Robust Identification of Quadrocopter Model for Control Purposes

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***Abstract*—The paper addresses a problem of quadrotor unmanned aerial vehicle (so-called X4-flyer or quadrocopter) utility model identification for control design purposes. To that goal the quadrotor model is assumed to be composed of two abstracted subsystems, namely a rigid body (plant) and four motors equipped with blades (actuators). The model of the former is acquired based on a well-established dynamic equations of motion while the latter is to be identified as a static relationship from laboratory experiments data. Moreover, the actuator model is to account for the on-flight battery power source voltage drop effects. The actuator parameter identification algorithm is kept in a set-membership framework. In addition a mechanism to reduce the conservativeness of the solution is proposed and applied. Numerical illustration of the results is provided.**

Keywords*—*quadrotor, robust, modelling, identification, X4-flyer.

I. Introduction

In recent years man-operated Unmanned Aerial Vehicles (UAVs) have become a tool of modern industry as well as commerce and constitute an ever growing branch of 'scale' modelling and the so called do-it-yourself (DIY) projects with a lot of the 'modelling community' support. One of the most popular construction in this field is a quadrotor UAV also addressed as a quadrocopter and/or X4-flyer [1].

The goal of this work is to deliver utility model of the quadrotor for robust control design purposes. In principal, the quadrotor flight controller synthesis is a non-trivial task due to the nonlinear plant dynamics and its structural instability [2]. These features make mathematical modelling an important step frequently utilised to propose a control system design solution. A variety of control schemes have been proposed in literature and they range from applications of classical regulators such as proportional-integral-derivative output compensators (PIDs) e.g. [3] up to optimal and model-based predictive control (MPC) based approach driven control solutions e.g. [4].

The problem of a quadrotor model identification for the control design purposes has been addressed in literature on many occasions. In general, in order to classify the available model structures let us assume that the model is composed of two subsystems, namely a rigid body (plant) and a motor (actuator) subsystems. Then it follows that the general approach to rigid body model construction follows the Euler-Lagrange modelling method [5] gives equations of motion in three-dimensional space (6 degrees in freedom) e.g. [6]–[8]. The main differentiation is in the manner the actuator system is addressed. Three general approaches can be distinguished. First ones deals with the 'ideal' models which neglect the effects of the actuators on the overall quadrotor dynamics e.g. [9], [10]. Second, is the static relation characterisation of the motor-blades pair e.g. [11]. Third, structurally most complex, is when the dynamics relation is pursued e.g. [12], [13]. The further categorisation can be made based on the fact of including/excluding the so-called gyro effects in the actuators due to the motor rotor axis movements and the consequent changes in rake angle of the propeller e.g. [14]. In this work a model of a second type is proposed, namely the Euler-Lagrange dynamic equations of motion with a static actuator system model formulated using a set-membership framework to account for the vast uncertainty impact. The key model feature is that it enables one to encompass the effects of the on-flight power source voltage drop due to the battery drain.

The proposed model identification method is as follows. Firstly, the general model identification problem is posed using a non-parametric framework. Secondly, by structuring the parameter space the general problem is decomposed into three independent tasks. Thirdly, assumptions on the quadrotor operational conditions are formulated and used to reintroduce the tasks in a more tractable parametric identification framework. Finally, an efficient method for solving these tasks is proposed. The method originates from the set-membership estimation theory [15]. A priori knowledge of the problem is exploited by using bounded models of uncertainty to construct a so-called Feasible System Set (FSS) in the model joint parameter and measurement space. Then, using the assumed available model structure and the measurements of model inputs and outputs contained in the measurement space the set is mapped (using optimisation tools) into the solution space from which the robust characterisation of the model parameters follow. The resulting set is a hypercube in the parameter space with edges parallel to the basis vectors. Henceforth, under unfavourable conditions this results in introducing a certain amount of conservatism into the obtained result which leads to enlargement of the parameter bounds [15]. The advantage of the proposed approach is that at each step a 'classical' optimisation tasks with a guarantee of finding a global solution in finite number of steps is obtained. In addition a conservatism reduction mechanism is proposed which in general is based on manipulating the structure of the parameter space.

The paper is organised in the following manner. In Section II the problem formulation is given. Section III addressed the main result. Section IV illustrates the obtained numerical results. Section V concludes the work.

II. Problem Formulation

Let us start that by taking Lagrange equation obtained from a difference between the kinetic and potential energies of the quadrotor mechanical structure (see Fig. 1), which in fact is reduced to main body (P), arms (Rs) and motors (Ss), the solution yields the Newton–Euler equations of motion. The P is assumed to be a rigid body of evenly distributed mass composed of i.e: chassis, battery (B), dedicated (ARM-based) flight controller. The connected Rs are assumed to be weightless beams of length l. A single S is mounted at the end of each R and powered by a dedicated converter (Z). Moreover, each S is equipped with a propeller. The rotary motion of each propeller is a source of thrust and moment of rotation: $(F_{\mathrm{c}\,i},\, M_{\mathrm{rot}\,i}) \overset{\mathrm{def}}{=} (F_{\mathrm{c}\,i}\,(\cdot)\,,\, M_{\mathrm{rot}\,i}\,(\cdot))$, $\forall i \in \overline{1,4}$, respectively.

