *TC2*

The transition curve *TC2* has length l_2 and is located in the system of coordinates $O_2x_2y_2$ (Fig. 2), but the exact location of point O_2 at this stage is still unknown. However, by the use of this system it is possible to model the transition curve itself and collect its basic data necessary to determine the coordinates of point K_2 (i.e. the values of l_{TC2} and Δy_{TC2}). The choice of the type of curve determines the function of its curvature $k_2(l)$, on the basis of which one

can work out the transition curve equation, and find the angle of inclination of tangent $\Theta_2 (l)$. This enables us to determine the tangent values at the end of curve *TC2* in the working system of coordinates *Oxy*.

$$s_2 = \tan \left[\Theta_2 (-l_2) - \frac{\propto}{2} \right] \qquad (12)$$

There is also a possibility of transforming the curve points into an auxiliary system of coordinates $O_2 x_3 y_3$ (Fig. 2). Since the axes of the system are parallel to the axes of the working system *Oxy*, the operation can be useful to determine l_{TC2} and Δy_{TC2}. The coordinates of the curve *TC2* points in the $O_2 x_3 y_3$ system are described by the parametric equations:

– in the case of *TC2* determined in an accurate way

$$x_3(l) = x_2(l) \cos \frac{\propto}{2} + y_2 (l) \sin \frac{\propto}{2} \qquad (13)$$

$$y_3 (l) = -x_2(l) \sin \frac{\propto}{2} + y_2 (l) \cos \frac{\propto}{2} \qquad (14)$$

– in the case of *TC2* determined approximately

$$x_3(x_2) = x_2 \cos \frac{\propto}{2} + y_2 (x_2) \sin \frac{\propto}{2} \qquad (15)$$

$$y_3 (x_2) = -x_2 \sin \frac{\propto}{2} + y_2 (x_2) \cos \frac{\propto}{2} \qquad (16)$$

In the above equations parameter $l \in \langle -l_2, 0 \rangle$ and $x_2 \in \langle -l_2, 0 \rangle$. The transition curve abscissa $x_3 \in \langle -l_{TC2}, 0 \rangle$, where

$$l_{TC2} = \left| x_2(-l_2) \cos \frac{\propto}{2} + y_2 (-l_2) \sin \frac{\propto}{2} \right| \qquad (17)$$

or

$$l_{TC2} = \left| -l_2 \cos \frac{\propto}{2} + y_2 (-l_2) \sin \frac{\propto}{2} \right| \qquad (18)$$

Value Δy_{TC2} is

$$\Delta y_{TC2} = y_3 (-l_2) = \left| -x_2(-l_2) \sin \frac{\propto}{2} + y_2 (-l_2) \cos \frac{\propto}{2} \right| \qquad (19)$$

or

$$\Delta y_{TC2} = y_3 (-l_{KP2}) = \left| l_2 \sin \frac{\propto}{2} + y_2 (-l_2) \cos \frac{\propto}{2} \right| \qquad (20)$$

3.4 Determination of the circular arc ordinates

If the position of the transition curve *TC1* and the value of tangent at the end of transition curve *TC2* are known one can inscribe the circular arc of radius R into the geometric system. Appropriate equations describe the situation shown in Figure 3.
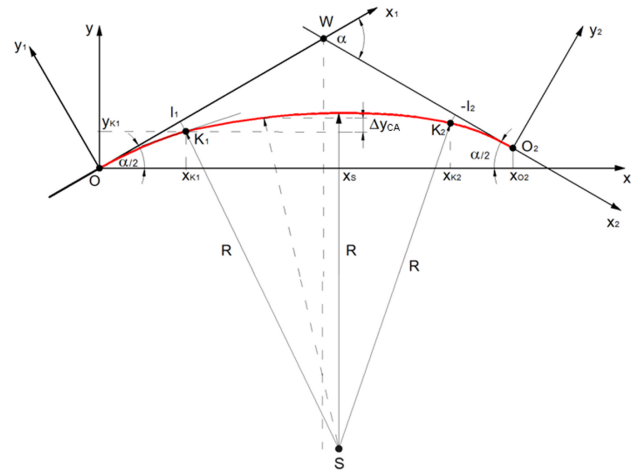

