

# An Analysis of Periodic Arrangements of Cylindrical Objects of Arbitrary Convex Cross Sections with the Use of Field Matching Method

Malgorzata Warecka, Rafal Lech, Piotr Kowalczyk  
Department of Microwave and Antenna Engineering,  
Faculty of Electronics, Telecommunications and Informatics,  
Gdansk University of Technology,  
Narutowicza 11/12, 80-233 Gdansk, Poland,  
e-mail: (see <http://mwave.eti.pg.gda.pl/>)

**Abstract**—A problem of electromagnetic wave scattering from multilayered frequency selective surfaces is presented. Each surface is composed of periodically arranged cylindrical posts of arbitrary convex cross-section. The method of analysis is based on the direct field matching technique for a single cell, and the transmission matrix method with the lattice sums technique for periodic arrangement of scatterers.

## I. INTRODUCTION

The multilayered frequency selective surfaces (FSSs) are utilized as an electromagnetic band gap structure (EBG) in microwave wavelength range or photonic band gap structure (PBG) in optical range [1]. They find application in e.g., filters, polarizers, substrates for radiating elements or optical switches [2]–[5], as well as in polarizers and polarization rotators to change the polarization state of an electromagnetic wave. Many methods of analysis have been utilized to investigate such structures, e.g. the cylindrical-harmonic expansion method [6], the finite element method [7], the finite-difference method [8] and Fourier modal method [9].

In this paper we calculate the scattering parameters of periodic arrangements of posts illuminated by a plane wave. The investigated structure is composed of multilayered FSSs, which are 1-D periodic arrays of uniformly spaced identical unit cells situated in a free space. Each unit cell is described by its impedance matrix  $\mathbf{Z}$  defined on a hypothetical circular cylindrical surface describing the scatterer, and each FSS is described by its multimodal scattering matrix  $\mathbf{S}$  relating the incident space-harmonics to the scattered, both reflected and transmitted ones. To calculate the impedance matrix of each unit cell we utilize the method based on the direct field matching technique involving the usage of projection of the fields at the boundary on a fixed set of orthogonal basis functions [10]. To calculate the multimodal scattering matrix of FSS the method based on the transmission matrix (T-matrix) approach [11] and the lattice sums technique [12] is utilized. The scattering parameters of the multilayered structure are derived by using cascading formulas to connect the scattering matrices of FSSs.

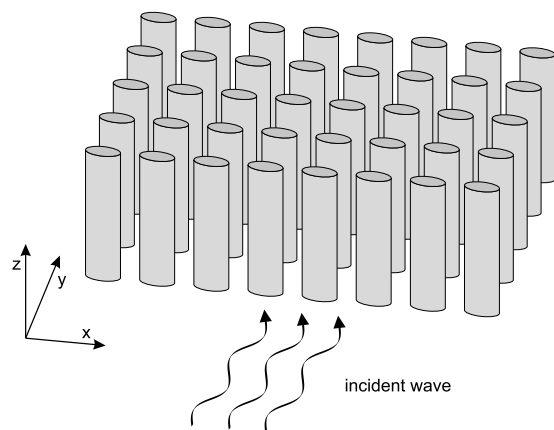


Fig. 1. Analyzed periodic configuration.

## II. FORMULATION OF THE PROBLEM

The investigated structure is presented in Fig. 1. It is composed of a two-dimensional array of uniformly spaced identical sections situated in a free space and illuminated perpendicularly by a harmonic wave. The sections are composed of scatterers spaced with distances  $h_x$  in each linear array, and  $h_y$  between FSSs.

A single FSS is composed of identical unit cells, however in multilayered structure each FSS can be different. Therefore, each FSS can be described by its own scattering matrix  $\mathbf{S}_i$  where  $i$  denotes the layer number. Assuming that the distances between layers are also arbitrary the schematic representation of multilayer structure is depicted in Fig. 4. The matrices  $\mathbf{D}_i$  represent the scattering matrices of free space between the FSSs.

In order to analyze the single FSS we utilize the procedure described in [13], [14]. This procedure allows us to calculate the multimodal scattering matrix, which relates the incident space-harmonics to the scattered, both reflected and transmitted ones. Utilizing the cascading formula, the scattering matrix of the multilayer periodic structure can be calculated.

