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Jakub Golik

Analysis and improvement attempt of prof. Alan Fowler's negotiation game*

Abstract. The main goal of the following article is to design an improved version of the negotiation game created by prof. Alan Fowler (1996). I have tried to achieve this by constructing four separate versions of the game which represent different approaches while preserving rules, chosen basic technical assumptions and the simplicity of the base game. Each version of the game is supposed to i.a. make it less obvious, create new negotiation possibilities (including potential cooperation), widen the course of action, make the game more enjoyable for the players and implement elementary mathematical foundation to the game's root (with regard to Game Theory).

Firstly, the article presents the description and the analysis of the base game. Afterwards, the four versions are presented with the follow-up summary and recommendations. Additionally, the article is supported by interactive Excel spreadsheet which enables the reader to have a more detailed view into the analysis and to carry out their own. Since the game has not been created as a mathematical exercise, the strictly mathematical notions are very limited and simplified to make the article reader-friendly for wider audience.

1. Introduction

Nowadays more and more pressure is put on teaching so called ”soft-skills” on various stages of education including university students. One of such soft-skills is negotiation which might be considered from the point of view of Game Theory. One of the negotiation exercises still being used by some lecturers is the negotiation game created by prof. Alan Fowler in his book “Negotiation: Skills and Strategies” (1996). Having observed such exercise in action I noticed some serious disadvantages and decided to make an attempt to improve the game. To my best

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knowledge no one has ever tried to improve it. Hence, the main goal of the following article is to design an improved version of the negotiation game created by prof. Alan Fowler. I have tried to achieve this by constructing four separate versions of the game which represent different approaches while preserving rules, chosen basic technical assumptions and the simplicity of the base game. Each version of the game is supposed to i.a. make it less obvious, create new negotiation possibilities (including potential cooperation), widen the course of action, make the game more enjoyable for the players and implement elementary mathematical foundation to the game's root (with regard to Game Theory). Hopefully this work will be used by lecturers to enrich their negotiation classes as well as other researchers who might consider further empirical testing and extensions to be made.

2. Description of the game

The following part describes the rules of the original game ”as is”, highlights the key aspects of it, discusses the purpose and context of the game and finally introduces standardised notation for further considerations.

2.1. Rules

The rules of the original game were presented as follows in configuration of two *Forms.*

Form 1

You are the employee of one of the departments in Firm A.

- Firm A
  - Department O
  - Department P
  - Department Q
  - Department R

The transactions between the departments involve the exchanges of X and Y. Some combinations of X and Y bring profit, some bring losses measured in points (see Table in Form 2).
You are required to make 6 transactions.
For more information look at Form 2.

Form 2

- There are 6 transactions
- Each transaction means giving X or Y by each department
- Each transaction result is a combination of X and Y. The result is calculated on the basis of Table 2
- Before the third and the fifth transaction the consultations between all the departments are required - one person from each department should be designated for this meeting

Table 1. Points

transaction	Your choice	Profit points	Loss points	Result
1				
2				
3				
4				
5				
6				

Table 2. Points calculation system

4 X	loss	1 point
3 X	loss	1 point
1 Y	profit	3 points
2 X	loss	2 points
2 Y	profit	2 points
1 X	loss	3 points
3 Y	profit	1 point
4 Y	profit	1 point

- The final result should be calculated in Table 1
- Profit or loss in three transactions should be multiplied: the 3^{rd} by 3, the 5^{th} by 5 and the 6^{th} by 10

2.2. Key aspects

Having presented the rules, there are some aspects of the game I would like to emphasise. For our purpose this game is usually played in teams of 3 to 5 people sitting together in the distance enabling each team to have a private discussion about the game. Furthermore, teams are not allowed to ask questions about the rules of the game and are provided only with the above-mentioned Forms. Each round, teams are given time to make a decision after which they are required to submit it on a piece of paper given to the person overseeing the game (not taking part in it). Afterwards, the overseer (in our case the lecturer) summarises the result and presents the final outcome of the round to all teams in such a way that each team knows only the final outcome but does not know the particular decisions of other teams. The process is repeated each round. The teams are not allowed to communicate with each other apart from the consultations before the third and the fifth round during which only representatives of each team are allowed to have a public conversation. For the time being the multiplication of the third, the fifth and the sixth round will not be discussed. Finally, I would like to point out the most important aspect of the game so far:

Table 3. Simplified points calculation system

4X0Y	-1
3X1Y	2
2X2Y	0
1X3Y	-2
0X4Y	1

Fact 1
Teams do not know if the given forms are the same for each team. In particular they are not given the information if Table 2 works the same for each team.