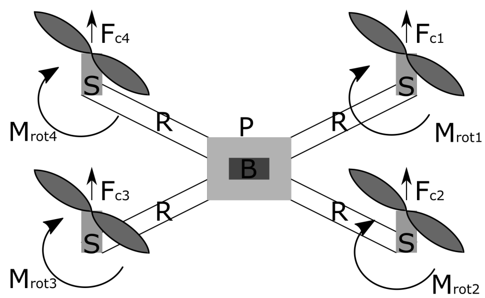

Fig. 1. Quadrotor

Suppose $(F_{\mathrm{c}\,i}, M_{\mathrm{rot}\,i})\,(u, e)$, $\forall i$, where u is the pulse point modulated (PPM) control signal (hence physically is represented by a quantity which has the interpretation of length of time period and is scaled in μs e.g. [7]) and e is the B voltage level. Then based on the Systems Theory the quadrotor's control oriented utility model structure abstraction is proposed — see Fig. 2. This system structure results form the low measurement information availability under the notion of reduction of quadrotor on-board mass.

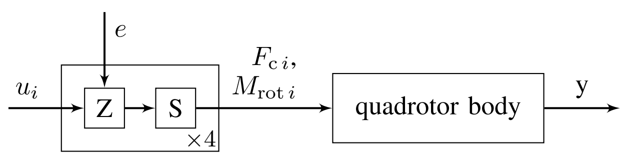

Fig. 2. Quadrotor block diagram

Let us now formulate the assumptions originating from the quadrotor operating conditions and structural characteristics.

Assumption 1. *Coordinates x, y and z are to be neglected in utility model structure.*

The justification for Assumption 1 is that the quadrotor is to be man-operated vehicle so that the operator is to be responsible for maintaining desired position in space (x, y and z coordinates adjustment).

Assumption 2. *The internal dynamics of actuators, namely: the power system, motor and propeller is considered negligible.*

The Assumption 2 is verified true from a simple fact that the dynamics of the actuator system is much faster in comparison to that of the plant.

Assumption 3. *The uncertainty due to the difference of physical structure in between the four acting actuator subsystems (motor and propeller) is considered negligible.*

The Assumption 3 results from a fact that the construction and means of manufacturing of motor and propeller guarantee high accuracy and reproducibility of their parameters. Therefore, the uncertainty due to these factors is relatively small (negligible) in comparison to the other acting sources of uncertainty addressed in the following paragraphs. This also has been verified experimentally during the laboratory experiments. Hence, it follows that: $\forall_{i \in \overline{1,4}} F_{\mathrm{c}\,i} = F_{\mathrm{c}}$, $\forall_{i \in \overline{1,4}} M_{\mathrm{rot}\,i} = M_{\mathrm{rot}}$.

Assumption 4. *It is assumed that the physical structure of the quadrotor is symmetric.*

From Assumption 4 it follows that the moment of inertia with respect to axis x and y coincide, thus: $I_{\mathrm{x}} \overset{\mathrm{def}}{=} I_{\mathrm{xy}}$, $I_{\mathrm{y}} \overset{\mathrm{def}}{=} I_{\mathrm{xy}}$.

Considering this setup, Assumptions 1–4 and including the forces and torques generated by the rotation of propellers (with respect to the generalised forces vector [5]) yields the following quadrotor model formulation:

$$\ddot{\phi} = \dot{\theta}\dot{\psi}\frac{I_{\mathrm{xy}} - I_{\mathrm{z}}}{I_{\mathrm{xy}}} + \frac{l}{I_{\mathrm{xy}}}\left(-F_{\mathrm{c}\,2} + F_{\mathrm{c}\,4}\right), \tag{1a}$$

$$\ddot{\theta} = \dot{\phi}\dot{\psi}\frac{I_z - I_{\mathrm{xy}}}{I_{\mathrm{xy}}} + \frac{l}{I_{\mathrm{xy}}}\left(F_{\mathrm{c}\,1} - F_{\mathrm{c}\,3}\right), \tag{1b}$$

$$\ddot{\psi} = \frac{l}{I_{\mathrm{z}}}\sum_{i=1}^{4}(-1)^i M_{\mathrm{rot}\,i}. \tag{1c}$$

