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Calculation of Resonance in Planar and Cylindrical Microstrip Structures Using a Hybrid Technique

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***Abstract*—A hybrid technique was employed for the analysis of the resonance frequency of thin planar and cylindrical microstrip structures with patches of arbitrary geometry. The proposed technique utilizes a combination of Galerkin's moment method and a finite element method. In this approach, a finite element method is adopted to calculate the patch surface current densities, and a method of moments is utilized to calculate the resonance frequencies of the microstrip structure. The technique allows the analysis of different shaped patches. To verify the validity of the approach, the results were compared with those obtained from commercial software and actual measurements of manufactured prototypes.**

***Index Terms*—Current density, Cylindrical structure, Finite element method, Galerkin's method, Microstrip structure, Resonance structure.**

I. Introduction

MICROSTRIP structures are very popular due to their thin profile, light weight, low cost and ease of production. They are commonly used in antennas. The thin profiles of microstrip structures allow them to be utilized in conformal microstrip structures that have many practical applications in airplanes, spacecraft, speedboats and other high-speed vehicles in which aerodynamics or hydrodynamics are important [1]. The resonant frequency problem of a microstrip patch has been studied and reported in many papers both for planar structures [2]–[4] as well as curved ones [5]–[12]. The most common full-wave technique utilized for the analysis of microstrip antennas is the method of moments (MoM) [2], [3], [5]–[12]. This technique is applied to the analysis of microstrip structures with patches of simple geometry such as rectangular, circular, triangular or elliptical, for which the analytical form of the current basis functions can be easily derived. For structures with arbitrary patch geometry, MoM employing the Rao-Wilton-Glisson triangular basis functions [13], a hybrid method employing the MoM and the finite-difference technique [12], or a commercial simulator employing discrete methods, can be applied. In the case of large structures, high frequency approaches based on the asymptotic techniques are utilized [1].

In this communication, the problem of determining the resonant frequencies of both planar and cylindrical microstrip structures with patches of arbitrary shapes, located on dielectric coated, conducting planar and cylindrical surfaces, was investigated. As an alternative to using commercial software, a hybrid technique employing a two-dimensional finite element method (2D FEM) and the MoM is proposed here. A

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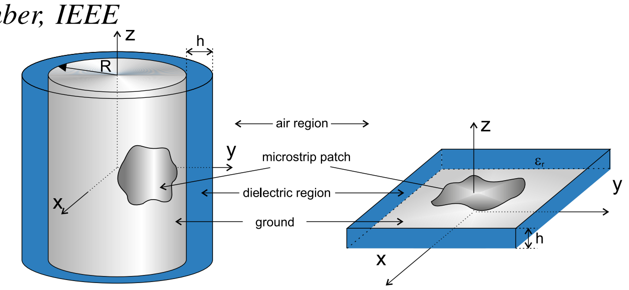

Fig. 1. The geometry of the investigated structures.

Galerkin's moment method for planar structures [2] and for cylindrical structures [5], [7] is utilized to calculate resonant frequencies. To determine the patch surface current basis functions (which are required in this procedure) for patches with arbitrary shapes, the 2D FEM is used in a cavity model. It is assumed that the patch is located on a thin substrate, which is common in conformal structure applications, and therefore the cavity problem can be treated as two-dimensional. The proposed technique utilizes the set of current basis functions defined on the entire surface of the patch of arbitrary shape. Therefore, only a few current basis functions, as opposed to MoM with subdomain current basis functions, are required to obtain sufficient convergence of the methods. Differently shaped patches are examined to verify the validity of the approach. The results are compared with results calculated using commercial software and measurements of manufactured prototypes.

II. Formulation of the Problem

The schematic views of the investigated structures are illustrated in Fig. 1. The structures are composed of a microstrip patch of arbitrary shape deposited on a single-layer dielectric substrate with a ground plane. The height of the substrate is h and its relative permittivity is ε_r. Considering a single-layer dielectric substrate, the structure can be divided into two regions: region 1, a dielectric layer with a relative permittivity ε_r; and region 2, outside of the structure.

