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Errors of a Linear Current Approximation in High-Speed PMSM Drives

Leszek Jarzebowicz

***Abstract*—Current sampling techniques and predictive algorithms used in the digital control of electric drives rely on a simple mathematical model that assumes linear current changes upon constant supplying voltages. This paper identifie rotor movement as a factor that makes this assumption invalid when the rotor covers an angular distance of a few tens of degrees during the control interval duration. The errors of the linear current approximation were quantifie both by simulation and by experiment for an exemplary high-speed permanent magnet synchronous motor drive.**

***Index Terms*—Current control, electric vehicles, pulse width modulation (PWM), predictive control, synchronous sampling.**

I. Introduction

CURRENT sampling techniques and predictive algorithms applied to the digital control of electric drives use a simple mathematical model of electric motor assuming that the electrical variables change much faster than the mechanical ones [1], [2]. This allows for approximating the rotor speed and position by constants when analyzing the electrical transients. In addition, the considered time intervals, related to digital control periods, are presumed to be short compared with the electrical time constant of motor circuits. As a result of the above simplifications, changes in motor phase currents are assumed to be linear upon constant supplying voltages [see Fig. 1]. This linear approximation applies both to current ripples related to the pulse width modulated (PWM) voltage and to a fundamental current component defined as a theoretical response to the mean voltage [1].

The linear approximation, for instance, allows for measuring the current mean value for a symmetrical PWM cycle by a single sample acquired in the middle of the cycle. This feature is widely used to provide feedback for digital current controllers without filtering or oversampling. Nevertheless, the rotational speed of electric drives has extended to improve their power-to-weight and power-to-volume ratios [3]. For instance, an eight-pole permanent magnet synchronous motor (PMSM) drive applied to the latest Toyota Prius exceeds 13 000 r/min. On the other hand,

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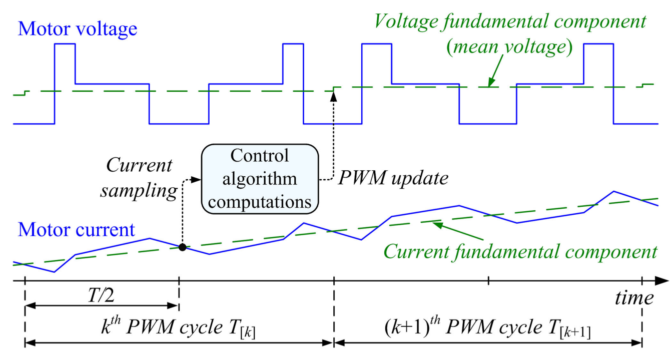

Fig. 1. Current response to the PWM supplying voltage according to the standard motor model using a linear approximation.

there are also high power applications, where rotational speeds are not so high, but PWM frequencies are typically kept below 1 kHz in order to reduce inverter switching losses [4]. As a result, the angular distance covered by the rotor during a control (PWM) cycle may approach 40° [5]. Furthermore, double-rotor motors, where the electrical speed between the rotors is double to that of standard solutions, are considered in a number of applications [6]. Moreover, six-step control algorithms are in use, where motor supplying voltages change only six times per rotor revolution. Until recently, these applications were open-loop, but since current control solutions have been proposed [7], accurate current feedback has gained importance.

The electric drives specified above require verifying if the assumptions related to the linear current approximation are still justified. Errors related to nonlinear current changes were analyzed by Wolf *et al*. [8]; however, they were associated with a short electrical time constant. This paper identifies rotor movement as a source of current nonlinear changes, which to the best of the author's knowledge has not been associated with current sampling or prediction algorithms. The impact of rotor movement on motor currents is discussed based on a PMSM drive, but similar dependences are expected in other inverter-supplied drives.

II. Impact of Rotor Movement on Motor Currents

Using vector notation, the PMSM model is given by the following:

$$\mathbf{u} = R_s\mathbf{i} + \mathbf{L}\frac{d\mathbf{i}}{dt} + \mathbf{e} \tag{1}$$

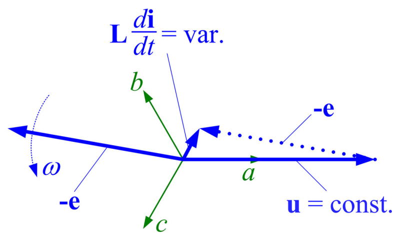

Fig. 2. Change in PMSM electromotive force **e** within the steady supplying voltage **u** interval and its impact on motor current **i**.

where **u**, **i**, **e** are the vector representations of motor supplying voltages, currents, and electromotive force, respectively; **L** is the inductances matrix; and R_s is the stator resistance.