where: $\phi, \theta, \psi \in \mathbb{R}$ are the Tait-Bryan angles typically addressed as yaw, pitch and roll angle, respectively; I_z denotes the moment of inertia with respect to the basis of quadrotor's coordinate frame. In order for the utility model to be well-defined it is crucial to identify the parameters of the derived structure, namely: $p_d \overset{\mathrm{def}}{=} [I_{\mathrm{xy}}, I_{\mathrm{z}}, l]^T$ as well as the shape and parameter of the *functions* F_{c}, M_{rot}. It is also prudent to emphasise that the model and the parameters are subject to the uncertainty i.e. due to the stated Assumptions. Indeed, the effect of errors in understanding the true nature of F_{c}, M_{rot} results in structural error. Determining the exact structure of F_{c}, M_{rot} is a nontrivial task due to cumbersome mathematical model underlying the principals of the motor – propeller pair and how it contributes to structure of F_{c}, M_{rot}. Henceforth, the problem at hand is a non-parametric identification task where the parameter 'sub-vectors' p_{F} and p_{M} that correspond to F_{c} and M_{rot}, respectively, have in fact infinite number of entries.

The goal of this work is to assess (robustly) the region containing all the parameters $p \overset{\mathrm{def}}{=} \left[p_{\mathrm{q}}^T, p_{\mathrm{F}}^T, p_{\mathrm{M}}^T\right]^T$ of quadrotor which in general can be done as:

$$\underset{i_{\mathrm{p}}}{\forall} \quad p_{i_{\mathrm{p}}}^{+} \overset{\mathrm{def}}{=} \max_{p \in FSS_{\mathrm{p}}} p_{i_{\mathrm{p}}}, \tag{2a}$$

$$\underset{i_{\mathrm{p}}}{\forall} \quad p_{i_{\mathrm{p}}}^{-} \overset{\mathrm{def}}{=} \min_{p \in FSS_{\mathrm{p}}} p_{i_{\mathrm{p}}}, \tag{2b}$$

where: FSS_{p} is FSS which aggregates all the information *a priori* that is available on p and $p_{i_{\mathrm{p}}}$ is the i_{p}th element of p.

Taking $FSS_{\mathrm{p}} \overset{\mathrm{def}}{=} FSS_{\mathrm{q}} \times FSS_{\mathrm{F}} \times FSS_{\mathrm{M}}$, the (2) can be decomposed into the three independent parameter identification tasks addressing: *construction (Task 1)*, *thrust force function (Task 2)* and *rotation moment function (Task 3)*.

Task 1: Similarly to (2) construction parameter identification task yields:

$$\underset{i_{\mathrm{q}}}{\forall} \quad p^{\pm}_{q\,i_{\mathrm{q}}}\left(\dot{x},u\right) \overset{\text{def}}{=} \pm \max_{p_{\mathrm{q}} \in FSS_{\mathrm{q}}} \pm p_{\mathrm{q}\,i_{\mathrm{q}}}, \tag{3}$$

where: $p_{\mathrm{q}\,i_{\mathrm{d}}}$ is the i_{q}th element of p_{q} and $i_{\mathrm{q}} \in \overline{1,3}$. Since the physical elements of the construction are measurement available (in terms of their mass and physical dimensions) thus the resulting parameters and the derived variables can be directly obtained from the measurements as:

$$\begin{aligned} FSS_{\mathrm{q}} \overset{\text{def}}{=} \big\{ (I_{\mathrm{xy}}, I_{\mathrm{z}}, l) : I_{\mathrm{xy}} = 2m_s l^2 + 1/12\, m_{\mathrm{M}} l_{\mathrm{M}}^2, \\ I_{\mathrm{z}} = 4m_{\mathrm{s}} l^2 + \frac{1}{6} m_{\mathrm{M}} l_{\mathrm{M}}^2, m_{\mathrm{M}}^- \le m_{\mathrm{M}} \le m_{\mathrm{M}}^+, \\ l_{\mathrm{M}}^- \le l_{\mathrm{M}} \le l_{\mathrm{M}}^+, l^- \le l \le l^+, m_{\mathrm{s}}^- \le m_{\mathrm{s}} \le m_{\mathrm{s}}^+ \big\}, \end{aligned} \tag{4}$$

where: $m_{\mathrm{M}}^{\pm}$, $l_{\mathrm{M}}^{\pm}$, $l^{\pm}$, $m_{\mathrm{s}}^{\pm}$ denote the physical structure parameter intervals due to measurement uncertainty of m_{M}, l_{M}, l, m_{s}, respectively (see Table I in Section IV).