Figure 3: The designed geometric lay-out applied to the local coordinates system

The circular arc should be tangential to the transition curve *TC1* at its end, i.e. at point K_1, and the circular arc radius R should be lying on the straight line perpendicular to the tangent at point K_1. The coordinates of the circular arc centre $S(x_S, y_S)$ are as follows:

$$x_S = x_{K1} + \frac{s_1}{\sqrt{1 + s_1^2}} R \qquad (21)$$

$$y_S = y_{K1} - \frac{1}{\sqrt{1 + s_1^2}} R \qquad (22)$$

Thus it is possible to write the circular arc equation (using appropriate rectangular triangles in Figure 3 and the Pythagorean theorem):

$$y_{CA} = y_{K1} + \left[R^2 - (x_S - x)^2 \right]^{\frac{1}{2}} - \left[R^2 - (x_S - x_{K1})^2 \right]^{\frac{1}{2}}, \qquad (23)$$
$$x \in \langle x_{K1}, x_{K2} \rangle$$

Since the value of tangent s_2 defined by Equation (12) is known, one can find the position of the end of the circular arc, that is, the coordinates of point $K_2(x_{K2}, y_{K2})$.

$$x_{K2} = x_S - \frac{s_2}{\sqrt{1 + s_2^2}} R \qquad (24)$$

Of course under this circumstances $y_{K2} = y_{CA}(x_{K2})$, and the length of the circular arc projected onto axis x is equal to $l_{CA} = x_{K2} - x_{K1}$.

3.5 Preparation of ordinates for the entire geometric lay-out

A knowledge of the coordinates of point K_2 enables consequently to consider an auxiliary system $O_2 x_2 y_2$ within

the local system Oxy in order to determine the position of point $O_2(x_{O2}, y_{O2})$.

$$x_{O2} = x_{K2} + l_{TC2} = l_{TC1} + l_{CA} + l_{TC2} \qquad (25)$$

$$y_{O2} = y_{K2} - \Delta y_{TC2} \qquad (26)$$

As soon as the position of point O_2 has been determined it is possible to set the whole geometric lay-out together and to insert the transition curve $TC2$. For this purpose values $x_3(l)$ and $y_3(l)$ or $x_3(x_2)$ and $y_3(x_2)$ should be added to the coordinates of point O_2 respectively. The use of Equations (13) and (14) provides parametric equations $x(l)$ and $y(l)$, whereas Equations (15) and (16) give us equations $x(x_2)$ and $y(x_2)$ in which $l \in \langle -l_2, 0 \rangle$ and $x_2 \in \langle -l_2, 0 \rangle$.

3.6 Application of the solution to system 2000

The determination of the position of the basic points in the total geometric solution relating to the local coordinate system finally enables us to find the outset of the system, that is, the point $O(0,0)$ on an appropriate main direction of the route (Fig. 3). The position is determined by the vertex coordinates $W(x_W, y_W)$ in the Oxy system, for the reason that it is possible to find easily its coordinates Y_W and X_W in terms of the system 2000, using the satellite measurement data and the aid of Equations (1) and (2).

The coordinates of point $W(x_W, y_W)$ are defined by the relations

$$x_W = \frac{y_{O2} + \tan \frac{\propto}{2} x_{O2}}{2 \tan \frac{\propto}{2}} \qquad (27)$$

$$y_W = \frac{1}{2} \left(y_{O2} + \tan \frac{\propto}{2} x_{O2} \right) \qquad (28)$$

while the coordinates of point $O(Y_O, X_O)$ in the system 2000

$$Y_O = Y_W - \sqrt{\frac{x_W^2 + y_W^2}{1 + B_1^2}} \qquad (29)$$

$$X_O = X_W \mp B_1 \sqrt{\frac{x_W^2 + y_W^2}{1 + B_1^2}} \qquad (30)$$

In Equation (30) the sign "–" is used for coefficient $B_1 > 0$, whereas $B_1 < 0$ is marked with sign "+". Of course the determined Y_O and X_O should conform to the equation of the straight 1.

