

# A Novel Microstrip Dual-Layer Rat-Race Coupler with Compact Size and Enhanced Bandwidth

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**Abstract**—Microwave hybrid couplers are crucial components of mixers, phase shifters, amplifiers and other high-frequency systems. Conventional couplers are characterized by large size which limits their usefulness in modern applications. In this work, a novel compact rat-race coupler with enhanced bandwidth has been proposed. The structure consists of six compact microstrip resonant cells. It is implemented on two separate layers which permits further size reduction as compared to single-layer topologies. The size of the optimized structure is only 15.3 mm × 19.5 mm (footprint of 297 mm<sup>2</sup>). The coupler is characterized by 300 MHz bandwidth (symmetrical w.r.t. 1 GHz operating frequency) and nearly perfect power split at the center frequency. The coupler has been favorably compared to state-of-the-art compact structures reported in the literature.

**Keywords**—EM-driven design; compact microstrip resonant cells; rat-race coupler; circuit synthesis; coupler optimization

## I. INTRODUCTION

Hybrid rat-race couplers (RRCs) are vital components of many devices including amplifiers, antenna feeding networks, mixers, phase shifters and others [1]-[3]. Conventional hybrid RRCs consist of three quarter-wavelength and one 270-degree sections characterized by 70.7 Ohm input impedance [2]. Large size makes conventional couplers of limited use in modern applications. Also, their operational bandwidth might be insufficient for use within broadband structures.

Design of compact and bandwidth enhanced couplers has gained significant attention in recent years [4]-[11]. The most straightforward but expensive technique that allows for achieving compact size is utilization of high-permittivity substrates. More popular, and significantly cheaper miniaturization techniques include modifications of conventional RRC topology through meandering [12], using space filling curves (e.g., von Koch [13], or Moore ones [14]), or non-uniform transmission lines [15]. However, usefulness of the mentioned changes for bandwidth enhancement is limited [13], [14].

Arguably, the most popular class of methods that permit size reduction of couplers while maintaining acceptable bandwidth is replacement of conventional transmission lines by a combination high- and low-impedance lines [4], [16]. Such structures are often referred to as compact microstrip resonant cells (CMRCs). Upon appropriate interconnection and suitably crafted topology, miniaturization rates of CMRC-based couplers may exceed 90% (with respect to conventional

topology [4], [17]), while ensuring acceptable performance characteristics. In [16], a CMRC-based coupler characterized by 92% size reduction and 20% bandwidth (defined at the level of  $S_{11}$  and  $S_{41}$  being both below  $-20$  dB) has been proposed. In [4], 91% size reduction and 28% bandwidth have been obtained using meandered high-impedance lines and stepped impedance stubs. In [5], 50% size reduction has been achieved along with enhanced bandwidth of 34 percent. Besides usefulness for size reduction, CMRC structures are suitable for achieving broadband harmonic suppression in compact couplers [10], [11].

Miniaturization of couplers is a problem with conflicting design objectives where size reduction is normally obtained at the expense of degraded performance characteristics [6], [7], [12]. Therefore, appropriate balance between these figures has to be sought. This class of problems cannot be effectively handled using conventional design approaches based on manual adjustment of parameters followed by visual inspection of the structure responses. Instead, numerical optimization is required to obtain high performance compact couplers topologies [12], [17]. On the other hand, tightly arranged layouts exhibit multiple cross couplings between individual sections [17]. Therefore, their accurate evaluation requires high-fidelity electromagnetic (EM) simulations which are computationally expensive. This becomes a bottleneck of EM-driven design procedures [17]. In particular, for practical matters, EM-based design tuning should be performed using robust algorithms. Furthermore, a reasonably good initial design should be provided.

