Determination of the minimal critical rotational speeds of the circular saw blades with the quasi-twin resonant frequencies

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ABSTRACT

To determine the minimal critical rotational speeds of the circular saw blades is the fundamental aspect of obtaining the range of the rotational operating speeds, by which the circular saw blade can work with required stability. While for the circular saw blades with full-homogeneous bodies the determination of such rotational speeds is the relatively low level of difficulty function, whereas for circular saw blades with more complex shapes (with any kinds of holes in the body, e.g. holes for cleaning knives, thermal stress compensators, indirect teeth etc.) that task becomes much more complicated.
The aim of this paper is to compare the results of the minimal critical rotational speeds' determination of the circular saw blades, which feature is the appearance the phenomenon of quasi-twin resonant frequencies, which results were obtained by the experimental method and with the FEM modeling.

Keywords: circular saw blade, minimal critical rotational speed, quasi-twin resonant frequencies, FEM modeling

INTRODUCTION

For circular saw blades clamped with collars the knowledge about phenomenon of critical rotational speeds is much more important, on the contrary to the machines with guided spline circular saws which are often used in the North America (Mohammadpanah and Hutton, 2016). Assuming that each circular saw blade in approximation is a thin circular plate, its resonant frequencies can be described as the result of correlation of its dimensions and corresponding to them nodes (Houli and Siekkinen, 1983; Kaczmarek et al., 2014, 2016; Schajer, 1986). Moreover, for such thin circular plates exists theory, which says that its resonance is a result of the interference of two component waves in which the first is traveling forward f_f and the second is traveling backward f_b (Schajer, 1986; Orłowski et al., 2007; Šteuček, 1971):

$$f_{f/b} = f_{s(n)} \pm \frac{k \cdot n}{60} \qquad \text{[Hz]} \qquad (1)$$

where: n – is the rotational speed of saw [rpm], k – is the number of nodal diameter [mm], f_s – is the natural frequency of the saw blade [Hz]:

$$f_{s(0)}^2 = f_{(n=0)}^2 + \lambda \cdot \left(\frac{n}{60}\right)^2 \qquad \text{[Hz]} \qquad (2)$$

where: $f_{(n=0)}$ – is the natural frequency of non-rotating saw (n = 0) [Hz], λ – centrifugal force coefficient (Šteuček, 1971), which can be determined experimentally or estimated with an equation:

$$\lambda = \frac{m_P - 1}{4 \cdot m_P} \cdot k^2 + \frac{3 \cdot m_P + 1}{4 \cdot m_P} \cdot k \qquad [-] \qquad (3)$$

where: m_P – is constant value of Poisson's process (Houli and Siekkinen, 1983):

$$m_P = \frac{1}{\nu} \qquad [-] \qquad (4)$$

where: ν – Poisson's ratio, and ν = 0,3 [-].

The use of the above equations gives a possibility to determine the minimal critical rotational speeds of the circular saw blades, which defines the maximum speed for which the saw stability is guaranteed (Orłowski et al., 2007; Stakhiev, 1998, 2000, 2003). The value of the minimal critical rotational speed may be calculated from the following equation:

$$n_{cr}{}^{min} = \frac{60 \cdot f_{(n=0)}}{\sqrt{k^2 - \lambda}} \qquad [\text{rpm}] \qquad (5)$$

Moreover, due to reduction of the probability of bringing circular saw blades info about the resonance, Stakhiev (1998, 2000, 2003) suggesteded that in practice circular saw blades should work with rotational speed below or equal to their permissible rotational speed:

$$n_p = 0.85 \cdot n_{cr}{}^{min} \qquad [\text{rpm}] \qquad (6)$$

There exist a wide range of methods which could be used to determine the resonant frequencies of the circular saw blades, and consequently also the minimal critical rotational speeds (Kaczmarek and Orłowski, 2016). The first group of that research methods are empirical methods like: the harmonic method (Houli and Siekkinen, 1983; Kaczmarek et al., 2014; Strzelecki, 1974; Šteuček, 1971), the impact test (Kaczmarek et al., 2014, 2016; Orlowski et al., 2007), the vision technique (Orlowski et al., 2007) and some methods with a rotating saw blade (Droba et al., 2015; Gogu, 1988; Houli and Siekkinen, 1983; Nishio and Marui, 1996).