Knowing the coordinates of point O, it is possible to transfer the obtained solution to the global system by using equations [21]:

$$Y = Y_0 + x \cos \beta - y \sin \beta \qquad (31)$$

$$X = X_0 + x \sin \beta + y \cos \beta \qquad (32)$$

4 Solution for the symmetric case

A particular case of the presented solution is the model situation, i.e. a symmetrical one with two identical transition curves [17, 18]. Since $l_1 = l_2$, therefore for $TC2$, angle $\Theta(-l_2) = -\Theta(l_1)$, while the tangent inclination $s_2 = -s_1$. In that situation the coordinates of point K_2 are as follows:

$$x_{K2} = x_{K1} + 2 \frac{s_1}{\sqrt{1 + s_1^2}} R \qquad (33)$$

$$y_{K2} = \Delta y_{TC2} = y_{K1} \qquad (34)$$

Point O_2 lies, of course, on axis x and has an abscissa

$$x_{O2} = 2 x_{K1} + 2 \frac{s_1}{\sqrt{1 + s_1^2}} R \qquad (35)$$

5 Calculation examples

5.1 Assessment of the geometric situation

In the calculation example advantage will be taken of the satellite measurements carried out in 2010 as shown in Figure 1. The equations chosen for the main directions of the route, on the left side of the geometric system under consideration (i.e. the straight 1) is:

$$X_1 = -25780782, 28763 + 4, 88229474Y \qquad (36)$$

Its angle of inclination $\phi_1 = \arctan B_1 = 1, 36876884$ rad.

The main direction of the route on the right side of the analyzed geometric lay-out (i.e. the straight 2) is described by the equation

$$X_2 = 5011989, 46931 + 0, 15432805Y, \qquad (37)$$

and the angle of inclination $\phi_2 = 0, 15312000$ rad. Thus, the intersection angle of the route amounts to $\alpha = \phi_1 - \phi_2 = 1, 21564884$ rad. Coordinates of the intersection point of the major route directions obtained by using Equations (1) and (2) are as follows: $Y_W = 6512899, 472$ m, $X_W = 6017112, 545$ m.

To make an appraisal of the design data it is necessary to make a turn of the coordinates system and to carry out a shift in it. On the basis of Equation (6) the angular displacement of the frame of reference is β = 0, 76094442 rad. The abscissa of the outset of the new system is assumed to be Y_0 = 6512652, 56600 m. Making use of Equation (36) one can note that it corresponds to ordinate X_0 = 6015907, 07880 m of point O situated on the straight *1*. The application of Equations (4) and (5) leads to the situation presented in Figure 4.
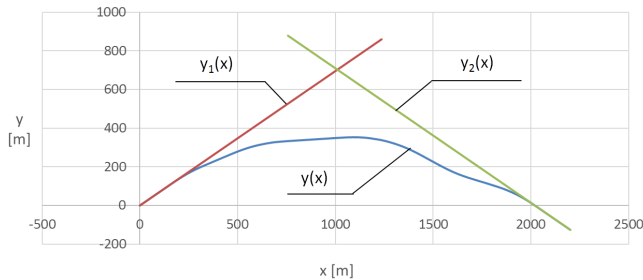
Figure 4: Illustrative section of the railway route in local coordinates system; $y(x)$ – the existing course of the route, $y_1(x)$ and $y_2(x)$ – determined main directions of the route.

From Figure 4 it follows that the length of the projection of the whole geometric lay-out onto axis x reaches about 2000 m, whereas the system itself is in fact a composition of five consecutive arcs. With respect to the analyzed case it has been acknowledged that the situation is incorrect and the evaluation of radii in particular circular arcs is useless. Quite the contrary an attempt will be made to apply one circular arc with two transition curves (of different lengths) and to inscribe it in the existing geometric lay-out.