Following the analysis for planar [2] and cylindrical [7] structures, the transforms of the transverse components of the electric field due to the surface currents on the patch can be expressed as:

$$\begin{bmatrix} \tilde{E}_\xi \\ \tilde{E}_\chi \end{bmatrix} = \begin{bmatrix} G_{\xi\xi} & G_{\xi\chi} \\ G_{\chi\xi} & G_{\chi\chi} \end{bmatrix} \begin{bmatrix} \tilde{J}_\xi \\ \tilde{J}_\chi \end{bmatrix} \tag{1}$$

where $\xi, \chi = \{x, y\}$ is for a planar structure and $\xi, \chi = \{\phi, z\}$ is for a cylindrical structure, $\tilde{J}_{(\cdot)}$ are the Fourier transforms of patch surface currents and $G_{(\cdot)}$ are the elements of the dyadic

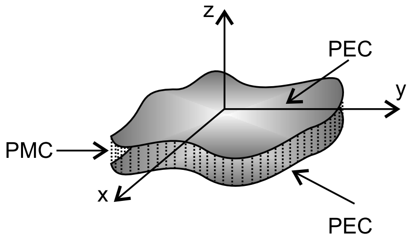

Fig. 2. The geometry of the cavity.

Green's function, which expressions for the planar structures can be found, e.g., in [2] and for cylindrical structure, e.g., in [7].

At the resonance, the field and the current can sustain themselves without the external source, therefore, assuming a perfectly conducting patch, the electric field (1) vanishes. To solve the resultant integral equations, the Galerkin's moment method is utilized. For this reason, the surface current distributions $\vec{J}$ are expanded in terms of linear combinations of basis functions as follows:

$$\vec{J}(\xi,\chi) = \sum_{n=1}^{N} \left(J_{n\xi}\vec{i}_\xi + J_{n\chi}\vec{i}_\chi \right) a_n \tag{2}$$

where index n specifies a single current mode (N - number of current modes) and a_n are the current mode coefficients. The most common choice for the basis functions is the cavity model. For simple cases, such as rectangular patches, the basis functions have an analytical form and can be found, e.g., in [2], [5]. For arbitrary patch shapes, the analytical forms are not available, therefore discrete methods such as finite-difference frequency-domain [9], [12] can be applied. It will be shown later how the finite element method can be applied.

Applying Galerkin's moment method, sets of homogeneous equations are obtained in the form:

$$(\mathbf{Z}_{\xi\xi} + \mathbf{Z}_{\xi\chi} + \mathbf{Z}_{\chi\xi} + \mathbf{Z}_{\chi\chi})\,\mathbf{a} = \mathbf{0} \tag{3}$$

where $\mathbf{Z}_{(\cdot)}$ are square matrices of size $N \times N$, whose terms are defined in [2] and [7] for planar and cylindrical structures, respectively.

Non-trivial solutions of (3) exist if the determinant of matrix $\mathbf{Z}$ equals zero. This is the eigenvalue equation, the roots of which are complex frequencies $f = Re(f) + jIm(f)$ for a particular mode [6]. This complex frequency gives the resonant frequency $Re(f)$ and the quality factor $Re(f)/2Im(f)$ for the microstrip patch. The imaginary part of the complex resonance frequencies accounts for the radiation losses [5]. Since the resonance solutions are satisfied by complex frequencies, a hybrid complex root-finding algorithm [14], which uses an adaptive mesh, was utilized.

To calculate the surface current distribution on patch of arbitrary geometry, an FEM with a second order of basis functions is utilized. The current basis functions are obtained from the cavity model approach [15]. The cavity of the patch shape is obtained by bounding the area at the top and bottom by a perfect electric conductor (PEC) and along its side by perfect magnetic conductor (PMC) as illustrated in Fig. 2.

