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Computer-aided evaluation of the railway track geometry on the basis of satellite measurements

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Abstract: In recent years, all over the world there has been a period of intensive development of GNSS (Global Navigation Satellite Systems) measurement techniques and their extension for the purpose of their applications in the field of surveying and navigation. Moreover, in many countries a rising trend in the development of rail transportation systems has been noticed. In this paper, a method of railway track geometry assessment based on mobile satellite measurements is presented. The paper shows the implementation effects of satellite surveying railway geometry. The investigation process described in the paper is divided on two phases. The first phase is the GNSS mobile surveying and the analysis obtained data. The second phase is the analysis of the track geometry using the flat coordinates from the surveying. The visualization of the measured route, separation and quality assessment of the uniform geometric elements (straight sections, arcs), identification of the track polygon (main directions and intersection angles) are discussed and illustrated by the calculation example within the article.

Keywords: Railway route; Geometric lay-out; Design method

1 Introduction

The classical tachymetry surveying methods based on the national geodetic network has always played a key role in shaping the geometry of the track, as well as in its subsequent maintenance. The propagation of the network's errors, along with often unsatisfactory and diverse accuracy of its points, results in difficulty in the adjustment of the measurements. These problems result from the fact that measurements of railway track cover long distances, therefore a visual assessment of the track shape becomes impossible. With this in view, the usage of uniform geodetic control network, in terms of accuracy, for this type of measurements is expected by their implementers.

A special feature preventing the use of satellite positioning systems GNSS in inventory measurements of railways was the lack of differential structures (geodetic reference station) covering vast areas which could provide determination of the positions with an accuracy on the centimeter level (phase measurements) and a uniform, high-precision geodetic network. It has been proved, that the mobile satellite surveying is fully suitable for the railway track geometric shape inventory in terms of measurement accuracy, however the key role of such surveying plays the network of reference stations [1–3].

Permanent GNSS observations carried out by large-scale satellite geodetic networks in the past few years have been transformed into complex telecommunication systems offering, in addition to the post-processing differentials service, also the real time corrections to the satellite measurements. The first stage of their development were passive national systems, created in the early 90s of the twentieth century. They evolved from single reference stations located in universities to national systems. They were characterized by autonomy of stations, the lack of standardization in the use of a uniform protocol of transferring data and, finally, the local character of utilization. As time passed, those passive systems have been successively upgraded on differential functions (GPS) of real time, becoming active structures, which allowed the provision of DGNSS (Differential GNSS) services in real time, thus providing geodetic realizations in a qualitatively new dimension in investments service. A significant expansion of that action, after marine radio beacons DGPS (Differential GPS) [4], has been associated with the appearing of new types of RTCM (Radio Technical Commission for Maritime) messages, starting from version 2.0 up to the current 3.0 [5]

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as well as with the development of mathematical modeling of surface corrections of GPS together with the methods of their transmission [6].

Joining in this process, the Polish Head Office of Geodesy and Cartography has taken on a serious challenge of the implementation of Active Geodetic Network ASG-EUPOS, which was finalized in April 2008, and was completed successfully by testing of services and IT infrastructure [7].

In this way, the opportunity to undertake research of the use of GNSS for the railway network inventory came about also in Poland and therefore the technique of mobile satellite measurements in 2009 was verified by the research team of Gdansk University of Technology, the Naval Academy in Gdynia, Department of Railways PKP PLK SA in Gdynia and company Leica Geosystems AG. The main essence of the research was to assess the new capabilities of the reference network with regard to the railway track geometry analysis. Already, the first measurements [8–12] allowed for the very precise determination of the basic data for the design and modernization of the railway line (the main directions of the route and its intersection angles), as well as, with a relatively small error, the coordinates of the existing axis of the track.

2 Methodology of GNSS measurements of railway track

The application of phase GNSS (surveying) for the inventory methods for railways, encounters a number of limitations. One of the most essential problems being the partial restriction of the reception of GNSS signals is the occurrence of the so-called field obstacles affecting the geometric accuracy of coefficients values – DOP (Dilution of Precision) [13, 14]. While in an open space, the present constellation of both GPS and GLONASS (Globalnaja Nawigacionnaja Sputnikowaja Sistiema) provides a very good geometry of the space segment, in urban, mountainous or wooded conditions periodic difficulties were observed in obtaining an accurate solution phase, or even a coding [15–17]. In conditions of unfavorable geometry of the space segment or the lack of a sufficient number of satellites, it is difficult to rely on the continuation of the measurements with the required accuracy and thus obtain good availability, reliability, continuity and integrity of the determinations.

