Uncertainty of the liquid mass flow measurement using the orifice plate

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Abstract. The article presents an estimation of measurement uncertainty of a liquid mass flow using the orifice plate. This subject is essential because of the widespread use of this type of flow meters. Not only the quantitative estimation but also the qualitative results of those measurements are important. To achieve these results the authors of the paper propose to use the theory of uncertainty. The article shows the analysis of the measurement uncertainty using two methods: one based on the 'Guide to the expression of uncertainty in measurement' (GUM) of the International Organization for Standardization by means of the law of propagation of uncertainty, and the second one by means of the Monte Carlo numerical method. The paper presents a comparative analysis of the results obtained by employing both of these methods for the centric and eccentric orifice plate. In both of the examples, the resulting uncertainty of flow measurement is approximately 1%. The uncertainty determined by the analytical method was higher than what was obtained from the Monte Carlo simulations with a difference of 0.04%.

Keywords: mass flow measurement, orifice, uncertainty, Monte Carlo.

Nomenclature

C – discharge coefficient [-]
c – sensitivity coefficient [-]
cl – confidence level [%]
D – pipe diameter [m]
d – orifice diameter [m]
k_p – coverage factor [-]
M – number of samples for Monte Carlo simulations [-]
N – number of input quantities for Monte Carlo simulations [-]
n – number of observations (measurements) [-]
p – pressure [Pa]
q – mass flow [kg/s]
s – standard deviation [kg/s]
T – temperature [K]
u – standard uncertainty
u_A – uncertainty Type A
u_B – uncertainty Type B
u_c – combined standard uncertainty
U_p – expanded uncertainty
u_{rel} – relative standard uncertainty [%]

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Δ – error
δp – difference pressure [Pa]
ε – compressibility of different fluids [-]
ν – number of degrees of freedom [-]
ρ – liquid density [kg/m^3]
$\overline{\upsilon}$ – mean velocity [m/s]

1. Introduction

Liquid flow is a critical parameter which contributes to the provision of a high-quality product in many processes including health, safety, production etc. It is often necessary to accurately measure the flow and pressure due to the security of the personnel and the process in industries such as chemicals or energy [1]. Experimental or accredited laboratories in particular should specially focus on the full and correct estimation of the measurement uncertainty of the mass flow.

There are various methods and measuring instruments which are used to measure the mass flow. The classification of flow meters is based on the principle of their operation, therefore the following types of flow meters can be distinguished: mechanical flow meters (such as: meter-piston, rotating piston meter, woftmann meter, variable area meter, turbine flow meter, nutating disk meter, single jet meter, oval gear meter, pelton wheel and multiple jet meter) and meters operating on the basis of pressure (Venturi, Pitot tube, orifice plate, Coriolis, optical, thermal, electromagnetic, ultrasonic, laser droplet and vortex flowmeter) [2-14].

Due to the nature of the analyzed issue, these methods continue to be developed. The reducing flow meter was selected for analysis because of it being the most common application thanks to its simple structure, its reliability and usability in a wide range of pressures and the temperatures of transported substances.

Flow meters must be calibrated because their accuracy changes over time [15]. In particular, this applies to instruments used for a long time after maintenance. The most common reasons for the change in the calibration of a flow meter are the presence of flushing or corrosion in the process, the slow degradation of the internal part of the instrument or the improper installation of the meter. Orifice plates are most often subject to wear. They get knocked out of position. That is why calibration of differential pressure flow meters is required.

What is more, accredited laboratories should have a procedure for estimating the uncertainty of flow measurement as they must work in accordance with standard 17025 and national and international guidelines (standards) [15]. Modern metrology [16-17] requires that in addition to the quantitative measure of the particular value, its quality should be ensured as well – preferably minimizing the uncertainty of measurement. For this reason, in each thematic area: in economics [18], physics [19], chemistry [20], environmental protection [21], fuel cell technology [22], medicine [23-24], in the measurement of power [25], mines [26] and, biomedical and electric values [27-28], an estimate of the uncertainty of the measured value is provided.

There is an ISO Standard [29] where general principles and procedures for evaluating the uncertainty of a fluid flow-rate or quantity are presented. There are also studies which address this significant issue [30-33], among others the analysis of the measurement uncertainty by numerically solving the Navier - Stokes equation (taking into account the formation of vortices behind the orifice, as shown in the article [34]). However there is still lack of elaboration of a methodology (in a step by step manner) to determine the uncertainty of the extended flow measurement.

It should be emphasized that flow measurement is an indirect measure which makes this analysis much more difficult. That is why the authors decided to take up this important topic and to develop a methodology of the procedure to determine the uncertainty of the extended liquid flow measurement [35].

This paper presents the complete methodology for calculating the uncertainty of the measurement of the mass flow of the liquid reducing flow meter using both the analytical method [16] and the Monte Carlo simulation – the numerical one [33, 36-38]. The analytical method is based on a convolution of the input distribution values, using a mathematical model. In this case, the designated measure of uncertainty is the expanded uncertainty, calculated as the product of the coverage factor k_p and the standard uncertainty value. The numerical method is based on the determining of the uncertainty of measurement based on the extension range, which is determined by the probability distribution of the measured value quantiles. The parameters of this distribution are the following: the expected value as the estimate of the output value, the standard deviation and the confidence interval for a given probability level.

The article presents two examples of the uncertainty analysis for fluid flow measurement using an orifice in order to improve the presentation of the methodology. The first example relates to the uncertainty of the liquid flow using a centric orifice. The data taken for analysis [39] are compatible with ISO standards. In this example, the authors have 40 measurement results. The second example shows the estimation of the accuracy of mass flow measurements using eccentric orifice. Only 6 results are available. The data was taken from the experiments of Kasprzak and Mrowiec [40]. The results of the analytical calculations (for both cases) were verified using the Monte Carlo simulation.

