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Modelling the time-dependent behaviour of soft soils

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Abstract: Time dependence of soft soils has already been thoroughly investigated. The knowledge on creep and relaxation phenomena is generally available in the literature. However, it is still rarely applied in practice. Regarding the organic soils, geotechnical engineers mostly base their calculations on the simple assumptions. Yet, as presented within this article, the rate-dependent behaviour of soft soils is a very special and important feature. It influences both the strength and the stiffness of a soil depending on time. It is, thus, significant to account for time dependence in the geotechnical design when considering the soft soils. This can result in a more robust and economic design of geotechnical structures. Hence, the up-to-date possibilities of regarding creep in practice, which are provided by the existing theories, are reviewed herein.

In this article, we first justify the importance of creep effects in practical applications. Next, we present the fundamental theories explaining the time-dependent behaviour of organic soils. Finally, the revision of the existing constitutive models that can be used in numerical simulations involving soft soils is introduced. Both the models that are implemented in the commercial geotechnical software and some more advanced models that take into account further aspects of soft soils behaviour are revised. The assumptions, the basic equations along with the advantages and the drawbacks of the considered models are described.

Keywords: creep; soft soil; normally consolidated soils; elasto-viscoplastic model.

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1 Introduction

Owing to infrastructural development, an increasing number of engineering structures is constructed on soft soil deposits. This specific soil type is usually considered regarding its strength and compressibility characteristics. The most significant property of soft soils is that they are highly compressible. In the design, it is mostly taken into account by assuming low values of the stiffness moduli. Engineers usually apply low strength parameters of soft soils. This does not always comply with the studies carried out so far, for example, den Haan and Feddema [1]. According to these studies, the effective friction angle in soft soils can reach the values of up to 20-40º or even 80º in the case of fibrous peats. This should not be understood as a high bearing capacity because the peak strength is mobilised at large strains exceeding serviceability limits of any engineering structures. However, high values of the friction angle remarkably affect the coefficient of lateral stress K_0 and the initial stress obliquity. Amongst the characteristic features of normally consolidated (NC) or slightly overconsolidated (OC) organic soils is not only the high compressibility. Another important aspect of soft soils is the time-dependent behaviour. It is often neglected in practice. Time dependence appears in the form of the rate-dependent both stiffness and strength as well as the creep deformations under the constant stress conditions. The creep phenomenon is, therefore, of high importance when considering the long-term behaviour and the settlements of soft soils.

Creep effects were reported in the past as deformations of ancient structures and natural slope movements. First attempts at investigating this phenomenon were undertaken in the 19th century as the result of the industrial development at this time. A classic example of creep deformation is the case of the Leaning Tower of Pisa. The creep process occurred in the clay lens that were present in the sandy subsoil of the tower. This resulted in the unexpected non-uniform settlement of the structure and led to its tilt [2].

Nowadays, creep phenomena can be observed, for example, when improving the soft ground with rigid columns or piles. Before installation of the concrete columns, the working platform is formed. Next, supporting elements are executed. Because of loading, the soil structure gets locally disturbed and remoulded. The overconsolidation ratio (OCR) is reduced and normal consolidation state may be reached. The soft soil begins to creep initiating downdrag on the pile shafts. This is due to the fact that the initial creep rate in soft soils is relatively low (Fig. 1). When the actual stress state approaches the yield surface, an increase in the deformation rate is observed. The deformation rate achieves its maximum value just after crossing the initial yield surface. In the subsequent phase, the deformation rate decreases, which is attributed to the primary creep. The creep deformation rate is, therefore, very sensitive to the OCR changes. After the resumption of primary creep process, the working or load transfer platform follows the settlements of the subsoil. This leads to the mobilisation of the negative skin friction that acts on the column or the pile shafts. In the case of organic soils with a significant content of sand, the surrounding strata may also cause a considerable negative friction.

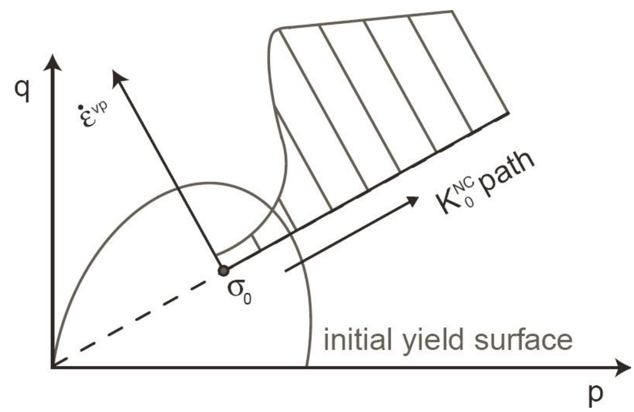

Figure 1: Influence of OCR on the creep rate in oedometric conditions.

When dealing with creeping slopes and landslides problems, the pile dowels are often applied. A dowel induces the passive resistance in the moving soil mass. In addition, the rotation of principal stress directions occurs. Considering the pure stress rotation, the soft soil undergoes further deformation [3, 4]. Most of the existing constitutive models do not allow the simulation of this process related to the stress-induced and inherent anisotropy. Besides, using the isotropic models, we obtain an overestimated value of the undrained shear strength c_u (Fig. 2). Both the real and the simulated undrained stress paths at passive side are illustrated. The real behaviour of a dowel in a creeping slope can only be represented by the anisotropic material models in which the rotational hardening law is introduced. The yield surface rotations are controlled by the back stress tensor that is associated with kinematic hardening. It controls the translation of the yield surface in the stress space. This allows a proper estimation of the passive undrained shear strength.

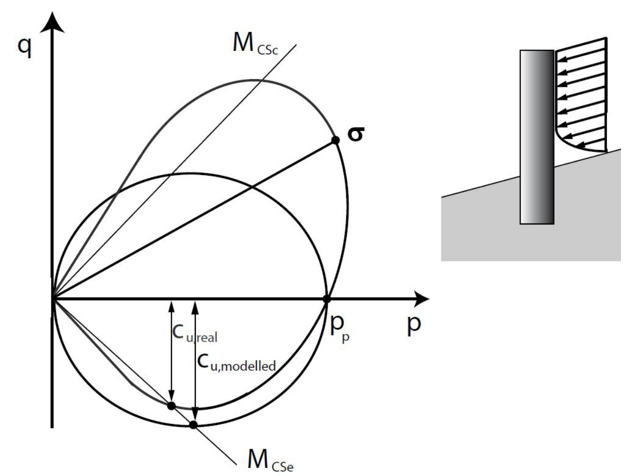

Figure 2: Undrained shear strength in triaxial extension resulting from the application of isotropic model and the real shear strength.

Rate dependence in soft soils has also been reported in the case of pile load tests. The results of a displacement-controlled load test on floating micropiles [5] are shown in Figure 3. It can be seen that the higher strain rate we apply on the pile, the higher soil stiffness we obtain. Rate-dependent effects have an important influence on the load-displacement behaviour of a pile during a load test.