Assuming high-speed operation, the resistive drop $R_s.\mathbf{i}$ is typically negligible when compared with the other terms of (1); thus, the model can be simplified as follows:

$$\mathbf{u} \cong \mathbf{L}\frac{d\mathbf{i}}{dt} + \mathbf{e} \Rightarrow \mathbf{L}\frac{d\mathbf{i}}{dt} \cong \mathbf{u} - \mathbf{e}. \quad (2)$$

According to (2), the motor current **i** changes at a rate depending on constant inductance **L** and on the imbalance between the supplying voltage **u** and electromotive force **e**. The voltage vector **u** changes discretely and these discrete changes correspond either to the control cycle T if the fundamental current component is modeled or to the particular subintervals of a PWM sequence if current ripples are analyzed. By contrast, the electromotive force vector **e** revolves smoothly as the rotor position $\theta(t)$ advances in a continuous manner

$$\mathbf{e} = (\omega \cdot \psi_f)\, e^{\mathbf{j}(\theta(t)+\pi/2)} \quad (3)$$

where ω is the rotor speed; ψ_f is the flux linkage produced by the permanent magnets.

As a result of the continuous rotor movement and discrete inverter operation, the angular displacement between the electromotive force vector **e** and supplying voltage vector **u** varies, as depicted in Fig. 2. This forces the voltage drop $\mathbf{L}(d\mathbf{i}/dt)$ to vary, so that the current changes are not linear.

III. Reference Model

In order to evaluate the errors of the linear current approximation, a reference model is proposed. It consists of the PMSM, a controller, and an inverter. The motor is modeled in continuous time using (1). The controller and inverter are modeled in a discrete manner and they can operate in two modes reflecting either fundamental or PWM voltage.

The reference model was developed in the Simulink and set up to simulate a laboratory drive whose parameters are given in Table I. According to these parameters, the angular distance $\Delta\theta$ covered by the rotor during control cycle T reaches 25° at the rated speed ω_r.

Example outcomes from the reference model operating in the fundamental component mode are presented in Fig. 3. The outcomes consist of values related to the A-phase. As a result of the periodically constant motor voltage u_a and continuously changing electromotive force e_a, the difference $u_a - e_a$ varies.

TABLE I
Parameters of the PMSM Drive in the Simulation and Experiment

Parameter	Value
Control (PWM) frequency $f = 1/T$	5 kHz
Rated phase current I_r	10 A r.m.s.
DC-bus voltage U_{DC}	300 V
Rated speed (electrical) ω_r	2200 rad/s
Stator resistance R_s	0.1 Ω
Stator inductance L	1 mH
Flux linkage produced by the permanent magnets ψ_f	75 mWb

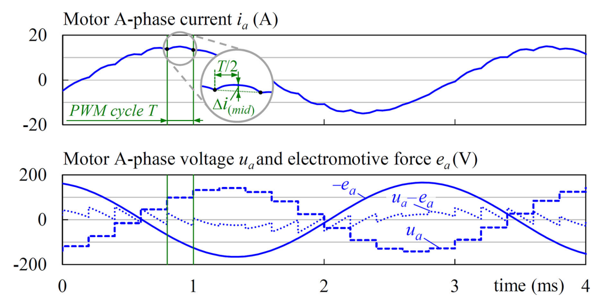

Fig. 3. Waveforms of motor phase current i_a, voltage u_a, and electromotive force e_a obtained from the reference model running in fundamental component mode; rotor speed $\omega = 2200$ rad/s.

Consequently, nonlinear current changes within control cycles are clearly visible in the current waveform.