Task 2 and 3: The non-parametric tasks set up in order to determine the influence of actuator system are as follows:

$$\underset{i_{\mathrm{F}}}{\forall} \quad p^{\pm}_{\mathrm{F}\,i_{\mathrm{F}}} \overset{\text{def}}{=} \pm \max_{p_{\mathrm{F}} \in FSS_{\mathrm{F}}} \pm p_{\mathrm{F}\,i_{\mathrm{F}}}, \tag{5}$$

$$\underset{i_{\mathrm{M}}}{\forall} \quad p^{\pm}_{\mathrm{M}\,i_{\mathrm{M}}} \overset{\text{def}}{=} \pm \max_{p_{\mathrm{M}} \in FSS_{\mathrm{M}}} \pm p_{\mathrm{M}\,i_{\mathrm{M}}}, \tag{6}$$

where: FSS_{F} and FSS_{M} denote the feasible system set and $p_{\mathrm{F}\,i_{\mathrm{F}}}$, $p_{\mathrm{M}\,i_{\mathrm{M}}}$ are the i_{F}th, i_{M}th element of p_{F}, p_{M}, respectively.

Now in order to rewrite the non-parametric identification tasks using (more tractable) parametric identification framework observations done during laboratory experiments and the knowledge from literature e.g. [6]–[8], [11]–[14] is used to formulate the following assumptions regarding F_{c} and M_{rot} structures.

Assumption 5. *Suppose F_{c} and M_{rot} have imposed structures:*

$$F_{\mathrm{c}}\left(u,e\right) = b\left(e\right)\left(u-c\right)^2, \tag{7}$$

$$M_{\mathrm{rot}}\left(u,e\right) = d\left(e\right)\left(u-c\right)^2, \tag{8}$$

where: $c = 1000\,\mu s$ corresponds to null speed and is a result of an offset in the PPM communication channel.

Assumption 6. *Assume that $b(e)$ and $d(e)$ have linear structure.*

Indeed, under Assumptions 5 – 6 the non-parametric tasks become the parametric identification problems to be handled by the proposed identification algorithm — see Fig. 3. The proposed procedure is a two step algorithm. At first, the problem feasible sets are to be constructed and the dedicated parametric optimisation problems are to be solved in order to deliver information on the optimised parameter bounds. The second step is to be applied conditionally. The condition term identifies the shape of the problem feasible set and based on its geometry with respect to the bounding hypercube the problem is either considered solved or reformulated and solved once more. The reformulation is the second step of the proposed method. It is based on the manipulation of the parameter space by applying in principal a translation and rotation transformations. Under these transformations a new set of decision variables and problem feasible sets are obtained. Solving the resulting new robust parameter identification problem yields a less conservative result. The proposed method is applied to each of the two remaining tasks. The technical details for the considered case of quadrotor are given in Section III.

Assumption 7. *It is assumed that the joint uncertainty due to measurement and structural error is as large as to guarantee the existence of the estimates (or $FSS_{(\cdot)} \neq \emptyset$).*

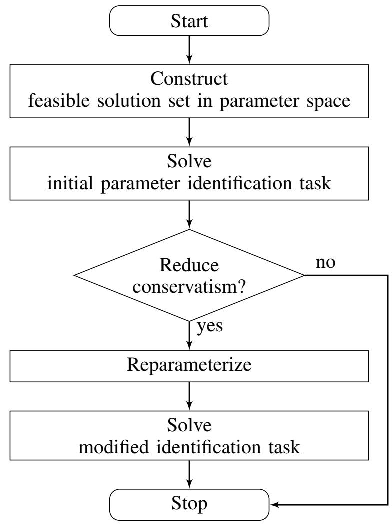

Fig. 3. Identification procedure

III. Approximate Solution

The result given in this Section is to be delivered in two steps. Firstly, an approximate solution is pursued by parametrisation of (5) and (6). Secondly, a conservatism reduction methods is invoked by manipulating the geometry of FSS.

A. Initial Parameter Identification Tasks

In both F_c and M_{rot} parameter identification cases the decision input space is two dimensional. In this work authors propose to discretise and sample this space in the dimension of e and for each $e_{(\cdot)}$ of the voltage scenario set a parameter identification task with respect to u is solved. Thereafter, based on the obtained results the model of the relation with respect to e is imposed to acquire the full information of the actuator model (in terms of both e and u).

First is the assessment of the impact of the control signal on the parameter bounding box. Second addresses the problem of power-source voltage drop in an analogous context.