Examining circular saw blades by the use of numerical methods, in the model the following assumptions ought to be taken into account:

- the circular saw blade is a full circular plate with given outer diameter (Droba et al., 2015; Ingielewicz and Wittbrodt, 1992);
- the circular saw blade is an annular plate with defined an inner clamping diameter (Gogu, 1988; Skoblar et al., 2016);
- the circular saw blade is viscoelastic and consist of solid "composite" plates (or shells) (Vasqueq and Cardoso, 2011);
- the circular saw blade is exposed to the influence of external forces (Gogu, 1988; Kaczmarek and Orlowski, 2016; Nishio and Marui, 1996; Vasueq and Cardoso, 2011);
- the circular saw blade rotates (Ingielewicz and Wittbrodt, 1992; Nishi and Marui, 1996).

The goal of this paper is a comparison of the results of the minimal critical rotational speeds' determination for the circular saw blades, of which the presence of the phenomenon of quasi-twin resonant frequencies was observed (Kaczmarek et al. 2016). The experimental method and the FEM modeling were compared.

MATERIALS AND METHODS

Materials

The objects of tests were six circular saw blades (Figure 1), prepared by the company ASPI TECH Sp. z o.o. Sp. K from Suwałki (PL), for which the maximal rotational speeds indicated on the saw blade are equal to 6000 rpm. These circular saw blades were divided into two series – SERIES A and SERIES B. Parameters of tested circular saw blades are shown in Tables 1 and 2.

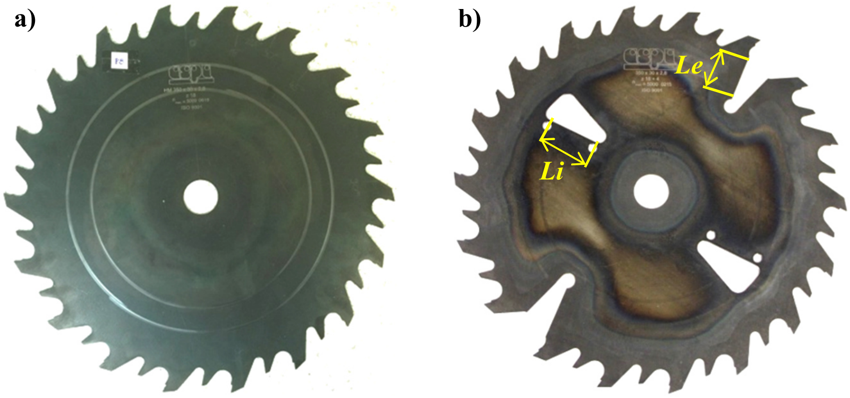

Figure 1: Tested circular saw blades: a) saw blade P8 with homogeneous blade, b) saw blade P1 with internal holes and external cuts for cleaning knives, *Le* – length of the external cleaning knifes of circular saw blades, *Li* – length of the internal cleaning knifes of circular saw blades

Table 1: Parameters of the SERIES A of the tested circular saw blades

Number of the circular saw blade	P1	P2	P3	P4	P5	P8
Outside diameter [mm]	350					
Internal diameter [mm]	30					
Clamping diameter [mm]	90					
Thickness [mm]	2.8					
Number of teeth	18					
Length of external cleaning knife [mm]	45	35	25	15	0	0
Length of internal cleaning knife [mm]	45	45	45	45	45	0

Table 2: Parameters of the SERIES B of the tested circular saw blades

Number of the circular saw blade	P1	P6	P7	P8
Outer diameter [mm]	350			
Inner diameter [mm]	30			
Clamping diameter [mm]	90			
Thickness [mm]	2.8			
Number of teeth	18			
Length of external cleaning knife [mm]	45	45	45	0
Length of internal cleaning knife [mm]	45	30	0	0

Methods

To obtain values of the minimal critical rotational speeds of the tested circular saw blades in this paper there were taken under consideration a comparison of two following methods:

1) a method based on the determination of the centrifugal force coefficient on the laboratory stand enabling the change values of the rotation speed of the circular saw blade's spindle;
2) a method based on the FEM modeling in the commercial software Autodesk Inventor Professional 2016.

