

# Independence in uniform linear triangle-free hypergraphs



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## ARTICLE INFO

### Article history:

Received 2 October 2014

Accepted 5 January 2016

Available online 1 February 2016

### Keywords:

Independence

Hypergraph

Linear

Uniform

Double linear

Triangle-free

## ABSTRACT

The independence number  $\alpha(H)$  of a hypergraph  $H$  is the maximum cardinality of a set of vertices of  $H$  that does not contain an edge of  $H$ . Generalizing Shearer's classical lower bound on the independence number of triangle-free graphs Shearer (1991), and considerably improving recent results of Li and Zang (2006) and Chishti et al. (2014), we show that

$$\alpha(H) \geq \sum_{u \in V(H)} f_r(d_H(u))$$

for an  $r$ -uniform linear triangle-free hypergraph  $H$  with  $r \geq 2$ , where

$$f_r(0) = 1, \quad \text{and}$$

$$f_r(d) = \frac{1 + ((r-1)d^2 - d)f_r(d-1)}{1 + (r-1)d^2} \quad \text{for } d \geq 1.$$

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## 1. Introduction

We consider finite hypergraphs  $H$ , which are ordered pairs  $(V(H), E(H))$  of two sets, where  $V(H)$  is the finite set of vertices of  $H$  and  $E(H)$  is the set of edges of  $H$ , which are subsets of  $V(H)$ . The order  $n(H)$  of  $H$  is the cardinality of  $V(H)$ . The degree  $d_H(u)$  of a vertex  $u$  of  $H$  is the number of edges of  $H$  that contain  $u$ . The average degree  $d(H)$  of  $H$  is the arithmetic mean of the degrees of its vertices. Two distinct vertices of  $H$  are adjacent or neighbors if some edge of  $H$  contains both. The neighborhood  $N_H(u)$  of a vertex  $u$  of  $H$  is the set of vertices of  $H$  that are adjacent to  $u$ . For a set  $X$  of vertices of  $H$ , the hypergraph  $H - X$  arises from  $H$  by removing from  $V(H)$  all vertices in  $X$  and removing from  $E(H)$  all edges that intersect  $X$ . If every two distinct edges of  $H$  share at most one vertex, then  $H$  is linear. If  $H$  is linear and for every two distinct non-adjacent vertices  $u$  and  $v$  of  $H$ , every edge of  $H$  that contains  $u$  contains at most one neighbor of  $v$ , then  $H$  is double linear. If there are not three distinct vertices  $u_1, u_2$ , and  $u_3$  of  $H$  and three distinct edges  $e_1, e_2$ , and  $e_3$  of  $H$  such that  $\{u_1, u_2, u_3\} \setminus \{u_i\} \subseteq e_i$  for  $i \in \{1, 2, 3\}$ , then  $H$  is triangle-free. A set  $I$  of vertices of  $H$  is a (weak) independent set of  $H$  if no edge of  $H$  is contained in  $I$ . The (weak) independence number  $\alpha(H)$  of  $H$  is the maximum cardinality of an independent set of  $H$ . If all edges of  $H$  have cardinality  $r$ , then  $H$  is  $r$ -uniform. If  $H$  is 2-uniform, then  $H$  is referred to as a graph.

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<http://dx.doi.org/10.1016/j.disc.2016.01.006>

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The independence number of (hyper)graphs is a well studied computationally hard parameter. Caro [4] and Wei [14] proved a classical lower bound on the independence number of graphs, which was extended to hypergraphs by Caro and Tuza [5]. Specifically, for an  $r$ -uniform hypergraph  $H$ , Caro and Tuza [5] proved

$$\alpha(H) \geq \sum_{u \in V(H)} f_{CT(r)}(d_H(u)),$$

where  $f_{CT(r)}(d) = \left(\frac{d+\frac{1}{r-1}}{d}\right)^{-1}$ . Thiele [13] generalized Caro and Tuza's bound to general hypergraphs; see [3] for a very simple probabilistic proof of Thiele's bound. Originally motivated by Ramsey theory, Ajtai et al. [2] showed that  $\alpha(G) = \Omega\left(\frac{\ln d(G)}{d(G)} n(G)\right)$  for every triangle-free graph  $G$ . Confirming a conjecture from [2] concerning the implicit constant, Shearer [11] improved this bound to  $\alpha(H) \geq f_{S_1}(d(G))n(G)$ , where  $f_{S_1}(d) = \frac{d \ln d - d + 1}{(d-1)^2}$ . In [11] the function  $f_{S_1}$  arises as a solution of the differential equation

