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THE USE OF FEM FOR DETERMINATION OF RESONANT FREQUENCIES OF CIRCULAR SAW BLADES WITH INDIRECT TEETH IN GULLETS

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Abstract

Understanding the dynamical properties of the circular saw blade is necessary, since, there is a really need for a stable work at working rotational speeds, which are determined by frequencies at which the circular saw blade tends to vibrate. It was observed that the resonant frequency depend on the circular saw blade shape, a collar diameter and on saw's teeth shape. In the presented work an attention was paid to the effect of indirect teeth in the saw blade gullets on resonant frequencies. This article presents the results of the FEM modeling carried out on examples the circular saw equipped with indirect teeth, which are used to eject the chips, and the circular saw without those teeth. Modeling was done with the use of the commercial software, due to a desire of testing capabilities of such programs.

Key words: *circular saw blades, resonant frequencies, FEM modeling, indirect teeth, quasi-twin resonant frequencies*

INTRODUCTION

The critical rotational speed of circular saw blade defines the highest speed in which circular saw can work with enquired stability (Kaczmarek et al. 2014, 2016; Orlowski et al. 2007; Stakhiev 1998, 2000; Strzelecki 1974). In order to determine this rotational speed it is necessary to know the value of resonant frequencies of a circular saw blade:

$$n_{cr}^{\min} = \frac{60 \cdot f_n(0)}{\sqrt{k^2 - \lambda}} \qquad \text{[rpm]} \qquad (1)$$

where: $f_n(0)$ – is the of natural frequency of the no running saw blade, k – is number of the nodal diameter, λ – is the centrifugal force coefficient.

Moreover, it was proved that certain shapes of teeth or a saw blade can have a direct effect on the value of the resonant frequencies of circular saw blades (Droba et al. 2015b; Kaczmarek et al. 2014, 2016; Nishio and Marui 1996; Yokochi et al. 1993).

There exist several methods of determination of resonant frequencies of circular saw blades. Among empirical methods there are well known: the harmonic method (Kaczmarek et al. 2014; Strzelecki 1974), the impact test (Kaczmarek et al. 2014; Orlowski et al. 2007), the vision technique (Orlowski et al. 2007) and some methods with a rotating saw blade (Droba et al. 2015a; Gogu 1988; Nishio and Marui 1996).

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Contemporary numerical methods have based on the FEM method, and they can be classified because of the applied models, e.g.:

- a model of full circular plate with given outer diameter (Droba et al. 2015a; Ingielewicz and Wittbrodt 1992),
- a model with an annular plate with defined an inner clamping diameter (Gogu 1988; Skoblar et al. 2016; Tian 1998),
- a model of spherical shells with the radius of curvature tending to the infinity (Thomas et al. 2005),
- viscoelastic models consisted of solid "composite" plates (or shells) (Vasqueq and Cardoso 2011),
- a model which includes the influence of external forces (Gogu 1988; Nishio and Marui 1996; Thomas et al. 2005; Tian 1998; Vasueq and Cardoso 2011),
- a model which takes into account rotational movement of the circular saw blade (Ingielewicz and Wittbrodt 1992; Nishi and Marui 1996).

Every of the mentioned FEM models are based on similar algorithm of a process of modeling (Fig. 1) (Ingielewicz and Wittbrodt 1992).

The real system responds to the exact shape of saw, including tooth shape. The physical model is used to obtain a mathematical simplification of the real object (Fig. 2a). The segment of the physical model can by defined in two ways: by describing it with the angle α between to neighboring nodal diameters (Fig. 2b) (Ingielewicz and Wittbrodt 1992), or by a mesh of the finite elements (e.g. triangular mesh – Fig. 2c) (Nishio and Marui 1996).