In 2009-2012 various configurations of phase GNSS receivers, both in terms of their number as well as their distribution on the measurement platform, were used to determine the coordinates of the investigated route. During the first measurements (in 2009) a system of four GPS devices was placed in a parallelogram directly above the wheels of the measuring vehicle. These studies showed, that the factor determining the accuracy of the coordinate designation were field obstacles (the availability of positions with errors of less than 5 cm was approximately 50%). In the next measurements, in 2010, seeking the optimal location of instruments, three receivers GPS were deployed diametrically in the measurement vehicle as shown in Fig. 1. Tests have shown similar availability and accuracy of GPS space segment for all measurement units, but still the achieved level of availability for the measurement error of less than 5 cm reached unsatisfactory values (60–70%).

Figure 1: Configuration of GNSS receivers on the PWM-15 platforms.

After a detailed analysis of the conditions of the measurements carried out in 2009-2012, it was decided to verify the methodology thoroughly. The verification resulted in:

- The abandonment of the implementation of real-time measurements using the ASG-EUPOS network, due to the existing breaks of GPS pseudo-range corrections transmission associated with, in the afternoon hours, the significant number of users resulted in the disconnection of users with a service packet of transmission data GPRS (General Packet Radio Service).
- The decision to carry out measurements in post-processing, which brought more possibilities to use the signals from various reference stations.
- To improve the accuracy of the coordinate designation, which is directly related to the number of available GPS satellites, it was decided to implement measurements using dual-mode GNSS receivers, thus utilizing the signals of two satellite systems: GPS and GLONASS.

- With the application of dual-mode receivers, it was necessary to use a local GPS/GLONASS Gdansk Technical University reference station, because ASG- EUPOS does not support the corrections for dual-mode receivers. It has been also assumed that the local reference station should be located in the area of conducted measurements (within 10 kilometers).

3 Measurement accuracy

Based on the above assumptions, in February 2012, a measurement campaign was carried out on the tram routes in Gdansk. The inventory measurements were carried out using two Leica Viva GS-15 and GS-12 receivers (Fig. 2). With the possibility of using an active satellite geodetic networks. Receiver Leica Viva GS-15 controller CS-15 and GS-12 receiver controller CS-15, characterized by accuracy in kinematic mode (phase measurement) horizontal: 10 mm + 1 ppm (rms) and vertically 20 mm + 1 ppm (rms). Despite such possibilities, as it has been assumed, the measurements were carried out using the reference station located in Gdansk University of Technology, which allows the transmission of differential correction of GPS / GLONASS. In addition, data recording was set up with a 30 cm distance between the points. The position calculations were realized in post-processing mode.

Figure 2: Measuring set with Leica Viva GS-15 and GS-12 receivers mounted on a tram bogie (photo by Jacek Szmaglinski).

The measurements on the tram lines in Gdansk urban areas positively verified the assumptions regarding to accuracy and availability. Fig. 3 presents the probability density function of the coordinates designations of two GNSS receivers (GPS/GLONASS) in 2D and 3D mode.

The fact that the receiver Leica Viva placed closer to the towing unit (GNSS1) marked the coordinates significantly more accurately than the other one - Leica System 1200 (GNSS2) – is astonishing. This undoubtedly proves the influence of the technical quality of the actual receiver (Leica Viva GNSS receiver was the newest product of the company) on the accuracy of the positions' determination.

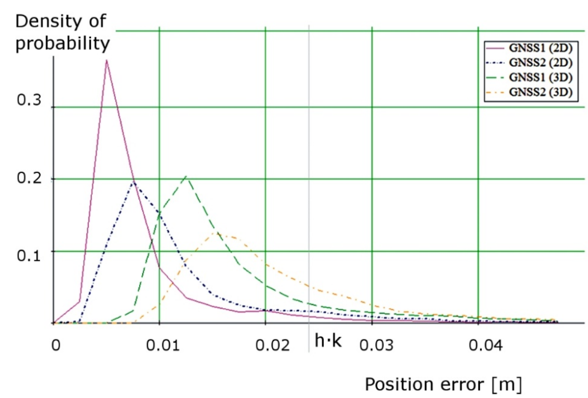

Figure 3: Probability density functions of GNSS position errors in 2D and 3D coordinate systems (measurements from 2012).