5.2 Variant I

Attention was concentrated on a number of variants related to the solution of the design problem for the assumed speed of v_p = 120 km/h. An assumption was made to use transition curves in the form of Cornu's spiral (i.e. clothoid). At first the paper will present a solution for the following design data:

- the intersection angle α = 1, 21564884 rad,
- the radius of circular arc R = 1500 m,
- the value of the superelevation of arc h_0 = 70 mm,
- the transition curve on the left of the system of length l_1 = 80 m,

- the transition curve on the right of the system of length l_2 = 120 m.

Consequently the following set of equations giving an overall description of the designed geometric lay-out was obtained:

- Transition curve *TC1* ($x \in \langle 0, \ 66, 073 \rangle$m)
  $x(l) = 0, 820892 \cdot l + 7, 93171 \cdot 10^{-7} \cdot l^3$
  $- 1, 42516 \cdot 10^{-12} \cdot l^5$
  $y(l) = 0, 571083 \cdot l - 1, 14013 \cdot 10^{-6} \cdot l^3$
  $- 9, 91463 \cdot 10^{-13} \cdot l^5$
  Parameter $l \in \langle 0, \ 80 \rangle$ m.
- Circular arc ($x \in \langle 66, 073, \ 1696, 260 \rangle$ m)
  $y_{CA} = -1208, 64175 + \left[1500^2 - (889, 56078 \right.$
  $\left. - \ x)^2 \right]^{\frac{1}{2}}$
- Transition curve *TC2* ($x \in \langle 1696, 260,$
  $1795, 674 \rangle$ m)
  $x(l) = 1795, 67389 + 0, 820892 \cdot l$
  $+5, 2878 \cdot 10^{-7} \cdot l^3 - 6, 33405 \cdot 10^{-13} \cdot l^5$
  $y(l) = -11, 2445 - 0, 571083 \cdot l$
  $+7, 6009 \cdot 10^{-7} \cdot l^3 + 4, 4065 \cdot 10^{-13} \cdot l^5$
  Parameter $l \in \langle -120, \ 0 \rangle$ m.

At this stage of executing the design procedure it is possible to transfer the measured points of the route to the local system provided with the new geometric conditions. This will enable us to make a comparison of the obtained solution with the existing run of the route. Of course, by retaining the same value of angle β as before, and by inserting the determined coordinates Y_0 = 6512649, 089 m and X_0 = 6015890, 103 m one can obtain the geometric situation presented in Figure 5.
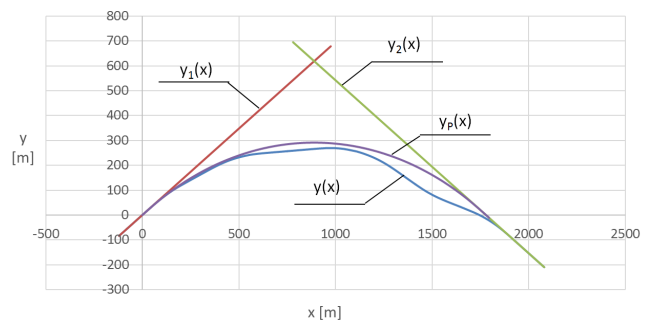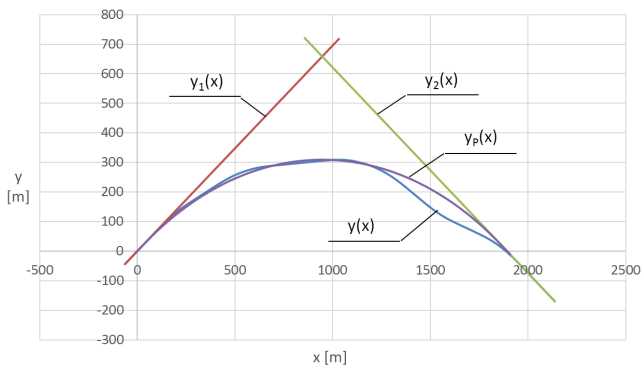

Figure 5: The existing and the designed geometric system in the local system of coordinates (variant I, using the non-isometric scales); $y(x)$ – the existing course of the route, $y_1(x)$ and $y_2(x)$ – determined main directions of the route, $y_P(x)$ – designed course of the route.