Two error measures are considered below to evaluate the impact of rotor movement on the linear current modeling accuracy. The error of reconstructing the midpoint current value, marked $\Delta i_{(\mathrm{mid})}$ in Fig. 3, is quantified as follows. First, the theoretical current value for the midpoint of the control cycle is computed as the average of the samples from the beginning $i_{t\,=\,0}$ and the end $i_{t\,=\,T}$ of this cycle, according to the assumption of linear current changes. Then, the difference between the theoretical outcome and the measured midpoint current value $i_{t\,=\,T/2}$ is derived as

$$\Delta i_{(\mathrm{mid})} = \left(i_{t=0} + i_{t=T}\right)/2 - i_{t=T/2}. \quad (4)$$

The other error, labeled $\Delta i_{(\mathrm{mean})}$, is used to quantify how the value of current sampled at the midpoint of the control cycle differs from the mean current value in the cycle

$$\Delta i_{(\mathrm{mean})} = \frac{1}{T}\int_0^T i(t)dt - i_{t=T/2}. \quad (5)$$

As similar errors are expected in all the three motor phases, the verification aims at determining the $\Delta i_{a(\mathrm{mid})}$ and $\Delta i_{a(\mathrm{mean})}$ errors corresponding to the A-phase.

IV. Verification

The verification is carried out in three stages. The first two stages are both based on the reference model but running in the fundamental component and PWM mode, respectively. The third stage is carried out using the laboratory PMSM drive.

Waveforms of i_a current and its errors $\Delta i_{a(\mathrm{mid})}$ and $\Delta i_{a(\mathrm{mean})}$ recorded for operating at a rated speed of 2200 rad/s

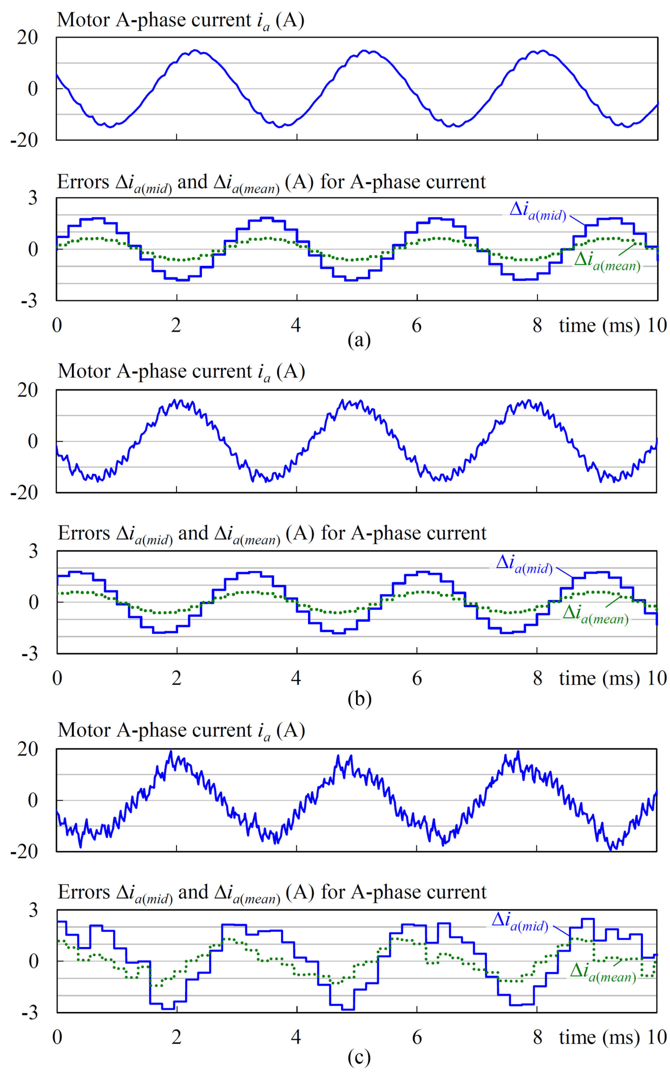

Fig. 4. Waveforms of motor current i_a and its errors $\Delta i_{a(\mathrm{mid})}$ and $\Delta i_{a(\mathrm{mean})}$ at $\omega = 2200$ rad/s recorded in the reference model running in the (a) fundamental component mode, (b) PWM mode, and (c) laboratory drive.

are shown in Fig. 4. The results obtained from the reference model running in the fundamental component mode [see Fig. 4(a)] show the explicit nonlinearity of the current waveform. Both the error waveforms are quasi sine-wave shaped, with amplitudes of 1.8 and 0.6 A for $\Delta i_{a(\mathrm{mid})}$ and $\Delta i_{a(\mathrm{mean})}$, respectively. In the simulation outcomes including PWM [see Fig. 4(b)], the nonlinearity is difficult to notice in the current waveform itself; however, the computed error waveforms prove that the modeling inaccuracy remains the same as in the previous case. The experimental results [see Fig. 4(c)] were obtained using a waveform recorder, which sampled the i_a current in parallel to discrete controller signals indicating midpoints and endpoints of each control cycle. This set of signals, sampled at 1 MS/s frequency, made it possible to compute both the errors. The obtained waveforms contain substantial disturbances, which are most likely related to the narrow zero-voltage intervals that force current sampling shortly after transistors' switching. Nevertheless, the general properties of the simulation outcomes are confirmed.