The study showed, that using GPS/GLONASS receivers the accuracy of determining the position coordinates in 2D measurements reaches a value below 1 cm. In the 3D solution the expected value is slightly higher by about 1 cm. In this way, new approaches for the implementation of measurement in the railway track were positively verified.

4 Computer aided data analysis

The planning of GNSS measurements, as well as working out the measurement data is a complex issue, which requires additional computer aided analysis. For that purpose, the authors used Leica GeoOffice [18], Mathsoft Mathcad ver. 14 [19] and Scilab [20] software.

4.1 Dilution of Precision analysis

Leica GeoOffice ver. 8.2 allows the user for planning the GNSS satellite constellation for the duration of the measurement in order to optimize the process from the point of view of the measurement accuracy. Moreover the transformation of the registered coordinates to the Cartesian coordinates is also possible in the software. In Poland, the National Spatial Reference System 2000 is used. The system 2000 is based on the Gauss-Kruger projection using central meridian of 18 degrees. The special property of this software is the ability to take into account the geoid model which allows the determination of the orthometric levels in Kronstadt vertical system 1986 (relative to Mean Sea Level). From the other hand, the Mathsoft Mathcad ver. 14 is an engineering software that allows to carry out complex mathematical calculations, and in this particular case, to analyze GNSS measurement results. GNSS data analyzing in a post-processing mode allows the configuration of various relative solutions of GPS and GPS/GLONASS systems. By offering statistical analysis of random possible variables, multi-dimensional array import of data and user-friendly interface, the above mentioned software becomes an important element of the geodetic elaborating of the GPS/GLONASS measurements.

Taking into account the influence of PDOP (Position 3D) value on the accuracy of the coordinates designation in 3D space, the comparison of both GPS and GPS/GLONASS systems is crucial. Based on Figs 4 and 5 it is clear that the average daily value of PDOP for GPS measurements and GPS/GLONASS differ from each other significantly. For GPS measurements the mean value of PDOP fluctuates around 2, whereas the parameter is equal to 1.5 in case of GPS/GLONASS measurements. Therefore, it can be postulated, that by the use of dual-mode receivers it is possible to increase the accuracy of determining the position coordinates by approximately about 25%.

In the above example attention should also be paid to the expected decrease in the measurements accuracy (represented by the PDOP) which is situated on the horizontal axis around 10.00 am (Fig. 4 and 5). On the other hand, the best time for implementation is around 3.00 pm. Fig. 6 shows the value of PDOP and the number of available satellites GPS/GLONASS between 2.00–5.00 pm. In analyzing the above charts it is appropriate to interrupt the implementation of measurements around 3.40 pm for 10 minutes due to an unfavorable value of PDOP = 1.9.

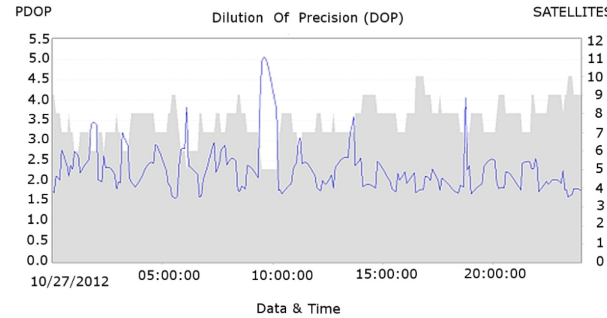

Figure 4: PDOF value and number of available satellites for GPS system obtained in Leica Geo Office 8.2.

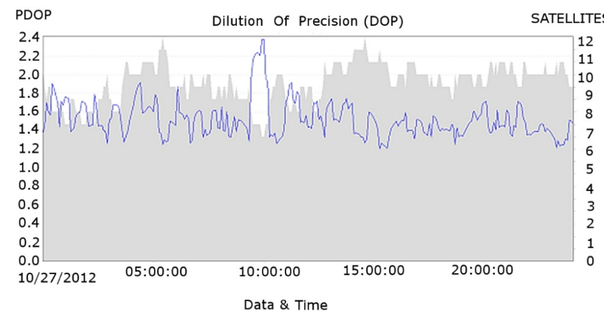

Figure 5: PDOF value and number of available satellites for GPS/GLONASS systems obtained in Leica Geo Office 8.2.