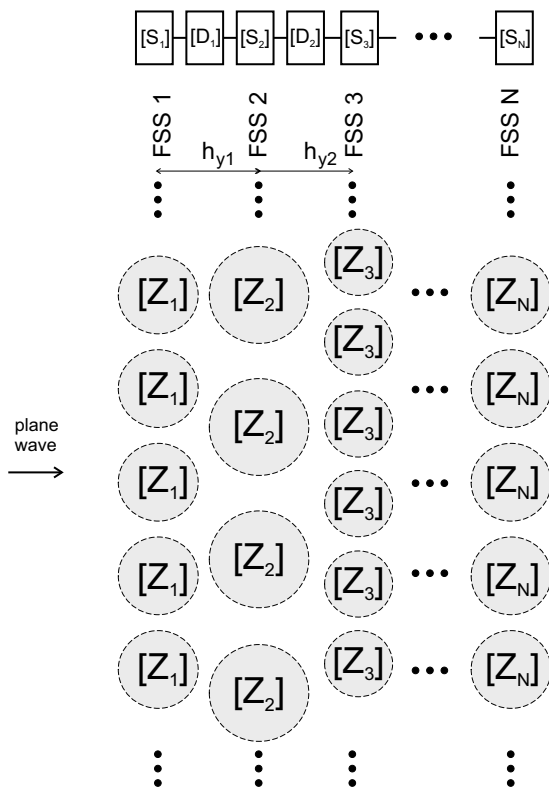


Fig. 2. S-parameter representation of multilayered structure.

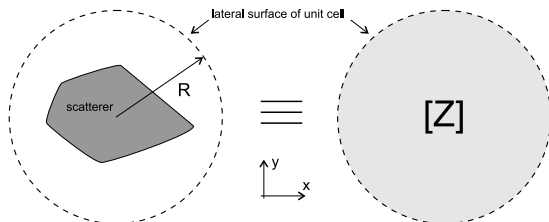


Fig. 3. The geometry of a scatterer in the unit cell and its impedance matrix representation.

Each unit cell in the periodic arrangement is composed of a single scatterer and it is described by its impedance matrix defined on the lateral cylindrical surface surrounding the object. The impedance matrix relates the total scattered electric to magnetic fields and can be obtained independently on the outer excitation. For the scatterer with simple geometry it is possible to utilize analytical solutions [15] to calculate the impedance matrix. For complex geometries the discrete methods such finite difference frequency domain technique also can be utilized [14]. However, when the post's surface is arbitrary and convex, there is no need to use discrete method and it is possible to utilize semi-analytic and simple approach based on direct field matching method to find the solution. The single investigated obstacle is schematically illustrated in Fig. 3.

Satisfying the continuity conditions for tangential components of electric and magnetic fields on the post's surface and using the projection of the fields at the boundary on a fixed set

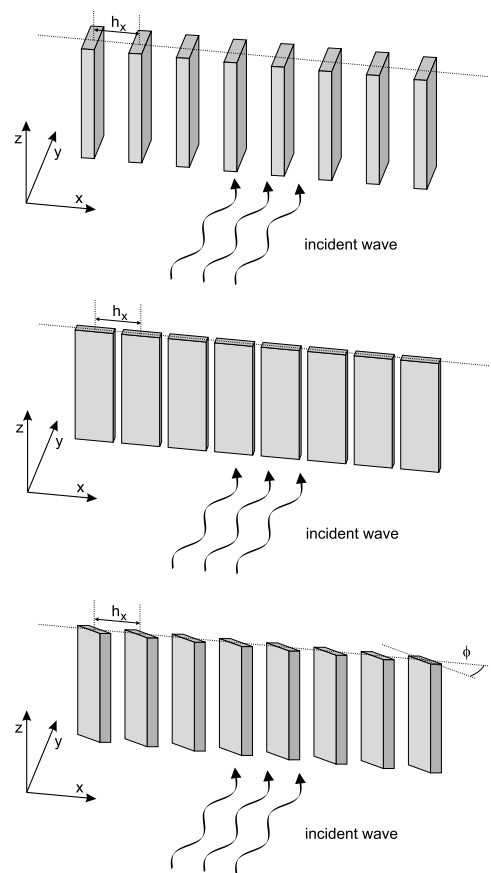


Fig. 4. Schematic representation plane wave scattering on linear array of metallic cylinders or rectangular cross section.

of orthogonal basis functions [10] it is possible to calculate the scattered field from the object. Then the relation between the electric and magnetic scattered fields calculated on the circular surface surrounding the object can be formulated as impedance matrix, which describes the object. This way, in the further analysis, the objects are treated as cylinders with circular cross-sections.