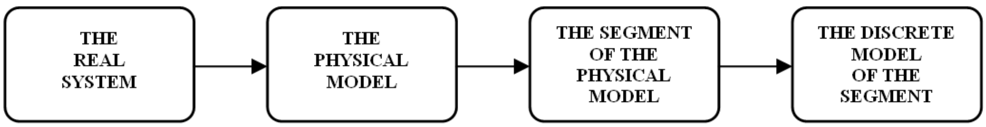


Figure 1 The algorithm of modeling of the circular saw blades (Ingielewicz and Wittbrodt 1992)

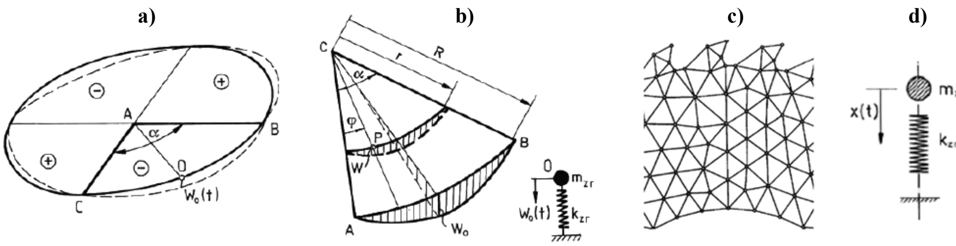


Figure 2 The physical model of the circular saw blade (a), the segment of the physical model (b), view of the triangular mesh (c) and the discrete model of the segment (d) (Ingielewicz and Wittbrodt 1992; Nishio and Marui 1996)

The discrete model of the segment (Fig. 2d) has one degree of freedom and might be described by the equation:

$$m_r\ddot{x} + k_r x = 0 \tag{2}$$

where: m_r– is reduced mass of the discrete model; k_r – is reduced stiffness of a spring element of the discrete model, x – is amplitude of vibration in Cartesian coordinates.

A solution of Eq. (2) would be presented as:

$$x = x_0 \cdot \sin 2\pi f t \tag{3}$$

where: x_0 – is an amplitude of the natural vibration calculated from initial conditions, f – in the natural frequency of the discrete model:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} \tag{4}$$

The reduced mass m_r can by calculated by comparing the kinetic energies of the physical model and the discrete model and the reduced stiffness k_r can by calculated by comparing the potential energies (Ingielewicz and Wittbrodt 1992) with the use of Hamilton's principle (Thomas et al. 2005; Tian 1998).

The displacement of the examined point P (fig. 2b) should be understand as (Ingielewicz and Wittbrodt 1992):

$$w = N(r,\varphi) \cdot w_0(t) \tag{5}$$

where: $w_0(t)$ – displacement of the point 0 on the free edge in time function, $N(r,\varphi)$ is the shape function in polar coordinates.

When the circular saw blade does not rotate and is only excited by simple sinusoidal vibrations, its deflection in function of radius r, an angular position φ and time t can be written as:

$$w(r,\varphi,t) = f(r)\sin k\varphi \cos 2\pi f_n t \tag{6}$$

where:$f(r)$ is the deflection function in the radial direction.

For circular saw blade which rotates with rotational speed n [rpm] the angular position will be equal $\varphi = 2\pi n t \times 60^{-1}$, so the equation (6) can be written as:

$$w(r,n,t) = \frac{f(r)}{2} 2\sin 2\pi\left(f_n + \frac{kn}{60}\right)t - \frac{f(r)}{2}\sin 2\pi\left(f_n - \frac{kn}{60}\right)t \tag{7}$$

in which the fist term refers to the forward wave (which rotates in the same direction as the circular saw blade), and the second term refers to the backward wave (Stakhiev 1998, 2000).

The shape function could be also calculated with the Bessel function (Skoblar et al. 2016).

MATERIAL AND METHODS

Two circular saw blades *S-I* (with indirect teeth in gullets) (Fig. 3c) and *S-II* (without indirect teeth in gullets) (Fig. 3d) were the objects of the FEM modeling. Technical data of circular saw blades: outside diameter $D = 350$ mm, internal diameter $d = 30$ mm, thickness $b = 2.8$ mm, number of teeth $z = 18$. Saws were clamped with collars $d_c = 90$ mm, hence, the clamping ratio was equal to $\alpha = 0.257$ (Kaczmarek et al. 2016; Stakhiev 1998). Modeling was performed with the use of the commercial software (Autodesk Inventor Professional 2015). Figures 3a-b show the created models.