This paper is an extended and revised version of the conference publication [35]. The authors extended the publication to justify the need for a detailed description of the methodology for estimating flow uncertainty and broaden the presentation of analytical results of estimating measurement uncertainty with the case of eccentric orifice.

2. The principle of the liquid mass flow measurement using the orifice plate

The most common reducer of a stream is the orifice. The principles of mass flow measurement using an orifice and the typical installation of the centric orifice are shown in Fig. 1.

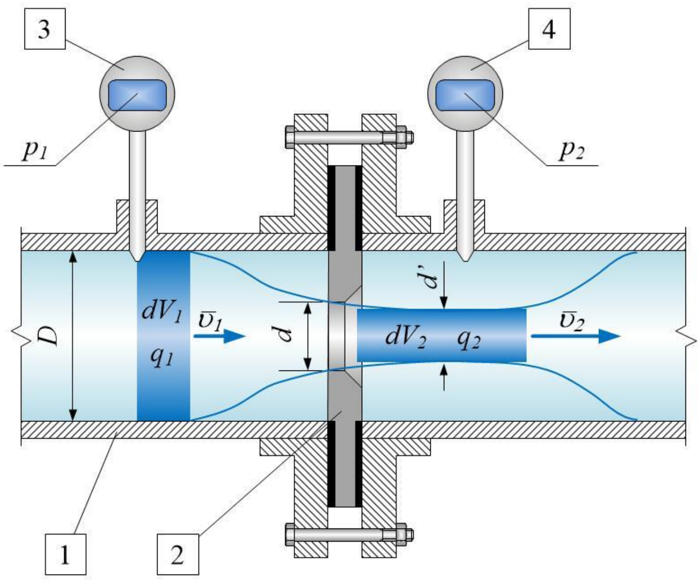

Fig. 1. The principle of mass flow measurement using a centric orifice: 1 – pipe, 2 – orifice, 3, 4 – pressure gauges.

Orifice 2 is a thin metal disk with an appropriately profiled hole of d diameter. It is mounted by means of flanges inside the D - diameter pipe 1. This setting of the orifice causes the liquid stream unit volume of dV_1 and average velocity $\bar{\upsilon}_1$. After passing through an orifice it behaves according to the law of the continuity stream of:

$$\frac{\pi}{4} D^2 \cdot \bar{\upsilon}_1 = \frac{\pi}{4} d'^2 \cdot \bar{\upsilon}_2. \tag{1}$$

This causes an increase in the flow average velocity $\bar{\upsilon}_2$ per unit volume by dV_2 at a slightly smaller diameter than the d' orifice.

It is assumed that liquids are not compressible and that the whole process takes place at a constant temperature. Hence, the density of the liquid ρ = const., which implies that $dV_1 = dV_2$ and $q_1 = q_2 = q$, where q is the mass flow rate defined as:

$$q = \frac{\pi}{4} D^2 \cdot \rho \cdot \bar{\upsilon} \tag{2}$$

Placing an orifice in the flow also means that the dynamic pressure p_1 before the orifice plate 2 measured by the pressure gauge 3 will be lower than the pressure p_2 measured by manometer 4. This system allows the measurement of mass flow rate q, which can be expressed by the following formula [17]:

$$q = \frac{C}{\sqrt{1-\beta^4}} \varepsilon \frac{\pi}{4} d^2 \sqrt{2 \cdot \delta p \cdot \rho} \tag{3}$$

where:
C – discharge coefficient; ε – compressibility of different fluids (for water and other liquids $\varepsilon = 1$); $\delta p = |p_2 - p_1|$ – difference pressure; β – relation between the orifice and the pipe diameter:

$$\beta = \frac{d}{D} \tag{4}$$

Substituting equation (4) by (3) we obtain the following:

$$q = \frac{C}{\sqrt{1-\left(\frac{d}{D}\right)^4}} \varepsilon \frac{\pi}{4} d^2 \sqrt{2 \cdot \delta p \cdot \rho} \tag{5}$$

The mass flow q is determined indirectly, and the function of the measurement is dependent on the following parameters: $q = f(C, d, D, \delta p, \rho)$.

The analysis of the uncertainty of measurement of the mass flow q presented further in this publication was carried out under the following assumptions:

- the medium is a fluid, that is a ratio $\varepsilon = 1$,
- at any point in any cross section perpendicular to the axis of the flow, a continuous stream of the fluid flow is maintained and is equal to the velocity,
- incompressible fluid without internal friction and a constant density ρ,
- a minimum of 30 measurements were made on the basis of which the n = 30 observations of mass flow q were achieved.

2.1. Analysis of the measurement uncertainty according to GUM

Complex uncertainty $u_c(q)$ determining the mass flow q is defined as follows [16]:

$$u_c(q) = \sqrt{u_A^2(q) + u_B^2(q)} \tag{6}$$

where: $u_A(q)$ – uncertainty Type A, $u_B(q)$ – uncertainty Type B.

To determine Type A uncertainty, the probability distribution of values of the observations was examined. Most frequently, the normal distribution is assumed *a priori* (especially when the number of measurements n is greater than 30).