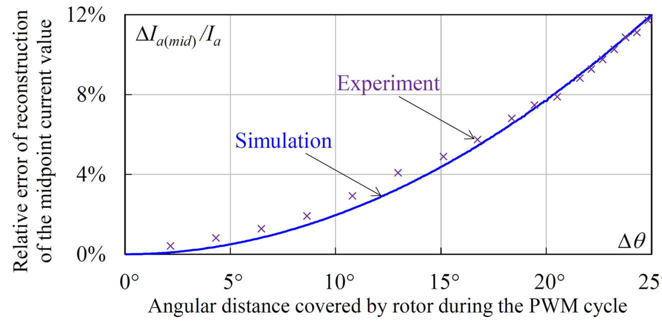

Fig. 5. Relative error of reconstruction of the midpoint current value versus the angular distance covered by the rotor during the PWM cycle.

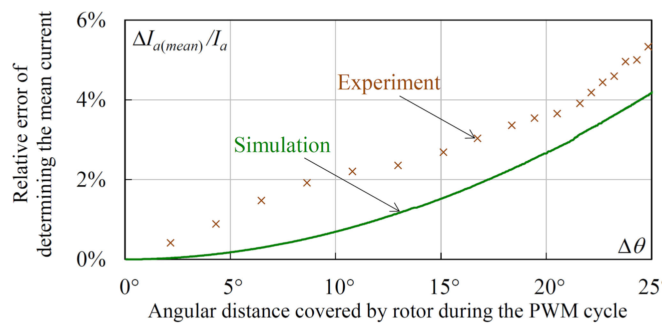

Fig. 6. Relative error of determining the mean current versus the angular distance covered by the rotor during the PWM cycle.

In order to evaluate how the errors are related to the angular distance $\Delta\theta$ covered by the rotor during a control cycle, a series of recordings was carried out at different rotor speeds. The PMSM drive operated with maximum torque, which corresponds to the phase current amplitude $I_a \cong 15$ A. The load torque, provided by a dc machine, was adjusted to settle the rotor speed at a desired value. A series of recordings was post processed to quantify the errors $\Delta i_{a(\mathrm{mid})}$ and $\Delta i_{a(\mathrm{mean})}$. Based on the measured speed ω and the constant PWM frequency f, the angular distance $\Delta\theta$ covered by the rotor during the control period was derived. As the error waveforms change over time, their amplitudes $\Delta I_{a(\mathrm{mid})}$ and $\Delta I_{a(\mathrm{mean})}$ were computed to represent the inaccuracies under a certain speed. The results are shown in Figs. 5 and 6. Identical graphs are obtained for both variants examined in the simulation. The experimental results follow the general trend of the simulation outcomes. Differences that are more distinctive appear in the graph associated with the mean current determination. This may be caused by the fact that the mean derived from experimental recordings includes samples corresponding to current disturbances caused by transistors' switching, and these disturbances are not reflected in the simulation model.

V. Conclusion

It is proven both by simulation and by experiment that the linear approximation of PMSM phase currents results in substantial errors when the rotor position advances by $\Delta\theta = 25°$ during a control period. As the error increases with $\Delta\theta$ and references report on drives featured by $\Delta\theta$ reaching 40°, the importance of the indicated problem is confirmed.

The verification carried out in this letter is aimed at proving the distinctive relation between the angular distance $\Delta\theta$ and the motor currents nonlinear changes, and some exemplary error measures are used for this purpose. As the nonlinearity may influence differently the accuracy of particular algorithms, evaluation of each algorithm should be approached individually using a specific error definition.

Future work may be aimed at mathematically describing the nonlinearity, which would enable the control algorithm to compensate for the error when measuring or predicting motor currents.

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