Remark 1
However, in the original version of the game each team is given exactly the same forms.

2.3. Purpose and context

The game has been created as a negotiation skills exercise aiming at exercising *internal and external negotiation skills*, *teamwork*, *leadership* and to some extent *decision making under uncertainty* (1996). Even though the game has not been created as a mathematical exercise, it implements some distinct mathematical notions which make it possible to be analysed from a little more technical perspective than psychology or sociology. The aim of this work is to slightly improve the mathematical root of the game while preserving its original character and, as a result, make the game more consistent, enjoyable and easier to analyse. In order to achieve this, in the next part I will introduce standardised notation which will simplify the game itself and make further analysis clearer.

2.4. Standardised notation

For the sake of further analysis I am going to simplify the Table 2: Points calculation system and team reference. Since each team has to make one consistent decision each round, it will be treated as a single player. Hence, from now on team representing *Department O* will be referred to as Player 1 ($P1$ in short), *Department P* as Player 2 ($P2$), *Department Q* as Player 3 ($P3$) and *Department R* as Player 4 ($P4$).
When it comes to Table 1: Points I suggest transforming *Profit points* and *Loss points* into single pay-off based on each possible combination (which will be defined later on). The following table presents the Table 2 after simplification.

This table will be extended in the next parts in a logical way without further explanation. Table 1 should be changed analogically.

3. Core analysis

With the standardised notation and the description of the game I can start the actual analysis. I will start with elementary definitions and very simple calculations to create a framework for attached Excel spreadsheet.

3.1. Assumptions and simplifications

In order to make a reliable analysis, it is crucial to understand the limitations, context and what we want to achieve. Since I aim only for a slight adjustment to the base game, many advanced mathematical notions will be omitted or very simplified. However, I will make my best to point out and explain every simplification or omission in order not to confuse advanced readers as well as not to mislead less experienced ones.

To begin with, I would like to point out that I have already made one important simplification i.e. treating the whole team as a single player. This simplification is essential to narrow the dimension of the game, which is already complex with four players (teams) as it must be considered as an *n-player game*.

Other limitations come from the way how I decide to perceive this game. For the sake of simplicity and as a result of the above simplification, I decided not to treat this game as a cooperative game in the sense of Game Theory (for further reading on cooperative games see Straffin (1993), Webb (2012), Maschler at al. (2013)). Hence, many notions associated with cooperation games like e.g. *Shapley value* as well as other complex notions, such as *risk aversion profiles* or *consistency of believes*, will not be considered in this work (for explanation of these terms see e.g. Aliprantis, Chakrabarti (2000); Maschler at al. (2013)). Any further reference to "cooperation" should be considered in the sense of social sciences.

So far I have decided to treat this game as a non-cooperative n-player game (4-player game to be precise). The question arises whether this game should be treated in an extensive or normal (strategic) form. The first impression suggests that this is an extensive form game. However, it is worth noticing that in each round players make decisions simultaneously (in the sense of game theory) and each and every round is the same up to the result multiplication (which will not be discussed at the moment). Furthermore, players do not know the decision their opponents have made even after the end of the round (they only know the overall outcome and personal pay-off). All of the above suggest that this game should be considered a normal game (one round) repeated several times (in our case $\rightarrow$ 6 times = number of rounds played).

The aspect of repeatability opens a completely new perspective for analysis. If we consider the fact that our game is repeated several times, it obviously gives our players the opportunity to "learn" and gather information (imperfect information) about previous rounds. That is why, the game can also be analysed in the view of *opponent modelling* and *opponent exploitation theory* as well as *adaptive learning* and *learning algorithms theory*. However, these theories are considered to be closer to computer science than mathematics. Moreover, they most likely have their equivalents in social sciences, where even different approaches are presented. Regrettably, these aspects will not be considered in this work either due to the high complexity of these subjects and a lack of resources.