The estimate of the mass flow was determined as [16]:

$$\bar{q} = \frac{1}{n} \sum_{i=1}^{n} q_i \tag{7}$$

The standard deviation s of an observation was determined from the dependency:

$$s = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^{n} (q_i - \bar{q})^2} \tag{8}$$

The standard uncertainty $u_A(q)$ of the measurement is evaluated as [16]:

$$u_A(q) = \frac{s}{\sqrt{n}} \tag{9}$$

Assuming no correlation between the uncertainties of measured quantities, according to the law of the uncertainty propagation [16], the one of Type B determining the mass flow q is defined as follows:

$$\begin{aligned} u_B(q) &= \sqrt{c_1{}^2 u^2(C) + c_2{}^2 u^2(d) + c_3{}^2 u^2(D) + c_4{}^2 u^2(\delta p) + c_5{}^2 u^2(\rho)} = \\ &= \sqrt{\left(\frac{\partial q}{\partial C}\right)^2 u^2(C) + \left(\frac{\partial q}{\partial d}\right)^2 u^2(d) + \left(\frac{\partial q}{\partial D}\right)^2 u^2(D) + \left(\frac{\partial q}{\partial \delta p}\right)^2 u^2(\delta p) + \left(\frac{\partial q}{\partial \rho}\right)^2 u^2(\rho)} \end{aligned} \tag{10}$$

where:
c_1 to c_5 – sensitivity coefficients; $u_B(q)$ – the estimated uncertainty of measurement of the mass flow q with the Type B method; $u(C)$ – the uncertainty in the discharge coefficient C; $u(d)$ – the uncertainty of orifice diameter measurement; $u(D)$ – the uncertainty of pipe diameter measurement; $u(\delta p)$ – the uncertainty of the differential pressure measurement; $u(\rho)$ – the uncertainty of the density of medium measurement.

In order to estimate the uncertainty Type B, the equations which describe the sensitivity coefficients which appear in equation (10) were established first. Then the values of sensitivity coefficients were summarized in Table 1 along with their units.

The weight coefficients in equation (10) (partial derivatives) were determined and were also included in Table 1.

Table 1. The weight coefficients arranged to equation (10) (partial derivatives).

Partial derivatives	**Formula**
$c_1 \left[\frac{kg}{s}\right]$	$\frac{1}{\sqrt{1-\left(\frac{d}{D}\right)^4}} \frac{\pi}{4} d^2 \sqrt{2 \cdot \delta p \cdot \rho}$ (11)
$c_2 \left[\frac{kg}{m \cdot s}\right]$	$\pi \cdot C \sqrt{2 \cdot \delta p \cdot \rho} \frac{2d^3}{D^4 \left(1-\left(\frac{d}{D}\right)^4\right)^{\frac{3}{2}}}$ (12)
$c_3 \left[\frac{kg}{m \cdot s}\right]$	$\pi \cdot C \cdot d^2 \sqrt{2 \cdot \delta p \cdot \rho} \frac{-d^4}{2 \cdot D^5 \left(1-\left(\frac{d}{D}\right)^4\right)^{\frac{3}{2}}}$ (13)
$c_4 \left[m \cdot s\right]$	$\frac{C}{\sqrt{(2 \cdot \delta p \cdot \rho) \cdot \left(1-\left(\frac{d}{D}\right)^4\right)}} \frac{\pi}{4} d^2 \cdot \rho$ (14)
$c_5 \left[\frac{m^3}{s}\right]$	$\frac{C}{\sqrt{(2 \cdot \delta p \cdot \rho) \cdot \left(1-\left(\frac{d}{D}\right)^4\right)}} \frac{\pi}{4} d^2 \cdot \delta p$ (15)

The next stage of the uncertainty analysis was to estimate the following variances:
a) $u^2(C)$ for discharge coefficient C,
b) $u^2(d)$ for orifice diameter,
c) $u^2(D)$ for pipe diameter,

d) $u^2(\delta p)$ for differential pressure,
e) $u^2(\rho)$ for the density of a medium.

The variance of the discharge coefficient $u^2(C)$ was estimated on the assumption of the normal probability distribution for estimation of the discharge coefficient C, as:

$$u^2(C)=\left(\frac{\Delta C}{2}\right)^2 \tag{16}$$

where ΔC is the estimation error of the coefficient C.

Consequently, the variances of the orifice diameter $u^2(d)$ and pipe diameter $u^2(D)$ were estimated on the assumption of the rectangular probability distribution of the orifice d and pipe D diameters tolerances, as:

$$u^2(d)=\left(\frac{\Delta d}{\sqrt{3}}\right)^2 \tag{17}$$

$$u^2(D)=\left(\frac{\Delta D}{\sqrt{3}}\right)^2 \tag{18}$$

where Δd and ΔD are tolerances of the appropriate diameters.

The differential pressure δp is usually measured by a differential pressure transducer, which is made with the error $\Delta(\delta p)$. The differential pressure variance $u^2(\delta p)$, resulting from the processing error at the pressure transducer was determined on the assumption of the rectangular probability distribution, as:

$$u^2(\delta p)=\left(\frac{\Delta(\delta p)}{\sqrt{3}}\right)^2 \tag{19}$$

The following discussion assumed that the density of the medium ρ is constant. The variance of this density $u^2(\rho)$ was determined also on the assumption of the rectangular probability distribution, as:

$$u^2(\rho)=\left(\frac{\Delta\rho}{\sqrt{3}}\right)^2 \tag{20}$$

After determining the combined standard uncertainty $u_c(q)$ according to formula (6), one should calculate the expanded uncertainty $U_p(q)$, which is a measure of the required quality of mass flow q. The expanded uncertainty $U_p(q)$ is determined on the basis of the following formula:

$$U_p(q)=k_p\cdot u_c(q) \tag{21}$$

The most commonly accepted value of the coverage factor k_p is equal to 1.96 for confidence level $cl = 95\%$.