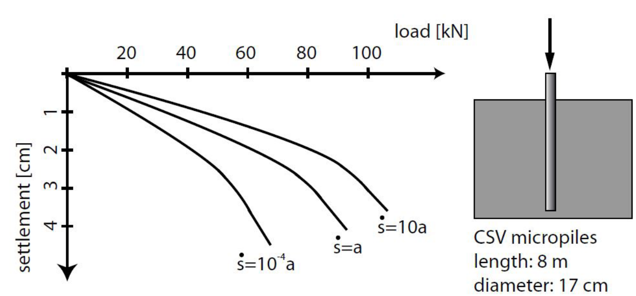

Figure 3: Results of displacement-controlled load tests on floating CSV (Combined Soil Stabilization with Vertical Columns) micropiles after Vermeer and Leoni [5]. $\dot{s}$ is the penetration rate and a is its reference value.

It is worth mentioning that the time-dependent processes also take place in granular soils. An example of such case was reported by Briaud and Gibbens [8]. In a load-controlled test on the footing on dense sand, the load increments were held for 24-h periods. The creep deformations could be observed (Fig. 4). Creep in sands, however, is related to different micro-mechanisms and motions, concerning the interfaces between the soil grains.

Considering the practical importance, the robust modelling of the time-dependent behaviour is crucial in the proper design of geotechnical structures on soft ground.

2 Creep theories

Creep phenomenon has already been investigated by many researchers (e.g. [7]). Numerous models allowing its simulations have been proposed. However, this knowledge is still rarely being involved in practice. Unlike granular soils, NC or slightly OC fine-grained soils usually perform significant creep deformations.

A special feature of soft soils is their complex microstructure. It is mainly constituted by fine clayey particles of relatively small size. This specific soil microstructure results in complicated physical, mechanical and physico-mechanical phenomena. These phenomena occur inside the material and are usually observed as creep. Creep behaviour is dependent on the stress-strain history of a soil. Determination of this relation is, therefore, important for the proper estimation of the creep effects in a particular situation. Given the stress state, we distinguish two basic types of creep: volumetric and deviatoric. Volumetric creep is usually observed in oedometer tests under the constant stress conditions. As oedometer tests provide solely control of only one stress component, we cannot observe the deviatoric creep. The true representative test to investigate the volumetric creep is the triaxial isotropic compression. Deviatoric creep can be identified under the constant deviatoric stress in both the drained and the undrained triaxial creep tests. It is more visible in the case of the undrained conditions because no volumetric strain occurs and only the shear strain is recorded. This classification was only introduced with respect to the laboratory tests. Creep behaviour is a complicated phenomenon that should be considered in a comprehensive way. We can as well classify the creep process into three stages depending on the shape of strain versus time curve, that is, the primary, the secondary and the tertiary stage (Fig. 5).

The primary stage is characterised by decreasing creep rate. During the secondary stage, the strain rate remains approximately constant. Tertiary creep is identified by increasing strain rate and leads to the creep rupture. Basically, in the case of volumetric creep, the strain rate tends to stabilise. This means that only the primary stage

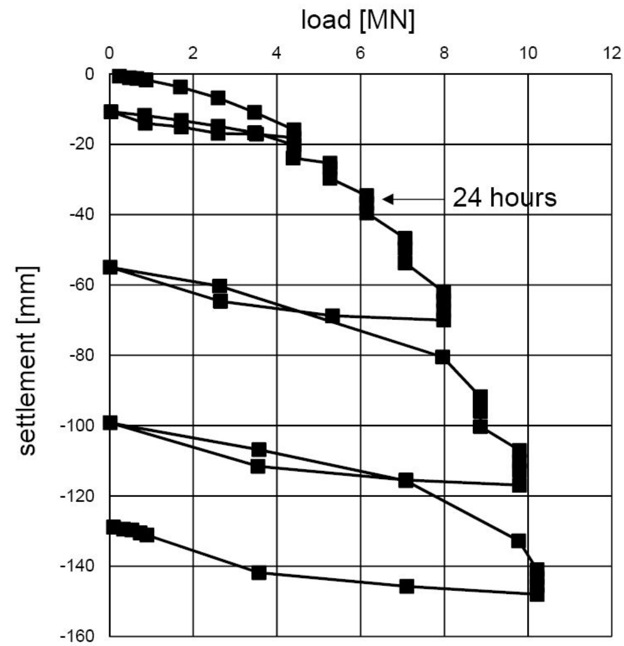

Figure 4: Results of the shallow foundation load test on dense sand after Briaud and Gibbens [8].

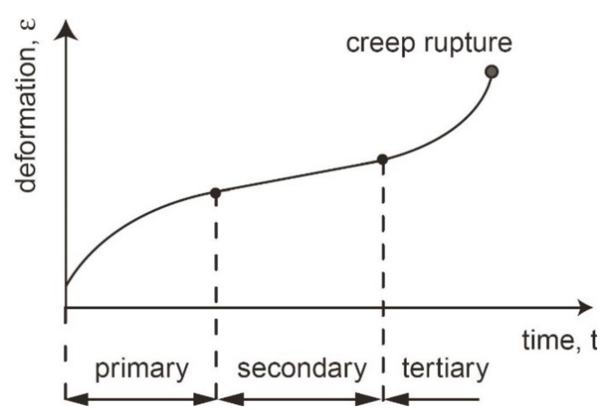

Figure 5: Creep stages based on the strain versus time curve.

is involved. Deviatoric creep may, however, perform all the three stages in terms of the shear strength mobilisation [2].

To present the history and the classification of creep theories in brief, it is convenient to introduce the definition of hypotheses A and B after Ladd et al. [9] at first. Deformations of soft soils are usually considered with respect to the two stages of consolidation process: primary and secondary. During the primary consolidation, the dissipation of excess pore water pressure takes place. The strain rate is then determined mainly by the soil permeability. On the other hand, the secondary consolidation relates to the reorganisation and better packing of soil grains because of the volumetric creep. The strain rate depends on the viscous properties of a soil.

