

# Design of optimal state feedback controller with observer for multidimensional electrical system

**Streszczenie.** Z punktu widzenia sterowania, system elektryczny RC z dwoma wejściami i dwoma wyjściami jest systemem liniowym i wielowymiarowym. W artykule zaprojektowano sterowanie całkujące tym system wraz z obserwatorem dołączonym do układu regulacji. W pracy wykorzystano obserwator Kalmana. Przedstawiono testy symulacyjne i testy wykonanego systemu regulacji.

**Abstract.** From the control point of view, two-input-two-output RC electrical system is a linear and multidimensional system. In this paper, integral control and closed-loop observer are verified whether they can be used and designed. To verify this, Kalman tests are made. These structures are presented and discussed. Integral Control is tested with some base disturbances impact and checked whether it can handle them. The estimated values from the observer are compared with the original states. Simulation tests and real tests of control systems are presented. (**Projektowanie optymalnego regulatora od stanu z obserwatorem dla wielowymiarowego systemu elektrycznego.**)

**Słowa kluczowe:** wielowymiarowy system elektryczny, układ regulacji od stanu, obserwator stanu, LQR.

**Keywords:** multidimensional electrical system, state feedback controller, state observer, LQR.

## Introduction

RC electrical systems are very interesting objects which give the possibility to implement various, even very complicated control algorithms and test them.

In this paper, a MIMO (Multiple Input Multiple Outputs) system with two inputs and two outputs is considered. In this kind of system, the best option to create a model of it is to use state space. The consequence of this decision is to use some version of a state feedback controller. Obtaining such a model and design of state feedback is presented in [1]. In this case, integral control is designed and presented. Such control could be designed not only for the system presented in this paper but also for some more complicated and advanced systems such as energy plants and inverters with only little changes. In the system the closed-loop observer is needed as there are some unmeasurable states. The mechanism is verified with Kalman test whether it can be used and then implemented in the control system.

The issue of using state space and controllers is still popular and is considered in many other research works. The integral controller is designed to control load frequency [2] and the servomotor control issue is taken into account [3]. The linear quadratic regulator (LQR) is used to improve the performance of an electrical power controlling system [4] and to control the twin-rotor MIMO system with predefined degree of stability [5]. The other examples of possible uses of integral controller were described in [6].

This paper is organised as follows: the mathematical and physical model are described in section 2. The structure of the control system is presented in section 3. Integral control is described and its settings are set and presented. The closed-loop observer is designed and presented. Verification of the observer mechanism and simulation results of the control system are presented and analysed in section 4. Section 5 concludes the paper.

## Description of electrical system

### Mathematical model

The electrical system is a linear MIMO model. The system was made out of resistors and capacitors. Capacitors are responsible for the dynamics of the system. It will require high current or high value of impedance to reach a time constant which is possible to implement in the control system. The designed system is shown in Figure 1. The values of elements are equal to:  $R_1 = 1\text{M}\Omega$ ,  $R_2 = 470\text{k}\Omega$ ,

$R_3 = 1\text{M}\Omega$ ,  $R_4 = 680\text{k}\Omega$ ,  $R_5 = 2.2\text{M}\Omega$ ,  $C_1 = 3\mu\text{F}$ ,  $C_2 = 1\mu\text{F}$ ,  $C_3 = 1\mu\text{F}$ .

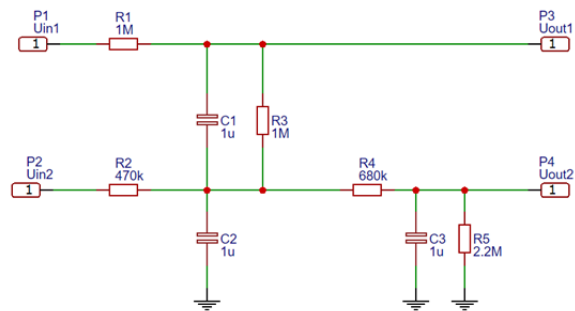


Fig.1. Electrical diagram

The mathematical model is given by three equations:

$$(1) \quad \frac{dU_{C1}(t)}{dt} = -\left(\frac{1}{R_1 C_1} + \frac{1}{R_3 C_1}\right) U_{C1}(t) - \frac{1}{R_1 C_1} U_{C2}(t) + \frac{1}{R_1 C_1} U_1(t)$$

$$(2) \quad \frac{dU_{C2}(t)}{dt} = \frac{1}{R_1 C_2} U_{C1}(t) - \frac{1}{C_2} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) U_{C2}(t) + \frac{1}{R_4 C_2} U_{C3}(t) + \frac{1}{R_1 C_2} U_1(t) + \frac{1}{R_2 C_2} U_2(t)$$

$$(3) \quad \frac{dU_{C3}(t)}{dt} = -\left(\frac{1}{C_3 R_5} + \frac{1}{C_3 R_4}\right) U_{C3}(t) + \frac{1}{R_4 C_3} U_{C2}(t)$$

where:  $R_1, R_2, R_3, R_4, R_5$  in the form of: resistance of an individual resistor,  $C_1, C_2, C_3$  is capacity of an individual capacitor,  $U_1$  and  $U_2$  is input voltage,  $U_{C1}, U_{C2}, U_{C3}$  is the voltage of an individual capacitor.

Furthermore,  $U_{out1} = U_{C1} + U_{C2}$  and  $U_{out2} = U_{C3}$ . For the purposes of designing the control system, there is a need for a linear model:

$$(4) \quad \dot{x}(t) = Ax(t) + Bu(t)$$

$$(5) \quad y(t) = Cx(t) + Du(t)$$

where:

$$(6) \quad A = \begin{bmatrix} -\left(\frac{1}{R_3C_1} + \frac{1}{R_1C_1}\right) & \frac{-1}{R_1C_1} & 0 \\ \frac{-1}{R_1C_2} & -\left(\frac{1}{R_1C_2} + \frac{1}{R_2C_2} + \frac{1}{R_4C_2}\right) & \frac{1}{R_4C_2} \\ 0 & \frac{1}{R_4C_3} & -\left(\frac{1}{R_4C_3} + \frac{1}{R_5C_3}\right) \end{bmatrix}$$

$$(7) \quad B = \begin{bmatrix} \frac{1}{R_1C_1} & 0 \\ \frac{1}{R_1C_2} & \frac{1}{R_2C_2} \\ 0 & 0 \end{bmatrix}$$

$$(8) \quad C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(9) \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Substituting values to matrixes (6) and (7) gives:

$$(10) \quad A = \begin{bmatrix} -0.6667 & -0.3333 & 0 \\ -1 & -4.5982 & 1.4706 \\ 0 & 1.4706 & -1.9251 \end{bmatrix}$$

$$(11) \quad B = \begin{bmatrix} 0.3333 & 0 \\ 1 & 2.1227 \\ 0 & 0 \end{bmatrix}$$

### Physical model

Figure 2 shows the electrical diagram of the system. Due to low current efficiency of outputs in a data acquisition card, voltage followers (unity buffer amplifier) were implemented. They were created from the operational amplifier LM741. Voltage followers work multiple times faster than this system so they can be skipped in the mathematic model. Using voltage followers requires using a voltage source. The system works in the range from -10V to 10V.

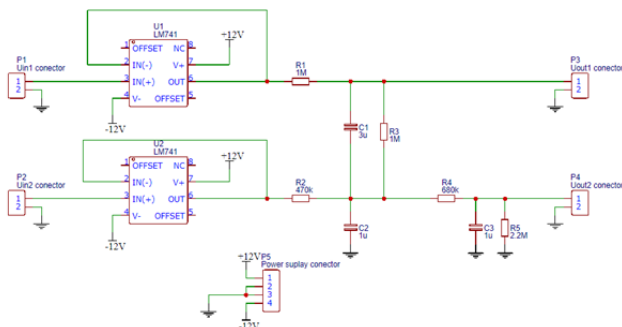


Fig.2. Electrical diagram

The electrical system was created on a prototype paper copper PCB with dimensions of 10cm x 5cm. All components were soldered using the THT (Through-Hole Technology) method. Figure 3 shows the final electrical circuit.