For thin dielectric layers, which are common in conformal structure applications, it can be assumed that there is no variation of the field along the height of resonator (it can be assumed that $\partial/\partial z = 0$ and only TM^z modes are considered). Therefore, the investigated problem can be analyzed as two-dimensional. The patch surface current distributions can be found by solving the scalar Helmholtz equation:

$$\nabla_t^2 E_z(x,y) + k_0^2 \varepsilon_r E_z(x,y) = 0 \tag{4}$$

assuming Neumann boundary condition on the cavity side, with the use of 2D FEM, e.g., [16], and calculating the magnetic field distributions on the surface of the patch, which correspond to the surface current distributions:

$$J_x = H_y \sim \partial E_z/\partial y, \qquad -J_y = H_x \sim \partial E_z/\partial x \tag{5}$$

The Fourier transforms of the current distributions are then calculated from:

$$\tilde{J}_{x(y)}(k_x,k_y) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\; e^{-jk_x x - jk_y y} J_{x(y)}(x,y) \tag{6}$$

$$\tilde{J}_{\phi(z),m}(k_z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \int_{-\infty}^{\infty} dz\; e^{-jm\phi - jk_z z} J_{x(y)}(\phi,z) \tag{7}$$

if $\phi = x/r_2$ and $z = y$ in the case of cylindrical structures. For the implementation of the Fourier transform calculations, the Gaussian quadrature method was used.

III. Results

The procedure for calculating the resonant frequencies of the investigated structure can be divided into the following steps:

1) Description of the patch shape and generation of the triangular mesh [17].
2) Calculation of the resonant frequencies of the cavity by solving the scalar Helmholtz equation with the use of 2D FEM.
3) Calculation of the current distribution on the patch surface for the resonant frequencies of the cavity.
4) Calculation of the Fourier transforms of the current distribution.
5) Search for the resonant frequencies of the microstrip structure from Fig. 1 by finding the roots of equation $det(\mathbf{Z}) = 0$. A complex root-finding algorithm is necessary for this task as the resonance solutions are satisfied by complex frequencies.

There are several parameters of the analysis that need to be determined for convergence of the methods, accuracy of results and the computation time. In addition to the MoM parameters, such as the shape of integration path and the integration step (see [2] and [7]) the maximum size of the triangular mesh q_{max} and the order of Gaussian quadrature method for the FEM and current distribution calculation O must be selected; these determine the accuracy of the result and the time of the Fourier transforms calculation as well as single frequency point calculation in the last step of the analysis procedure.

The calculations for the proposed model were performed in the MATLAB environment on a Pentium i5-2450M 2.5-GHz laptop computer. Following the recommendation for the MoM

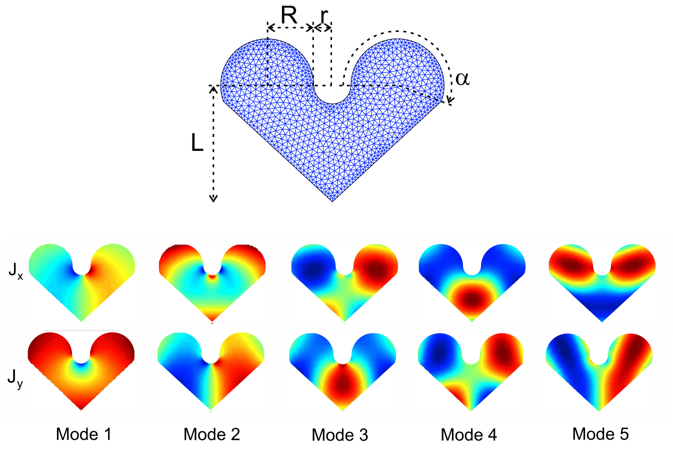

Fig. 3. The geometry of a heart-shape patch and current densities J_x and J_y for five investigated modes on its surface. Patch dimensions: $R = 10$ mm, $r = 4$ mm, $L = 25$ mm and $\alpha = 200°$.

parameters given in [2] and [7], and assuming the order of Gaussian quadrature $O = 3$, the calculation of the first step of the procedure took about 0.15 s, second step about 1.5 s, third step about 0.7 s, fourth step about 10 s and the single frequency point calculation in the fifth step about 1.5 s.