The above considerations indicate that design of high-performance compact couplers involves determination of appropriate structure topology, as well as its computationally expensive fine tuning. In this work, a miniaturized microstrip rat-race coupler with enhanced operational bandwidth has been proposed. The structure consists of high- and low-impedance transmission line sections. Their shape has been adjusted to ensure compactness of the design. The CMRC sections have been designed individually at a low computational cost using a circuit simulator. Dimensions of the optimized cells have been used as a starting point for EM-based tuning of the assembled coupler. Besides of using CMRC structures, size reduction has been increased by shifting its 270-degree section to the separate layer. Moreover, the number of degrees of freedom has been increased by representing the reallocated section using separate

set of geometry parameters. The optimized coupler is characterized by 30% bandwidth, whereas its size is only 15.3 mm × 19.5 mm with the overall footprint of 297 mm<sup>2</sup>. The structure has been favorably compared to the state-of-the-art couplers reported in the literature.

## II. DESIGN OF COUPLER SECTIONS

In this section, a design procedure of compact microstrip resonant cells for miniaturized coupler is discussed. Specifically, we formulate design problem and cell performance specifications. Moreover, we explain the cell synthesis procedure and describe their final geometry. EM-based tuning of the coupler is discussed in Section III.

### A. Problem Formulation and Cells Specifications

The problem related to design of compact cells and the miniaturized coupler can be defined as the following minimization task

$$\mathbf{x}^* = \arg \min_{\mathbf{x}} U(\mathbf{R}(\mathbf{x})) \quad (1)$$

where  $\mathbf{R}(\mathbf{x})$  represents response characteristics of the structure (circuit- or EM-model based, depending on the design stage) for the given adjustable parameters  $\mathbf{x}$ ,  $U$  is the scalar objective function, and  $\mathbf{x}^*$  is the optimal design to be found.

The design specifications for CMRC structures are as follows (for the center frequency  $f_0$ ):

- Minimization of reflection;
- Maintaining electrical length of 90 degrees.

Compact cells are implemented in Keysight ADS and evaluated using its circuit simulator [18]. The substrate is a 0.762 mm thick Taconic RF-35 ( $\epsilon_r = 3.5$ ,  $\tan \delta = 0.0018$ ).

### B. Design of Compact Quarter-Wavelength Sections

Conventional coupler consists of six quarter-wavelength ( $\theta_C = 90^\circ$ ) transmission line sections with characteristic impedance of  $Z_C = 70.7$  Ohm [2]. Size reduction of the structure can be achieved by replacing conventional lines with the structures that exhibit low reflection and 90-degree phase shift, both at the center frequency  $f_0$ . Here, this is achieved using a high-impedance transmission line interconnected with a stepped-impedance stub as shown in Fig. 1.

The CMRC sections design procedure can be summarized as follows: (i) synthesize an unconventional quarter-wavelength section, (ii) use the obtained electrical parameters to determine physical dimensions of the section, and (iii) use the calculated dimensions as a starting point for constructing two complementary CMRC structures that, upon assembly, ensure compact dimensions of the coupler.

Electrical parameters of the structure can be obtained from ABCD matrix representations as follows [16]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_c = \begin{bmatrix} \cos(\theta_C) & jZ_C \sin(\theta_C) \\ j \frac{\sin(\theta_C)}{Z_C} & \cos(\theta_C) \end{bmatrix} = \begin{bmatrix} 0 & jZ_C \\ j \frac{1}{Z_C} & 0 \end{bmatrix}_{f_0} \quad (2)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_u = \begin{bmatrix} \cos(\theta_1) & jZ_1 \sin(\theta_1) \\ j \frac{\sin(\theta_1)}{Z_1} & \cos(\theta_1) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_{S1}} & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\theta_1) & jZ_1 \sin(\theta_1) \\ j \frac{\sin(\theta_1)}{Z_1} & \cos(\theta_1) \end{bmatrix}_{f_0} \quad (3)$$

where (1) and (2) are chain matrix representations of conventional transmission line section and its high- low-impedance equivalent. The input impedance  $Z_{S1}$  of stub section in (2) is given by [16]

$$Z_{S1} = Z_1 \frac{Z_{S2} + jZ_1 \tan(\theta_2)}{Z_1 + jZ_{S2} \tan(\theta_2)} \quad (4)$$