However, it is more accurate to determine the coverage factor $k_p = t_p(\nu_{eff})$ on the adopted probability distribution, on the basis of the combined uncertainty values $u_c(q)$, and on the basis of the knowledge concerning the number of degrees of freedom ν_A for the uncertainty Type A and ν_B for the uncertainty Type B.

ν_{eff} signifies effective degrees of freedom that can be obtained from the expanded Welch-Satterthwaite formula [16]:

$$\nu_{eff}=\frac{u_c^4(q)}{\frac{u_A^4(q)}{\nu_A}+\frac{u_B^4(q)}{\nu_B}} \tag{22}$$

The number of degrees of freedom ν_A was designated as:

$$\nu_A = n - 1 \tag{23}$$

Consequently, the number of degrees of freedom ν_B can be estimated from the squared relative uncertainty $u^2_{rel,B}$ according to the following formula [16]:

$$\nu_B = \frac{1}{2 \cdot u^2_{rel,B}} \tag{24}$$

The values of ν_{eff} were read out from t-Student's table as corresponding to coverage factors k_p. Those results should be presented as the mass flow estimate q with the expanded uncertainty $U_p(q)$:

$$\left(q \pm U_p(q)\right) \ \frac{kg}{s} \tag{25}$$

and with information about the confidence level cl, the effective degrees of freedom ν_{eff}, coverage factor k_p and probability distribution.

2.2. Analysis of the measurement uncertainty using the Monte Carlo method

The Monte Carlo numerical method [33, 36-38] is used for simulations of different kinds of phenomena. It is particularly effective in the case when the analytical description of the phenomenon is complicated.

This method is a numerical tool which generally simulates an unlimited number of unique measurements by random sampling from the known probability density function of all input quantities and propagates their distributions for the measurement model as the output.

The Monte Carlo procedure is conducted as follows:

1) selection of the number M of the trials,
2) generation of M vectors by random sampling from the probability density function for the (set of N) input quantities,
3) evaluation (for each vector) of the model to give the corresponding output quantity,
4) estimation of the output of the model,
5) sorting the model values into non-decreasing order,
6) use of the sorted values to estimate the uncertainty for the output.

In metrology, it is used to verify the estimates of analytical uncertainty, especially in the cases when the researcher deals with an indirect measurement of the measured value [36, 41] and when the measuring function is non-linear. In order to better illustrate the methodology given above for estimating the uncertainty of measurement, two examples of the mass flow q measurement are presented.

3. Examples of uncertainty analysis

3.1 Centric orifice

The first example is carried out as shown in Fig. 1. The data given in Table 2 were adopted. Measurement conditions are compliant with the ISO Standards [42-43].

Table 2. Data which pertains to the first example.

Parameter	**Unit**	**Value**
C	[-]	0.605070

d	[m]	0.073648
D	[m]	0.100051
ε	[-]	1.000000
δp	[Pa]	2753.400
ρ	[kg/m^3]	1.109800
β	[-]	0.736100

Assumptions raised in paragraph 2 are still applied.
A series of 40 mass flow rate q results were obtained and they were summarized in Table 3. An estimated value of the mean flow rate q was calculated according to the relation (5), on the basis of which the value of $239.57 \cdot 10^{-3}$ kg/s was reached.

Table 3. Data of the mass flow q.

i	q_i [kg/s]	i	q_i [kg/s]	i	q_i [kg/s]	i	q_i [kg/s]
1	0.237787	11	0.238927	21	0.237290	31	0.239063
2	0.238122	12	0.238068	22	0.237893	32	0.239783
3	0.240419	13	0,240917	23	0.241714	33	0.237751
4	0.240966	14	0.238402	24	0.236430	34	0.242054
5	0.241060	15	0.240914	25	0.240820	35	0.238818
6	0.237631	16	0.238900	26	0.241837	36	0.241229
7	0.239521	17	0.240062	27	0.241761	37	0.239436
8	0.239253	18	0.243598	28	0.238990	38	0.238690
9	0.239305	19	0.240143	29	0.239754	39	0.241107
10	0.236826	20	0.238266	30	0.238643	40	0.240567

3.1.1. Results of the measurement uncertainty analysis according to GUM

The first step in order to determine the mass flow q measurement uncertainty was to choose uncertainty Type A. According to the model equation (7), the value $u_A(q)$ is equal to $0.25 \cdot 10^{-3}$ kg/s.

The next step was to estimate uncertainty Type B, on the basis of the sensitivity coefficients which appear in equation (10). Their values are summarized in Table 4.

Table 4. The values of the partial derivatives.

Partial derivative	**Value**
$\left(\frac{\partial q}{\partial C}\right)\left[\frac{kg}{s}\right]$	$3.96 \cdot 10^{-1}$
$\left(\frac{\partial q}{\partial d}\right)\left[\frac{kg}{m \cdot s}\right]$	9.22
$\left(\frac{\partial q}{\partial D}\right)\left[\frac{kg}{m \cdot s}\right]$	-1.99

$\left(\frac{\partial q}{\partial \delta p}\right)$ $[m \cdot s]$	$4.35 \cdot 10^{-5}$
$\left(\frac{\partial q}{\partial \rho}\right)$ $\left[\frac{m^3}{s}\right]$	$1.08 \cdot 10^{-1}$

The next step in the estimation of measurement uncertainty was to estimate the following variances: discharge coefficient $C - u^2(C)$, orifice diameter $u^2(d)$, pipe diameter $u^2(D)$, differential pressure $u^2(\delta p)$, and density of the medium $u^2(\rho)$.

Table 5 shows the values of the variance of the assumed probability distributions and the relative errors of the various parameters measurements.

Table 5. The values of the partial variances.