A detailed analysis of the convergence of the FEM and timing of the calculations was performed for a planar structure composed of a heart-shape patch of dimensions presented in Fig. 3. Figs. 4(a) and (b) illustrate the convergence of resonant frequency of the FEM calculation for the first and second order of basis functions, respectively. The following error criterion was assumed:

$$Error = \frac{|f - f_{\mathrm{ref}}|}{|f_{\mathrm{ref}}|} 100\% \tag{8}$$

where f_{ref} is a reference resonance frequency calculated for very dense mesh (with 19614 elements). Fig. 4(c) shows the time of calculation of the FEM analysis, the patch current distributions and the patch current Fourier transforms. The assumption of the maximum size of the triangular mesh $q_{max} = 1$ mm produces 1832 triangular elements, using the second order of FEM basis functions, the calculated resonant frequency differs less than 1 MHz from that calculated for very dense mesh. For the chosen mesh, the calculation time of the FEM analysis and patch current distributions takes less than 1 s each, and the calculation of the patch current Fourier transforms takes about 6 s. The calculation time for denser meshes increases considerably and, in these cases, the resonant frequencies do not change significantly.

To verify the validity of the proposed approach, two examples of microstrip structures (one for planar and one for cylindrical structures) have been analyzed. The first example is a circular patch with slits as presented in Fig. 5. The obtained resonance frequencies of the microstrip structure for the investigated modes from the presented approach, HFSS simulations and measurements are presented in Table I. The prototype has been manufactured on the ISOLA I-TERAMT3.45 substrate and its photo is presented in Fig. 5. A satisfactory agreement between the obtained results, the calculations of alternative method and measurements was achieved.

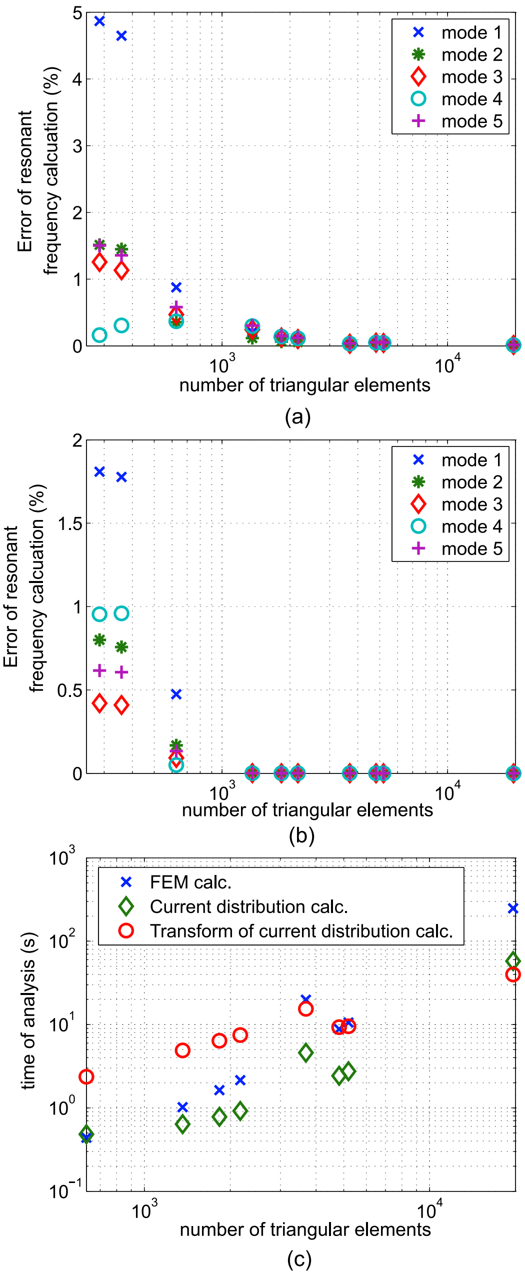

Fig. 4. Convergence of resonant frequencies for the structure from Fig. 3: (a) assuming first order of basis functions in FEM; (b) assuming second order of basis functions in FEM; (c) calculation time of different stages of the analysis.

TABLE I
RESONANCE FREQUENCIES IN GHZ FOR THE STRUCTURE FROM FIG. 5.