$$(d + 1)f(d) = 1 + (d - d^2)f'(d) \quad \text{and} \quad f(0) = 1.$$

In [12] Shearer showed that

$$\alpha(G) \geq \sum_{u \in V(G)} f_{S_2}(d_G(u))$$

for every triangle-free graph  $G$ , where  $f_{S_2}$  solves the difference equation

$$(d + 1)f(d) = 1 + (d - d^2)(f(d) - f(d - 1)) \quad \text{and} \quad f(0) = 1.$$

Since  $f_{S_1}(d) \leq f_{S_2}(d)$  for every non-negative integer  $d$ , and  $f_{S_1}$  is convex, Shearer's bound from [12] is stronger than his bound from [11].

Li and Zang [10] adapted Shearer's approach to hypergraphs and obtained the following.

**Theorem 1** (Li and Zang [10]). *Let  $r$  and  $m$  be positive integers with  $r \geq 2$ .*

*If  $H$  is an  $r$ -uniform double linear hypergraph such that the maximum degree of every subhypergraph of  $H$  induced by the neighborhood of a vertex of  $H$  is less than  $m$ , then*

$$\alpha(H) \geq \sum_{u \in V(H)} f_{LZ(r,m)}(d_H(u)),$$

where

$$f_{LZ(r,m)}(x) = \frac{m}{B} \int_0^1 \frac{(1-t)^{\frac{a}{m}}}{t^b(m - (x-m)t)} dt,$$

$$a = \frac{1}{(r-1)^2}, \quad b = \frac{r-2}{r-1}, \quad \text{and} \quad B = \int_0^1 (1-t)^{\left(\frac{a}{m}-1\right)} t^{-b} dt.$$

Note that for  $r \geq 2$ , an  $r$ -uniform linear hypergraph  $H$  is triangle-free if and only if it is double linear and the maximum degree of every subhypergraph of  $H$  induced by the neighborhood of a vertex of  $H$  is less than 1. Therefore, since  $f_{S_1} = f_{LZ(2,1)}$  and  $f_{S_1}$  is convex, Theorem 1 implies Shearer's bound from [11]. Nevertheless, since  $f_{S_1}(d) < f_{S_2}(d)$  for every integer  $d$  with  $d \geq 2$ , Shearer's bound from [12] does not quite follow from Theorem 1.

In [6] Chishti et al. presented another version of Shearer's bound from [11] for hypergraphs.