Variance	**Maximum relative error [%]**	**Distribution**	**Value**
$u^2(C)$ $[-]$	0.73	normal	$4.88 \cdot 10^{-6}$
$u^2(d)$ $[m^2]$	0.136	rectangular	$4.11 \cdot 10^{-5}$
$u^2(D)$ $[m^2]$	0.50		$8.34 \cdot 10^{-8}$
$u^2(\delta p)$ $[Pa^2]$	0.40		$3.30 \cdot 10^{-9}$
$u^2(\rho)$ $\left[\frac{kg^2}{m^6}\right]$	1.00		$4.04 \cdot 10^{-1}$

Finally, Type B uncertainty $u_B(q)$ was $1.39 \cdot 10^{-3}$ kg/s, which is more than 5 times greater than the Type A uncertainty $u_A(q)$.

Figure 2 shows the share of the particular elements of Type B mass flow uncertainty $u_B(q)$, according to formula (10).

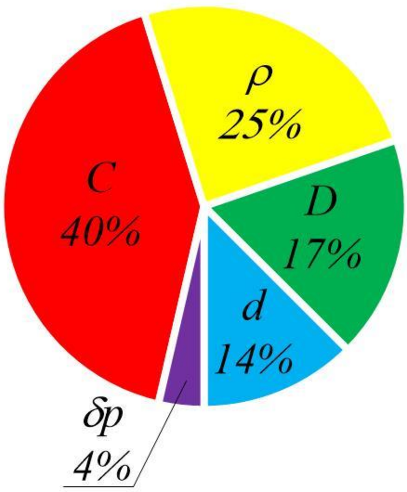

Fig. 2. The share of the particular components of the mass flow uncertainty $u_B(q)$.

To summarize the above, one of the essential uncertainty factors is the discharge coefficient C. It provides the greatest amount of information about the precision of the measurement. The contribution of the remaining parameters in the q uncertainty in descending order of importance is as follows: density of the medium ρ, pipe diameter D and orifice diameter d.

Using the formula (6), the combined standard uncertainty $u_c(q)$ was $1.41 \cdot 10^{-3}$ kg/s.

The last step in estimating the uncertainty of measurement of the mass flow q is the calculation of the expanded uncertainty.

The expanded uncertainty of the mass flow $U_p(q)$ measurement for the coverage factor k_p = 1.96 (which corresponds to approximately 95% probability of expansion) is:

$$U_p(q) = 2 \cdot u_c(q) = 2.76 \cdot 10^{-3} \ \frac{kg}{s} \tag{26}$$

On the other hand, assuming a coverage factor $k_p = t_p(\nu_{eff}) = 2.01$ and that the relative uncertainty $u_{rel,B}$ of estimated uncertainty Type B is 10%, the estimated expanded uncertainty of the mass flow $U_p(q)$ measurement (at the same probability) is $2.88 \cdot 10^{-3}$ kg/s. Table 6 shows the uncertainty values collected during the mass flow estimation on the basis of the attached data.

In conclusion, by using the analytical method in accordance with the guidelines contained in [16] for the case under consideration, the following results were obtained:

$$q = (239.6 \pm 2.8) \cdot 10^{-3} \frac{kg}{s},$$

for cl = 95%, coverage factor, respectively k_p = 1.96 and

$$q = (239.6 \pm 2.9) \cdot 10^{-3} \frac{kg}{s},$$

for $k_p = t_p(\nu_{eff})$ and t-Student distribution.

A greater value of the uncertainty of measurement of the mass flow q was achieved when the coverage factor k_p was calculated according to the formula (22) – that is, the range in which 95% of the measurement results are wider.

The relative uncertainty of the mass flow q measurement in the analyzed example did not exceed 0.5%. However, this value is highly dependent on the relative uncertainty $u_{rel,B}$.

Table 6. Uncertainty budget of mass flow q estimate.

Source of uncertainty	**Value**	**Standard uncertainty**	**Probability distribution**	**Sensitivity coefficient** $\boldsymbol{c}$	**Variance** $[kg^2/s^2]$
q	0.2396 [kg/s]	$2.54 \cdot 10^{-4}$ [kg/s]	t-Student	1.00 [-]	$6.45 \cdot 10^{-8}$
ΔC	0	$2.21 \cdot 10^{-3}$ [-]	normal	$3.96 \cdot 10^{-1}$ [kg/s]	$7.66 \cdot 10^{-7}$
Δd	0	$5.74 \cdot 10^{-5}$ [m]	rectangular	9.22 [kg/(m·s)]	$2.80 \cdot 10^{-7}$
ΔD	0	$2.89 \cdot 10^{-4}$ [m]		-1.99 [kg/(m·s)]	$3.31 \cdot 10^{-7}$
$\Delta(\delta p)$	0	6.36 [Pa]		$4.35E \cdot 10^{-5}$ [m·s]	$7.67 \cdot 10^{-8}$
$\Delta \rho$	0	$6.41 \cdot 10^{-3}$ [kg/m³]		$1.08 \cdot 10^{-1}$ [m³/s]	$4.79 \cdot 10^{-7}$
			Combined standard uncertainty $u_c(q)$ [kg/s]		$1.41 \cdot 10^{-3}$
			Expanded uncertainty $U_p(q)$ [kg/s]		$2.83 \cdot 10^{-3}$

3.1.2. Results of the measurement uncertainty analysis using the Monte Carlo method

In order to verify the results of the estimated uncertainty obtained in the previous section, the Monte Carlo simulation was performed [33]. This approach is based on the definition of the range expansion, and limits for them were defined by the quantile of the probability distribution associated with the measured values.