5.3 Variant II

To bring the designed geometric system closer to the existing lay-out the circular arc radius was increased to value $R = 1600$ m and the following set of equations was obtained:

- Transition curve *TC1* ($x \in \langle 0, \; 66,048 \rangle$ m)
  $x(l) = 0,820892 \cdot l + 7,43598 \cdot 10^{-7} \cdot l^3$
  $-1,25258 \cdot 10^{-12} \cdot l^5$
  $y(l) = 0,571083 \cdot l - 1,06887 \cdot 10^{-6} \cdot l^3$
  $-8,71403 \cdot 10^{-13} \cdot l^5$
  Parameter $l \in \langle 0, \; 80 \rangle$ m.
- Circular arc ($x \in \langle 66,048, \; 1810,511 \rangle$ m)
  $y_{CA} = -1290,72181 + \left[1600^2 - (946,6628 - x)^2 \right]^{\frac{1}{2}}$
- Transition curve *TC2* ($x \in \langle 1810,511, \; 1909,869 \rangle$ m)
  $x(l) = 1909,86912 + 0,820892 \cdot l$
  $+4,9673 \cdot 10^{-7} \cdot l^3 - 5,56703 \cdot 10^{-13} \cdot l^5$
  $y(l) = -11,25528 - 0,571083 \cdot l$
  $+7,1258 \cdot 10^{-7} \cdot l^3 + 3,8729 \cdot 10^{-13} \cdot l^5$
  Parameter $l \in \langle -120, \; 0 \rangle$ m.

The use of coordinates $Y_0 = 6512649,089$ m and $X_0 = 6015890,103$ m, chosen for this variant provides the situation illustrated in Figure 6.

Figure 6: The existing and the designed geometric system in the local system of coordinates (variant II, using the non-isometric scales); $y(x)$ – the existing course of the route, $y_1(x)$ and $y_2(x)$ – determined main directions of the route, $y_P(x)$ – designed course of the route.
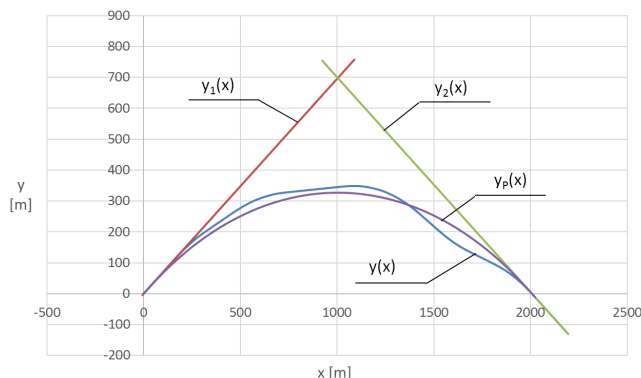
5.4 Variant III

The solution presented in Figure 6 to a great extent is compatible with the existing course of the route. However, the final part of the route significantly deviates from the original course (reaching even more than 50 m). For this reason the effects of the application of radius $R = 1700$ m were tested. The results are expressed by the set of equations:

- Transition curve *TC1* ($x \in \langle 0, \; 66,026 \rangle$ m)
  $x(l) = 0,820892 \cdot l + 6,99856 \cdot 10^{-7} \cdot l^3$
  $-1,10955 \cdot 10^{-12} \cdot l^5$
  $y(l) = 0,571083 \cdot l - 1,006 \cdot 10^{-6} \cdot l^3$
  $-7,71901 \cdot 10^{-13} \cdot l^5$
  Parameter $l \in \langle 0, \; 80 \rangle$ m.
- Circular arc ($x \in \langle 66,026, \; 1924,759 \rangle$ m)
  $y_{CA} = -1372,80294 + \left[1700^2 - (1003,766 - x)^2 \right]^{\frac{1}{2}}$
- Transition curve *TC2* ($x \in \langle 1924,759, \; 2024,067 \rangle$ m)
  $x(l) = 2024,0669 + 0,820892 \cdot l$
  $+4,66571 \cdot 10^{-7} \cdot l^3 - 4,9314 \cdot 10^{-13} \cdot l^5$
  $y(l) = -11,26481 - 0,571083 \cdot l$
  $+6,70664 \cdot 10^{-7} \cdot l^3 + 3,43067 \cdot 10^{-13} \cdot l^5$
  Parameter $l \in \langle -120, \; 0 \rangle$ m.