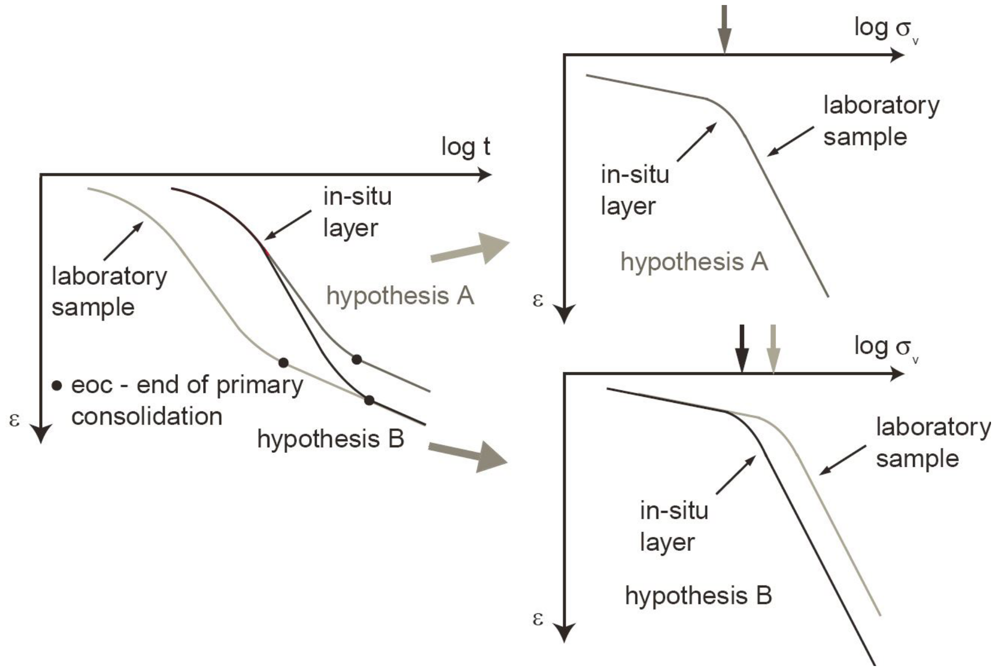

Figure 6: Creep during primary consolidation according to hypotheses A and B after Degago [10].

Deformations produced during the primary consolidation are the result of both the increase in the effective stress and the creep process. The secondary compression involves the creep deformations. Since the introduction of hypotheses A and B, the relation between the creep and the primary consolidation has been a controversial issue. The creep phenomenon can be considered as an independent process concomitant with the excess pore water dissipation. This is represented by the elasto-viscoplastic constitutive models (hypothesis B). On the other hand, creep can be interpreted as a process that begins at the end of the primary consolidation (hypothesis A). In the first case, the final strain at the end of primary consolidation (corresponding to the effective stress increase) is determined by the duration of this stage. Hence, it depends on the thickness of the consolidating soil layer. The hypothesis A assumes that relation between the effective stress and the strain is independent of the duration of the primary consolidation. The comparison between the two hypotheses based on two different soil thicknesses (a laboratory sample and an *in situ* layer) is shown in Figure 6.

The curve A represents a result of the simple assumptions according to the hypothesis A. The curve B corresponds to the use of a viscous model. It results in the higher strain value at the end of the excess pore water dissipation. Many laboratory and *in situ* investigations confirm the validity of hypothesis B. This hypothesis postulates the presence of creep during the primary consolidation and describes the overall time dependence of soft soils. To solve initial boundary value problems (BVP) using Finite Element (FE) software, solely hypothesis B is used. However, the hypothesis A still has its followers, for example, Mesri and Kane [11].

One of the first to take up the creep phenomenon was Buisman [12]. The settlement theory of that time assumed that soft soils were characterised by the finite compressibility. This means that the load increment resulted, after some time, in a certain decrease in the porosity. This decrease was said to be dependent on the soil characteristics and the magnitude of a load increment. Responsibility for the situations, when the consolidation process was prolonged, was assigned to the low permeability of a soil. Oedometric tests were basically carried out until the settlement rate achieved a sufficiently low value. After analysing the settlement versus time curve in a semi-logarithmic scale, Buisman discovered that secondary effects cannot be ignored anymore. He proposed the following creep law for one dimensional (1D) consolidation:

$$\varepsilon = \varepsilon_c - C_B \log\left(\frac{t}{t_c}\right) \text{ for } t > t_c \qquad (1)$$

where ε_c is the strain at the end of the primary consolidation, t is the time from the beginning of loading, t_c is the duration of primary consolidation and C_B is the creep index.

Bjerrum [13], based on the settlement measurements of a few buildings in Drammen, Norway, described different phenomena typical of NC Norwegian clays occurring since their deposition and influencing their geotechnical properties. According to Bjerrum, the compressibility characteristics of soils can be represented by the system of curves (Fig. 7). These curves relate the void ratio, the effective stress and the time.

Each of the curves, called the isochrones, describes the equivalent void ratio e corresponding to different effective stress levels. The effective stress is caused by the overlying soil layer at a certain time after deposition. Each of the curves is related with a specific strain rate. The curves are parallel, which means that the strain rate corresponding to each of them changes in the same way. Besides, because of their curvilinearity, the strain rate decreases with increasing stress. The deformations are divided into two components (Fig. 8). The first component is the instant strain ε_i occurring along with the effective stress increase according to Eq. 2. The second component is the delayed strain and represents the void ratio reduction under the constant stress conditions. The total (instant and delayed) strain ε_t after 3000 years, according to Fig. 7, is given by Eq. 3.

$$\varepsilon_i = \frac{C_c}{1+e_0} \ln \frac{\sigma_{vp}+[\Delta\sigma_v-(\sigma_{vp}-\sigma_{v0})]}{\sigma_{vp}} \quad (2)$$

$$\varepsilon_t = \frac{C_c}{1+e_0} \ln \frac{\sigma_{v0}+\Delta\sigma_v}{\sigma_{v0}} \quad (3)$$

σ_{v0} is the initial vertical effective stress, σ_{vp} is the preconsolidation stress, $\Delta\sigma_v$ is the additional load, e_0 is the initial void ratio and C_c is the compression index.

The above classification is contrary to the definition of the primary and the secondary consolidation. This definition considers the end of the excess pore water pressure dissipation. It refers to the situation in which the load is directly transferred to the soil skeleton. It complies with the hypothesis A, which, however, was not yet proposed at this time. Bjerrum also stated that reduction in the water content during creep results in a better stability of soil structure by improving the interface between particles. This leads to the increase in the preconsolidation pressure. The additional load below the preconsolidation pressure would cause only the elastic strains. The closer the values of the additional load to the preconsolidation pressure, the higher the values of the creep rate would be obtained. However, if the additional load was applied with the strain rate value corresponding to the current curve, the effect of the preconsolidation pressure would vanish. The consolidation process would then continue according to the current curve and both the elastic and the plastic strains would be produced. It can be said that the preconsolidation pressure is a function of the strain rate. Yet, it should be emphasised that the curves were derived considering the behaviour of a certain type of NC clays. The other effects specific of such soils, for example, bonding, cementation and leaching, were not taken into account.

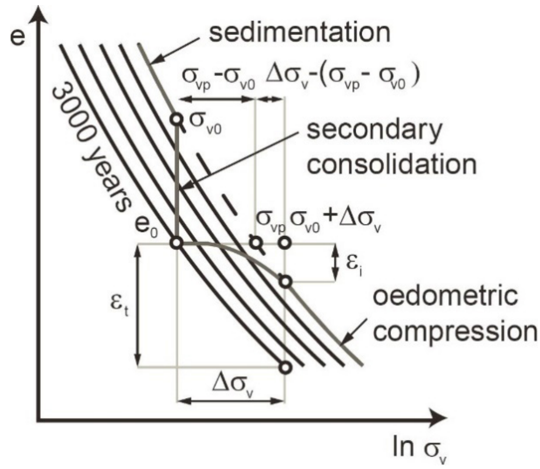

Figure 7: System of isochrones describing the compressibility characteristics of soft soils after Bjerrum [13].