For this purpose, the random number generator from Microsoft Excel was used. It is assumed that the function of measuring mass flow, according to equation (5), is:

$$q = \overline{q} + c_0 u_A(q) + c_1 u(C) + c_2 u(d) + + c_3 u(D) + c_4 u(\delta p) + c_5 u(\rho) \tag{27}$$

where c_1 to c_5 marks sensitivity coefficients, and the coefficient c_0 is equal to 1. The rest of the coefficients are defined in Table 1, so their values for the considered example are reported in Table 4.

Following the probability, distributions for the input quantities were assumed. For discharge coefficient C the normal distribution was chosen, while for the other parameters, d, D, δp *and* ρ, the rectangular distributions were determined.

Estimation of uncertainty using Monte Carlo was performed in Microsoft Excel for the number of samples M equal to 10^4. The value of the expected mass flow q and its density function for a confidence level cl= 95% and $k_p = t_p(\nu_{eff}) = 2.01$ was determined and applied to the following presentation.

Figure 3 shows the probability density function of the simulated numerical values of the mass flow q. Based on these results a histogram established with channel $0.4 \cdot 10^{-3}$ kg/s was plotted in Fig. 4.

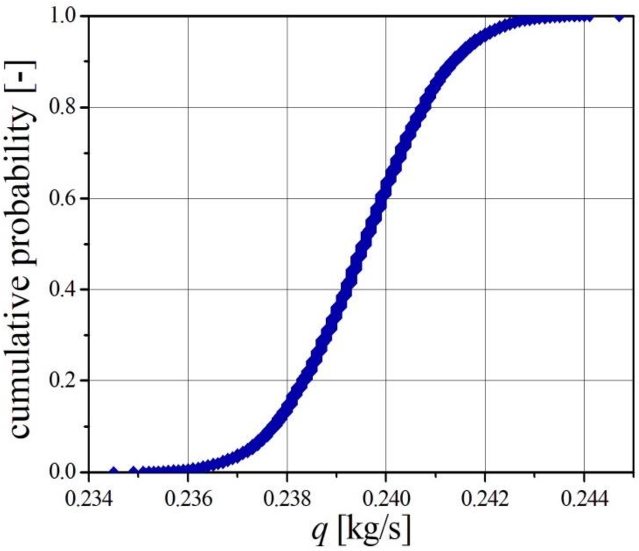

Fig. 3. Simulation of the outflow distribution of the q (first example – centric orifice).

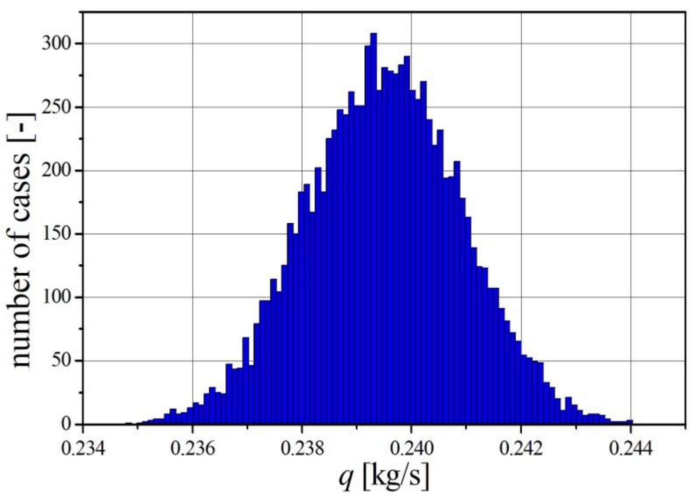

Fig. 4. A histogram of the observed values of the mass flow q (first example – centric orifice).

Table 7 summarizes the results obtained for estimation of the value of mass flow q and the expanded uncertainty $U_p(q)$ estimated with both methods of propagation of uncertainties (the traditional one, and the Monte Carlo).

Table 7. The results of mass flow estimation.

Mass flow estimation **q**			
cl [%]	k_p [-]	**Traditional method** 10^{-3}[kg/s]	**Monte Carlo** 10^{-3} [kg/s]
95	2.01	(239.6±2.9)	(239.5±2.8)

It can be observed that the results of measurement uncertainty of the mass flow by both methods are similar. The relative uncertainty is equal to 1.21% by the traditional method and 1.17% by the Monte Carlo method.

3.2 Eccentric orifice

The scheme of measurement using the eccentric orifice is shown in Fig. 5.

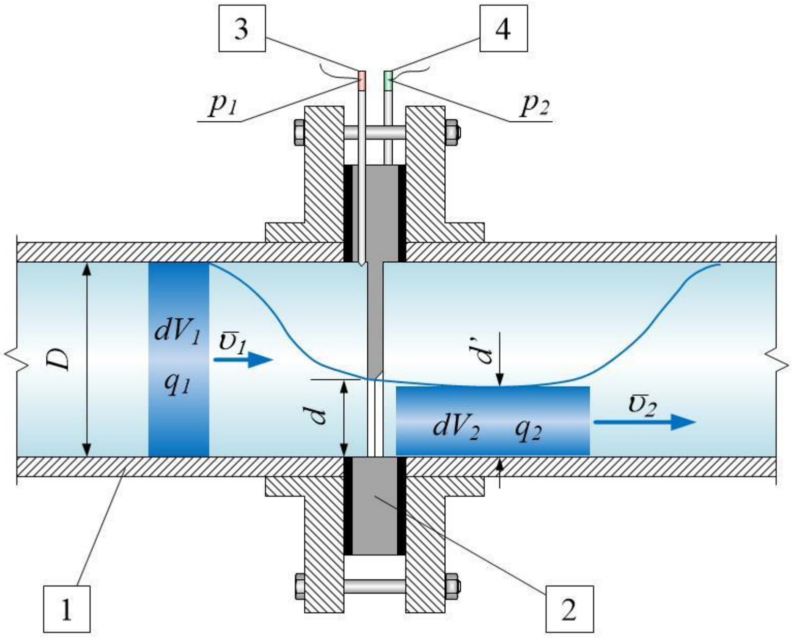

Fig. 5. Scheme of mass flow measurement, by means of the eccentric orifice: 1 – pipe, 2 – orifice, 3, 4 – pressure gauges.