The laboratory stand (Figure 2) of the experimental method is located at the Technical University in Zvolen (SK). The algorithm of the measurements on this laboratory stand is as follows:

- measurements of values of the natural frequency of non-rotating saw (n = 0) for the number of nodal diameter from k = 1 up to k = 4 and as well for the quasi-twin resonant frequencies for the number of nodal diameter from k = 1' up to k = 4' (Kaczmarek et al., 2014, 2016; Kaczmarek and Orlowski, 2016);
- measurements of the forward traveling wave f_f and the backward traveling wave f_b resonant frequencies values for circular saw blade which was rotated with rotational speeds in the range 2400–3600 rpm with a step of 200 rpm;
- calculation of the centrifugal force coefficient with the equation's (2) transformation and determination of its average value for each nodal diameter;
- calculation of the critical rotational speeds for the number of nodal diameter from k = 2 up to k = 4 and k = 2' up to k = 4' with equation (5);
- selection from the above values the minimal critical rotational speed of each tested circular saw blade;
- performing calculations of the permissible rotational speed from equation (6).

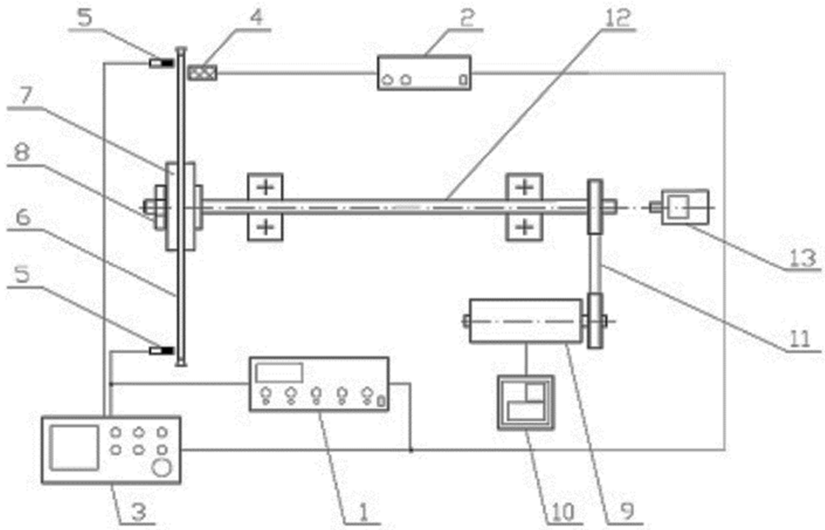

Figure 2: The scheme of the laboratory stand used for determination of the centrifugal force coefficient: 1 – function generator FG-506 (f. American Reliance Inc.), 2 – signal amplifier QSA260 (f. Q'Sound), 3 – digital oscilloscope PicoScope 2205 (f. Pico Technology), 4 – electromagnetic exciter, 5 – contactless sensors MDS-45-M30-SA (f. Micro-Epsylon), 6 – circular saw blade, 7 – clamping collars, 8 – clamping nut, 9 – electric motor, 10 – frequency converter, 11 – belt drive, 12 – spindle, 13 – contactless tachometer (Svoreň, 2012)

The second method was the FEM modeling (Figure 3) and its aim was to determine the resonant frequencies values also for the number of nodal diameter from k = 1 up to k = 4 and k = 1' up to k = 4'. Using equations (5) and (3) it was possible to calculate the minimal critical rotational speeds for each of tested circular saw blades. Values of mentioned centrifugal force coefficient, which were calculated from the equation (3), and are shown in Table 3. Moreover, the presented method was based on the following assumptions:

- the circular saw blades were fixed on the horizontal spindle (the gravity has taken into consideration);
- the model includes a small vibration exciter with spherical surface, through which the saw was excited by the horizontal force F = 0.1 N with a varied frequency in the range from 100 to 1700 Hz;
- average size of the mesh element was equal to 0.1 mm and the minimal size was 0.02 mm;
- the contacts have been added between flat surfaces of the circular saw blade, clamping collars (∅90 mm), a spindle and a clamping nut.

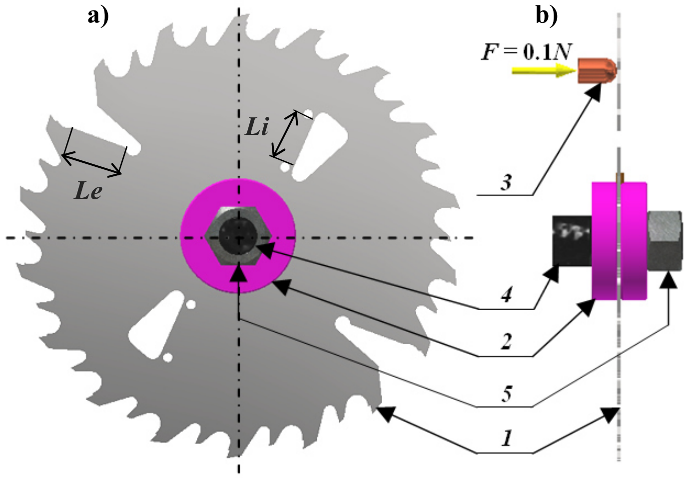

Figure 3: Model of the testes circular saw blade *P1* (a) with side view (b), where: 1 – circular saw blade, 2 – clamping collars, 3 – inductor with applied force, 4 – grounded shaft, 5 – clamping nut, *Le* – length of the external cleaning knifes of circular saw blades, *Li* – length of the internal cleaning knifes of circular saw blades

Table 3: Values of the centrifugal force coefficient λ calculated from equation (3)

Number of nodal diameter *k*	2	3	4
Coefficient λ	2.350	4.050	6.099

RESULTS AND DISCUSSION

The experimental method allowed to determine values of the centrifugal force coefficient (Table 4). Only for circular saw blade P8 it was impossible to recognize experimentally the quasi-twin resonant frequencies, so that it was decided to omit the influence of existence of quasi-twin resonant frequencies of this circular saw blade for this method (Figure 4, Figure 6). This method also made a difficulties in determination of the quasi-twin resonant frequencies for the first nodal diameter for all of tested circular saw blades – in Figure 4 and Figure 6 these values for k = 1 and k = 1' are equal and have been duplicated only to remain the clarity of these charts.

Table 4: Values of the centrifugal force coefficient λ determined in the experimental method

k	**2'**	**2**	**3'**	**3**	**4'**	**4**
P1	2.884	2.312	3.218	3.057	8.210	5.839
P2	1.614	2.357	3.257	3.668	4.545	5.256
P3	2.564	2.158	4.386	4.229	5.795	6.791
P4	2.971	2.392	4.156	4.608	5.524	6.286
P5	2.800	2.467	5.264	4.582	5.692	7.852
P6	2.637	2.211	3.076	3.533	3.915	7.852
P7	1.835	1.984	2.859	2.952	4.566	5.069
P8	2.671		4.163		6.515	

The FEM modeling allowed to obtain the quasi-twin resonant frequencies' values for cases which were problematic in the experimental method – i.e. for all the quasi-twin resonant frequencies of the circular saw blade P8, as well as the quasi-twin resonant frequencies' values for the nodal diameter k = 1' for all saws (Figure 5, Figure 7). The above proves that in these cases the occurrence of the quasi-twin resonant frequencies' values should be expected, but their values are more difficult to obtain with experimental methods.