3.2. Base analysis

Having established the form of the game i.e. normal 4-player game, every possible outcome should be considered. Knowing that each player has exactly two possible choices (X or Y) and that there are four players it is easy to calculate the number of outcomes and the number of possible combinations (of X's and Y's). These will be calculated with the use of elementary formulas from combinatorics (for further reading on introduction to probability theory see (2010)).

Definition 1 (Variation with repetition)
The number of all different k-element variations with repetition of n-element set is given by: Zwillinger (2002)

$$\overline{V}_n^k = n^k; \quad where \;\; n, k \in \mathbb{N}^+ \tag{1}$$

Definition 2 (Combination with repetition)
The number of all different k-element combinations with repetition of n-element set is given by: Zwillinger (2002)

$$\overline{C}_n^k = \binom{n+k-1}{n-1} = \binom{n+k-1}{k}; \quad where \;\; n, k \in \mathbb{N}^+ \tag{2}$$

Thanks to the formulas above, it is easy to calculate the number of all possible outcomes using equation 1 with $n = 2$ representing number of possible choices (X or Y) and $k = 4$ representing number of decisions to be made (which equals the number of players as every player has to make a decision). Hence, the number of all possible outcomes equals:

$$\overline{V}_n^k = n^k = 2^4 = 16$$

Due to the fact that the game does not distinguish the order of decisions, several of the above 16 outcomes will be treated as equal e.g. $XXXY \equiv XYXX$ as both of them correspond to $3X1Y$ in Table 3. That is why, equation 2 is used to show that the number of combinations in Table 3 is correct as:

$$\overline{C}_n^k = \binom{n+k-1}{n-1} = \binom{n+k-1}{k} = \binom{2+4-1}{4} = \binom{5}{4} = \frac{5!}{4!1!} = 5$$

The table below summarises the considerations above.

3.3. Preservation of core assumptions

With a wider view on the game, thanks to the Table 4, we can now focus on more detailed characteristics. Before that, however, I would like to point out several seemingly unimportant aspects of the pay-off set.

Definition 3 (Pay-off set)
Let P be the totally ordered set such as $P = \{-2, -1, 0, 1, 2\}$

Fact 2
The set of all possible pay-offs $P = \{-2, -1, 0, 1, 2\}$ is constructed in such a way that:

Table 4. Outcomes summary

outcome	P1	P2	P3	P4	combination
1	X	X	X	X	4X0Y
2	X	X	X	Y	3X1Y
3	X	X	Y	X	3X1Y
4	X	Y	X	X	3X1Y
5	Y	X	X	X	3X1Y
6	X	X	Y	Y	2X2Y
7	X	Y	Y	X	2X2Y
8	Y	Y	X	X	2X2Y
9	X	Y	X	Y	2X2Y
10	Y	X	Y	X	2X2Y
11	Y	X	X	Y	2X2Y
12	X	Y	Y	Y	1X3Y
13	Y	X	Y	Y	1X3Y
14	Y	Y	X	Y	1X3Y
15	Y	Y	Y	X	1X3Y
16	Y	Y	Y	Y	0X4Y

- *each and every pay-off from the set P has its opposite pay-off which belongs to the same set P*
- *hence, the sum of all elements (pay-offs) of the set P equals 0*
- *the difference between all neighbouring pay-offs is constant and equal to 1*

This fact will be treated as a prime baseline for creating new versions of the game later on.

Remark 2
For the further analysis one important assumption should be made. To simplify the analysis and separate the human factor from it, I assume constant probability $p = \frac{1}{16}$ of occurrence for each out of 16 possible outcomes.