**Theorem 2** (Chishti et al. [6]). *Let  $r$  be an integer with  $r \geq 2$ .*

*If  $H$  is an  $r$ -uniform linear triangle-free hypergraph, then*

$$\alpha(H) \geq f_{CZPI(r)}(d(H))n(H),$$

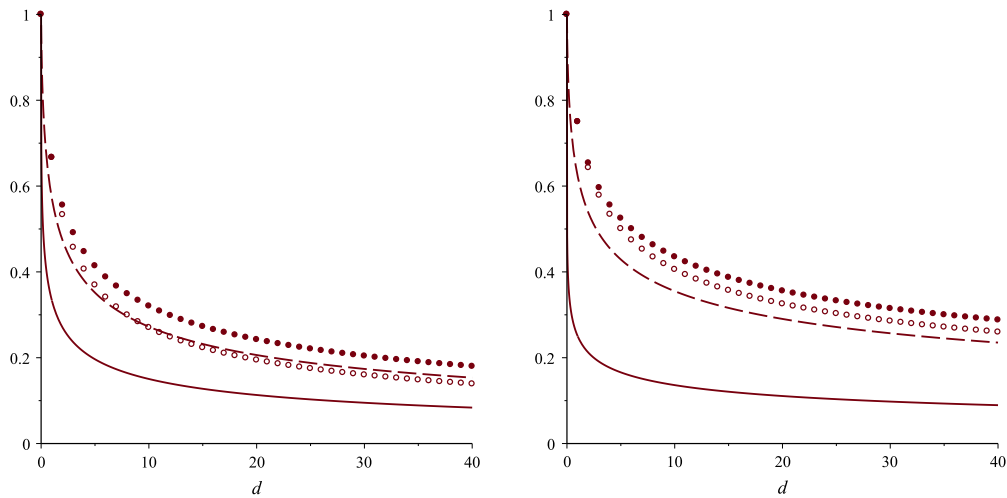
where

$$f_{CZPI(r)}(x) = \frac{1}{r-1} \int_0^1 \frac{1-t}{t^b(1 - ((r-1)x-1)t)} dt$$

$$\text{and } b = \frac{r-2}{r-1}.$$

Since  $f_{S_1} = f_{CZPI(2)}$ , for  $r = 2$ , the last result coincides with Shearer's bound from [11].

A drawback of the bounds in Theorems 1 and 2 is that they are very often weaker than Caro and Tuza's bound [5], which holds for a more general class of hypergraphs. See Fig. 1 for an illustration.



**Fig. 1.** The values of  $f_{LZ(r,1)}(d)$  (line),  $f_{CZP(r)}(d)$  (dashed line),  $f_{CT(r)}(d)$  (empty circles), and  $f_r(d)$  (solid circles) for  $0 \leq d \leq 40$  and  $r = 3$  (left) and  $r = 4$  (right).

In the present paper we extend Shearer’s approach from [12] and establish a lower bound on the independence number of a uniform linear triangle-free hypergraph that considerably improves Theorems 1 and 2 and is systematically better than Caro and Tuza’s bound.

For further related results we refer to Ajtai et al. [1], Duke et al. [7], Dutta et al. [8] and Kostochka et al. [9]. Note that our main result provides explicit values when applied to a specific hypergraph but that we do not completely understand its asymptotics. In contrast to that, results as in [1,7,8] are essentially asymptotic statements but are of limited value when applied to a specific hypergraph.

**2. Results**

For an integer  $r$  with  $r \geq 2$ , let  $f_r : \mathbb{N}_0 \rightarrow \mathbb{R}_0$  be such that

$$f_r(0) = 1 \quad \text{and}$$

$$f_r(d) = \frac{1 + ((r - 1)d^2 - d)f_r(d - 1)}{1 + (r - 1)d^2}$$

for every positive integer  $d$ .

**Lemma 3.** *If  $r$  and  $d$  are integers with  $r \geq 2$  and  $d \geq 0$ , then  $f_r(d) - f_r(d + 1) \geq f_r(d + 1) - f_r(d + 2)$ .*

**Proof.** Substituting within the inequality  $f_r(d) - 2f_r(d + 1) + f_r(d + 2) \geq 0$  first  $f_r(d + 2)$  with

$$\frac{1 + ((r - 1)(d + 2)^2 - (d + 2))f_r(d + 1)}{1 + (r - 1)(d + 2)^2}$$

and then  $f_r(d + 1)$  with

$$\frac{1 + ((r - 1)(d + 1)^2 - (d + 1))f_r(d)}{1 + (r - 1)(d + 1)^2},$$

and solving it for  $f_r(d)$ , it is straightforward but tedious to verify that it is equivalent to  $f_r(d) \geq L(r, d)$  where

$$L(r, d) = \frac{(2r - 1)d + 3r}{r(d^2 + 5d + 5)}.$$

Therefore, in order to complete the proof, it suffices to show  $f_r(d) \geq L(r, d)$ . For  $d = 0$ , we have  $f_r(0) = 1 > \frac{3}{5} = L(r, 0)$ . Now, let  $f(d) \geq L(r, d)$  for some non-negative integer  $d$ . Since  $(r - 1)(d + 1)^2 - (d + 1) \geq 0$ , we obtain by a straightforward yet tedious calculation

$$f(d + 1) - L(r, d + 1) = \frac{1 + ((r - 1)(d + 1)^2 - (d + 1))f(d)}{1 + (r - 1)(d + 1)^2} - L(r, d + 1)$$

$$\begin{aligned} &\geq \frac{(1 + ((r - 1)(d + 1)^2 - (d + 1))L(r, d)}{1 + (r - 1)(d + 1)^2} - L(r, d + 1) \\ &= \frac{2(1 + (r - 1)(d + 2)^2)}{r(d^2 + 7d + 11)(d^2 + 5d + 5)}, \end{aligned}$$

which is positive for  $r \geq 2$ . Therefore,  $f(d + 1) \geq L(r, d + 1)$ , which completes the proof by an inductive argument.  $\square$

The following is our main result.