1) Impact of the Control Signal: Under Assumption 4 it follows that the identification task becomes structured and the parametric approach can be applied if and only if e is amended as it still remains a free variable in the task. For that reason multiple optimisation tasks are to be solved for the different values of e due to the battery discharge during quadrotor flight. This yields:

$$\underset{i_{\mathrm{e}} \in \overline{1,n_{\mathrm{e}}}}{\forall} \quad b^{\pm}_{i_{\mathrm{e}}} \overset{\text{def}}{=} \pm \max_{b_{i_{\mathrm{e}}} \in FSS_{\mathrm{F}\,i_{\mathrm{e}}}} \pm b_{i_{\mathrm{e}}}, \tag{9}$$

$$\underset{i_{\rm e}\in\overline{1,n_{\rm e}}}{\forall} \quad d_{i_{\rm e}}^{\pm} \overset{\rm def}{=} \pm \max_{d_{i_{\rm e}}\in FSS_{M\,i_{\rm e}}} \pm d_{i_{\rm e}}, \tag{10}$$

where: $b_{i_{\rm e}}^{\pm}$ and $d_{i_{\rm e}}^{\pm}$ are the parameters related to $i_{\rm e}$th battery voltage level, $n_{\rm e}$ is the total number of voltage level scenarios,

$$FSS_{{\rm F}\,i_{\rm e}} \overset{\rm def}{=} \Big\{ b\in\mathbb{R}: \underset{i_{\rm u}\in\overline{1,n_{\rm u}}}{\forall} b\geq 0, \\ -\epsilon_{\rm F} \leq F_{{\rm c}\,u_{i_{\rm u}},e_{i_{\rm e}}} - F_{\rm c}\left(u_{i_{\rm u}},e_{i_{\rm e}}\right) \leq \epsilon_{\rm F} \Big\}, \tag{11}$$

$$FSS_{{\rm M}\,i_{\rm e}} \overset{\rm def}{=} \Big\{ d\in\mathbb{R}: \underset{i_{\rm u}\in\overline{1,n_{\rm u}}}{\forall} d\geq 0, \\ -\epsilon_{\rm M} \leq M_{{\rm rot}\,u_{i_{\rm u}},e_{i_{\rm e}}} - M_{\rm rot}\left(u_{i_{\rm u}},e_{i_{\rm e}}\right) \leq \epsilon_{\rm M} \Big\}, \tag{12}$$

where: $F_{{\rm c}\,i_{\rm u},i_{\rm e}}$, $M_{{\rm c}\,i_{\rm u},i_{\rm e}}$ are the measured thrust and torque with respect to $i_{\rm u}$th control signal and $i_{\rm e}$th voltage level scenario, respectively, $\epsilon_{\rm F}$, $\epsilon_{\rm M}$ denote thrust and torque uncertainty level, respectively, $n_{\rm u}$ is the total number of control signal scenarios. It follows from (11) that $FSS_{{\rm F}\,i_{\rm e}}$ is a set of feasible non-negative values of parameter b and there exists an element in $FSS_{{\rm F}\,i_{\rm e}}$ for which the experimental data (input–output data pairs, namely $\left(\left(u_{i_{\rm u}},e_{i_{\rm e}}\right), F_{{\rm c}\,u_{i_{\rm u}},e_{i_{\rm e}}}\right)$) is explained by $F_{\rm c}\left(u_{i_{\rm u}},e_{i_{\rm e}}\right)$ with accuracy of no less than $\epsilon_{\rm F}$. Analogous reasoning holds for $d\in FSS_{{\rm M}\,i_{\rm e}}$. Notice that both (11) and (12) are structurally linear due to $F_{\rm c}(\cdot)$ and $M_{\rm rot}(\cdot)$.

Remark 1. *The solutions of* (9) *and* (10) *are not equivalent to* (5) *and* (6)*.*

2) Impact of Power-source Voltage drop: At this point the domain of $F_{\rm c}$ and $M_{\rm rot}$ is discrete due to the way identification procedures were conducted. In this Subsection the continuous relation of $F_{\rm c}$ and $M_{\rm rot}$ are to be proposed by approximating the results obtained from solving (9) and (10) under Assumption 6. To that goal an analogous method to the one given in Subsection II is to be applied. Firstly, a structure of $b\left(e\right)$ and $d\left(e\right)$ is to be imposed. Secondly, a parameter identification task is to be solved as:

$$\underset{i_{\rm b}\in\overline{1,n_{\rm b}}}{\forall} \quad p_{b\,i_{\rm b}}^{\pm} \overset{\rm def}{=} \pm \max_{p_{\rm b}\in FSS_{\rm b}} \pm p_{{\rm b}\,i_{\rm b}}, \tag{13}$$

$$\underset{i_{\rm d}\in\overline{1,n_{\rm d}}}{\forall} \quad p_{{\rm d}\,i_{\rm d}}^{\pm} \overset{\rm def}{=} \pm \max_{p_{\rm d}\in FSS_{\rm d}} \pm p_{{\rm d}\,i_{\rm d}}, \tag{14}$$

where:

$$FSS_{\rm b} \overset{\rm def}{=} \left\{ p_{\rm d}: \underset{i_{\rm e}\in\overline{1,n_{\rm e}}}{\forall} b_{i_{\rm e}}^{-} \leq b\left(e_{i_{\rm e}}\right) \leq b_{i_{\rm e}}^{+} \right\}, \tag{15}$$

$$FSS_{d} \overset{\rm def}{=} \left\{ p_{\rm d}: \underset{i_{\rm e}\in\overline{1,n_{\rm e}}}{\forall} d_{i_{\rm e}}^{-} \leq d\left(e_{i_{\rm e}}\right) \leq d_{i_{\rm e}}^{+} \right\}, \tag{16}$$

$p_{\rm b}\in\mathbb{R}^{n_{\rm b}}$ and $p_{\rm d}\in\mathbb{R}^{n_{\rm d}}$ are the parameters of $b\left(e\right)$ and $d\left(e\right)$, respectively.