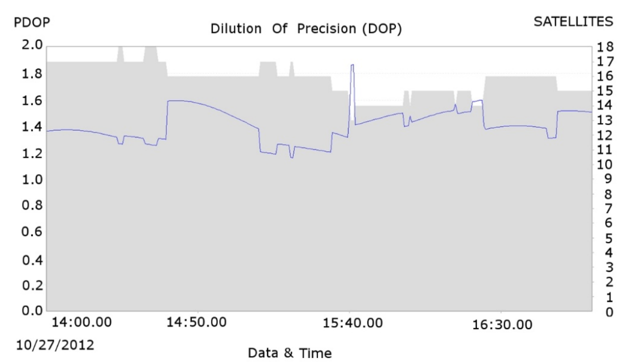

Figure 6: PDOP value and number of available satellites for GPS/GLONASS systems in optimal time interval obtained in Leica Geo Office 8.2.

5 The analysis of the railway track geometry

In relation to the CAD technique, designing of the railway routes, especially in case of upgrading or renewal projects regarding existing lines, the process is not just working on the graphical materials (drawings, plans) in which the convergence with the real features of the line is never fully guaranteed.

Actually, the work consists primarily in the usage of appropriate numerical data. These data, in presented approach, consists of the measurement's results obtained during the inventory of the railway line. With regard to satellite measurements, such data constitute the set of coordinates which represents the axis position of railway, as well as a whole range of information which the designer receives during analyzing the measurement's data. It follows that effective computer aided design process will connect both operations on the large sets of numerical data and the ability of the system to quickly present the effects of the work, especially the following variants of designed route.

In this paragraph the method of track geometry analysis together with the design process is presented in details. The analysis is performed on the previously prepared and elaborated data, *i.e.* flat coordinates in national system of references. The general algorithm consist of following stages [12]:

- Visualization of the railway line,
- Assessment of straight sections of the route,
- The creation of the main directions of the polygon,
- Assessment of route sections located in the circular arc,
- Design of the horizontal curve in the main directions intersection area.

5.1 Visualization of the railway course

The satellite measurements offer the possibility of a qualitative assessment of railway route on the basis of flat coordinates Y_i, X_i in the national reference system 2000. For the purpose of fast visual assessment, the discussed algorithm offers:

– Automatic chainage creation along the track geometric layout,
– Visual representation of coordinates Y_i, X_i on a grid of the Cartesian coordinate system,
– Separation and extraction of the selected range of the route for the individual analysis.

For better clarity and also to avoid the need of large operating values (occurring in the system 2000), the origin is shifted to the point of the lowest values of Y and X. An example of a visualization of such extracted data is shown in Fig. 7.

In general, the proposed algorithm should provide:

– Data loading (from text files) and defining the data tables,
– Operating on the matrixes greatly facilitates their analysis and shortens operating duration,
– Extracting of track's fragments identified by the users and creating files that serve as an output for further, detailed analysis,

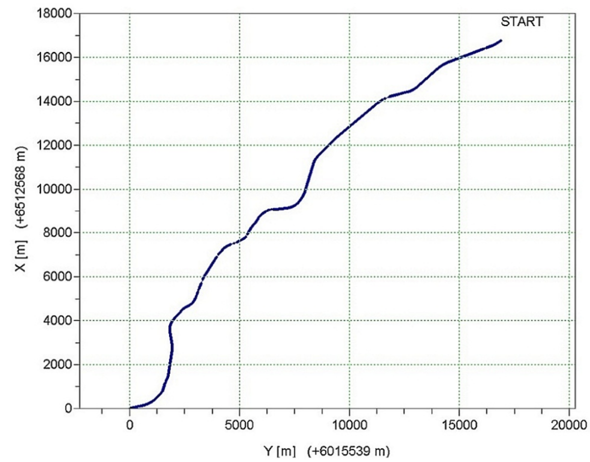

Figure 7: Example of visualization of the railway section.

– The possibility of visual (qualitative) assessment of existing lines planned for modernization by displaying points on a grid of coordinates system of 2000 in isometric scale,
– The possibility of zooming indicated fragments of an analysed line,
– The possibility of quick identification of the location of the route's indicated area (with respect to the chainage of the railway line).