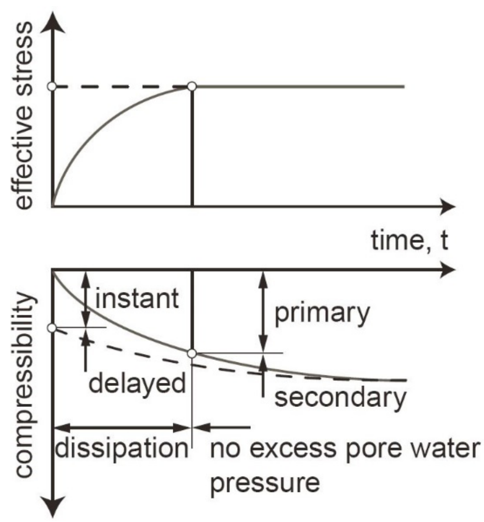

Figure 8: Instant and delayed deformations according to Bjerrum [13].

On the basis of Bjerrum's conception, Garlanger [14] introduced the following creep equation:

$$e = e_c - C_\alpha \log\left(\frac{\tau_c+t'}{\tau_c}\right) \text{ with } C_\alpha = C_B(1+e_0) \text{ for } t' > 0 \quad (4)$$

where e_c is the void ratio at the end of the primary consolidation, τ_c is the intrinsic time, t' is the effective creep time and C_α is the creep index. Contrary to Bjerrum, Garlanger considered the process of the excess pore water dissipation. He combined the compressibility and the creep laws with the seepage equation and, hence, conducted 1D coupled analysis consistent with hypothesis B.

After analysing the results of the oedometer tests of three different types of soft soils, Mesri and Godlewski [15] proposed the time- and stress-compressibility interrelationship (Fig. 9).

It describes the relation between void ratio related to the effective stress changes concerning both the primary and the secondary compressibility and the creep index connected with time. As a consequence of the interrelationship, we can predict the compressibility and the consolidation curves. It is to be noted that Mesri and Godlewski assumed the beginning of the creep process at the end of consolidation. This is in agreement with hypothesis A. Besides, the interrelationship appeared not to be constant in the case of sensitive soils.

Sensitive soils undergo softening after the effective stress exceeds preconsolidation pressure in the process called the destructuration [16]. Natural sedimentary clays have mechanical characteristics that differ significantly from their reconstituted equivalents. The most important differences are related to the existence of inter-particle bonds that develop during the diagenesis of natural clay deposits. Owing to these bonds, natural clays possess a considerable strength anisotropy, giving yield surfaces far beyond the critical state line (CSL).

Den Haan [17] defined the compressibility and the creep indices in a different way. He introduced the structure parameter. It is a measure of the soil sensitivity resulting from cementation or leaching. The structure parameter allowed a robust description of the compressibility curve even in the case of sensitive soils. Den Haan also incorporated the logarithmic strain into the creep law (Eq. 4). He proved that the relation between the void ratio and the compressibility indices may be constant for sensitive soils as well when using the indices in the proposed form. His work is consistent with hypothesis B.

The isotaches concept was first proposed by Šuklje [18]. Isotaches, same as the isochrones, correspond to the constant strain rates but in the pure mathematical terms. They are not related to the time after deposition.

Figure 10 depicts the relation between the void ratio e and the ratio of the current vertical effective stress σ_v to its reference value σ_{ve} in a natural logarithmic scale. The reference vertical effective stress σ_{ve} is related to the current void ratio and the reference isotache. The isotaches correspond to certain strain rates and remain in a certain relation to the reference isotache. The reference isotache

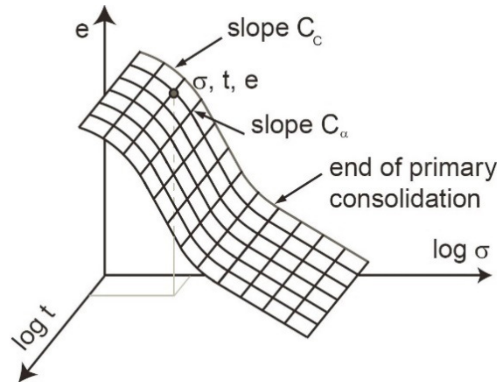

Figure 9: Time- and stress-compressibility interrelationship after Mesri and Godlewski [15].

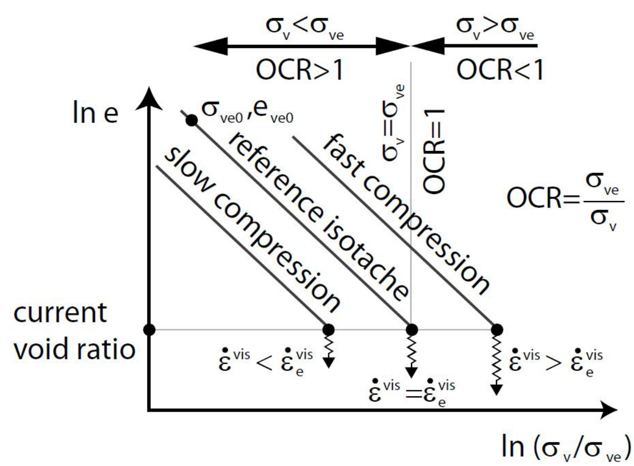

Figure 10: Isotaches concept.

is consistent with the NC line, which is usually determined in a 24-h test. The viscous strain rate corresponding to the reference isotache is equal to the reference creep rate $\dot{\boldsymbol{\varepsilon}}_e^{vis}$. For strain rates higher than the reference creep rate, the preconsolidation pressure is exceeded (OCR<1). For a given value of void ratio, different values of the vertical effective stress are obtained. On the basis of the isotaches concept, we can also derive the expression for the creep rate.

3 Constitutive modelling

Creep process is mostly considered from the phenomenological point of view. Besides the already mentioned theories, there exist other efficient empirical models, for example, Mitchell [19]. Rheological models can also be used, for example, Feda [20]. There are some physically based models in which the long-term deformations of soft soils are regarded as the water transfer from microstructure to macrostructure. Such models involve both the chemical approach and thermodynamics, for example, Cosenza and Korošak [21] and Navarro and Alonso [22].

In the literature, many models that regard the rate dependence are proposed [6]. In the design of geotechnical structures, we usually apply numerical models that are based on the phenomenological concept. For the simulation of soft soils behaviour, the constitutive models should provide the following issues.

The barotropy that relates the soil stiffness to the current stress state, for example, in the form of Eq. 5. In the case of organic soils, the soil stiffness changes linearly with depth, and the exponential power m in the power law (Eq. 5) is equal to one, which leads to the logarithmic law (Eqs. 6 and 7). The power law in the following form describes the change in a stiffness modulus, for example, the Young modulus E, depending on the change in the effective mean stress p:

$$E = E_{ref}\left(\frac{p}{p_0}\right)^m, \tag{5}$$

where E_{ref} is the reference stiffness modulus and p_0 is the reference effective mean stress. The logarithmic law for the virgin isotropic and the isotropic unloading/reloading compression respectively can be written as

$$\Delta e = -\lambda \ln\left(\frac{p}{p_0}\right) \tag{6}$$

$$\Delta e = -\kappa \ln\left(\frac{p}{p_0}\right), \tag{7}$$

where λ is the virgin compression index and κ is the swelling index.