B. Conservatism Reduction

Due to the estimated parameter set geometry the identification tasks yield conservative results. To reduce the conservativeness of the results a transformation of base in the parameter space is proposed as:

$$p_{\rm b}\left(p_{\rm bb}\right) \overset{\rm def}{=} \frac{p_{\rm b}^{+}+p_{\rm b}^{-}}{2} + R_{\rm b}\,p_{\rm bb}, \tag{17}$$

$$p_{\rm d}\left(p_{\rm dd}\right) \overset{\rm def}{=} \frac{p_{\rm d}^{+}+p_{\rm d}^{-}}{2} + R_{\rm d}\,p_{\rm dd}, \tag{18}$$

where: $p_{\rm bb}\in\mathbb{R}^{n_{\rm b}}$ i $p_{\rm dd}\in\mathbb{R}^{n_{\rm d}}$ are the new parameters with corresponding matrices $R_{\rm b}\in\mathbb{R}^{n_{\rm b}\times n_{\rm b}}$ and $R_{\rm d}\in\mathbb{R}^{n_{\rm d}\times n_{\rm d}}$. In consequence the identification tasks (13) and (14) are reformulated as:

$$\underset{i_{\rm bb}\in\overline{1,n_{\rm b}}}{\forall} \quad p_{{\rm bb}\,i_{\rm bb}}^{\pm} \overset{\rm def}{=} \pm \max_{p_{\rm bb}\in FSS_{\rm bb}} \pm p_{{\rm bb}\,i_{\rm bb}}, \tag{19}$$

$$\underset{i_{\rm dd}\in\overline{1,n_{\rm dd}}}{\forall} \quad p_{{\rm dd}\,i_{\rm dd}}^{\pm} \overset{\rm def}{=} \pm \max_{p_{\rm dd}\in FSS_{\rm dd}} \pm p_{{\rm dd}\,i_{\rm dd}}, \tag{20}$$

where:

$$FSS_{\rm bb} \overset{\rm def}{=} \left\{ p_{\rm bb}: p_{\rm b}\left(p_{\rm bb}\right)\in FSS_{\rm b} \right\}, \tag{21}$$

$$FSS_{\rm dd} \overset{\rm def}{=} \left\{ p_{\rm dd}: p_{\rm b}\left(p_{\rm dd}\right)\in FSS_{\rm d} \right\}, \tag{22}$$

$p_{{\rm bb}\,i_{\rm bb}}$ and $p_{{\rm dd}\,i_{\rm dd}}$ are the $i_{\rm bb}$th and $i_{\rm dd}$ th element of $p_{\rm bb}$ and $p_{\rm dd}$, respectively.

IV. Results

The results of the quadrotor's physical dimensions measurements have been given in Table I.

In order to measure the thrust and moment of rotation the laboratory stand has been equipped with two working nests. In both of the addressed cases the measurement of the quantity of interest has been done in terms of the pressure force as presented in [7]. The detailed description and results of the laboratory measurement campaign for both thrust and torque have been given in Tables 2 and 3 in [16].

The optimisation tasks (3) are characterised by nonlinear constraints. However, since they can be solved separately one can neglect one of the nonlinear constraints (in case of I_{xy} and I_z identification) or two (in case of l identification). This can be done by utilising the values at the beginning or end of the uncertainty interval as:

$$p_{\rm d\,1}^{-} = 2m_{\rm s}^{-}\left(l^{-}\right)^{2} + \frac{1}{12}m_{\rm M}^{-}\left(l_{\rm M}^{-}\right)^{2}, \tag{23a}$$

$$p_{\rm d\,2}^{-} = 4m_{\rm s}^{-}\left(l^{-}\right)^{2} + \frac{1}{6}m_{\rm M}^{-}\left(l_{\rm M}^{-}\right)^{2}, \tag{23b}$$

$$p_{\rm d\,3}^{-} = l^{-}, \tag{23c}$$

TABLE I. QUADROTOR'S PHYSICAL DIMENSIONS MEASUREMENTS

Element	Items	Parameter	Value	Min	Max	SI
body	1	$m_{\rm M}$	0.465 ± 0.001	0.464	0.466	kg
		$l_{\rm M}$	0.10 ± 0.01	0.09	0.11	m
arm	4	l	0.24 ± 0.01	0.23	0.25	m
motor	4	$m_{\rm s}$	0.066 ± 0.001	0.065	0.067	kg