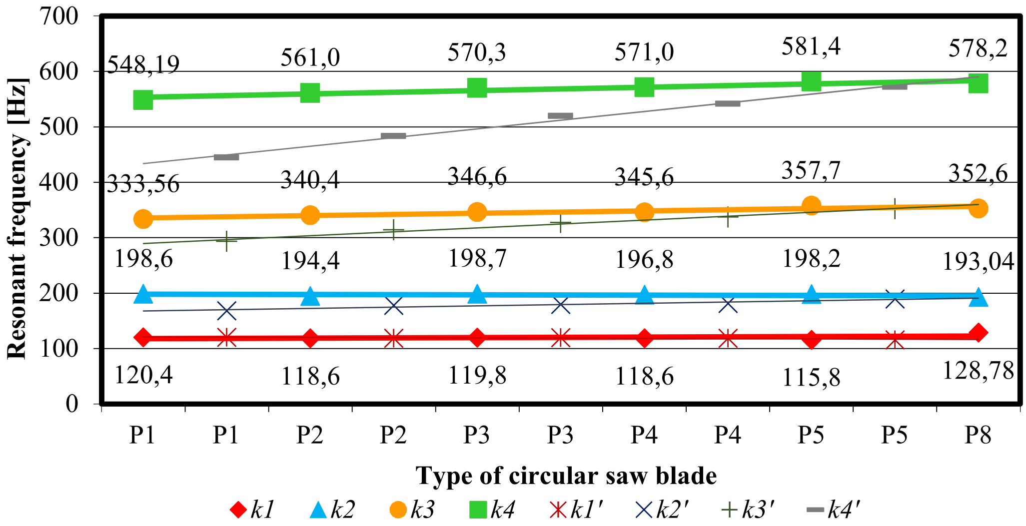

Figure 4: Values of the resonant frequencies for SERIES A obtained in the experimental method with mode trend lines

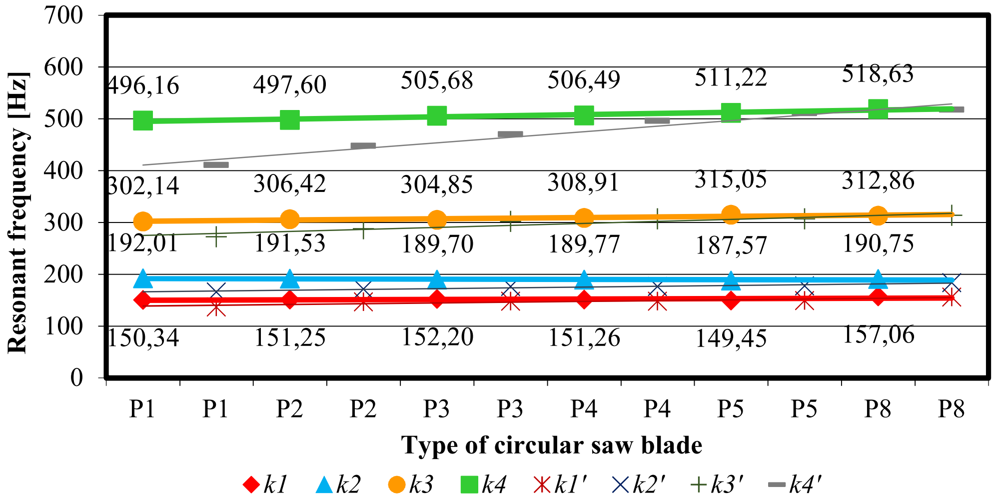

Figure 5: Values of the resonant frequencies for SERIES A obtained in the FEM modeling with mode trend lines

Moreover, analyzing the data presented in Figure 4, Figure 5, Figure 6 and Figure 7, the following features of the tested circular saw blades' resonant frequencies have been noticed:

- their values for $k = 1$, $k = 3$ and $k = 4$ mostly pretended to create a linear rising correlation for further circular saw blades;
- only for $k = 2$ the above correlation is decreasing;
- for SERIES A for circular saw blade P1 its resonant frequencies' values for $k = 1'$, $k = 2'$, $k = 3'$ and $k = 4'$ have the biggest negative difference in comparison to $k = 1$, $k = 2$, $k = 3$ and $k = 4$, and ranges of these differences are decreasing for further circular saw blades – it is related to the reduction of the external cleaning knife length *Le*;
- for SERIES B in the experimental method have been noticed that differences of resonant frequencies' values for $k = 1'$, $k = 2'$, $k = 3'$ and $k = 4'$ in comparison to $k = 1$, $k = 2$, $k = 3$ and $k = 4$ tend to hold on for further circular saw blades, however for the FEM modeling they decrease – therefore it can be considerate as a consequence of the internal cleaning knife length *Li* reduction;

- the range between values of $k = 1'$ and $k = 4$ is definitely smaller for data from the experimental method then the FEM modeling – it may be indicated because of insufficient recognition of boundary conditions for performing FEM calculations.

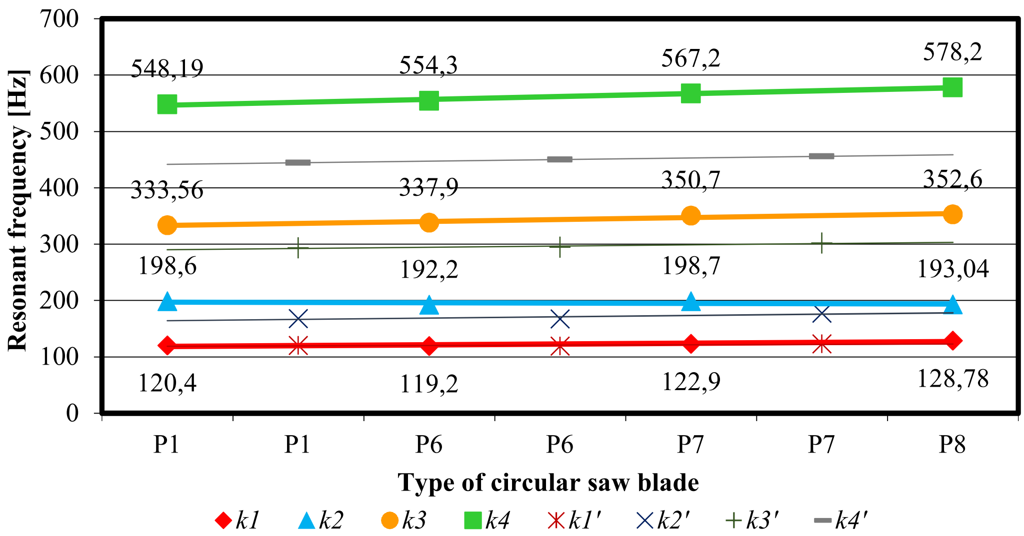

Figure 6: Values of the resonant frequencies for SERIES B obtained in the experimental method with mode trend lines