5.2 Evaluation of straight sections of the route and creation of the polygon

Continuous satellite measurements offer the possibility of a detailed assessment of straight sections of a railway track. The measured coordinates of the straight track are used to determine – by means of the least squares method - the equation in Y, X coordinate system as $X = A + BY$. From the point of view of searching for the actual direction of the route, the slope coefficient $B = \tan \phi$ is a key parameter. Having determined the equations of all straight sections of the route in the system 2000, it is possible to calculate the coordinates of the main points of the route together with the intersecting angles.

In order to assess the actual shape of the track in the chosen straight section the algorithm transforms the data to the local coordinate system. Considering the equation of a main direction X, the algorithm translates the Y axis by the value of the intercept A and then makes the proper rotation.

Coordinate system transformation is performed using the following formulas [21]

$$Y_1 = Y \cos\varphi + (X - A)\sin\varphi \tag{1}$$

$$X_1 = -Y \sin\varphi + (X - A)\cos\varphi \tag{2}$$

The $\sin\varphi$ and $\cos\varphi$ are equal to:

$$\sin\varphi = \pm\frac{B}{\sqrt{1+B^2}} \qquad \cos\varphi = \pm\frac{1}{\sqrt{1+B^2}} \tag{3}$$

After that transformation, the horizontal axis corresponds to the direction of our route. In the Y_i, X_i system the ordinates can be interpreted as a deviation from this direction, which results from horizontal misalignments of the track and measurement error. Therefore, the values of vertical axis show a deviation of the GPS signal from the direction of the measured line. In terms of navigation, the location of an object in a distance from the designated course (assumed direction) is termed *XTE* (Cross Track Error) and is a measure of the error of a moving object position. As can be seen, a similar phenomenon can observed on the railway. So the horizontal misalignment of a track can also be described by the function of *XTE* [9].

In the analyzed case, on the *XTE* the uncertainty associated with the measurement technique is also discussed. Therefore, the received signal must be analyzed in order to verify the possibility of filtering out certain components, which can be regarded as caused by phenomena having no direct relation to the shape of the measured track.

To analyze the measured signal in the frequency domain, the Fast Fourier Transformation was applied. The transformation is described by the formula:

$$P_k = \sum_{n=0}^{N-1}\left(p_n\, e^{-\frac{2\,\pi\, i}{N}\, n\, k}\right), \quad k = 0, \ldots, N-1 \tag{4}$$

Where:
P – transformation result,
p – samples of the signal.

The filtered by low pass filter signal brings us closer to the actual shape of the track, and the differences between the input signal and the filtered one could be treated as the measurement error. As a result of that procedure a set of lateral movements of the track axis to horizontal alignment project are obtained.

Coming back to the straight section assessment – the local non-isometric coordinate system turned out to be very good reference for the analysis of the lateral track deterioration. Moreover it is very easy to detect whether the separated section is too long, which means that the signal range includes additional curvilinear parts of the route (transition curves or horizontal arcs or nonlinear shape of random deformation). For this reason, the presented method allows for cutting off those parts of the signal, which clearly do not belong to the straight track section. The algorithm was implemented to the Scilab in such way to support the user in the evaluation process. The assigned range is highlighted in red color and the R^2 coefficient for the actual and further straight range is displayed in the chart. This situation is shown in Fig. 8, where the blue set of points represents the current range, and red set - the range adopted in the next step. The horizontal axis x represents the main direction of the analyzed track section.

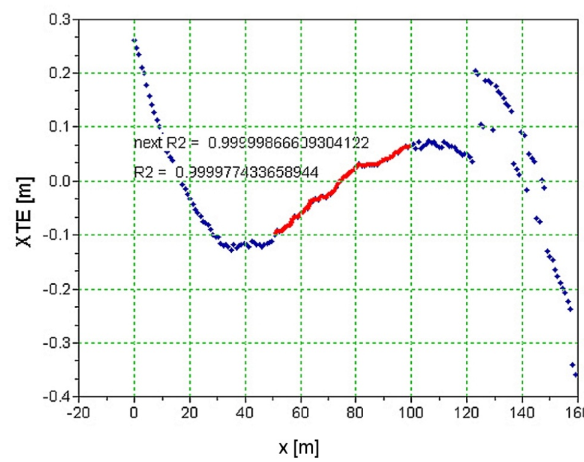

Figure 8: Separated straight section in local coordinate system.