Here,  $Z_{S2} = -jZ_2/\tan(\theta_3)$ , whereas  $Z_1$ ,  $Z_2$ ,  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ , represent impedances and electrical lengths of the high- and low-impedance sections (see Fig. 1(a), respectively). It should be noted that due to a large number of unknowns, solution to (2)-(4) is not unique. On the other hand, to ensure compactness of the unconventional section, the parameter  $Z_1$  should be large, whereas value of  $\theta_3$  should be small. Based on technological limitations for microstrip line fabrication (being around 0.17 mm), the aforementioned variables can be fixed to  $Z_1 = 120$  Ohm and  $\theta_3 = 0.4^\circ$ . Then  $Z_2 = 19$  Ohm,  $\theta_1 = 30.4^\circ$ ,  $\theta_2 = 9.9^\circ$  can be calculated from (2)-(4).

The obtained electrical parameters have been used to determine dimensions of the unconventional microstrip section (cf. Fig. 1(b)). Subsequently, the obtained geometry has been used as a starting point for design of the two CMRC structures shown in Figs. 1(c)-(d). Topologies of the cells are complementary which ensures small dimensions of the coupler [19]. Parameter vector of the first cell (Fig. 1(c)) is  $\mathbf{x}_I = [w_1 \ w_2 \ w_3 \ l_1]^T$ , whereas  $l_3 = w_1 + w_2 + 3d_1$ . Adjustable parameters of the second cell (Fig. 1(d)) are  $\mathbf{x}_{II} = [w_3 \ d_2]^T$ , whereas  $l_3 = l_2$ . The variables  $w_2$  and  $l_1$  are the same as for the first structure. The dimensions of the optimized cells are  $\mathbf{x}_I^* = [4.96 \ 0.28 \ 0.23 \ 1.27]^T$  and  $\mathbf{x}_{II}^* = [1.46 \ 0.39]^T$ . Reflection and phase characteristics of the synthesized structure and the optimized CMRCs have been shown in Fig. 2. Note that the phase responses of the structures are virtually the same.

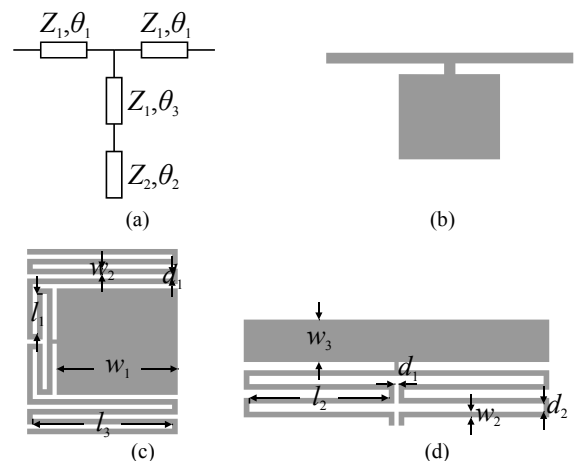


Fig. 1. Development of CMRC structures: (a) circuit model representation, (b) microstrip realization, (c) the first, and (d) the second complementary structure.



### III. COUPLER DESIGN AND EM-BASED TUNING

In this section, the design procedure of a compact multi-layer coupler structure with enhanced bandwidth is discussed. In particular, we define the circuit specifications, briefly explain the optimization algorithm, and provide description of the assembled coupler structure. Numerical results and benchmarking of the circuit are provided in Section IV.