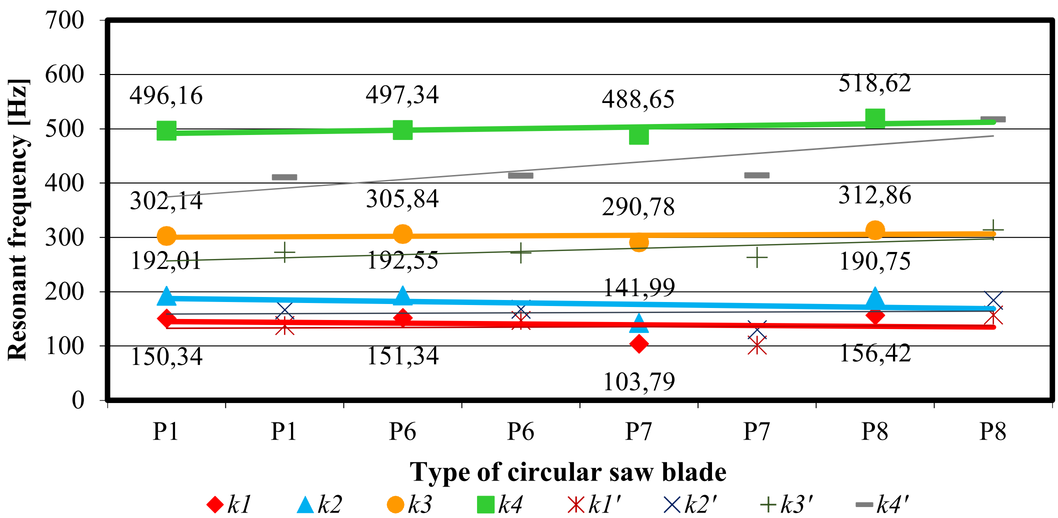

Figure 7: Values of the resonant frequencies for SERIES B obtained in the FEM modeling with mode trend lines

Analyzing the data from Table 5, Table 6, Table 7 and Table 8, the following features of the tested circular saw blades' critical rotational speeds have been noticed:

- for the FEM modeling for SERIES A for most circular saw blade minimal critical rotational speed have been determined for nodal diameter $k = 3'$ – only for circular saw blade P8 it was $k = 3$, but differences between $n_{cr}(k3)$ and $n_{cr}(k3')$ can be considered as negligibly small;
- for the experimental method for SERIES A for each circular saw blade minimal critical rotational speed have been determined for different nodal diameter, what was no expected and causes difficulties with analyzing this case – the most alarming is fact that for the circular saw blade P2 the calculated permissible rotational speed is lower about 130 rpm than the maximum rotational speed proposed by the producer – hence, for this saw blade design the maximum rotational speed indicated on the saw blade should be modified, e.g. 5800 rpm;

- for SERIES B for each circular saw blade their minimal critical rotational speeds were determined for the same nodal diameter as well for the experimental method as for the FEM modeling;
- for SERIES B for the FEM modeling it have been noticed that for circular saw blade P7 value of its calculated permissible rotational speed unfortunately is lower about 860 rpm than the maximum rotational speed given by the producer, what is equal to 14% of this maximum speed – it was considered as a result of the circular saw blade design type, which has cuts only for the external cleaning knives, while circular saw blade with cuts only for internal cleaning knives P5 (Table 5, Table 6) does not shows this kind tendency of properties;
- for SERIES A values of minimal critical rotational speeds (and permissible rotational speeds) tend to increase for further circular saw blades, what means that smaller length of external cleaning knife *Le* provide increase of the circular saw blade dynamic stiffness;
- having omitted measurements of the circular saw blade P8 and this large deviation of the circular saw blade P7 (from the FEM modeling) it can be considered that for the SERIES B changing the length of the internal cleaning knife *Li* has only slight effect on the minimal critical rotational speeds' values and the permissible rotational speeds' values are similar to maximum rotational speeds' values given by the producer.

Table 5: Values of the critical rotational speeds for SERIES A obtained in the experimental method

Critical rotational speeds [rpm]								
k	**2'**	**2**	**3'**	**3**	**4'**	**4**	n_{cr}^{min}	n_p
P1	9552.26	9171.35	**7313.31**	8209.80	9554.56	10318.64	**7313.31**	**6216.31**
P2	**6906.03**	9098.63	7866.63	8844.76	8562.49	10269.09	**6906.03**	**5870.13**
P3	8981.68	**8783.63**	9133.71	9520.38	9759.11	11276.05	**8783.63**	**7466.09**
P4	10731.43	9312.28	**9192.45**	9894.92	10036.26	10992.31	**9192.45**	**7813.58**
P5	10383.42	**9605.16**	10923.73	10210.23	10684.19	12220.81	**9605.16**	**8164.38**
P8	-	10047.41	-	**9619.38**	-	11264.62	**9619.38**	**8176.47**