After approval a new range of x coordinate, the program displays the final set of points in the local coordinate system. Such operation of limiting the scope of points the user can make as many times as is needed. Of course, a quasi-optimal range which provides a high value of both R^2 and number of samples is searched during this analyze. In Fig. 9 an example of the finally adopted *XTE* signal (after filtering) is presented. The dot line shows the measurement points together with the interpolation which was necessary for the filtering (Fourier Transformation) process. Therefore, the continuous line is an approximation of the track axis position together with its misalignments, while the x axis represents a theoretical main direction.

And finally, in Fig. 10 the values of the differences between the interpolated original signal and the filtered one are presented. These values (in absolute terms) reflect the measurement error. On this chart also the arithmetic mean (MEAN) and standard deviation (SD) of obtained differences are displayed.

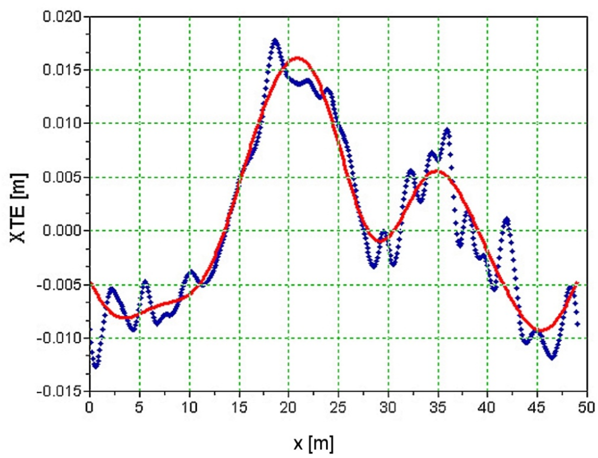

Figure 9: Finally approved signal of *XTE* representing the selected straight section.

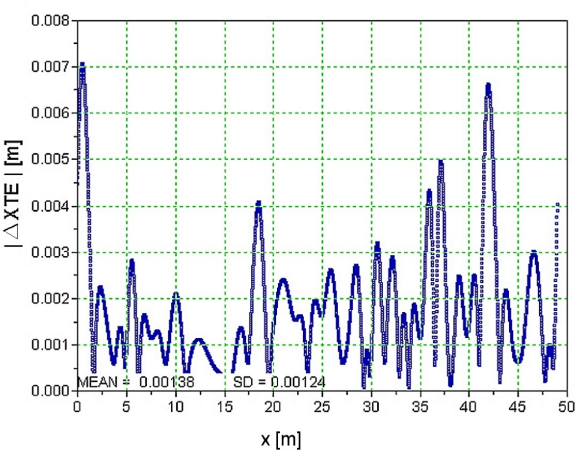

Figure 10: The absolute values of ΔXTE for assessed straight track section.

One of the main aim of straight section analysis is the problem of the polygon identification. The polygon – understood as a system of main directions is the essential determinant in designing of the geometrical layout in a horizontal plane. The issue is much more critical in the designing of track upgrading or renewal as well as during adjusting the existing track alignment. The presented method of the track assessment was prepared also for this purpose. Designated equations of identified straight sections, coordinates of the intersections and angles between the main directions are the fundamental factors in designing process. Therefore, those elements are established on the way of analysis like described above (evaluation of straight sections). On the base of the parameters the algorithm calculates the other ones, *i.e.* intersection angles and intersection coordinates. And finally, the established polygon is presented in the plot of track positions.

The input data covering six following straight sections is shown in Table 1 and the graphical interpretation is presented in Fig. 11.

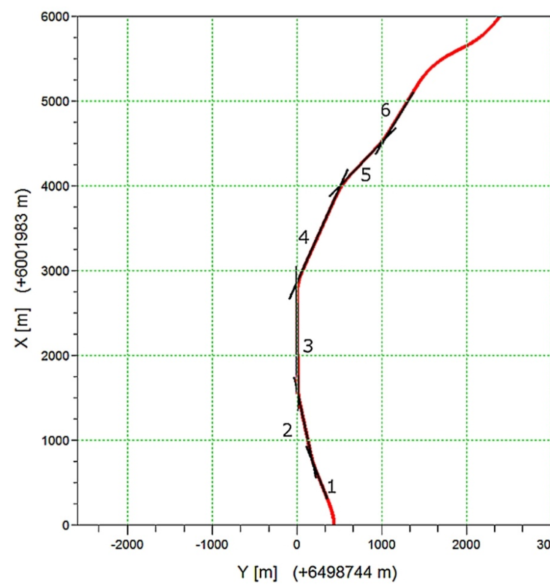

Figure 11: Fragment of the polygon of main directions. Straights no. 1-6 from Table 1.