*This assumption might be considered "vague" or "far-fetched" since it implies total randomness of choices made by the players, however, it is necessary in order to consider expected value without analysing notions mentioned earlier in sub-chapter **3.1.** on page 57.*

Let us consider the following characteristics of the base game with the assumption of constant probability mentioned in Remark 2:

- the sum of all pay-offs representing each of the 16 possible outcomes
- Expected value under constant probability assumption ($EV_{const.p=\frac{1}{16}}$)
- maximum pay-off
- minimum pay-off

- mode (modal value) of pay-offs considering all 16 possible outcomes

The following table presents the summary of these characteristics:

Table 5. Game characteristics

	combination	P1
1	4X0Y	-1
2	3X1Y	2
3	3X1Y	2
4	3X1Y	2
5	3X1Y	2
6	2X2Y	0
7	2X2Y	0
8	2X2Y	0
9	2X2Y	0
10	2X2Y	0
11	2X2Y	0
12	1X3Y	-2
13	1X3Y	-2
14	1X3Y	-2
15	1X3Y	-2
16	0X4Y	1
	sum	0
	$EV_{const.p=\frac{1}{16}}$	0
	max. pay-off	2
	min. pay-off	-2
	mode	0

From now on I will mainly refer to the lower part of the Table 5.

Table 6. Game characteristics

	P1
sum	0
$EV_{const.p=\frac{1}{16}}$	0
max. pay-off	2
min. pay-off	-2
mode	0

In terms of the characteristics mentioned above, it is easy to conclude that the game is consistent and most importantly worth playing. First and foremost, it should be pointed out that the *expected value* (EV) is non-negative, under constant probability assumption, which implies that the game might be considered worth-playing. Furthermore, in the original form of the game, the *expected value* is equal to 0 which, in our case, should be considered an advantage as it indicates that the players must find their strategy to make the game profitable for them and that they should refrain from randomly choosing option X or option Y.

When it comes to the consistency of the game, it should be noticed that the game reaches maximum and minimum pay-off. Mode value is equal to 0 which supports the argument above about finding the strategy for making profit and refraining from random choices.

By superficially analysing only these characteristics one might think that the game is well-constructed, fair and balanced. However, before reaching such conclusions let us look a bit deeper into the game by considering two perspectives of either choosing X or Y.

3.4. Choice X vs Choice Y perspective

In order to compare choice X and choice Y I shall remove respective choices from the perspective of one player in the pay-off table. Since the original game has the same pay-off table for each player, I shall only present the summary from $P1$'s perspective. The full table should be found in attached Excel spreadsheet. The following table is just an excerpt:

Table 7. Choice X vs Choice Y game characteristics

	$P1$ if X	$P1$ if Y
sum	3	-3
$EV_{const.p=\frac{1}{8}}$	0,375	-0,375
max. pay-off	2	2
min. pay-off	-2	-2
mode	2	0

Looking at the table above, now it can be easily seen that it is more profitable to choose X than Y under the assumption that other players would make their choices randomly. Having said that, it is obvious that all players should try to achieve the combination $3X1Y$ as it gives the highest payout to all of them. At this point it is a matter of getting to know if other players have the same pay-off table and deciding on which player should select Y.

Assuming everything goes well, in order to achieve the highest payout as fast as possible the players should:

1. find out that they were given exactly the same forms
2. realise that there is only one combination which gives them the same and the highest possible pay-off
3. decide on which player selects Y
4. stick to their choices until the end of the game

It is fairly easy to achieve the highest possible outcome in the game in its original form. Even achieving $3X1Y$ combination in the first round seems quite possible. It is easy to notice that the combination $3X1Y$ is an equilibrium state of the game as having achieved it no player is able to improve his pay-off any further and the potential change of strategy may only result in worse pay-off.

3.5. Advantages and disadvantages of the base game

Before presenting potential improvements of the game I shall point out its advantages and disadvantages. One of the strongest points of the game is the construction of the pay-off set which has been described in the previous parts. It is very clear, simple and leads to the neutrality of the game with regard to *expected value* and mode of pay-offs. The advantages of these characteristics have already been described. All of the above will be treated as a baseline for any further improvements as I aim to preserve as much as possible from the original version of the game.

The biggest disadvantage of the game is the fact of having only one pay-off table for each player. As a result players can easily achieve equilibrium state which is in fact at the maximum pay-off. When the players achieve the equilibrium, the game is practically over. It makes the game predictable and most of the time leads to similar results with each player earning the same number of points.