#### A. Coupler Structure

Consider a compact multi-layer rat-race coupler shown in Fig. 3. The structure consists of six complementary compact microstrip resonant cells. The 270-degree section of the coupler has been implemented on a separate layer in order to obtain more compact dimensions compared to single-layer designs. Furthermore, the number of degrees of freedom of the coupler has been increased by using a separate set of design parameters for its 270-degree section. This allows us to achieve bandwidth-enhanced operation of the proposed structure. The vector of coupler design parameters is  $\mathbf{x} = [w_{1,1} \ w_{1,2} \ w_{1,3} \ d_{1,1} \ d_{1,2} \ l_{2,1} \ w_{2,1} \ w_{2,2} \ w_{2,3} \ d_{2,1} \ d_{2,2} \ l_{2,1}]^T$ , whereas parameters  $l_{k,2} = w_{k,1} + w_{k,2} + 3d_{k,1}$  and  $l_{k,3} = l_{k,2}$ ,  $k = 1, 2$ , are relative. Dimension  $w_0 = 1.7$  to ensure 50 Ohm input impedance (all in mm). Size of the structure is defined as  $\max(A_1, A_2) \times \max(B_1, B_2)$  where  $A_k = 2(l_{k,2} + 2w_{k,2} + d_{k,1})$  and  $B_k = w_{k,3} + 12w_{k,2} + 5d_{k,2} + 11d_{k,1} + 2l_{k,1} + 4w_{k,2} + 2w_0$ . The structure EM model is implemented in CST Microwave Studio and simulated using its finite-element method solver [20]. The lower and upper design bounds for structure optimization are  $\mathbf{l} = [2 \ 0.15 \ 1.5 \ 0.15 \ 0.15 \ 0.5 \ 2 \ 0.15 \ 1.5 \ 0.15 \ 0.15 \ 0.5]^T$  and  $\mathbf{u} = [7 \ 0.45 \ 2.5 \ 0.45 \ 0.45 \ 3 \ 7 \ 0.45 \ 2.5 \ 0.45 \ 0.45 \ 3]^T$ , respectively.

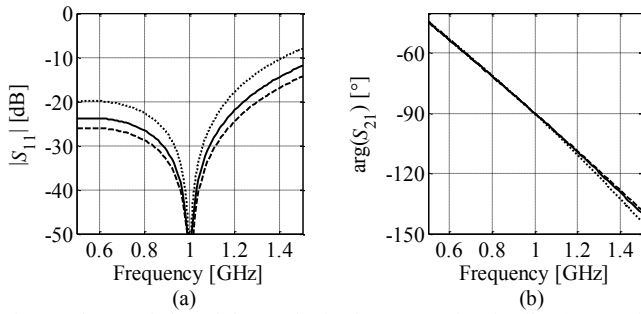


Fig. 2. Characteristics of the synthesized unconventional cell (···), as well as the first (—) and the second (---) optimized CMRC structure: (a) reflection, and (b) phase.

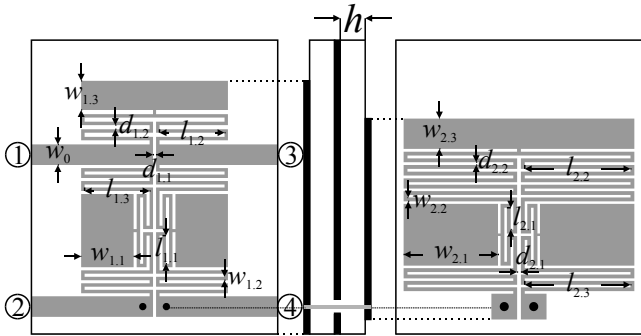


Fig. 3. Geometry of the proposed coupler structure.

#### B. Coupler Design Specifications

The following specifications are considered for the design of compact coupler ( $f_0$  is the center frequency)

- Maximization of  $f_{ds,f}$ , i.e., symmetric part of the frequency range (w.r.t.  $f_0$ ) for which  $S_{11}$  and  $S_{41}$  are both below  $-20$  dB;
- Maintaining  $S_m = \max(\min\{S_{11}(\mathbf{x}), S_{41}(\mathbf{x})\}) \leq -30$  dB (defined at  $f_0$ );
- Maintaining  $d_{s0} = |S_{21}(\mathbf{x}) - S_{31}(\mathbf{x})| \leq 0.1$  dB (at  $f_0$ ).