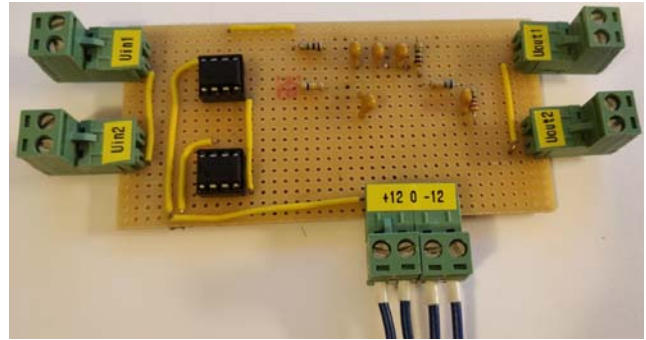


Fig.3. Real electrical circuit

### Design of state feedback controller

#### Structure of the control system

Although integral control is universal, to implement it, as it was said before, the state space model of the object is required. The biggest advantage of using this type of control is the fact, that it allows an engineer to observe any changes or abnormalities inside the system, not only the outputs. The structure of the whole system is presented in Figure 4. It consists of a closed-loop observer, an integral controller, the object, state feedback and estimated controlled outputs feedback [7].

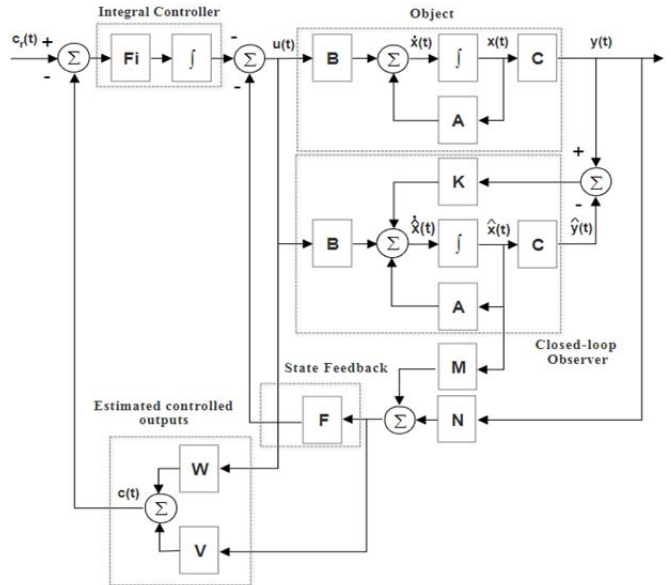


Fig.4. Structure of the control system

where:  $F_i$  – integral matrix,  $K$  – observer matrix,  $F$  – state feedback matrix,  $W, V$  – matrices which provide appropriate estimated control outputs,  $M, N$  – matrices which provide appropriate states (from observer or from measured outputs),  $c_r(t)$  – reference values,  $c(t)$  – estimated controlled outputs,  $u(t)$  – control signals,  $y(t)$  – measured outputs,  $\hat{x}(t)$  – estimated states,  $\hat{y}(t)$  – estimated measured outputs.

#### Integral control

At first, the applicability of the controller has to be checked. In this case, Kalman control matrix was designed as [7]:

$$(12) \quad M_c = [B \quad AB \quad A^2B]$$

Using (9) and (10):

$$(13) \quad M_c = \begin{bmatrix} 0.333 & 0 & -0.555 & -0.709 & 2.014 & 3.734 \\ 1 & 2.1277 & -4.931 & -9.783 & 25.394 & 50.297 \\ 0 & 0 & 1.470 & 3.128 & -10.083 & -20.411 \end{bmatrix}$$