**Theorem 4.** Let  $r$  be an integer with  $r \geq 2$ .

If  $H$  is an  $r$ -uniform linear triangle-free hypergraph, then

$$\alpha(H) \geq \sum_{u \in V(H)} f_r(d_H(u)).$$

Before we proceed to the proof, we compare our bound to the bounds of Caro and Tuza [5], Li and Zang [10], and Chishti et al. [6]. Fig. 1 illustrates some specific values. An inspection of Li and Zang’s proof in [10] reveals that they actually prove a lower bound on the so-called *strong independence number*, which is defined as the maximum cardinality of a set of vertices that does not contain two adjacent vertices. Therefore, especially for large values of  $r$ , Theorem 1 is much weaker than Theorem 2. In fact, it is quite natural that it is worse by a factor of about  $r - 1$ .

As we show now, our bound is systematically better than Caro and Tuza’s bound [5].

**Lemma 5.** If  $r$  and  $d$  are integers with  $r \geq 3$  and  $d \geq 2$ , then  $f_r(d) > f_{CT(r)}(d)$ .

**Proof.** Note that  $f_r(0) = f_{CT(r)}(0) = 1$ ,  $f_r(1) = f_{CT(r)}(1) = \frac{r-1}{r}$ , and  $f_{CT(r)}(d) = \frac{d}{d+r-1}f_{CT(r)}(d-1)$  for  $d \in \mathbb{N}$ , which immediately implies that  $f_{CT(r)}(d) < \frac{r-1}{r}$  for  $d \geq 2$ . Now, if  $f_r(d-1) \geq f_{CT(r)}(d-1)$  for some  $d \geq 2$ , then

$$\begin{aligned} f_r(d) - f_{CT(r)}(d) &= \frac{1 + ((r - 1)d^2 - d)f_r(d - 1)}{1 + (r - 1)d^2} - f_{CT(r)}(d) \\ &\geq \frac{1 + ((r - 1)d^2 - d)f_{CT(r)}(d - 1)}{1 + (r - 1)d^2} - f_{CT(r)}(d) \\ &= \frac{1 + ((r - 1)d^2 - d)\frac{1+(r-1)d}{(r-1)d}f_{CT(r)}(d)}{1 + (r - 1)d^2} - f_{CT(r)}(d) \\ &= \frac{1 - \frac{r}{r-1}f_{CT(r)}(d)}{1 + (r - 1)d^2} \\ &> 0, \end{aligned}$$

that is,  $f_r(d) > f_{CT(r)}(d)$ , which completes the proof by an inductive argument.  $\square$

For  $r = 2$ , Lemma 5 would state that Shearer’s bound [12] is better than Caro [4] and Wei’s bound [14], which is known. We proceed to the proof of Theorem 4.

**Proof of Theorem 4.** We prove the statement by induction on  $n(H)$ . If  $H$  has no edge, then  $\alpha(H) = n(H)$ , which implies the desired result for  $n(H) \leq r - 1$ . Now let  $n(H) \geq r$ . If  $H$  has a vertex  $x$  with  $d_H(x) = 0$ , then  $f_r(d_H(x)) = 1$  and, by induction,

$$\alpha(H) \geq 1 + \alpha(H - x) \geq f_r(d_H(x)) + \sum_{u \in V(H) \setminus \{x\}} f_r(d_{H-x}(u)) = \sum_{u \in V(H)} f_r(d_H(u)).$$

Hence we may assume that  $H$  has no vertex of degree 0.

Since  $H$  is  $r$ -uniform and linear, for every two edges  $e_1$  and  $e_2$  with  $e_1 \cap e_2 = \{u\}$  for some vertex  $u$  of  $H$ , the sets  $e_1 \setminus \{u\}$  and  $e_2 \setminus \{u\}$  are disjoint and of order  $r - 1$ . Therefore, for every vertex  $u$  of  $H$ , there is a set  $\mathcal{R}(u)$  of  $r - 1$  sets of neighbors of  $u$  such that every neighbor of  $u$  belongs to exactly one of the sets in  $\mathcal{R}(u)$ , and  $|e \cap R| = 1$  for every edge  $e$  of  $H$  with  $u \in e$  and every  $R \in \mathcal{R}(u)$ .