Coupler reflection, transmission, coupling, and isolation are denoted as  $S_{11}$ ,  $S_{21}$ ,  $S_{31}$ , and  $S_{41}$ , respectively. The objective function is defined as

$$U(\mathbf{R}(\mathbf{x})) = -2 \min\{f_0 - \min(f_{d_s, f}(\mathbf{x})), \max(f_{d_s, f}(\mathbf{x}) - f_0)\} + \beta_1 \left( \max\{(d_{s0}(\mathbf{x}) - 0.1) / 0.1, 0\} \right)^2 + \beta_2 \left( \max\{(S_m(\mathbf{x}) + 30) / 30, 0\} \right)^2 \quad (5)$$

It should be noted that small power split-error and low  $|S_{11}|$  and  $|S_{41}|$  are enforced using the penalty terms in (5). The penalty coefficients  $\beta_1$  and  $\beta_2$  are set to 10.

#### C. Optimization Algorithm

The optimization process is performed using a standard gradient-based algorithm embedded in the trust-region framework [21]. The algorithm generates a series of approximations  $\mathbf{x}^{(i)}$  to  $\mathbf{x}^*$ ,  $i = 0, 1, \dots$  as follows [22]

$$\mathbf{x}^{(i+1)} = \arg \min_{\mathbf{x} \|\mathbf{x} - \mathbf{x}^{(i)}\| \leq \delta^{(i)}} U(\mathbf{G}_s^{(i)}(\mathbf{x})) \quad (6)$$

where  $\mathbf{G}_s^{(i)}(\mathbf{x}) = \mathbf{R}(\mathbf{x}^{(i)}) + \mathbf{J}(\mathbf{x}^{(i)})(\mathbf{x} - \mathbf{x}^{(i)})$  is the linear expansion model and  $\mathbf{J}$  is the Jacobian of  $\mathbf{R}$  obtained through finite differentiation. The trust-region radius  $\delta^{(i)}$  is updated based on the gain ratio (actual versus predicted objective function improvement) using the standard trust-region rules [21]. The initial radius is  $\delta^{(0)} = 1$ . It is worth mentioning that due to utilization of the linear model  $\mathbf{G}_s^{(i)}$ , the computational cost of the algorithm is only  $N + 1$  (where  $N$  is the number of design parameters) evaluations of the EM model per iteration. Extra simulations are required for unsuccessful iterations [22]. More detailed discussion of the considered algorithm can be found e.g., in [21], [22].

### IV. RESULTS AND COMPARISONS

The initial design of the proposed structure  $\mathbf{x}^{(0)} = [4.97 \ 0.27 \ 1.46 \ 0.24 \ 0.39 \ 1.31 \ 4.97 \ 0.27 \ 1.46 \ 0.24 \ 0.39 \ 1.31]^T$  has been obtained from the dimensions of the optimized CMRC cells (cf. Section II.B). The final design  $\mathbf{x}^* = [4.34 \ 0.28 \ 2.41 \ 0.25 \ 0.34 \ 2.33 \ 6.25 \ 0.25 \ 1.99 \ 0.17 \ 0.19 \ 1.45]^T$  has been found in 11 iterations of the algorithm of Section III.C. Figure 4 shows comparison of structure characteristics at the initial and the final designs. The optimized coupler design is characterized by 30% bandwidth (symmetric w.r.t.  $f_0$ ) and 0.1 dB power split imbalance at the center frequency. At the same time, the size of the structure is  $15.3 \text{ mm} \times 19.5 \text{ mm}$  with

overall footprint of only 297 mm<sup>2</sup>. It should be noted that even though the phase shift was not considered as the design objective, the relative phase difference between coupler outputs (at the center frequency) is below 10°.

The structure has been compared to other state-of-the-art compact couplers in terms of bandwidth (symmetric w.r.t.  $f_0$ ; cf. Section II.B), power split imbalance at the center frequency, as well as size. For the sake of reliable comparison, the latter has been expressed in terms of the guided wavelength (calculated at the center frequency of the structure at hand and its substrate parameters). The results collected in Table I indicate that the proposed structure outperforms other couplers in terms of size. At the same time, it offers from 7% to 50% broader bandwidth compared to structures featuring comparable sizes. Note that coupler from [5] offers broader bandwidth compared to the proposed one, yet it is two orders of magnitude larger.