All assumptions about the flowing liquid are the same as for the case discussed in Section 3.1 for the centric orifice. Experimental research was conducted with an eccentric orifice of $\beta = 0.4$ [40]. Measurements were made at temperature T = (294 ± 1) K. The data that were adopted are presented in Table 8 [40]. The analyzed data concern the case for a small Reynolds numbers range from 3000 to 10000, which does not affect in any way the presented methodology for estimating the flow uncertainty.

Table 8. Data which pertain to the second example (eccentric orifice) [40].

Parameter	Unit	Value
C	[-]	0.610100
d	[m]	0.020000

D	[m]	0.050000
ε	[-]	1.000000
δp	[Pa]	2080.900
ρ	[kg/m^3]	998.000
β	[-]	0.40000

A series of 6 mass flow rate q results were obtained and they were summarized in Table 9.

Table 9. Data of the mass flow q (second example - eccentric orifice).

i	**q_i [kg/s]**	***i***	**q_i [kg/s]**
1	0.395271	4	0.395556
2	0.395905	5	0.395407
3	0.395433	6	0.395724

An estimated value of the mean flow rate q was calculated according to relation (5), on the basis of which the value of $395.55 \cdot 10^{-3}$ kg/s was reached.

3.2.1. Results of the measurement uncertainty analysis according to GUM

The uncertainty Type A $u_A(q)$ was calculated according to the model equation (9). This value is equal to $9.45 \cdot 10^{-5}$ kg/s.

The next step was to estimate the uncertainty Type B. The sensitivity coefficients which appear in equation (8) are summarized in Table 10.

Table 10. The values of the partial derivatives (second example – eccentric orifice).

Partial derivative	**Value**
$\left(\frac{\partial q}{\partial C}\right)\left[\frac{kg}{s}\right]$	0.65
$\left(\frac{\partial q}{\partial d}\right)\left[\frac{kg}{m \cdot s}\right]$	40.61
$\left(\frac{\partial q}{\partial D}\right)\left[\frac{kg}{m \cdot s}\right]$	-0.42
$\left(\frac{\partial q}{\partial \delta p}\right)[m \cdot s]$	$9.51 \cdot 10^{-5}$
$\left(\frac{\partial q}{\partial \rho}\right)\left[\frac{m^3}{s}\right]$	$1.98 \cdot 10^{-4}$

Another stage in the estimation of measurement uncertainty was to estimate the variances. Table 11 shows the values of these parameters.

Table 11. The values of the partial variances (second example – eccentric orifice).

Variance	Maximum relative error [%]	Distribution	Value
$u^2(C)\ [-]$	0.73	normal	$2.23\cdot10^{-3}$
$u^2(d)\ [m^2]$	0.25	rectangular	$2.89\cdot10^{-9}$
$u^2(D)\ [m^2]$	0.10		$2.89\cdot10^{-9}$
$u^2(\delta p)\ [Pa^2]$	0.075		0.90
$u^2(\rho)\ \left[\frac{kg^2}{m^6}\right]$	0.50		2.88

Finally, Type B uncertainty $u_B(q)$ was $1.95\cdot10^{-3}$ kg/s, which is more than 2 orders of magnitude larger than the Type A uncertainty $u_A(q)$. The combined standard uncertainty $u_c(q)$ was $1.95\cdot10^{-3}$ kg/s, using the formula (6). The expanded uncertainty of the mass flow $U_p(q)$ measurement for the coverage factor $k_p = 2.57$ (which corresponds to approximately 95% probability of expansion for Student's t-distribution with $\nu = 5$ degrees of freedom) is:

$$U_p(q) = 2.57\cdot u_c(q) = 5.01\cdot10^{-3}\ \frac{kg}{s} \tag{28}$$

For the coverage factor $k_p = t_p(\nu_{eff}) = 2.01$ (assuming the uncertainty value of type B as in the previous example) the expanded uncertainty of the mass flow $U_p(q)$ measurement is $3.92\cdot10^{-3}$ kg/s. Table 12 shows the budget of mass flow q estimate.

Table 12. Uncertainty budget of mass flow q estimate (second example – eccentric orifice).

Source of uncertainty	Value	Standard uncertainty	Probability distribution	Sensitivity coefficient *c*	Variance [kg²/s²]
q	0.3955 [kg/s]	$9.47\cdot10^{-5}$ [kg/s]	t-Student	1.00 [-]	$89.68\cdot10^{-10}$
ΔC	0	$2.23\cdot10^{-3}$ [-]	normal	0.65 [kg/s]	$2.09\cdot10^{-6}$
Δd	0	$2.89\cdot10^{-5}$ [m]	rectangular	40.61 [kg/(m·s)]	$1.37\cdot10^{-6}$
ΔD	0	$2.89\cdot10^{-5}$ [m]		-0.42 [kg/(m·s)]	$1.44\cdot10^{-10}$
$\Delta(\delta p)$	0	0.90 [Pa]		$9.51\cdot10^{-5}$ [m·s]	$7.34\cdot10^{-9}$
$\Delta\rho$	0	2.88 [kg/m³]		$1.98\cdot10^{-4}$ [m³/s]	$3.26\cdot10^{-7}$
			Combined standard uncertainty $u_c(q)$ [kg/s]		$1.95\cdot10^{-3}$
			Expanded uncertainty $U_p(q)$ [kg/s]		$5.01\cdot10^{-3}$

In conclusion, the final result can be written as:

$$q = (395.6 \pm 5.0)\cdot10^{-3}\ \frac{kg}{s},$$

for *cl* = 95%, coverage factor, respectively $k_p = 2.57$ and

$$q = (395.6 \pm 3.9)\cdot10^{-3}\ \frac{kg}{s},$$

for $k_p = t_p(\nu_{eff})$.