Mode	1	2	3	4	5
This method	0.959	1.044	1.616	2.3406	2.902
HFSS	0.957	1.047	1.589	2.333	2.904
Measurement	0.960	1.043	1.637	2.307	2.900

The second example is a rectangular patch with slits deposited on dielectric substrate covering a metallic cylindrical core. The schematic view of the structure with its dimensions and substrate parameters and the obtained resonant frequencies for TM_{10} and TM_{01} modes as a function of slit length are

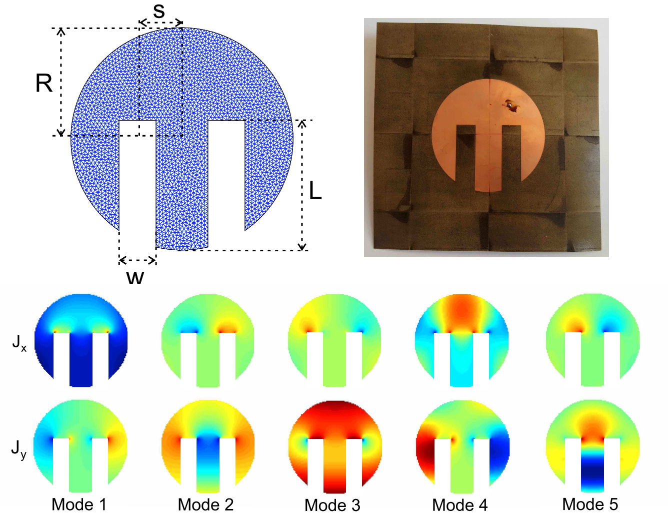

Fig. 5. The geometry of a circular patch with slits, a photo of a manufactured prototype and current densities J_x and J_y for five investigated modes on its surface. Patch dimensions: $R = 30$ mm, $w = 4$ mm, $L = 35$ mm and $s = 12$ mm. Substrate thickness $h = 0.254$ mm and permittivity $\varepsilon_r = 3.45$.

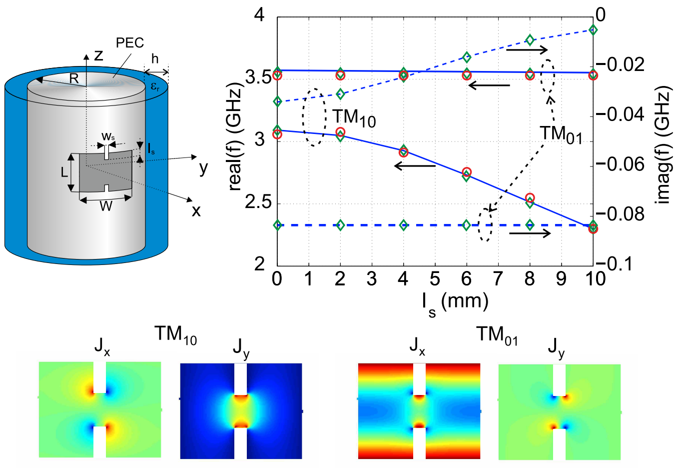

Fig. 6. Resonance frequencies of the rectangular patch with slits with dimensions $W = 30$ mm, $L = 26$ mm, $w_s = 2$ mm, placed on the dielectric substrate of permittivity $\varepsilon_r = 2.32$ and thickness $h = 2$ mm in function of slit lengths [9] and current densities J_x and J_y for TM_{01} and TM_{10} modes on its surface. Solid and dashed lines - this method, Diamonds - FDFD/MoM, Circles - HFSS.

shown in Fig. 6. The increase in the length of the slits causes a decrease in the resonant frequency of TM_{10} mode and does not affect TM_{01} mode. The situation would be reversed if the slits were introduced at vertical edges of the patch. The obtained results agree with the calculations obtained from alternative methods.

Considering the assumed limitations of the method, it does produce accurate results for the presented examples, and at least 30 times faster than currently used commercial software (the simulation of the investigated examples in HFSS takes about 400 s per single frequency point). Therefore, this method could be used with optimization procedures to design complex shapes resonant microstrip structures.

IV. Conclusion

A new procedure for calculating the resonance frequencies of planar and cylindrical microstrip structures with patches of arbitrary shapes was proposed. A hybrid technique based on the finite element method (to calculate patch surface currents) and MoM (to solve the microstrip resonance problem) was employed. Examples of microstrip patches with different geometries were analyzed, and the obtained results were verified by comparing them with calculations made by alternative methods, commercial software and measurements. A good agreement was achieved, proving the applicability of the newly proposed method.