## V. CONCLUSION

In this work, a structure and design procedure of a novel miniaturized hybrid rat-race coupler with enhanced bandwidth has been discussed. Compact dimensions of 15.3 mm × 19.5 mm and a small footprint of 297 mm<sup>2</sup> have been obtained using complementary compact microstrip resonant cells implemented in multi-layer topology. The proposed coupler is characterized by 300 MHz bandwidth and power split imbalance at the 1 GHz frequency of only 0.1 dB. The structure has been favorably compared with other state-of-the-art hybrid rat-race couplers in terms of bandwidth, power-split error and size. The future work will focus on fabrication and measurements of the proposed circuit, as well as its utilization for compact amplifier design.

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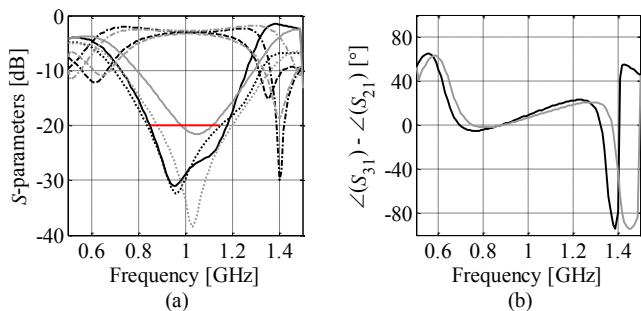


Fig. 4. Characteristics of the proposed compact coupler at the initial (gray) and optimized (black) designs: (a)  $|S_{11}|$  (—),  $|S_{21}|$  (---),  $|S_{31}|$  (····),  $|S_{41}|$  (— · — ·), and (b) relative phase difference.

TABLE I COUPLER BENCHMARKING

Design	$f_0$ [GHz]	BW [%]	$d_{s0}$ [dB]	Dimensions [mm × mm]	Dimensions [ $\lambda_g \times \lambda_g$ ]	Size [ $\lambda_g^2$ ]
[16]	1	20	0.01	22.8 × 17.0	0.13 × 0.09	0.012
[4]	0.9	28	0.65	26.3 × 14.9	0.13 × 0.07	0.010
[5]	3.5	34	0.10	25.1 × 25.1	0.41 × 0.41	0.168
[12]	1	27	0.02	58.5 × 9.20	0.32 × 0.05	0.016
This work	1	30	0.10	19.5 × 15.3	0.11 × 0.08	0.009