$$p_{\mathrm{d}\,1}^{+} = 2m_{\mathrm{s}}^{+}\left(l^{+}\right)^{2} + \frac{1}{12}m_{\mathrm{M}}^{+}\left(l_{\mathrm{M}}^{+}\right)^{2}, \tag{24a}$$

$$p_{\mathrm{d}\,2}^{+} = 4m_{\mathrm{s}}^{+}\left(l^{+}\right)^{2} + \frac{1}{6}m_{\mathrm{M}}^{+}\left(l_{\mathrm{M}}^{+}\right)^{2}, \tag{24b}$$

$$p_{\mathrm{d}\,3}^{+} = l^{+}. \tag{24c}$$

Since tasks (9) and (10) share analogous structures, thus the same methods can be applied in order to solve them. Since under Assumption 5 the (7) and (8) are linear in their parameters thus (11) and (12) are convex and in consequence (9) and (10) are in fact linear programming (LP) problems. In consequence the task can be solved in the finite number of steps by applying simplex algorithm. Under Assumption 7 it follows that $e_{\mathrm{F}} = 65\,[g]$, $e_{\mathrm{M}} = 12\,[g]$ in order to account for not only the measurement but also the structural uncertainty. Assuming such a setup the results of measurement reconstruction accuracy have been illustrated in Figs. 4 and 5. The bounding boxes of the parameters b and d obtained with respect to the i_{e}th power source voltage level scenario, namely: $b_{i_{\mathrm{e}}}^{+}$, $b_{i_{\mathrm{e}}}^{-}$ and $d_{i_{\mathrm{e}}}^{+}$, $d_{i_{\mathrm{e}}}^{-}$ have been depicted in Figs. 6 and 7.

Remark 2. *In general, very small values of e_{F}, e_{M} can yield an empty set of feasible solutions which justifies Assumption 7.*

It can be seen that the uncertainty related to the *a priori* knowledge of the problem results in situation in which the effects of power-source voltage drop are negligible under low values of u and comparable to the effects of uncertainty in the u's upper part of admissible range. Henceforth, at this point it cannot be decided if the effects of power-source voltage drop have significant impact on the overall plant performance. However, the observed differences (see Figs. 4 and 5) indicate that the matter requires more attention for the decisive conclusion to be made.

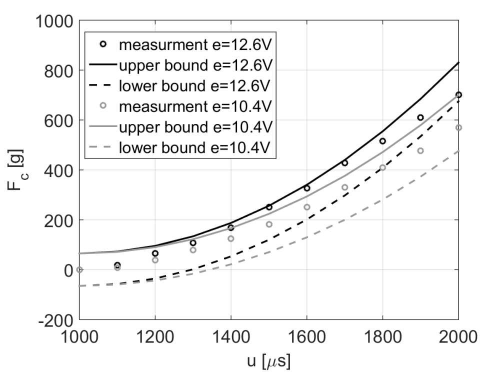

Fig. 4. Thrust characteristics

The estimated parameter sets have been illustrated in Figs. 6 and 7. These results tend to justify the Assumption 6 thus it follows that:

$$b\,(e) = p_{\mathrm{b}\,1}e + p_{\mathrm{b}\,2}, \tag{25}$$

$$d\,(e) = p_{\mathrm{d}\,1}e + p_{\mathrm{d}\,2}. \tag{26}$$

In consequence the solution to (13) and (14) can be obtained by applying LP. Unfortunately this yields a conservative parameter bounding set estimates:p_{b}^{+}, p_{b}^{-} and p_{d}^{+}, p_{d}^{-}, as depicted in Figs. 6 and 7. This is a result of a certain conservativeness in approximation of FFS_{b} and FFS_{d} (see Figs. 8 and 9).