Table 6: Values of the critical rotational speeds for SERIES A obtained in the FEM modeling

Critical rotational speeds [rpm]								
k	**2'**	**2**	**3'**	**3**	**4'**	**4**	n_{cr}^{min}	n_p
P1	7761.44	8968.37	**7342.51**	8147.74	7823.57	9460.97	**7342.51**	**6241.13**
P2	8020.20	8945.95	**7761.31**	8263.16	8535.01	9488.43	**7761.31**	**6597.11**
P3	8242.06	8860.47	**8144.50**	8220.82	8961.95	9642.50	**8144.50**	**6922.83**
P4	8237.39	8863.74	**8276.37**	8330.30	9447.43	9657.95	**8237.39**	**7001.78**
P5	8306.52	8760.98	**8280.42**	8495.88	9738.41	9748.14	**8280.42**	**7038.36**
P8	8582.09	8909.52	8459.21	**8436.82**	9859.88	9889.44	**8436.82**	**7171.30**

Table 7: Values of the critical rotational speeds for SERIES B obtained in the experimental method

Critical rotational speeds [rpm]								
k	**2'**	**2**	**3'**	**3**	**4'**	**4**	n_{cr}^{min}	n_p
P1	9552.26	9171.35	**7313.31**	8209.80	9554.56	10318.64	**7313.31**	**6216.31**
P6	8612.54	8622.06	**7259.86**	8671.20	7763.37	10403.47	**7259.86**	**6170.88**
P7	**7238.59**	8395.61	7306.94	8556.45	8086.07	10293.20	**7238.59**	**6152.80**
P8	-	10047.41	-	**9619.38**	-	11264.62	**9619.38**	**8176.47**

Table 8: Values of the critical rotational speeds for SERIES B obtained in the FEM modeling

Critical rotational speeds [rpm]								
k	**2'**	**2**	**3'**	**3**	**4'**	**4**	n_{cr}^{min}	n_p
P1	7761.44	8968.37	**7342.51**	8147.74	7823.57	9460.97	**7342.51**	**6241.13**
P6	7829.16	8993.59	**7310.42**	8247.52	7878.29	9483.47	**7310.42**	**6213.86**
P7	**6044.92**	6632.04	7080.93	7841.40	9317.77	9317.77	**6044.92**	**5138.18**
P8	8582.09	8909.52	8459.21	**8436.82**	9859.88	9889.25	**8436.82**	**7171.30**

CONCLUSIONS

1 - The external cleaning knife length *Le* affects in values of the resonant frequencies and minimal critical rotational speeds of testes circular saw blades. Shortening of *Le* causes that resonant frequencies move to higher values and the circular saw blade dynamic stiffness increases;

2 - Changing the internal cleaning knife length *Li* causes more difficult to identify the change in saw properties. Shortening of *Le* causes that resonant frequencies moves slightly to higher values, but the general dynamic stiffness in very tiny percentage decreases;

3 - Comparing results from the both methods it has been noticed that for the experimental method it have been obtained a much higher divergences of the permissible rotational speeds' values in both series of circular saw blade than for the FEM modeling;

4 - Unfortunately, for the experimental method it was not achieved the repeatability on the issue of obtaining the minimum critical rotational speeds from the resonant frequencies' values for the same nodal diameter, what had been expected;

5 - For both methods and simultaneously for both series the minimal critical rotational speeds were determined as well for both of quasi-twin resonant frequencies – omission of the existence of quasi-twin resonance frequencies could cause determination of the minimal critical rotational speed assigned to the wrong value of the resonant frequency of tested circular saw blade;

6 - Taking into consideration calculated values of permissible rotational speed it have been noticed that two cases for which value of rpm are below values of maximum rotational speed indicated by the producer. This phenomenon occurred for the circular saw blade P2 in the experimental method, and the circular saw blade P7 in the FEM modeling.

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