There are two conditions which have to be fulfilled to use integral control. The first one is to verify whether rank  $n_{Mc}$  of  $M_c$  matrix is equal to rank  $n$  of state matrix  $A$ . The second one tells that the number of control signals has to be at least equal to the number of controlled outputs. In this case, the number of control signals is the same as the number of controlled outputs and  $n_{Mc}=n=3$  so both conditions are fulfilled. Thus, the controller is applicable. The controller was designed in the Matlab environment. In this case, to use the integral controller on the specified outputs:  $c_1(t)=U_{C2}(t)$ ,  $c_2(t)=U_{R1}(t)=U_{C1}(t)+U_{C2}(t)-U_1(t)$ , matrices  $W$  and  $V$  were needed to provide them in the integral feedback:

$$(14) \quad W = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$$

$$(15) \quad V = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

#### Parameter choice

Designing of an integral controller can be done with pole placement. However, using this method does not give optimal results. To get the optimal parameters, LQR was used which is an automatic way of finding a proper controller. The regulator minimizes the cost function and also allows to prioritize the most important state from the control point of view. The parameters' selection of matrices  $Q$  and  $R$  is crucial and only proper selection guarantees expected performance of the system. In the beginning, the impact of each element of matrices  $Q$  and  $R$  was researched. Only the main diagonal of each matrix was under contemplation because it corresponds to the pole placement. To verify the impact, only a single element was changed, while the other elements were equal to 1. The first three items of  $Q$  matrix correspond to states, the next ones to integral variables. The  $R$  matrix corresponds to the inputs. After receiving the knowledge how each item impacts the system, the best values of each element were chosen and put together. Then, they were modified according to the criteria of the control system. The criteria were a compromise of integral of squared error, the character of regulation, the time of regulation and overshoot and such  $Q$ ,  $R$  matrices and  $F$ ,  $F_i$  as they result, were received as is presented below:

$$(16) \quad Q = \begin{bmatrix} 9 & 0 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}$$

$$(17) \quad R = \begin{bmatrix} 0.6 & 0 \\ 0 & 35 \end{bmatrix}$$

$$(18) \quad F = \begin{bmatrix} -0.4148 & 4.8211 & 0.2539 \\ -0.0465 & 0.4871 & 0.1926 \end{bmatrix}$$

$$(19) \quad F_i = \begin{bmatrix} 5.3999 & 2.5620 \\ 0.8019 & -0.2598 \end{bmatrix}$$

#### Closed-loop observer

In the system, there is no possibility for direct measuring of states  $x_1$  and  $x_2$ , and in consequence, there is no

possibility of measuring controlled variables. In this case, a closed-loop observer is needed. To implement the mechanism, observability has to be checked. It was done with designing a Kalman observability matrix:

$$(20) \quad M_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

Using dependencies (8) and (10) one gets:

$$(21) \quad M_o = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ -1.666 & -4.931 & 1.470 \\ 0 & 1.470 & -1.925 \\ 6.042 & 25.394 & -10.083 \\ -1.470 & -9.593 & 5.868 \end{bmatrix}$$

Then, the rank  $n_{M_o}$  of  $M_o$  was verified and checked whether it is the same as rank  $n$  of the state matrix. In this case,  $n_{M_o}=n=3$  so the system is observable. When using the observer, states are replaced with estimated states and the correction of the error has to be used. According to these facts, the observer equation can be written in the following form:

$$(22) \quad \frac{d\hat{x}(t)}{dt} = A\hat{x}(t) + Bu(t) + K[y(t) - C\hat{x}(t)]$$

where:  $\hat{x}$  – vector of estimated states,  $K$  – observer matrix,  $y(t)$  – measured output vector  $A$ ,  $B$ ,  $C$ ,  $D$  – matrices of system.