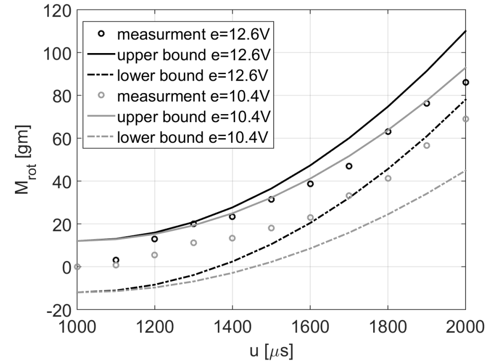

Fig. 5. Rotation moment characteristics

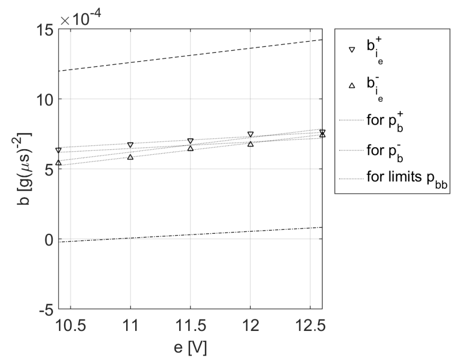

Fig. 6. Thrust case

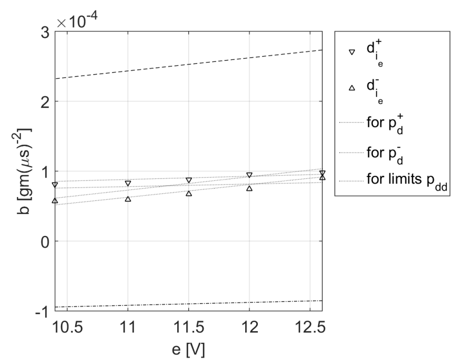

Fig. 7. Rotation moment case

This conservativeness can be reduced by re-parametrisation (see Subsection III-B) by applying (17) and (18) with:

$$R_{\mathrm{b}} = \begin{bmatrix} p_{\mathrm{b}\,1}^{+} - p_{\mathrm{b}\,1}^{-} & p_{\mathrm{b}\,2}^{-} - p_{\mathrm{b}\,2}^{+} \\ p_{\mathrm{b}\,2}^{-} - p_{\mathrm{b}\,2}^{+} & -(p_{\mathrm{b}\,1}^{+} - p_{\mathrm{b}\,1}^{-}) \end{bmatrix}, \tag{27a}$$

$$R_{\mathrm{d}} = \begin{bmatrix} p_{\mathrm{d}\,1}^{+} - p_{\mathrm{d}\,1}^{-} & p_{\mathrm{d}\,2}^{-} - p_{\mathrm{d}\,2}^{+} \\ p_{\mathrm{d}\,2}^{-} - p_{\mathrm{d}\,2}^{+} & -(p_{\mathrm{d}\,1}^{+} - p_{\mathrm{d}\,1}^{-}) \end{bmatrix}. \tag{27b}$$

The result is the four lines corresponding to the corners (limits) of estimated parameter set — see Figs. 8 and 9. The only drawback of the approach is that the new parametrisation is less intuitive during identification procedure as the direct physical interpretation is somehow 'lost'.

This task can still be solved using LP tools, since the affine transforms (17) and (18) did not modify the class

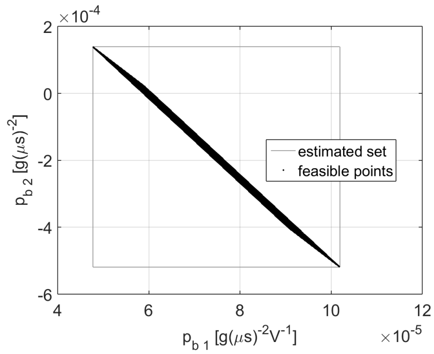

Fig. 8. Thrust case

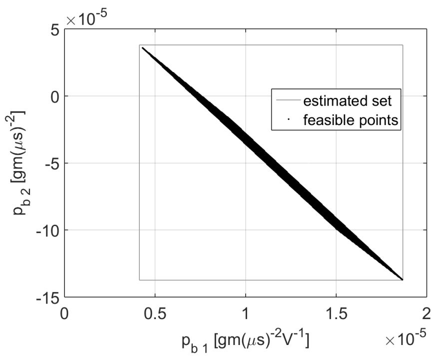

Fig. 9. Rotation moment case

of the problem. The obtained results of robust parameter identification illustrate the impact the model structure can exert on the results of the identification procedure. It has been shown that the most direct approach to set membership parameter estimation can lead to an empty feasible set of solutions and/or conservative results. However, when problem is properly addressed, constructive results can be obtained using simple and well defined tool-set. This, however, requires (comes at a price) of certain amount of assumptions that have to be made.

V. Conclusions

In principal the key aspects in the quadrotor design is to keep low mass and power consumption in order to extend flight time under limited battery capacity. The consequence i.a. is a reduced amount of measurement information available. Considering such a problem setup a utility model (dedicated for control design purposes) identification problem has been posed. The parameter estimation has been carried out in a set membership framework in order to find the quadrotor model parameter set as well as the model of the power-source voltage drop. To reduce the conservativeness of the results obtained in the latter case the parameter space transformation has been proposed. The obtained decrees in the uncertainty impact on the identification solutions have been found very promising. However, in order to take full advantage of this result a new actuator system model structure is required on the one hand as the amount of uncertainty in *a priori* knowledge at this point propagate in a manner that disables one to track the effects of the power-source voltage drop. On the other it is not excluded that the overall effects of power-source voltage drop will remain in comparison to the acting uncertainty. The future work in the field is to investigate this matter.

VI. Acknowlegments

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