- The consideration of the difference between the loading and the unloading-reloading stiffness.
- The viscosity allowing modelling of the time-dependent phenomena (i.e. creep and relaxation).
- The ability of simulation of the processes characteristic of organic soils, such as bonding, ageing or cementation.
- The proper reproduction of the K_0^{NC} path in the case of 1D compression.
- Both the stress-induced and the microstructural anisotropy as well as the stiffness and the strength anisotropy.

We can classify the material models used in geotechnical practice into two groups: considering the time-dependent behaviour (rate-dependent constitutive models) and neglecting it (rate-independent constitutive models).

Amongst the latter, there are elastic, elasto-plastic and hypoplastic models [23-25, 27, 28, 35]. They do not regard the time dependence, so that the simulation of creep and relaxation effects is not possible. However, these models are worth to be mentioned because some of them are often used in practice to consider solely the high compressibility of soft soils. They are also the basis for the rate-dependent models.

Rate-dependent models allow the simulation of creep and relaxation. Their main application is modelling the soft soil behaviour. They include the elasto-viscoplastic models [36, 37, 39, 41] and the visco-hypoplastic model [40].

3.1 Rate-independent constitutive models

3.1.1 Elastic models

The elastic models incorporate the linear isotropic elastic Hooke's law with two or, in the case of the transverse isotropy, five material parameters. These models can additionally take the barotropy into account (hyperelastic and hypoelastic models).

3.1.2 Elasto-plastic models

The elasto-plastic models consider the limit stress state in the form of one or more yield surfaces. Yield surfaces can either be constant (elasto-perfectly plastic models) or they can evolve (models with a hardening or a softening law). The elasto-plastic models are generally characterised by the variety of formulation and the wide range of application.

Modified Cam Clay model

One of the basic soil material models is the Modified Cam Clay (MCC) model [23-25]. The yield surface is an ellipse, and the no-tension criterion is also incorporated (Fig. 11). Drucker-Prager (D-P) criterion is mostly used as the failure criterion. The ellipse is, hence, symmetrical about the mean effective stress p-axis. The elastic domain inside the ellipse is described by the isotropic Hooke's law with the stress-dependent stiffness moduli based on the logarithmic compression law (hypoelastic) (Eqs. 6 and 7). The ellipse is divided by the point of intersection of the CSL and the yield surface into two parts. The left side of the ellipse is called the dry side. It concerns the OC soils and the softening. The right side, known as the wet side, regards the NC soils and the hardening. The MCC model has an isotropic hardening that is dependent on the volume changes. The initial location of the yield surface is determined by the OCR (OCR = p_p/p_{eq}, where p_p is the preconsolidation pressure and p_{eq} is the equivalent pressure that corresponds to the current stress state). The MCC model simulates the effects of unloading and reloading. However, the shape of the yield surface on the wet side does not allow a proper reproduction of the K_0^{NC} value in oedometric conditions. To realistically simulate the K_0^{NC} path, the surface should be steeper on the wet side. Besides, loading on the dry side results in the softening [24]. This means the shear strength reduction and leads to the strong dependence of a FE model on the discretisation when standard local formulations are used. The CSL is based on the D-P criterion that does not correspond to the real characteristics of the shear strength of soils. It also overestimates the elastic area below the p-axis. In some up-to-date implementations, the shape of the yield surface of MCC model in the deviatoric plane is not circular (e.g. [26]).

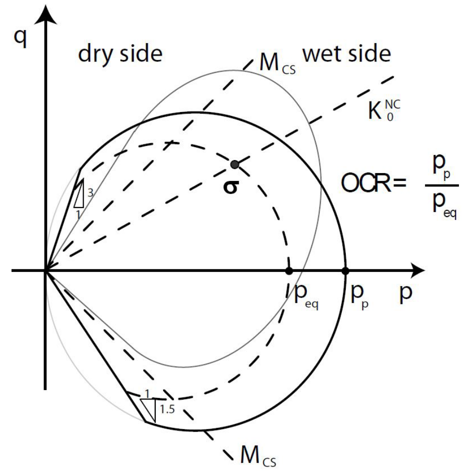

Figure 11: MCC model – the real yield surface known from high accuracy experiments, for example, [38, 43], is indicated with grey colour. M_{CS} is the slope of the CSL [24].

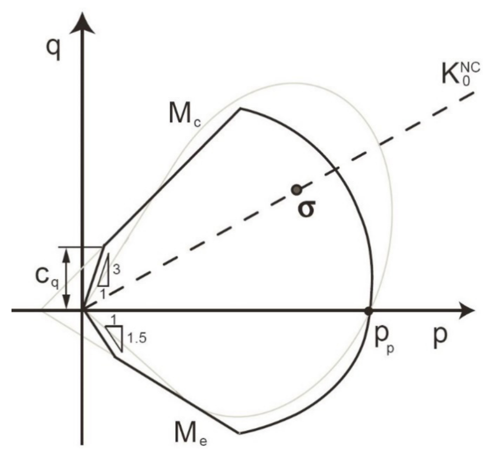

Figure 12: Cap models. M_c and M_e are the slopes of M-C criterion in the case of triaxial compression and extension, respectively.

Soft Soil model

In the case of cap models, for example, the Soft Soil (SS) model [35] and the Hardening Soil (HS) model [27, 28], the softening is neglected. The yield surface in the form of an ellipse is replaced with the combination of the Mohr-Coulomb (M-C) criterion and an elliptically shaped cap yield surface (Fig. 12). The M-C criterion is unsymmetrical about the p-axis and controls the shear strength. The hardening law for the cap surface is isotropic and based on the stress-dependent oedometric modulus. In such models, the CSL influences only the shape (slope) of the cap yield surface. The slope of the cap surface provides a proper reproduction of the K_0^{NC} path.

With the SS model, for the stress states within the above-mentioned boundaries, the elastic Hooke's law in the form of logarithmic law with the modified swelling index κ^* applies. κ^* is determined using the relationship between the strain ε and natural logarithm of the mean effective stress p. The barotropy as well as the unloading and the reloading effects are provided. In addition, the effective cohesion parameter c_q, Figure 12, in the M-C criterion is introduced. Owing to the introduction of the M-C criterion together with the cap surface, the proper shear strength in both the drained and the undrained conditions is obtained. Yet, the SS model considers neither the strength anisotropy nor the stiffness anisotropy.

3.1.3 Hypoplastic models

In the case of hypoplastic models [29], the strain rate that consists of the elastic and the plastic parts is replaced by the total strain rate. Besides, these models require neither a yield surface, a plastic potential nor a flow rule. This results in their simple framework. However, they are less robust in terms of numerical efficiency than elasto-plastic models. This concerns the stress return procedures [31]. The hypoplastic models have a potentially wide range of application and, yet, no commercial implementation.