If  $x$  is a vertex of  $H$  and  $R \in \mathcal{R}(x)$  is such that

$$1 + \sum_{u \in V(H) \setminus (\{x\} \cup R)} f_r(d_{H - (\{x\} \cup R)}(u)) \geq \sum_{u \in V(H)} f_r(d_H(u)),$$

then the statement follows by induction, because  $\alpha(H) \geq 1 + \alpha(H - (\{x\} \cup R))$ . Therefore, in order to complete the proof, it suffices to show that the following term is non-negative:

$$P = \sum_{x \in V(H)} \sum_{R \in \mathcal{R}(x)} \left( 1 + \sum_{u \in V(H) \setminus (\{x\} \cup R)} f_r(d_{H - (\{x\} \cup R)}(u)) - \sum_{u \in V(H)} f_r(d_H(u)) \right).$$

Since  $H$  is linear and triangle-free, we have  $d_{H - (\{x\} \cup R)}(z) = d_H(z) - |N_H(z) \cap R|$  for every vertex  $z$  in  $V(H) \setminus (\{x\} \cup R)$ . Trivially,  $d_{H - (\{x\} \cup R)}(z) = d_H(z)$  for  $z \notin N_H(R)$ , and hence  $P$  equals  $P_1 + P_2$ , where

$$P_1 = \sum_{x \in V(H)} \sum_{R \in \mathcal{R}(x)} \left( 1 - f_r(d_H(x)) - \sum_{y \in R} f_r(d_H(y)) \right) \quad \text{and}$$

$$P_2 = \sum_{x \in V(H)} \sum_{R \in \mathcal{R}(x)} \sum_{z \in N_H(R) \setminus \{x\}} \left( f_r(d_H(z) - |N_H(z) \cap R|) - f_r(d_H(z)) \right).$$

Since for every vertex  $u$  of  $H$ , there are exactly  $(r - 1)d_H(u)$  many vertices  $v$  of  $H$  such that  $u$  belongs to exactly one of the sets in  $\mathcal{R}(v)$ , we have

$$P_1 = \sum_{x \in V(H)} \left( (r - 1) - (r - 1)(d_H(x) + 1)f_r(d_H(x)) \right).$$

Since  $f_r(d - 1) - f_r(d)$  is decreasing by Lemma 3, we have  $f_r(d - n) - f_r(d) \geq n(f_r(d - 1) - f_r(d))$  for all positive integers  $d$  and  $n$  with  $n < d$ . Therefore,

$$\begin{aligned} P_2 &\geq \sum_{x \in V(H)} \sum_{R \in \mathcal{R}(x)} \sum_{z \in N_H(R) \setminus \{x\}} |N_H(z) \cap R| \left( f_r(d_H(z) - 1) - f_r(d_H(z)) \right) \\ &= \sum_{x \in V(H)} \sum_{R \in \mathcal{R}(x)} \sum_{z \in N_H(R) \setminus \{x\}} \sum_{y \in R} |N_H(z) \cap \{y\}| \left( f_r(d_H(z) - 1) - f_r(d_H(z)) \right) \\ &= \sum_{x \in V(H)} \sum_{R \in \mathcal{R}(x)} \sum_{y \in R} \sum_{z \in N_H(R) \setminus \{x\}} |N_H(z) \cap \{y\}| \left( f_r(d_H(z) - 1) - f_r(d_H(z)) \right) \\ &= \sum_{x \in V(H)} \sum_{R \in \mathcal{R}(x)} \sum_{y \in R} \sum_{z \in N_H(y) \setminus \{x\}} \left( f_r(d_H(z) - 1) - f_r(d_H(z)) \right). \end{aligned}$$