5.3 Assessment of route located in arc section

Apart from the assessment of straight sections, the implemented program allows also for the assessment of horizontal curves in the analysed region of the railway line. The main purpose of the assessment process is focused on the circular part of an arc. It should be noticed, that the key parameters for the transition design is the radius of the constant arc and its length [22].

The analysis of the arc begins by extracting a proper section containing the arc together with straights located in its both sides. Then, the isolated area of the layout is transformed to the local coordinate system in which two main directions are inclined to the horizontal axis at the same angle what is shown in the Fig. 12.

In order to pre-estimate the value of the radius R, the program presents a values calculated from the relationship between the radius of the arc and the versine for a variable-length chord. This information allows the user to quickly locate the non-linear section in the analysed part

Table 1: Exemplary input data for polygon analysis. B – tangent inclination, A – intersection with vertical axis, R^2 – fitness coefficient, Y_W and X_W – coordinates of tangents intersection, α – intersection angle.

Ordinal	B	A [m]	R^2	Y_w	X_w	α [deg]
1	−2.482	22133438	0.9999997	6498931	6002727	-
2	−4.636	36129409	0.9999996	6498758	6003528	9,770793
3	−84.63	5.560D+08	0.9998354	6498743	6004832	11,4963
4	2.218	−8409312	0.9999998	6499269	6006000	24,94566
5	1.052	−830026	0.9999999	6499774	6006531	19,28476
6	1.631	−4591577	0.9999999	6499657	6006408	12,0328

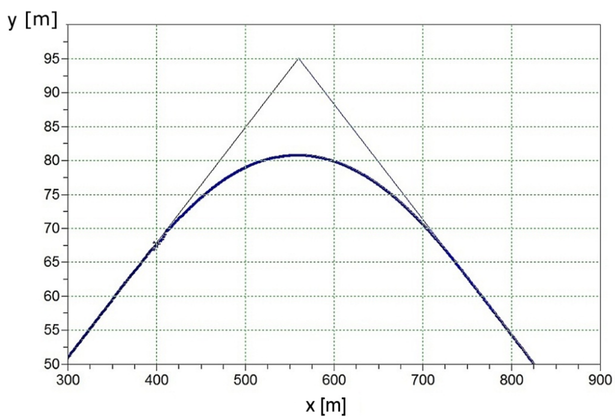

Figure 12: View of the arc area between the main directions in a local coordinate system.

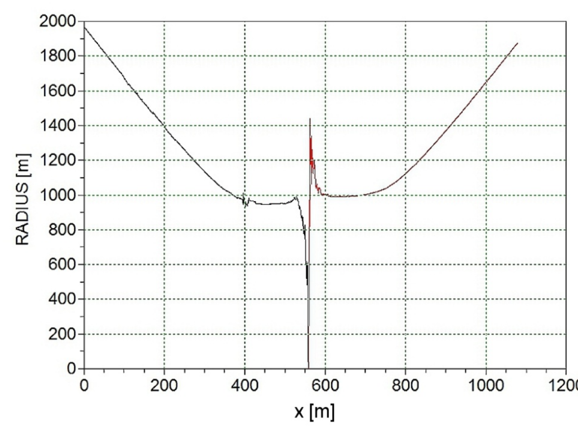

Figure 13: The value of radius R calculated for the left and right half of the arch.

of the arc. Additionally, the user gets visual information, which is helpful to estimate the average value of the radius of the circular arc (as a middle part of the whole isolated section). The final step of the algorithm is determination both the best fit radius value and the range of coordinate x, *i.e.* circular arc's location. As the calculations are conducted on the measurement results some geometrical imperfections are expected. Those imperfections result from the measurement error as well as tracks' deformations. Therefore, the algorithm of the radius fit skips the central part of the arc. The result of the radius estimation is presented in Fig. 13.