3.2 The meaning of strength and stiffness in modelling of soft soils

It is worth explaining the reason for the use of a simple hypoelastic description (Hooke's law, with the barotropy based on κ^*, or the power law) in the case of soft soils. In general, this type of soil is NC or slightly OC. It means that the *in situ* stress state is close to the yield surface, refer to the left hand side of Figure 13.

When introducing the load on soft soil, the stress state achieves the yield surface almost immediately. In the case of soft soils, the yield surface is the viscoplastic potential surface (see Section 3.3.1). Obtaining this surface means that the soil begins to perform the viscoplastic behaviour. Yet, this relation does not hold for the OC soils, which is illustrated in the right hand side of Figure 13. Considering the OC soils, the OCR that describes the size of the yield surface is of a higher value. Hence, the elastic behaviour involves a relatively wide range of the stress, so that it deserves a more extensive description than that in the case of soft soils. In the models dedicated for soft soils, the barotropy is based solely on the modified swelling index κ^*. It is a simplification that gives reliable results in practice.

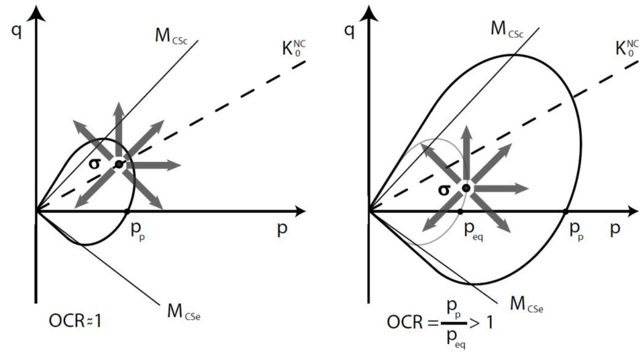

Figure 13: *In situ* stress state in NC (or slightly OC) and OC soil.

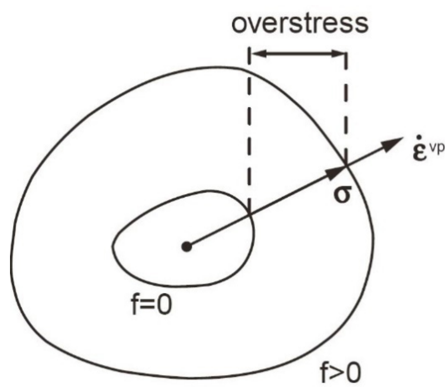

Figure 14: Overstress concept.

3.3 Rate-dependent constitutive models

3.3.1 Elasto-viscoplasticity

In general, according to the small strain theory, the total strain rate $\dot{\boldsymbol{\varepsilon}}$ in the case of elasto-viscoplasticity is divided as follows:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{e} + \dot{\boldsymbol{\varepsilon}}^{vp} \tag{8}$$

where $\dot{\boldsymbol{\varepsilon}}^{e}$ is the elastic strain rate and $\dot{\boldsymbol{\varepsilon}}^{vp}$ is the viscoplastic strain rate. The relation between the stress rate and the elastic strain rate is determined by a constitutive law:

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{D} : \dot{\boldsymbol{\varepsilon}}^{e} \tag{9}$$

where $\dot{\boldsymbol{\sigma}}$ is the stress rate and $\boldsymbol{D}$ is the stiffness matrix. The viscoplastic strain rate $\dot{\boldsymbol{\varepsilon}}^{vp}$ is described by the flow rule:

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \dot{\lambda}\frac{\partial g}{\partial \boldsymbol{\sigma}} \tag{10}$$

where $\dot{\lambda}$ is the viscoplastic multiplier that determines the magnitude of $\dot{\boldsymbol{\varepsilon}}^{vp}$ and $g = g(\boldsymbol{\sigma},\boldsymbol{\Phi})$ is the viscoplastic potential function with $\boldsymbol{\Phi}$ representing the state variables. The partial derivative $\partial g/\partial\boldsymbol{\sigma}$ determines the direction of the viscoplastic flow.

Viscoplasticity

To account for the time-dependent behaviour of soils, the viscoplastic models were developed. These models can be divided into the models based on the so-called overstress concept (e.g. Perzyna, Duvaut-Lions [31-33]; Fig. 14) and the consistency models [30, 32-34].

Overstress concept

In the overstress concept, the yield function f can obtain the values greater than zero, $f>0$ (the overstress). The yield function is a function of the current stress state and the state variables and is independent of the strain rate $f = f(\boldsymbol{\sigma},\boldsymbol{\Phi})$.

In the Perzyna model, the viscoplastic multiplier $\dot{\lambda}$ determines the magnitude of the flow. It is dependent on the overstress and has the following form:

$$\dot{\lambda} = \frac{<\zeta(f)>}{\eta} \tag{11}$$

where η is the viscosity parameter, ζ is the overstress function that depends on both the yield function and the derivative of the viscoplastic potential, and $\langle\cdot\rangle$ denotes the Macaulay brackets. The yield surface must be overridden for the viscoplastic effects to occur. In the Duvaut-Lions model, the viscoplastic strain rate is defined as

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \frac{1}{\eta}\boldsymbol{C}:(\boldsymbol{\sigma} - \overline{\boldsymbol{\sigma}}) \tag{12}$$

and the hardening law is written as follows:

$$\dot{\boldsymbol{\Phi}} = \frac{1}{\eta}(\boldsymbol{\Phi} - \overline{\boldsymbol{\Phi}}) \tag{13}$$

where $\boldsymbol{C}$ is the compliance matrix, $\overline{\boldsymbol{\sigma}}$ is the projection of the current stress state on the yield surface and $\overline{\boldsymbol{\Phi}}$ denotes the state variables for purely elasto-plastic model. In this case, the viscoplastic strain rate is defined directly in the stress space. Same as in the Perzyna model, the generation of the viscoplastic strains requires the overstress.