The modeling was based on the following assumptions:

- on the horizontal spindle the circular saw was fixed on,
- the model includes a small vibration exciter with spherical surface, through which the saw was excited by the horizontal force $F = 0.1$ N with a varied frequency in the range from 100 to 1700 Hz,

- the average size of the mesh element was equal to 0.1 mm and the minimal size was 0.02 mm,
- the contacts have been added between flat surfaces of the circular saw blade, clamping collars, a spindle and a nut,
- gravity has taken into consideration.

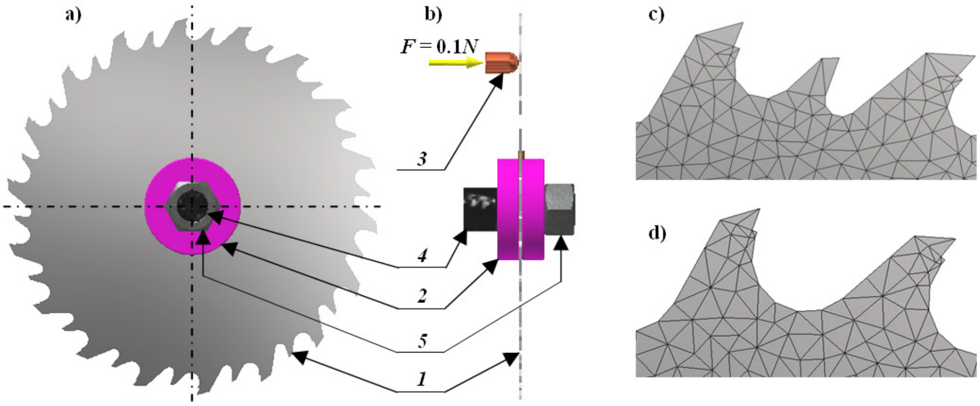


Figure 3 Model of the examined circular saw blades (a) with side view (b) and fragments of the blade of the base saw *S-I* (c) and for saw without indirect teeth *S-II*, where: 1 – circular saw blade, 2 – clamping collars, 3 – vibration exciter with applied force $F = 0.1$ N, 4 – grounded spindle, 5 – clamping nut

RESULTS AND DISCUSSION

As it was expected, the "quasi-twin" resonant frequencies have been observed (tab. 1), which are a phenomenon of occurrence, for different values of the frequencies of the same mode shapes, however, rotated relatively to each other with some angle (Kaczmarek et al. 2014, 2016), e.g. $f_{(j0,\ k2)}$ = 183.74 Hz and 190.75 Hz (tab. 1).

For circullar saw blade *S-I* differences between resonant frequencies of compatible modal shapes reached values approximately between 0.6 Hz for $k = 1$ up to 76 Hz for $k = 7$.