### III. RESULTS

To verify the method the example from [14] of a linear array of copper posts with rectangular cross-section with dimensions  $15 \times 3$  mm was considered first. The posts were arranged with distance  $h_x = 26.6$  mm and the calculations were performed for three different angles of post rotations (see Fig. 4) with the assumption of perpendicular plane wave excitation. The calculated and measured transmission coefficients (measured results from [14]) are shown in Fig. 5(a) for TM polarization and in Fig. 5(b) for TE polarization. As can be observed the satisfactory agreement was obtained.

The second example considers the multilayer structure composed of arrays from the previous example. When a periodic surface is multilayered it constitutes a 2D electromagnetic band gap structure. In a multilayered system, the interactions of space harmonics scattered from each surface layers modify the

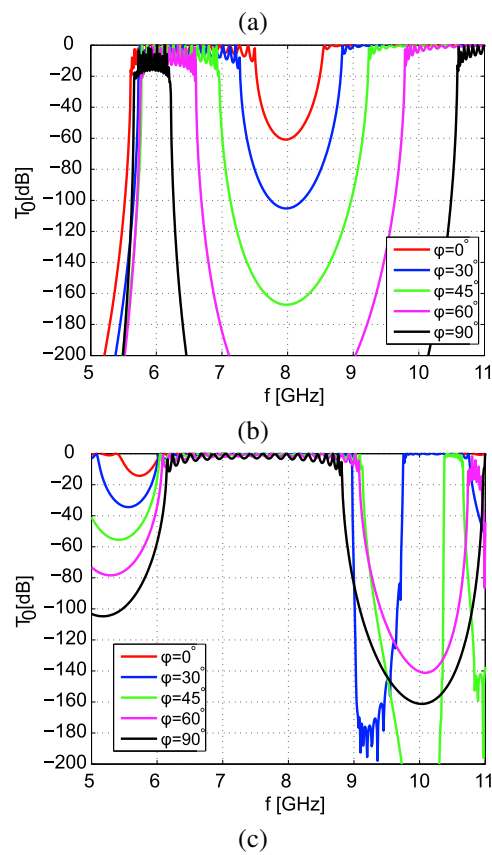
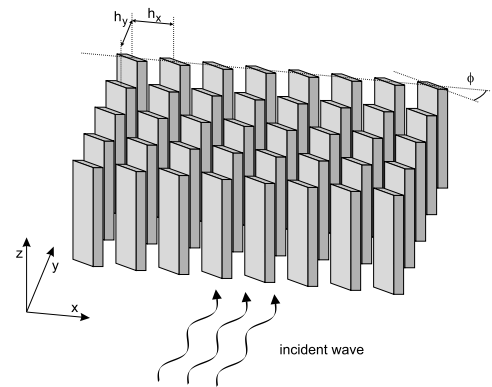
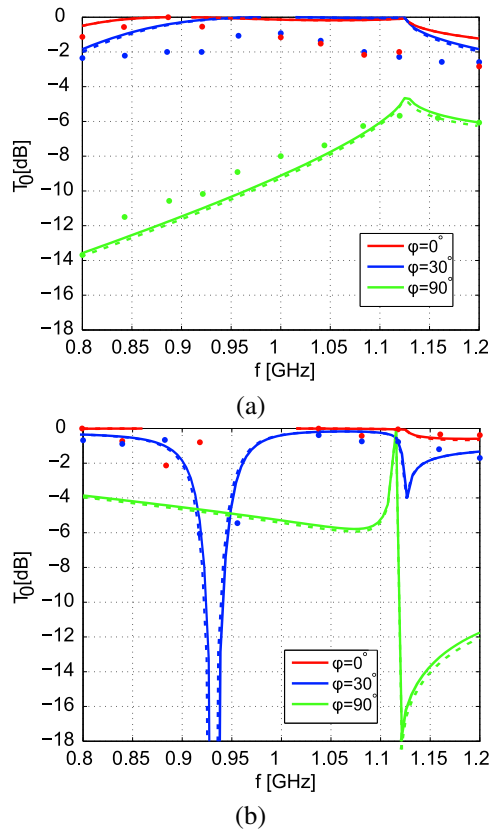


Fig. 5. Transmission coefficients for normal plane wave incidence on periodic structures composed of long metallic cylinders with rectangular cross-section with dimensions  $15 \times 3$  mm arranged with period  $h_x = 26.6$  mm for several angles of posts rotation in the array (solid line - this method, dashed line - HFSS, circles - measurements [14]). (a) Numerical and measured results for TM polarization. (b) Numerical and measured results for TE polarization.

frequency response, and bandgaps or stop bands are formed. The periodic structure composed of 20 layers, as schematically presented in Fig. 6(a), are analysed. The scattering coefficients for different posts rotation are presented in Fig. 6. As can be observed from the obtained results the rotation of the post allows to control the width of the stop and the pass bands.