3.6. Aim of improvements

In order to improve the game, I shall create different pay-off tables which will diversify the game. Moreover, I shall aim at balancing EV from both Choice X and Choice Y perspective. Finally, I shall move the equilibrium from the maximum pay-off and make it Pareto optimal Maschler at al. (2013), Shor (2016). All of these improvements shall be made with the preservation of core assumptions mentioned in the previous parts.

4. Proposed improved versions

The following parts presents four different improved versions of the game. While every version represents different approach and idea behind improvement, each one tries to preserve as much of the original game features as possible. The following versions are sorted from the most balanced to the least balanced with each of them having their own characteristic features and advantages.

4.1. Version 1: Zero-Sum Game

This version modifies the base game in such a way to make it a standard zero-sum game. It feels like the easiest and the best modification for improvement. It preserves almost every core assumption while improving most aspects of the original version:

1. It creates two versions of pay-off table diversifying the game
2. It perfectly balances EV from Choice X and Choice Y perspective

3. It preserves every other characteristic of the original game i.e. $EV_{const. p=\frac{1}{16}}$, min. and max. pay-offs and mode
4. It can be reduced to quasi bimatrix game and solved by single mixed strategy Nash equilibrium (MSNE)

This version preserves the original pay-off table for the $P1$. The other pay-off table has been constructed by making opposite pay-offs for each combination in pay-off table for the $P1$. That is how two different pay-off tables have been created. Next, the original pay-off table was assigned to $P1$ and $P3$ while the new one was assigned to $P2$ and $P4$ which shows the table below.

Table 8. Version 1: Zero-Sum Game points calculation system

	$P1$	$P2$	$P3$	$P4$
4X0Y	-1	1	-1	1
3X1Y	2	-2	2	-2
2X2Y	0	0	0	0
1X3Y	-2	2	-2	2
0X4Y	1	-1	1	-1

Remark 3
More detailed information about this version such as EV can be found in the Excel spreadsheet.

4.1.1. Reduction to Bimatrix game

If we assume that players with the same pay-off tables i.e $P1$ & $P3$ and $P2$ & $P4$ can together decide if they want to choose one of the three strategies: XX or XY (which is equal to YX) or YY we can convert this version to zero-sum 3x3 bimatrix game as described in the tables 9 and 10 below:

Table 9. Bimatrix reduction 1

			$P2P4$	
		XX	XY	YY
	XX	$4X0Y$	$3X1Y$	$2X2Y$
$P1P3$	XY	$3X1Y$	$2X2Y$	$1X3Y$
	YY	$2X2Y$	$1X3Y$	$0X4Y$

Definition 4 (Pure Strategy Nash Equilibrium - PSNE)
Pure strategy Nash equilibrium in a bimatrix game is a pair of unequivocally selected strategies by both players (each player selects exactely one strategy with probability $p_i = 1$) such that assuming this choice none of them has any incentive to deviate from this vector of strategies (Nash equilibrium).

Table 10. Bimatrix reduction 2

			$P2P4$	
		XX	XY	YY
	XX	$(-1,1)$	$(2,-2)$	$(0,0)$
$P1P3$	XY	$(2,-2)$	$(0,0)$	$(-2,2)$
	YY	$(0,0)$	$(-2,2)$	$(1,-1)$

Table 11. Mixed strategy Nash equilibrium

MSNE	XX	XY	YY
$P1P3$	0,4	0,2	0,4
$P2P4$	0,4	0,2	0,4

Definition 5 (Mixed Strategy)
A mixed strategy (or a probability profile) for the row player is any vector $p = (p_1, p_2, ..., p_m)$ *such that* $p_i \geq 0$ *for each strategy* i *and* $\sum_{i=1}^{m} p_i = 1$. *Similarly, a mixed strategy for the column player is a vector* $q = (q_1, q_2, ..., q_n)$ *such that* $q_j \geq 0$ *for each strategy* j *and* $\sum_{j=1}^{n} q_j = 1$. *A mixed strategy* p *for the row player is said to be a pure strategy, if for some strategy* i *we have* $p_i = 1$ *and* $p_k = 0$ *for* $k \neq i$. *That is, the pure strategy* i *for the row player is the strategy according to which the row player plays his or her original strategy* i *with probability 1 and every other strategy with probability 0 (Aliprantis, Chakrabarti 2000.*

Definition 6 (Mixed Strategy Nash Equilibrium - MSNE)
Mixed strategy Nash equilibrium is any vector of player's strategies which includes his or her probability profile $p = (p_1, p_2, ..., p_m)$ *(specifies the probability with which each of the pure strategies is used) such that there exist such* $p_i \neq 0 \wedge p_i \neq 1$.