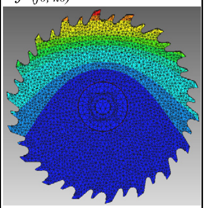
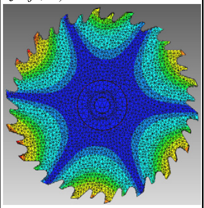
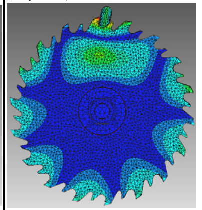
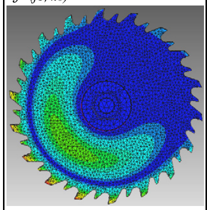
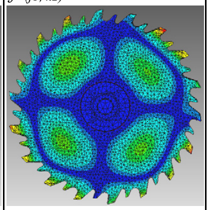
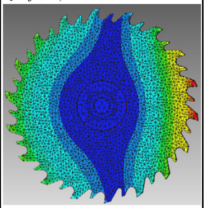
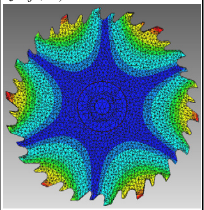
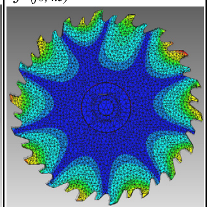
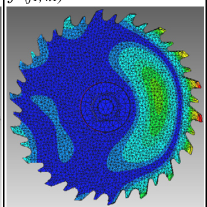
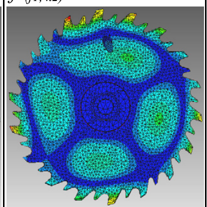
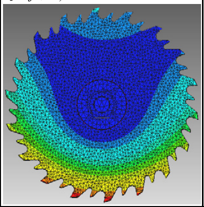
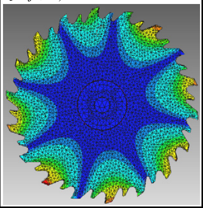
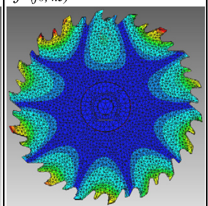
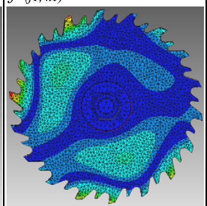
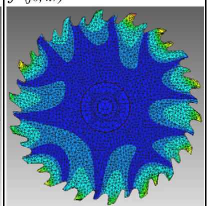
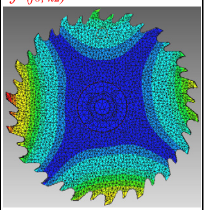
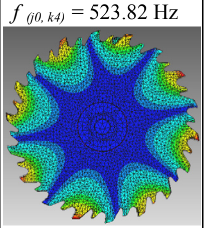
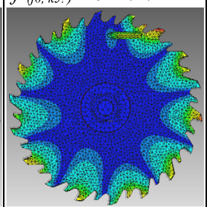
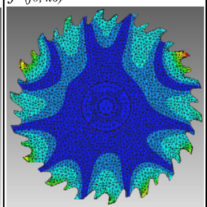
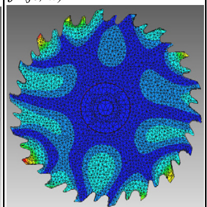
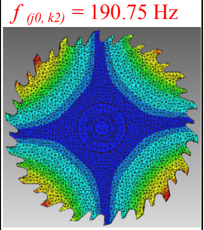
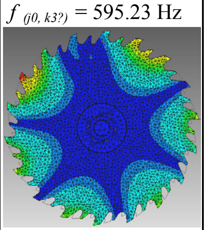
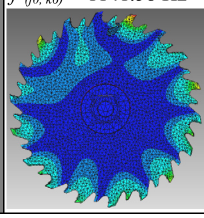
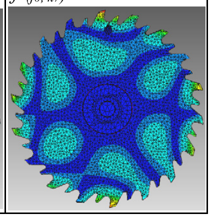
In practice, it is sufficient to measure the resonant frequency of the circular saw blade including nodal diameter form $k = 2$ up to $k = 4$, because in range of the applied rotational speeds the minimal critical rotational speed usually does not occur for higher and lower frequencies than $2 \leq k \leq 4$.

The comparison of the FEM results for circular saw blades *S-I* and S-*II* is shown in tab. 2:

- mostly, for circular saw blade *S-II* without indirect teeth in gullets, for the corresponding modal shapes were obtained higher resonant frequencies. This kind of the saw tends to have a greater dynamic stiffness than saw *S-I,*
- for nodal diameter in range up to $k \leq 4$, the difference between values of resonant frequencies for circular saw blades *S-I* and *S-II* were 15-20 Hz,
- for higher modal shapes, for both saw blades resonant frequencies were not appearing for all subsequent modes, what usually happens also for some other research methods,

such as experimental methods like the impact test or the harmonic method (Kaczmarek et al. 2014, 2016).