After transformation:

$$(23) \quad \frac{d\hat{x}(t)}{dt} = (A - KC)\hat{x}(t) + Bu(t) + Ky(t)$$

where:  $A-KC$  represents the inside dynamic of the observer. Using  $K$  matrix form as it is presented (24) and dependencies (8) and (10), the following matrix was received:

$$(24) \quad K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \\ K_{31} & K_{32} \end{bmatrix}$$

$$(25) \quad A - KC = \begin{bmatrix} -0.666 - K_{11} & -0.333 - K_{11} & -K_{12} \\ -1 - K_{21} & -4.598 - K_{21} & 1.470 - K_{22} \\ -K_{31} & 1.470 - K_{31} & -1.925 - K_{32} \end{bmatrix}$$

In the last step, coefficients of  $K$  matrix were designated for desirable poles  $q = [-30, -31, -32]$  which ensured the fact, that the observer is faster than an object (the eigenvalues of state matrix  $A$  are  $[-5.309, -0.534, -1.347]$ ):

$$(26) \quad K = \begin{bmatrix} 206.58 & -179.44 \\ -158.46 & 161.79 \\ -8.46 & 37.69 \end{bmatrix}$$

To use the measured state  $x_3$  and estimated states  $x_1$  and  $x_2$  some additional matrices were necessary:

$$(27) \quad M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(28) \quad N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

## Simulation tests and results analysis

### Estimated state values from the closed-loop observer

Quality verification of the estimated states from the closed-loop observer was performed on the mathematical model. In this case, there is a possibility to compare estimated state values from the observer to the values which come from the original mathematical model of the object. The results of the comparison are shown in Figure 5.

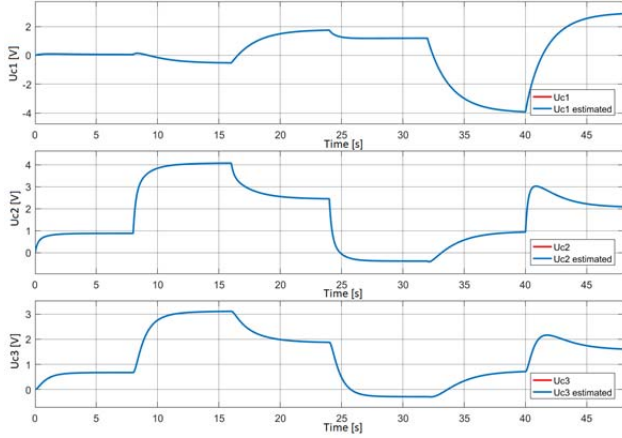


Fig.5. Estimated values – original states (red), estimated (blue)

The results indicate that the observer works perfectly and estimate states received from it are the same as the original states.

### Control results of the electrical system – mathematical model

Setpoints which were chosen to test the system are presented in Figure 6.

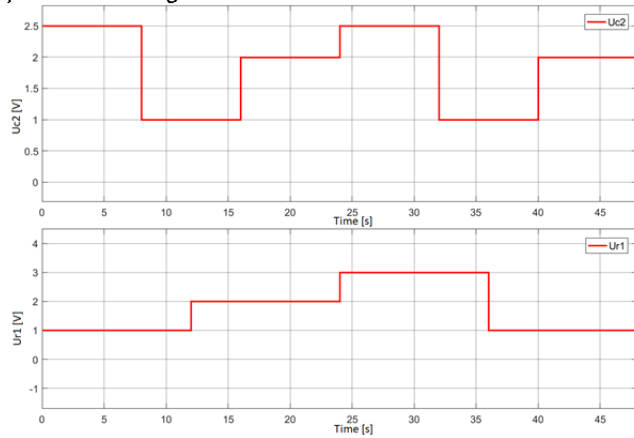


Fig.6. Setpoint –  $U_{C2}$ ,  $U_{R1}$

The setpoints are constant in compartments so the condition of integral control applicability is fulfilled. In addition, there is a change of values in both directions. Fig. 7 shows how the system reacts to the reference values.