## REFERENCES

- [1] S. Gruszczynski, K. Wincza, and J. Borgosz, "Application of a rat-race coupler in low-cost load and source pull transistor amplifier design," *Microw. Opt. Tech. Lett.*, vol. 51, no. 11, pp. 2537–2541, 2009.
- [2] H.-X. Xu, G.-M. Wang, and K. Lu, "Microstrip rat-race couplers," *IEEE Microwave Magazine*, vol. 12, no. 4, pp. 117–129, 2011.
- [3] D. de Castro Galan, L.E. Garcia Munoz, and D. Segovia Vargas, "Diversity monopulse antenna based on a dual-frequency and dual mode crlh rat-race coupler," *Prog. Electromagnetics Res. B*, vol. 14, pp. 87–106, 2009.
- [4] C.-H. Tseng, and C.-L. Chang, "A rigorous design methodology for compact planar branch-line and rat-race couplers with asymmetrical T-structures," *IEEE Trans. Microw. Theory Tech.* vol. 60, no. 7, pp. 2085–2092, 2012.
- [5] M.A. Salari, O. Manoochehri, and S. Abbasniazare, "Miniaturized microstrip ring hybrid with defected microstrip structure," *Microwave Opt. Tech. Lett.*, vol. 55, no. 10, pp. 2245–2248, 2013.
- [6] P. Kurgan and M. Kitlinski, "Doubly miniaturized rat-race hybrid coupler," *Microwave. Opt. Tech. Lett.*, vol. 53, no. 6, pp. 1242–1244, 2011.
- [7] C.-H. Tseng and H.-J. Chen, "Compact rat-race coupler using shuntstub-based artificial transmission lines," *IEEE Microwave Wireless Comp. Lett.*, vol. 18, no. 11, pp. 734–736, 2008.
- [8] J. Wang, B.-Z. Wang, Y.-X. Guo, L.C. Ong, and S. Xiao, "Compact slow-wave microstrip rat-race ring coupler," *Electronics Lett.*, vol. 43, no. 2, pp. 111–113, 2007.
- [9] S. Opozda, P. Kurgan, and M. Kitlinski, "A compact seven-section rat-race hybrid coupler incorporating PBG cells," *Microwave Opt. Tech. Lett.*, vol. 51, no. 12, pp. 2910–2913, 2009.
- [10] W. Shao, J. He, and B.-Z. Wang, "Compact rat-race ring coupler with capacitor loading," *Microwave Opt. Tech. Lett.*, vol. 52, no. 1, pp. 7–9, 2010.
- [11] C.F. Zhang, "Planar rat-race coupler with microstrip electromagnetic bandgap element," *Microwave Opt. Tech. Lett.*, vol. 53, no. 11, pp. 2619–2622, 2011.
- [12] S. Koziel, A. Bekasiewicz, P. Kurgan, and J.W. Bandler, "Rapid multi-objective design optimization of compact microwave couplers by means of physics-based surrogates," *IET Microwaves, Ant. Prop.*, vol. 10, no. 5, pp. 479–486, 2015.
- [13] K. Annaram, S. Deepak Ram Prasath, S. Raju, V. Abhaikumar, "Reduced size koch fractal hybrid coupler for IEEE 802.11b/g wireless applications," *Int. J. Microwave Opt. Tech.*, vol. 3, no. 4, pp. 426–431, 2008.
- [14] M. Caillet, M. Clenet, A. Sharaiha and Y. M. M. Antar, "A compact wide-band rat-race hybrid using microstrip lines," *IEEE Microwave Wireless Comp. Lett.*, vol. 19, no. 4, pp. 191–193, 2009.
- [15] F. Hosseini, M.K.-A. Hosseini, and M. Yazdany, "To compact ring branch-line coupler using nonuniform transmission line," *Microw. Opt. Technol. Lett.*, vol. 51, no. 11, pp. 2679–2682, 2009.
- [16] A. Bekasiewicz and P. Kurgan, "A compact microstrip rat-race coupler constituted by nonuniform transmission lines," *Microwave Opt. Tech. Lett.*, vol. 56, no. 4, pp. 970–974, 2014.
- [17] A. Bekasiewicz, S. Koziel, and B. Pankiewicz, "Accelerated simulation-driven design optimisation of compact couplers by means of two-level space mapping," *IET Microwaves, Ant. Prop.*, vol. 9, no. 7, pp. 618–626, 2015.
- [18] Agilent (Keysight) ADS, ver. 2011.10, Agilent Technologies, 1400 Fountaingrove Parkway, Santa Rosa, CA 95403-1799, 2011.
- [19] S. Koziel and A. Bekasiewicz, "Novel structure and size-reduction-oriented design of microstrip compact rat-race coupler," *Int. Rev. Prog. Applied Comp. Electromagnetics*, pp. 1–2, Honolulu, 2016.
- [20] CST Microwave Studio, ver. 2013, Dassault Systems, 10 rue Marcel Dassault, CS 40501, Vélizy-Villacoublay Cedex, France, 2013.
- [21] A. Conn, N.I.M. Gould, P.L. Toint, *Trust-region methods*, MPS-SIAM Series on Optimization, Philadelphia, 2000.
- [22] S. Koziel and A. Bekasiewicz, "Expedited simulation-driven design optimization of UWB antennas by means of response features," *Int. J. RF Microwave CAE*, vol. 27, no. 6, pp. 1–8, 2017.