Consistency concept

The consistency models allow the consideration of the viscoplastic effects based on the strain rate-dependent yield surface $f = f(\boldsymbol{\sigma},\boldsymbol{\Phi},\dot{\boldsymbol{\Phi}})$. In the case of consistency models, during the viscoplastic flow, the current stress state must remain on the yield surface. The yield surface is now dependent not only on the state variables but also on their rates (Eq. 14). This way, it is able to change both the size and the shape depending on the viscoplastic strain. In the Perzyna model, the viscoplastic multiplier is explicitly defined by the overstress function. In the consistency models, it is described by the so-called consistency condition that depends on the strain rate. The consistency condition has the form of a differential equation:

$$\dot{f}\left(\boldsymbol{\sigma},\boldsymbol{\Phi},\dot{\boldsymbol{\Phi}}\right) = \frac{\partial f}{\partial \sigma}:\dot{\boldsymbol{\sigma}} + \frac{\partial f}{\partial \boldsymbol{\Phi}}\cdot\ \dot{\boldsymbol{\Phi}} + \frac{\partial f}{\partial \dot{\boldsymbol{\Phi}}}\cdot\ddot{\boldsymbol{\Phi}}\ = 0 \quad \text{with } \dot{\boldsymbol{\Phi}} = \dot{\lambda}g(\boldsymbol{\sigma}). \tag{14}$$

3.3.2 Elasto-viscoplastic models

Soft Soil Creep model

The elasto-viscoplastic Soft Soil Creep (SSC) model [36, 37] is the most frequently used constitutive model that allows the simulation of the time-dependent behaviour of soils and is implemented in the commercial geotechnical software. In this model, the M-C criterion remains elasto-plastic. The barotropy in the form of the logarithmic law is incorporated and the realistic K_0^{NC} value in oedometric conditions is provided. The effects of unloading and reloading are also considered. The cap surface is replaced with the viscoplastic potential surface to account for creep and relaxation. This surface can now be overridden by the stress state (the overstress concept). The viscoplastic potential function evolves with the volumetric strain and time, so that solely the volumetric deformations are considered here. Yet, it is possible to account for the deviatoric deformations in an indirect way. This requires the incorporation of the undrained conditions to eliminate the volumetric deformation. The model is based on the isotache concept, and the volumetric viscoplastic strain rate $\dot{\varepsilon}_{vol}^{vp}$ is described as follows:

$$\dot{\varepsilon}_{vol}^{vp} = \frac{\mu*}{\tau}\left(\frac{p_p}{p_{eq}}\right)^{\frac{\lambda*-\kappa*}{\mu*}} \tag{15}$$

where λ^*, κ^* and μ^* are the modified virgin compression index, the modified swelling index and the modified creep index, respectively. τ is a parameter known as the reference time. It corresponds to the total time of creep during both the primary and the secondary consolidation. The incremental loading (IL) oedometer tests are mostly performed with the 24-h load steps. Owing to that, the reference time τ is usually assumed equal to 1 day. During a 24-h test, the primary consolidation time is insignificant compared to the creep time, which is difficult to determine. This justifies the use of the reference time instead of the total creep time. In the SSC model, λ^* and κ^* indices are assumed to be constant in time. On the basis of Eq. 15, we can see that the creep rate is strongly dependent on OCR. The SSC model accounts for creep within the primary consolidation; it is consistent with hypothesis B. It offers the rate-dependent stiffness and strength. However, it is an isotropic model. It does not allow considering the destructuration effects. It involves the creep index that is determined for the stage after the excess pore water dissipates.

Creep index

In reality, the creep rate in the primary and the secondary consolidation phases may vary. Besides, the creep index value may change with time. The strain-time relationship plotted in a semi-logarithmic scale is usually linear in the considered time range (Fig. 15). Thus, in practice, the constant creep index is generally used. The creep index can be determined based on different relationships between the strain and time, which is depicted in Fig. 15.

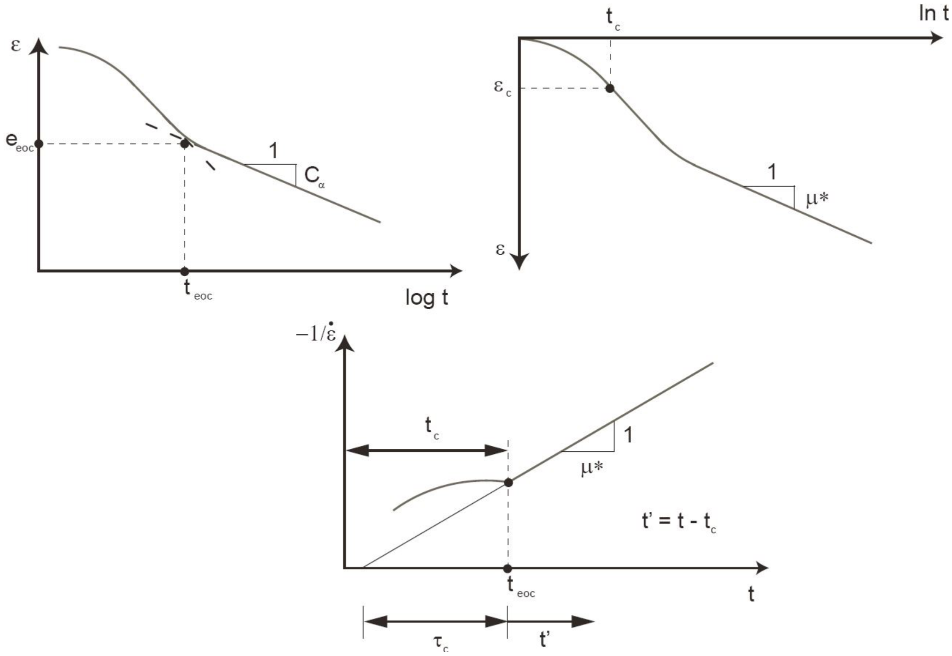

Figure 15: Determination of the creep index in oedometric conditions.

Anisotropic effects

The anisotropic yield surfaces are often observed in the case of organic soils (e.g. [38, 43]). Such soils are characterised by the initial anisotropy that results from the deposition, the characteristic soil skeleton and the stress state. This influences the rate-dependent behaviour of soft soils and should be considered in the constitutive modelling.

Leoni model

The model developed by Leoni et al. [39] is shown in Figure 16.

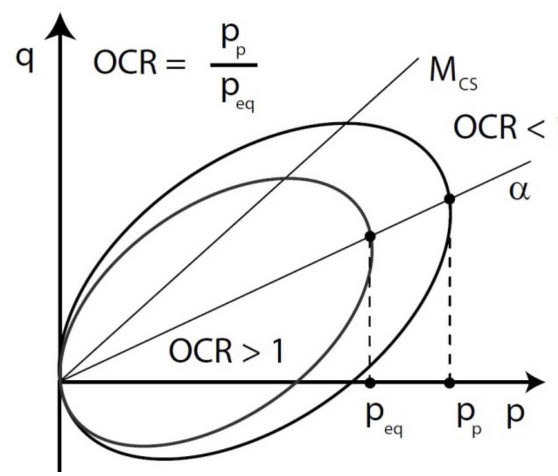

Figure 16: The Leoni model [39].

The Leoni model is based on the SSC model. However, as it introduces the anisotropy, it exceeds the abilities of the SSC model. The anisotropic hardening law, which is related to the rotation of the yield surface, is applied:

$$\dot{\boldsymbol{\alpha}} = \omega\left[\left(\frac{3q}{4p} - \alpha\right)\dot{\boldsymbol{\varepsilon}}_{vol}^{vp} + \omega_d\left(\frac{3q}{4p} - \alpha\right)\dot{\boldsymbol{\gamma}}^{vp}\right] \qquad (16)$$

where α is the scalar anisotropic hardening parameter that evolves with the creep deformation, q is the deviatoric stress, $\dot{\boldsymbol{\gamma}}^{vp}$ is the deviatoric creep strain rate and ω, ω_d are the soil constants. Here, the creep deformation regards not only the volumetric but also the deviatoric part. Thus, compared to the SSC model, in the Leoni model, the evolution of the yield surface depends on both the volumetric and the deviatoric deformation. The anisotropic hardening law allows the consideration of the change in anisotropy because of the viscous deformations. This is of high importance regarding the proper values of the undrained shear strength in both the triaxial compression and the triaxial extension. As already said, the isotropic models overestimate the undrained shear strength. The rotational law also provides the proper reproduction of the K_0^{NC} path in the case of 1D compression. This model does not allow consideration of the destructuration effects. It also involves the softening behaviour on the dry side of the yield surface. To manage this problem, nonlocal or gradient plasticity may be incorporated as regularisation [24, 44]. Yet, its most significant drawback is that it is based on the D-P criterion.