#### IV. CONCLUSION

A simple field matching technique has been utilized to investigate electromagnetic wave scattering from multilayered periodic structure. The asymmetry of the scatterers, from which the structure is composed allows for tuning the width of stopbands and passbands. The results have been verified by comparison with measurement and alternative numerical methods found in literature.

#### ACKNOWLEDGMENT

This work was supported from sources of project "EDISON - Electromagnetic Design of flexible Sensors" carried out within the TEAM-TECH programme of the Foundation for Polish Science co-financed by the European Union under the European Regional Development Fund, Smart Growth Operational Programme 2014-2020.

Fig. 6. Transmission coefficients for normal plane wave incidence on multilayer periodic structures composed of 20 layers of metallic cylinders with rectangular cross-section with dimensions from Fig. 5 and period  $h_y = 26$  mm. (a) Schematic representation of multilayer structure. (b) Numerical for TM polarization. (c) Numerical results for TE polarization.

#### REFERENCES

- [1] V. Jandieri, K. Yasumoto, and Y.-K. Cho, "Rigorous analysis of electromagnetic scattering by cylindrical EBG structures," *Prog. Electromagn. Res.*, Vol. 121, 317–342, 2011.
- [2] Y. W. Kong and S. T. Chew, "EBG-based dual mode resonator filter," *IEEE Microwave and Wireless Components Letters*, Vol. 14, No. 3, 124–126, Mar. 2004.
- [3] R. Coccioli, F.-R. Yang, K.-P. Ma, and T. Itoh, "Aperture coupled patch antenna on uc-pbg substrate," *IEEE Trans. Microw. Theory Techn.*, Vol. 47, No. 11, 2123–2130, Nov. 1999.
- [4] S. Y. Lin and J. G. Fleming, "A three-dimensional optical photonic crystal," *J. Lightwave Technol.*, Vol. 17, 1944–1947, 1999.

- [5] X. Wang, M. Zhang, and S.-J. Wang, "Practicability analysis and application of PBG structures on cylindrical conformal microstrip antenna and array," *Prog. Electromagn. Res.*, Vol. 115, 495–507, 2011.
- [6] T. Kushta and K. Yasumoto, "Electromagnetic scattering from periodic arrays of two circular cylinders per unit cell," *Prog. Electromagn. Res.*, Vol. 29, 69–85, 2000.
- [7] G. Pelosi, A. Cocchi, and A. Monorchio, "A hybrid FEM-based procedure for the scattering from photonic crystals illuminated by a gaussian beam," *IEEE Trans. Antennas Propag.*, Vol. 48, 973–980, Jun. 2000.
- [8] H. Y. D. Yang, "Finite difference analysis of 2-D photonic crystals," *IEEE Trans. Microw. Theory Techn.*, Vol. 44, No. 12, 2688–2695, Dec. 1996.
- [9] F. Frezza, L. Pajewski, and G. Schettini, "Characterization and design of two-dimensional electromagnetic band-gap structures by use of a full-wave method for diffraction gratings," *IEEE Trans. Microw. Theory Techn.*, Vol. 51, No. 3, 941–951, Mar. 2003.
- [10] R. Lech, P. Kowalczyk, A. Kusiek, "Scattering from a cylindrical object of arbitrary cross section with the use of field matching method," *IEEE Trans. Antennas Propag.*, vol. 64, no. 11, pp. 4883–4887, Nov. 2016.
- [11] L. Tsang, J. A. Kong, and K.-H. Ding, "Scattering of Electromagnetic Waves: Theories and Applications", John Wiley and Sons, New York, 2000.
- [12] K. Yasumoto and K. Yoshitomi, "Efficient calculation of lattice sums for free-space periodic Greens function," *IEEE Trans. Antennas Propag.*, Vol. 47, No. 6, 1050–1055, Jun. 1999.
- [13] A. Kusiek, R. Lech, J. Mazur, "Analysis of Electromagnetic Plane Wave Scattering from 2-D Periodic Arrangements of Posts", *Prog. Electromagn. Res.*, PIER 129, 2012, pp. 69–90.
- [14] A. Kusiek, R. Lech, J. Mazur, "Hybrid Technique for the Analysis of Scattering from Periodic Structures Composed of Irregular Objects", *Prog. Electromagn. Res.*, PIER 135, pp. 657–675, 2013.
- [15] R. Lech, M. Mazur, J. Mazur, "Analysis and Design of a Polarizer Rotator System," *IEEE Trans. Antennas Propag.*, Vol. 56, No. 3, March 2008, pp. 844–847.