Basing on the Fig. 13, the radius could be initially set as R = 1000 m, while the range of abscissa will be determined by the boundaries of G_L = 400 m and G_R = 700 m. In Fig. 14 the generated graph of Δy indicator is shown. Δy is defined as a difference of ordinates measured and theoretical ordinates of identified arc.

Fig. 14 clearly shows that in the range of abscissa x from 400 m up to 520 m, a theoretical position of arc deviates from the measured geometries. Therefore, in the next step, new parameters of the abscissa, *i.e.*, G_L = 520 m and G_R = 620 m were chosen, leaving the radius equal

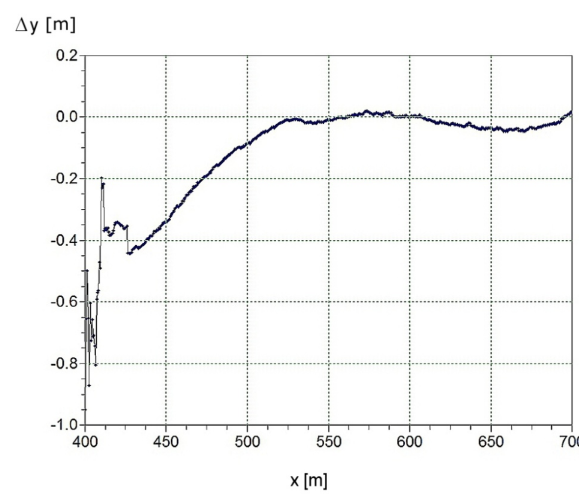

Figure 14: Matching of the circular arc. Radius R = 1000 m; adopted bounds of the measurement points: G_L = 400 m and G_R = 700 m.

to R = 1000 m. For new range of abscissa x the obtained matching is presented in Fig. 15.

The obtained differences of ordinates in Fig. 15 are already much lower, but some asymmetry of the layout is

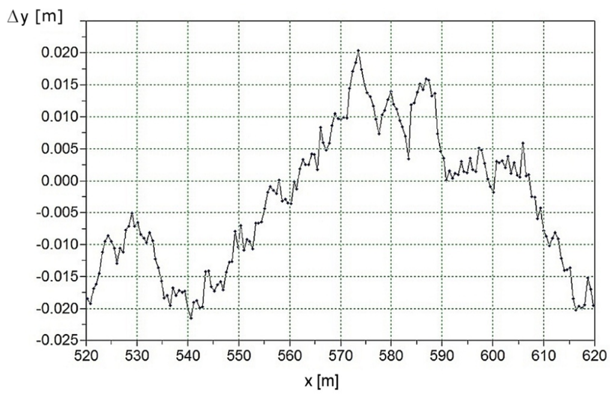

Figure 15: Matching of the circular arc. Radius $R = 1000$ m; adopted bounds of the measurement points: $G_L = 520$ m and $G_R = 620$ m.

evident. That asymmetry could already be forecast on the basis of Fig. 13, where the estimated value of the radiuses was different on both sides of the arc. When the user accepts the final range abscissa of the arc in the set of measurement points, the program displays the average of the differences of ordinates and the new value of the final radius R, for which the ordinate differences are minimal is calculated in the algorithm. For the present case, the final value of the radius generated by the program is $R = 994$ m, with an average of ordinate difference $\Delta y = 0.009$ m.

6 Summary

In the measurements carried out for the purpose of railway inventory the uniform, in terms of accuracy, geodetic reference system plays a key role. The implementation of continuous satellite measurements using receivers installed on a moving rail vehicle enable identification of a railroad axis position in the absolute reference system. Modern satellite measurements provide a huge amounts of data, that need to first be archived, and then subjected to a relevant analysis in order to obtain information useful from a practical point of view. Therefore, for the purpose of implementation that procedure it is necessary to create an appropriate support in a form of efficient algorithms. In the paper the authors have presented a complex method for evaluation the GNSS measurements for the purpose of track geometry assessment. It was indicated, that the whole process should be preceded by the planning and optimization of GNSS surveying. This approach minimizes the difficulties of a resultant measurement error.

The application results of satellite measurements presented in this paper have been obtained by the use of algorithms implemented by the authors. Those algorithms support the process of assessment the railway geometry by the functions for visualizing of the route, evaluating the track's polygon and assessing the curve geometric characteristics. According to the authors, the presented method could bring an efficient support for investments of railway geometry adjustment, upgrading or renewal as well as in a process of railway maintenance.

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