Let  $T$  be the set of all 4-tuples  $(x, R, y, z)$  with  $x \in V(H)$ ,  $R \in \mathcal{R}(x)$ ,  $y \in R$ , and  $z \in N_H(y) \setminus \{x\}$ . Note that  $y \in N_H(z)$  for every  $(x, R, y, z)$  in  $T$ . Since  $H$  is linear, for a given vertex  $z$  of  $H$  and a given neighbor  $y$  of  $z$ , there are  $(r - 1)d_H(y) - 1$  many vertices  $x$  of  $H$  with  $y \in R$  for some  $R$  in  $\mathcal{R}(x)$  and  $z \in N_H(y) \setminus \{x\}$ . Furthermore, by the properties of  $\mathcal{R}(x)$ , given  $x$  and  $y$ , the set  $R$  in  $\mathcal{R}(x)$  with  $y \in R$  is unique. Therefore,

$$\begin{aligned} P_2 &\geq \sum_{x \in V(H)} \sum_{R \in \mathcal{R}(x)} \sum_{y \in R} \sum_{z \in N_H(y) \setminus \{x\}} \left( f_r(d_H(z) - 1) - f_r(d_H(z)) \right) \\ &= \sum_{z \in V(H)} \sum_{y \in N_H(z)} \left( (r - 1)d_H(y) - 1 \right) \left( f_r(d_H(z) - 1) - f_r(d_H(z)) \right). \end{aligned}$$

Let  $\mathcal{E}$  be the edge set of the graph that arises from  $H$  by replacing every edge of  $H$  by a clique, that is,  $\mathcal{E}$  is the set of all sets containing exactly two adjacent vertices of  $H$ .

We obtain

$$\begin{aligned} P_2 &\geq \sum_{z \in V(H)} \sum_{y \in N_H(z)} \left( (r - 1)d_H(y) - 1 \right) \left( f_r(d_H(z) - 1) - f_r(d_H(z)) \right) \\ &= \sum_{\{y,z\} \in \mathcal{E}} \left( h_1(y)h_2(z) + h_1(z)h_2(y) \right), \quad \text{where} \end{aligned}$$

$$h_1(x) = (r - 1)d_H(x) - 1 \quad \text{and}$$

$$h_2(x) = f_r(d_H(x) - 1) - f_r(d_H(x)).$$

If  $d_H(y) \geq d_H(z)$ , then  $h_1(y) \geq h_1(z)$  and, by Lemma 3,  $h_2(z) \geq h_2(y)$ , which implies

$$\left( h_1(y) - h_1(z) \right) \left( h_2(z) - h_2(y) \right) \geq 0.$$

Therefore,  $h_1(y)h_2(z) + h_1(z)h_2(y) \geq h_1(y)h_2(y) + h_1(z)h_2(z)$ .

Since, for every vertex  $y$  of  $H$ , there are exactly  $(r - 1)d_H(y)$  many vertices  $z$  of  $H$  with  $\{y, z\} \in \mathcal{E}$ , we obtain

$$\begin{aligned} P_2 &\geq \sum_{\{y,z\} \in \mathcal{E}} \left( h_1(y)h_2(z) + h_1(z)h_2(y) \right) \\ &\geq \sum_{\{y,z\} \in \mathcal{E}} \left( h_1(y)h_2(y) + h_1(z)h_2(z) \right) \\ &= \sum_{x \in V(H)} (r - 1)d_H(x)h_1(x)h_2(x) \\ &= \sum_{x \in V(H)} (r - 1)d_H(x) \left( (r - 1)d_H(x) - 1 \right) \left( f_r(d_H(x) - 1) - f_r(d_H(x)) \right). \end{aligned}$$

Combining these estimates, we see that

$$\begin{aligned} P &= P_1 + P_2 \\ &\geq \sum_{x \in V(H)} \left( (r - 1) - (r - 1)(d_H(x) + 1)f_r(d_H(x)) \right. \\ &\quad \left. + (r - 1)d_H(x) \left( (r - 1)d_H(x) - 1 \right) \left( f_r(d_H(x) - 1) - f_r(d_H(x)) \right) \right), \end{aligned}$$

which is 0 by the definition of  $f_r$ . This completes the proof.  $\square$

It seems a challenging task to extend the presented results to non-uniform and/or non-linear triangle-free hypergraphs.

### Acknowledgment

The first author has been partially supported by National Science Centre under contract DEC-2011/02/A/ST6/00201.

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