References

[1] L. Josefsson and P. Persson, *Conformal Array Antenna Theory and Design*. Hoboken: John Wiley and Sons, Inc., 2006.

[2] J.-S. Row and K.-L. Wong, "Resonance in a superstrate-loaded rectangular microstrip structure," *IEEE Trans. Microw. Theory Techn.*, vol. 41, no. 8, pp. 1349-1355, Aug. 1993.

[3] W. C. Chew and Q. Liu, "Resonance frequency of a rectangular microstrip patch," *IEEE Trans. Antennas Propag.*, vol. 36, no. 8, pp. 1045-1056, Aug 1988.

[4] J. Wang, Q. Liu and L. Zhu, "Bandwidth Enhancement of a Differential-Fed Equilateral Triangular Patch Antenna via Loading of Shorting Posts," *IEEE Trans. Antennas Propag.*, vol. 65, no. 1, pp. 36-43, Jan. 2017.

[5] S. M. Ali, T. M. Habashy, J. F. Kiang and J. A. Kong, "Resonance in cylindrical-rectangular and wraparound microstrip structures," *IEEE Trans. Microw. Theory Techn.*, vol. 37, no. 11, pp. 1773-1783, Nov 1989.

[6] K.-L. Wong, Y-T Cheng, and J-S Row, "Resonance in a superstrate-loaded cylindrical-rectangular microstrip structure," *IEEE Trans. Microw. Theory Techn.*, vol. 41, no. 5, pp. 814819, May 1993.

[7] R. Lech, W. Marynowski, A. Kusiek and J. Mazur, "An Analysis of Probe-Fed Rectangular Patch Antennas With Multilayer and Multipatch Configurations on Cylindrical Surfaces," *IEEE Trans. Antennas and Propag.*, vol. 62, no. 6, pp. 2935-2945, June 2014.

[8] Mang He and Xiaowen Xu, "Closed-form solutions for analysis of cylindrically conformal microstrip antennas with arbitrary radii," *IEEE Trans. Antennas Propag.*, vol. 53, no. 1, pp. 518-525, Jan. 2005.

[9] A. Kusiek and R. Lech, "Resonance Frequency Calculation of Microstrip Structure Located on Cylindrical Surface Using Hybrid Technique," *2016 International Symposium on Antennas and Propagation (ISAP2016)*, Okinawa, Japan, 24-28 October 2016.

[10] R. Lech, W. Marynowski and A. Kusiek, "An Analysis of Elliptical-Rectangular Multipatch Structure on Dielectric-Coated Confocal and Nonconfocal Elliptic Cylinders," *IEEE Trans. Antennas and Propag.*, vol. 63, no. 1, pp. 97-105, Jan. 2015.

[11] G. Amendola, "Analysis of the rectangular patch antenna printed on elliptic-cylindrical substrates," *IEE Proceedings - Microw., Antennas Propag.*, vol. 147, no. 3, pp. 187-194, Jun 2000.

[12] A. Kusiek and R. Lech, "Resonance Frequency Calculation of a Multilayer and Multipatch Spherical Microstrip Structure Using a Hybrid Technique," *IEEE Trans. Antennas Propag.*, vol. 64, no. 11, pp. 4948-4953, Nov. 2016.

[13] S. K. Khamas, "Electromagnetic Radiation by Antennas of Arbitrary Shape in a Layered Spherical Media," *IEEE Trans. Antennas Propag.*, vol. 57, no. 12, pp. 3827-3834, Dec. 2009.

[14] P. Kowalczyk, "Complex Root Finding Algorithm Based on Delaunay Triangulation," *ACM Transactions on Mathematical Software*, vol. 41, no. 3, pp. 19:1-19:13, June 2015.

[15] C. Balanis, Ed., *Antenna Theory and Design*. Hoboken, NJ, USA: Wiley, 2005.

[16] C. J. Reddy, M. D. Deshpande, C. R. Cockrell and F. B. Beck, "Finite element method for eigenvalue problems in electromagnetics," NASA Langely Res. Ctr., Dec. 1994.

[17] D. Engwirda, *Mesh2d - delaunay-based unstructured mesh generation* 2009. online: https://www.mathworks.com/matlabcentral/fileexchange/25555-mesh2d-automatic-meshgeneration.