The control results are satisfying. The  $U_{C2}$  reaches the setpoint in 4 seconds in the worst case. In a more optimistic case, the value stabilizes in less than 2 seconds. Controls of  $U_{R1}$  are a little faster than  $U_{C2}$ . In the worst case, 90% of the reference value is reached in 1.9 seconds. Both responses have inertial character. It is important to note that controls values are heavily connected. The change of one of them has a large impact on the second one. It can be noticed that at every change of setpoints the biggest change is in the 32 seconds.

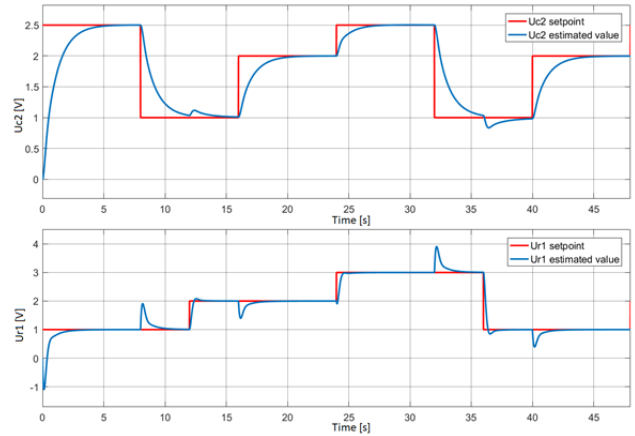


Fig.7. Comparison: set point –  $U_{C2}$  (red),  $U_{R1}$  (red) and control results –  $U_{C2}$  (blue),  $U_{R1}$  (blue)

However, real objects are rarely isolated – in most cases, there are some disturbances which impact the system. The theory of Integral control states that the controller could handle with some slowly changeable or small noises and would minimize the error to zero in steady-state. To verify this aspect, the noises were implemented and added to the input of the object. The noises are presented in Fig. 8.

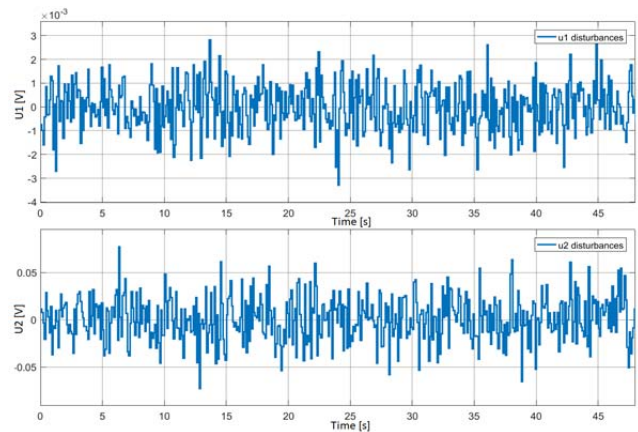


Fig.8. Implemented noises for  $U_1$  and  $U_2$

With such disturbances, the simulation was repeated. The control results are presented in Figure 9.

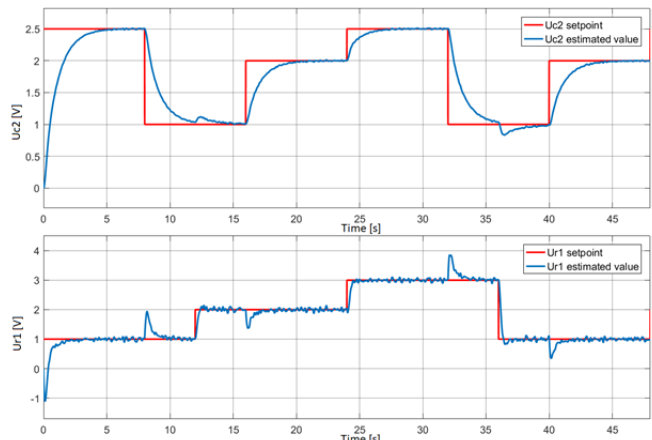


Fig.9. Comparison: set point –  $U_{C2}$ ,  $U_{R1}$  (red) and control results –  $U_{C2}$ ,  $U_{R1}$  (blue) with disturbances

The noises are especially visible in the control process of  $U_{R1}$ . However, they do not impact the system in the way which could destabilize it. The system still minimizes the

error to zero in steady-state and if there is a bigger peak of noise, it returns to the reference value. The results proved that the integral control can handle the disturbances.