3.2.2. Results of the measurement uncertainty analysis using the Monte Carlo method

In order to verify results of the estimated uncertainty that were obtained, a Monte Carlo simulation was performed [33]. This simulation was conducted with the same assumptions as in the first example and also using the Microsoft Excel application.

The value of the expected mass flow q and its density function for a confidence level $cl = 95\%$ and $k_p = t_p(\nu_{eff}) = 2.01$ was determined and applied to the following presentation.

Figure 6 shows the probability density function of the simulated numerical values of the mass flow q. On the basis of these results, a histogram established with channel $0.4 \cdot 10^{-3}$ kg/s was plotted in Fig. 7.

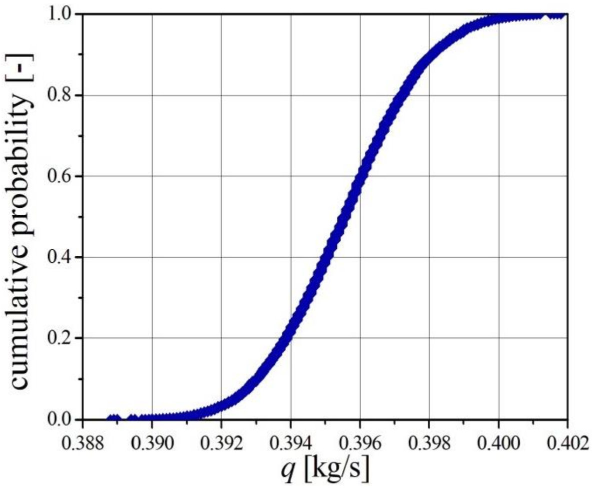

Fig. 6. Simulation of the outflow distribution of q (second example – eccentric orifice).

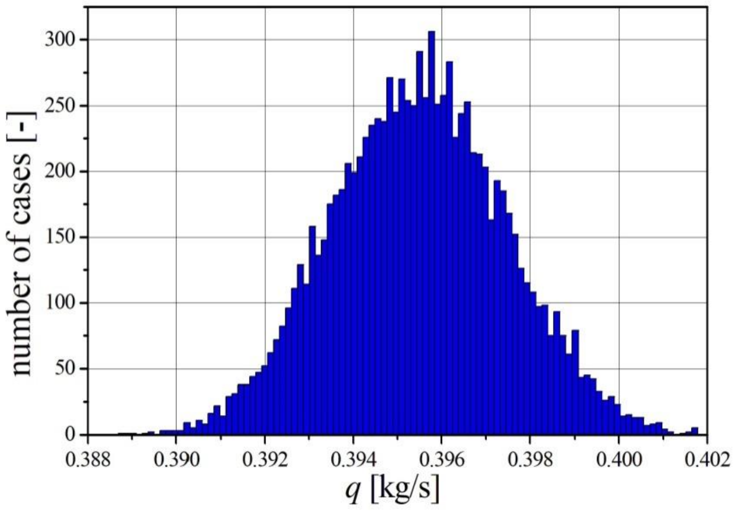

Fig. 7. A histogram for observed values of the mass flow q (second example – eccentric orifice).

Table 13 summarizes the results obtained for estimation of the value of mass flow q and the expanded uncertainty $U_p(q)$ estimated with both methods of propagation of uncertainties (the traditional one and the Monte Carlo method).

Table 13. The results of mass flow estimation (second example – eccentric orifice).

Mass flow estimation *q*			
cl [%]	k_p [-]	**Traditional method** 10^{-3}[kg/s]	**Monte Carlo** 10^{-3} [kg/s]
95	2.01	(395.6±3.9)	(395.6±3.8)

It can be observed that the results of measurement uncertainty of the mass flow by both methods are very similar. The relative uncertainty is equal to 0.99% by the traditional method, and 0.96% by the Monte Carlo method.

The relative uncertainty value presented in the second case is at the same level as the first case that was analyzed.

4. Conclusions

The article presents an estimation of measurement uncertainty of a liquid mass flow using the orifice plates. Two methods of uncertainty analysis were considered: the method based on the GUM Guide using the law of propagation of uncertainty and the Monte Carlo numerical method. The authors presented a comprehensive methodology for estimating the uncertainty of measurement of the fluid flow through the orifice plate and calculations were carried out for the sample data. Two examples were considered: the centric orifice and the eccentric orifice. In both of the examples the resulting uncertainty of flow measurement is approximately 1%. The uncertainty of flow measurement obtained by the analytical method was verified by the Monte Carlo method. The value of the uncertainty of flow determined by the analytical method was higher than that obtained from the Monte Carlo simulations, with a difference of 0.04%. The uncertainty budget demonstrated that regardless of the type of orifice, the major part of the standard uncertainty is the variance of discharge coefficient *C*.

The methodology of uncertainty estimation described in the paper can be adapted for use in the measuring stations with freely selected parameters, not only those presented in the article.

Standard estimation of measurement uncertainty is essential in scientific research. Fulfilling these conditions enables the assessment of measurements made in various institutions and it also enables the experimental verification of the announced hypotheses in any place in the world.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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