Table 1 Modal shapes and resonant frequencies of circular saw blade *S-I* modeling

$f_{(j0,\,k0)}$ = 144.6 Hz 	$f_{(j0,\,k3)}$ = 313.69 Hz 	$f_{(j0,\,k4?)}$ = 737.23 Hz 	$f_{(j1,\,k0)}$ = 998.84 Hz 	$f_{(j1,\,k2)}$ = 1191.27 Hz 
$f_{(j0,\,k1)}$ = 156.43 Hz 	$f_{(j0,\,k3)}$ = 321.86 Hz 	$f_{(j0,\,k5)}$ = 758.73 Hz 	$f_{(j1,\,k1)}$ = 1026.60 Hz 	$f_{(j1,\,k2)}$ = 1249.48 Hz 
$f_{(j0,\,k1)}$ = 157.06 Hz 	$f_{(j0,\,k4)}$ = 521.97 Hz 	$f_{(j0,\,k5)}$ = 788.92 Hz 	$f_{(j1,\,k1)}$ = 1072.69 Hz 	$f_{(j0,\,k7)}$ = 1423.48 Hz 
$f_{(j0,\,k2)}$ = 183.74 Hz 	$f_{(j0,\,k4)}$ = 523.82 Hz 	$f_{(j0,\,k5?)}$ = 941.17 Hz 	$f_{(j0,\,k6)}$ = 1089.64 Hz 	$f_{(j0,\,k7)}$ = 1426.76 Hz 
$f_{(j0,\,k2)}$ = 190.75 Hz 	$f_{(j0,\,k3?)}$ = 595.23 Hz 		$f_{(j0,\,k6)}$ = 1141.58 Hz 	$f_{(j0,\,k7)}$ = 1502.88 Hz 

CONCLUSIONS

The obtained results seem to be on a satisfactory accuracy level. The software Autodesk Inventor Professional 2015 did not cause any major difficulties in conducting the FEM modeling for the tested cases of circular saw blades. This software gives the ability of

3D models regulation in a desired manner, ie. it gives the possibility to change bonds between components, to set an appropriate size of the mesh elements and to adjust types of preset load, as well as their occurrence frequencies.

The circular saw blade *S-I* with indirect teeth in gullets has generally a lower dynamic stiffness than saw *S-II*. For nodal diameter up to $k \leq 4$ differences between values of resonant frequencies were in the range 15-20 Hz. For the higher modal shapes comparison of frequencies' values are rather difficult because of random occurrence of the corresponding vibrations modes for both circular saw blades.

Table 2 Resonant frequencies of circular saw blades *S-I* and *S-II* FEM modeling

Modal shape	***S-I***	***S-II***	**Difference**	**Modal shape**	***S-I***	***S-II***	**Difference**
j0, k0	144.60	162.17	17.57	*j1, k0*	998.84	-	-
j0, k1	156.43	169.73	13.30	*j1, k1*	1026.60	1060.43	33.83
j0, k1	157.06	174.46	17.40	*j1, k1*	1072.69	1070.36	2.33
j0, k2	183.74	202.01	18.27	*j0, k6*	1089.64	1089.71	0.07
j0, k2	190.75	208.21	17.46	*j0, k6?*	-	1128.32	-
j0, k3	313.69	329.65	15.96	*j0, k6*	1141.58	-	-
j0, k3	321.86	340.83	18.97	*j1, k2?*	-	1141.99	-
j0, k4	521.97	540.24	18.27	*j1, k2*	1191.27	1265.93	74.66
j0, k4	523.82	543.92	20.10	*j1, k2*	1249.48	1309.31	59.83
j0, k3?	595.23	-	-	*j0, k7*	1423.48	1331.24	92.24
j0, k4?	737.23	748.89	11.66	*j0, k7*	1426.76	1385.13	41.63
j0, k5	758.73	777.18	18.45	*j0, k7*	1502.88	1501.45	1.43
j0, k5	788.92	845.94	57.02	*j1, k3*	-	1506.83	-
j0, k3?	-	869.97	-	*j1, k3*	-	1525.28	-
j0, k5?	941.17	945.49	4.32	*j0, k9?*	-	1592.88	-
-	-	-	-	*j0, k9?*	-	1640.62	-
Legend: *"red color"* – range of resonant frequencies of modal shapes appropriate to determination of minimal critical rotational speeds; sign "*?*" – modal shapes difficult to recognize							

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