#### Control results of the electrical system – physical model

For testing of the physical model, the same setpoints were used (see Fig. 6). The results are almost identical to the mathematical model. The main difference is in the measuring noise. It does not impact the quality of control. Figure 10 shows the control results.

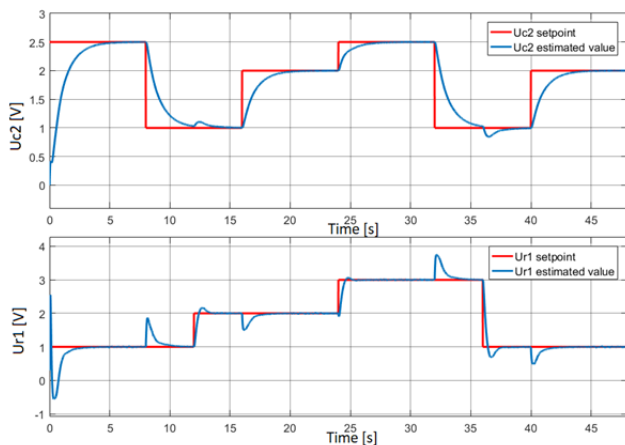


Fig.10. Comparison: set point –  $U_{C2}$ (red),  $U_{R1}$ (red) and control results of the physical model –  $U_{C2}$ (blue),  $U_{R1}$ (blue)

The main difference that can be observed is at the beginning. It is related to initial values. In the simulation, initial values equal to 0V (voltage on capacitors) were adopted. While testing the real system, the voltages were different and as of consequence, the closed-loop observer estimated wrong values of states but it has been corrected fast and the rest of the control was correct.

The test with disturbances was repeated with the same noises as they are presented in Figure 8. The results are presented in Figure 11. As it is shown in Figure 11, the control is correct. The result indicates that the control system with physical model of the object behaves even better than the one with the mathematical model. However, the difference is visible only after very precise investigation. Such a situation is caused because the elements of physical model are not ideal and its dynamic properties may be a little different than the mathematical one.

#### Conclusions

In this paper, an electrical system was presented and control results analysis was done. The implementation of integral control and a closed-loop observer for the electrical system was successful. The results of estimated states given by the closed-loop observer are the same as the original ones.

The control results are satisfying – the controller reacts correctly to the reference values and the error in steady states is heading to zero. Even noises added to the input to the object do not destabilize the control system. The physical realization of the model was done and the system reacts almost identically for control signals – the controlled outputs are similar.

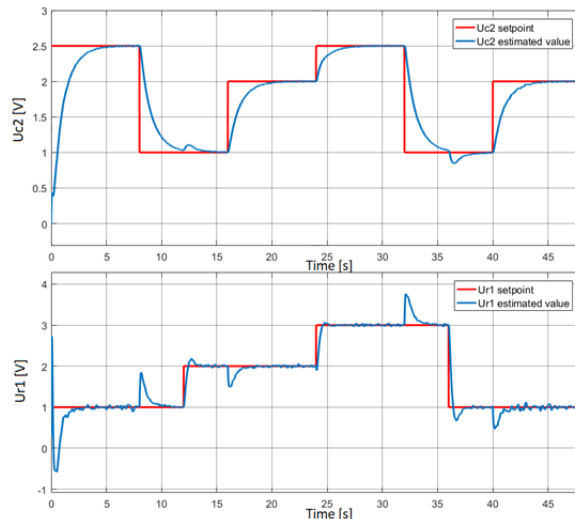


Fig.11. Comparison: set point –  $U_{C2}$ (red),  $U_{R1}$ (red) and control results of the physical model –  $U_{C2}$ (blue),  $U_{R1}$ (blue) with disturbances

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