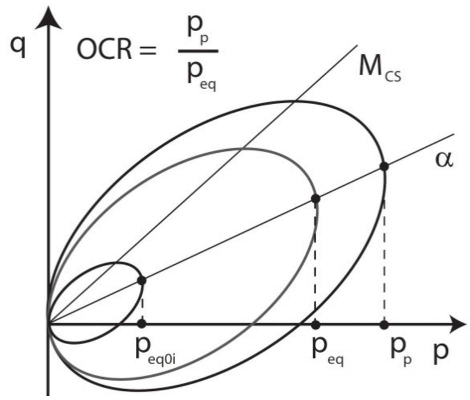

Figure 17: The Sivasithamparam et al.'s model [41].

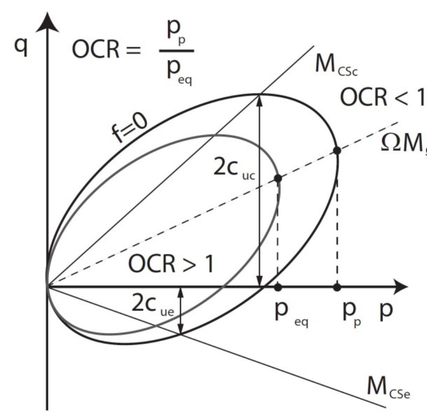

Figure 18: The Niemunis and Grandas-Tavera model [40]. M_{CSc} and M_{CSe} are the slopes of the CSL for triaxial compression and extension, respectively. M_Ω denotes the slope of the CSL by the current $\boldsymbol{\Omega}$.

Sivasithamparam et al.'s model

The next example of the anisotropic models was developed by Sivasithamparam et al. [41]. It constitutes a further extension of the Leoni model. In addition, it provides the simulation of destructuration effects. The destructuration process was introduced in the form of the intrinsic yield surface (Fig. 17).

The relation between the NC surface and the intrinsic surface is determined by the value of the initial amount of bonding χ_0 in the following form:

$$p_{eq0} = (1 + \chi_0) p_{eq0i} \tag{17}$$

where p_{eq0i} corresponds to the intrinsic yield surface. The initial amount of bonding χ_0 is determined by the sensitivity of a soil. The incorporated destructuration law is dependent on both the volumetric creep rate $\dot{\boldsymbol{\varepsilon}}_{vol}^{vp}$ and the deviatoric creep rate $\dot{\boldsymbol{\varepsilon}}_d^{vp}$. It holds

$$\dot{\boldsymbol{\chi}} = -\xi\chi\left(\left|\dot{\boldsymbol{\varepsilon}}_{vol}^{vp}\right| + \xi_d\left|\dot{\boldsymbol{\varepsilon}}_d^{vp}\right|\right) \tag{18}$$

where ξ and ξ_d are the soil constants. Although this model considers the anisotropy as well as the destructuration process, it is based on the D-P criterion, similar to the Leoni model. The softening behaviour related to strong mesh dependency is also involved.

3.3.3 Niemunis and Grandas-Tavera visco-hypoplastic model

The visco-hypoplastic model is a combination of viscosity and hypoplasticity [40]. It combines the elasticity and the flow rule, as incorporated in the hypoplastic models, with the Norton law [42]. This model also accounts for the anisotropy in the form of the anisotropic yield surface (Fig. 18).

The yield surface is based on the Matsuoka-Nakai (M-N) criterion. The basic element of the model is the anisotropic NC surface that estimates the realistic values of the undrained shear strength in the triaxial compression as well as in the triaxial extension. Here, the strain-induced anisotropy is described by the structure tensor $\boldsymbol{\Omega}$ (Fig. 18). The NC surface is used to determine the magnitude (with the Norton law) and the direction of the viscous flow. The surface can be overridden by the stress state (the overstress concept), which results in an increase in the creep rate. The constitutive equation incorporated in the model holds

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{D} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{vis} - \dot{\boldsymbol{\varepsilon}}^{Hp}\right) \tag{19}$$

$$\text{with } \dot{\boldsymbol{\varepsilon}}^{Hp} = C_1 \boldsymbol{m}\sqrt{\dot{\boldsymbol{\varepsilon}}\dot{\boldsymbol{\varepsilon}}} \text{ and } \dot{\boldsymbol{\varepsilon}}^{vis} = \boldsymbol{m}\dot{\varepsilon}_r \mathrm{OCR}^{-1/I_v} \tag{20}$$

where $\dot{\boldsymbol{\varepsilon}}^{Hp}$ is the hypoplastic strain rate dependent on the strain rate $\dot{\boldsymbol{\varepsilon}}$, C_1 is a material constant, $\boldsymbol{m}$ describes the flow rule, $\dot{\varepsilon}_r$ is the creep rate and I_v is the viscosity index from the Norton law. When the yield function is equal to zero, $f = 0$, the OCR is equal to one, OCR = 1, and the stress state remains on the NC surface. The viscous strain rate has then the reference value. In this case, the hardening law regards the total volumetric strain and not only its viscous part. Besides, this model includes the rotational law, so that it allows the realistic simulation of the K_0^{NC}

path in oedometric conditions. However, the problem of softening occurs again.

4 Conclusions

Rate-dependent effects are commonly present in practical cases when NC or slightly OC soils are involved. These effects occur mostly in the form of creep process. In general, the time-dependent behaviour of organic soils determines their stiffness and strength parameters. It, thus, proves to be a significant issue of soft soils and means that these effects should not be neglected. Geotechnical design would be more robust and economic if the rate dependence was more frequently regarded. Moreover, creep effects are often observed after a geotechnical structure is performed. This results mostly in additional costs and states another important reason for considering creep behaviour in practice.

There are constitutive models that are capable of more or less advanced simulation of the soft soils behaviour. Some of them, the basic ones, are available in the commercial geotechnical software. The more advanced models are increasingly being implemented, but their complexity may lead to some practical problems, for example, with setting the parameters in a given case. It is, then, important to know the theories which the models are based on. Besides, one should be aware of both their advantages and their drawbacks.

Time-dependent behaviour of soft soils should be incorporated in the design reasonably. Each practical case involving organic soils should be thoroughly investigated. Finally, the proper design methods, considering safe and economic aspects, should be chosen.

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