For the complete conceptualisation of pure and mixed strategies it is adviseable to see e.g. Aliprantis, Chakrabarti (2000) or Maschler at al. (2013).

The game above does not have pure strategy Nash equilibrium (PSNE) but it has one mixed strategy Nash equilibrium (MSNE). This mixed strategy equilibrium is presented in the table 11 below:

Theorem 1
Every matrix game has a Nash equilibrium in mixed strategies (Aliprantis, Chakrabarti 2000).

4.2. Version 2: 4 different pay-offs

The idea behind this version of the game was to create four different pay-off tables in such a way that each possible combination would give different pay-off for each player apart from the combination $2X2Y$ which would be treated as neutral and give 0 for all the players.

The result is similar to Version 1 as this version:

1. creates four versions of pay-off table diversifying the game

2. balances EV from Choice X and Choice Y perspective

3. preserves every other characteristic of the original game i.e. $EV_{const.p=\frac{1}{16}}$, min. and max. pay-offs and mode

Table 12. Version 2: 4 different pay-offs points calculation system

	$P1$	$P2$	$P3$	$P4$
4X0Y	-1	1	-2	2
3X1Y	2	-2	1	-1
2X2Y	0	0	0	0
1X3Y	-2	2	-1	1
0X4Y	1	-1	2	-2

Remark 4
More detailed information about this version such as EV can be found in the Excel spreadsheet.

4.3. Version 3: Polarised roles

The following version is much more polarised as it focuses on predestining each player for certain role. It is characterised by four completely different pay-off tables which represent certain roles:

- *for X*
- *for Y*
- *Balancer*
- *Polariser*
  - Role "for X" gains the most from having as much X'es in the final combination and losses the most when having only Y's in the final combination
  - Role "for Y" is the exact opposite of the role "for X" - it gains the most from Y's and looses the most from X'es
  - Balancer gains the biggest profit from neutral combination i.e. $2X2Y$ and looses the most from each extreme combination ($4X0Y$ and $0X4Y$)
  - Polariser is the opposite of the Balancer as it gains the most profit from extreme combinations ($4X0Y$ and $0X4Y$) and looses most from the neutral one ($2X2Y$)

This version results in the following:

1. It creates four different versions of pay-off table polarising the game

Table 13. Version 3: Polarised roles points calculation system

	for X	*for Y*	*Balancer*	*Polariser*
	$P1$	$P2$	$P3$	$P4$
4X0Y	2	-2	-2	2
3X1Y	1	-1	1	-1
2X2Y	0	0	2	-2
1X3Y	-1	1	1	-1
0X4Y	-2	2	-2	2

2. It gives unfair advantage to $P3$ (Balancer) and makes the game not profitable for $P4$ (Polariser)

Remark 5
More detailed information about this version such as EV can be found in the Excel spreadsheet.

4.4. Version 4: Quasi Bimatrix Game - PSNE Pareto optimal

The idea behind the last version was to create a game with single pure strategy Nash equilibrium (PSNE) which is Pareto optimal. This version is similar to Version 1 as it is also possible to reduce it to quasi Bimatrix game. However, this version alters the core assumptions to the largest extent from all proposed versions. As a result, the following version:

1. creates two versions of payoff table diversifying the game and making it possible to be reduced to Bimatrix game

2. has single pure strategy Nash equilibrium (PSNE) which is Pareto optimal

3. has 4 strictly dominated strategies and is possible to be solved by the elimination method