As a result of taking advantage of the determined coordinates $Y_0 = 6512649,089$ m and $X_0 = 6015890,103$ m the situation takes the form as illustrated in Figure 7.
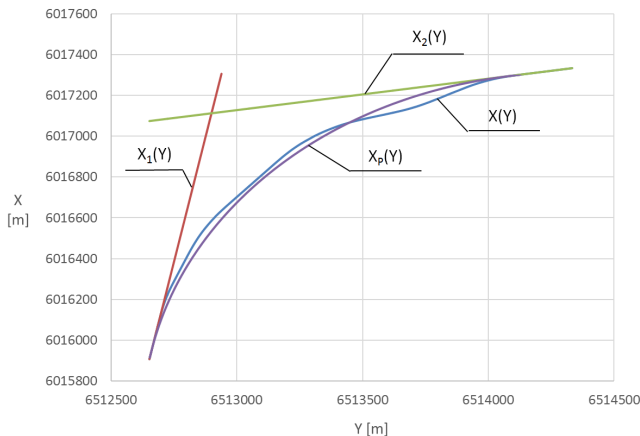

Figure 7: The existing and the designed geometric system in the local system of coordinates (variant III, using the non-isometric scales); $y(x)$ – the existing course of the route, $y_1(x)$ and $y_2(x)$ – determined main directions of the route, $y_P(x)$ – designed course of the route.

5.5 Choice of solution and application of the project to system 2000

Out of the three variants presented, variant III (Fig. 7) seems to be most advantageous, in fact, requiring some transverse shifts in the track along its whole length, but, of course, shorter than in the case of the other two variants

at the final part of the route. Therefore this variant will be chosen for use.

The determined coordinates of points along the route in the local coordinate system should now be transferred to the global system. In the transformation, use will be made of Equations (31) and (32), as well as values β, Y_0 and X_0 obtained in the course of carrying out the calculations. On transferring the solution to the national space reference system 2000, the solution takes the form as shown in Figure 8.

Figure 8: The existing and the designed geometric lay-out in the national system 2000; $X(Y)$ – the existing course of the route, $X_1(Y)$ and $X_2(Y)$ – determined main directions of the route, $X_P(Y)$ – the designed course of the route.

6 The summing-up

The use of continuous satellite measurements with antennas installed on mobile rail carriage makes it possible to reproduce the position of the rail track axis, by using the absolute frame of reference, while the number of the exploited coordinates depends only on the assumed signal sampling frequency. Owing to this, as one can expect, a radical improvement in the sphere of shaping the rail track geometry will take place in the nearest future.

Under conditions of the existing situation there arises the necessity for working out a new method of designing the geometric railway track lay-outs. The shaping of straight route directions by the use of satellite measurements can be carried out as part of an analysis of the obtained measuring results. However, the design of sections situated in bends is more complex. To take advantage of the obtained measuring data it is necessary to separate

the area of the route direction change that is of interest to us, from the whole geometric lay-out, and to make an adequate transformation (a shift and a turn) of the coordinates. The new system of coordinates x, y creates an opportunity to symmetrically position the geometrical lay-out with the plotted main directions of the route.

The conception of how to design the area of the route direction change, presented in the paper, leads to obtaining an analytical solution, with the use of some special mathematical formulae, which is most suitable for practical application. The design procedure is of an universal character and offers a possibility for diversifying the type and length of the applied transition curves. The whole study is provided with calculation examples making use of data obtained from the railway line in operation.

As soon as an appropriate computer-aided system is ready it will immediately be possible to generate a set of coordinates for the design of the route, practically with an indefinite number of variants. Under such circumstances, the only problem to solve, will become the question of choosing the most optimal solution.

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