4. significantly alters the assumptions of the base game

Table 14. Version 4: Quasi Bimatrix Game points calculation system

	$P1$	$P2$	$P3$	$P4$
4X0Y	-2	0	-2	0
3X1Y	-2	2	-2	2
2X2Y	1	1	1	1
1X3Y	2	-1	2	-1
0X4Y	1	-2	1	-2

4.4.1. Reduction to Bimatrix game

The assumptions in this reduction are exactly the same as in the sub-chapter **4.1.1.** on page 63. The form of the Bimatrix game is presented below in the configuration of two tables:

Table 15. Bimatrix reduction 1

			$P2P4$	
		XX	XY	YY
	XX	$4X0Y$	$3X1Y$	$2X2Y$
$P1P3$	XY	$3X1Y$	$2X2Y$	$1X3Y$
	YY	$2X2Y$	$1X3Y$	$0X4Y$

Table 16. Bimatrix reduction 2

			$P2P4$	
		XX	XY	YY
	XX	$(-2,0)$	$(-2,2)$	$(1,1)$
$P1P3$	XY	$(-2,2)$	$(1,1)$	$(2,-1)$
	YY	$(1,1)$	$(2,-1)$	$(1,-2)$

FACT 3
This version of the game is not a zero-sum game as it was in Version 1. It can be easily observed by looking at payoff vector as its payouts do not sum to 0.
Example: Vector representing combination $4X0Y \rightarrow (-2,0) \Rightarrow -2+0 \neq 0$.

Version 4 of the game is solvable by using the easiest possible method i.e. elimination of strictly dominated strategies.
It has only one pure strategy Nash equilibrium (PSNE) which is: $YYXX \equiv 2X2Y \Rightarrow (1,1)$

Having identified the Nash equilibrium, it can be concluded that this equilibrium is Pareto optimal. Below can be found the definition of Pareto optimality in the sense of Game Theory.

DEFINITION 7 (PARETO OPTIMALITY)
Named after Vilfredo Pareto, Pareto optimality is a measure of efficiency. An outcome of a game is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto Optimal outcome cannot be improved upon without hurting at least one player. Often, a Nash Equilibrium is not Pareto Optimal implying that the players' pay-offs can all be increased Shor (2016).
For more definitions of Pareto optimality see also: Webb (2012), Maschler at al. (2013).

Remark 6
More detailed information about this version such as EV can be found in the Excel spreadsheet.

4.5. Overview and recommendations

All presented versions try to preserve as many as possible features and characteristics of the original game. However, some of them deviates more from the original than the others. The fourth version deviates the most, as it is not easy to create a new game with presented assumptions and at the same time preserve all characteristics of the original version. The closest to the original game are versions 1 and 2 while at the same time making some significant changes.

The most important change for each version is introducing more pay-off tables. It was achieved without major deviations in both the first and the second version. The third version might be the most fun to play as it heavily diversifies the roles between players. However, it is necessary to point out that this version also makes the game "unfair" as it gives the advantage to $P3$ and makes the game unprofitable for $P4$.

Taking all into account, I would suggest using the first version as it is the closest to the base game but at the same time introduces some interesting changes. If possible, I would also suggest trying the third version as it may bring some unexpected results and be the subject of further analysis not only from the technical but also psychological perspective.

5. Summary

I hope this analysis gives better insight into the original game and creates new opportunities to explore and research during the negotiation classes. As it was mentioned at the beginning of this article, it was mainly the technical analysis. That is why all psychological aspects of the game have been omitted. Nevertheless, the improved versions should not diminish any psychological value of the core game. That is the reason why rules concerning multiplication of the scores after certain rounds and the possibility of discussion between players were preserved in all versions of the game. Finally, I would like to point out that it is just a very simple analysis which requires further research and consideration of other factors which have not yet been analysed.

6. Appendix description

The whole article is based on and fully supported with interactive Excel spreadsheet. It contains more detailed results and is a powerful, ready to use tool for carrying out further analysis. Five separate sheets represent the base game together with four improved versions of the game. Features of the spreadsheet:

- Fully intractable
- Formatted in easy-to-understand way

- Formatted in clear and colourful way
- Protected from entry mistakes
- Consistent with the article

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Gdansk University of Technology
Faculty of Management and Economics
email: jakub.golik@pg.edu.pl