

## ENERGY LOSS COEFFICIENTS $k_i$ IN A DISPLACEMENT PUMP AND HYDRAULIC MOTOR USED IN HYDROSTATIC DRIVES

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### ABSTRACT

*The article aims at defining and analysing the energy loss coefficients in design solutions of rotating displacement machines, with a piston machine as an example. The energy losses observed in these machines include mechanical loss, volumetric loss, and pressure loss. The scale and relations between these losses in different machines depend on machine design and manufacturing quality, and on operating parameters. The operating parameters, in turn, which affect directly or indirectly the above losses depend on whether the machine works in pump or hydraulic motor regime. The article is also a contribution to the development of a library of  $k_i$  coefficients which define the losses in displacement machines, as the knowledge about these coefficients makes it possible to assess fast and easily the energy efficiency of a machine or drive system at each point of its working area.*

**Keywords:** hydrostatic drive, energy loss rate, pump, hydraulic motor, energy loss coefficients

### INTRODUCTION

The motivation for developing a library of  $k_i$  coefficients defining energy losses in displacement machines was creating the opportunity for fast and easy efficiency assessment of a machine or drive system at each point of its working area (Fig. 1).

In articles [4 ÷ 7], the author has analysed the structure of losses generated in a hydraulic motor as a function of other, loss independent parameters. Fig. 2 presents the loss increase in the direction opposite to that of power flow. It was mentioned in these articles that assessing the energy efficiency of a motor requires prior accurate assessments of mechanical loss, volumetric loss, and pressure loss. In turn, to assess accurately these losses, they should be made depended on relevant parameters which directly determine their scale.

The drive system is not decisive for conditions in which the driven device works. On the contrary, it is the driven device or mechanism which determines the real load or speed of the hydraulic motor used in the hydrostatic drive system. The control structure (of volumetric or throttling type) used for

controlling the hydraulic motor speed should allow the motor to reach the speed required by the driven device.

Working conditions of the device change in time, as a result of loss changes, among other causes [10, 11, 13–15]. Consequently, the working conditions of the hydrostatic system used as the drive also change.

Changes of working parameters of the hydrostatic system are limited by maximal parameter values. In Fig. 1, these maximal values are marked as boundaries of the working area of the system (illustrated by maximal values of motor speed coefficient  $\bar{\omega}_{Mmax}$  and motor load coefficient  $\bar{M}_{Mmax}$ ).

Working conditions of the device can change within the full range of changes of the motor speed coefficient  $\bar{\omega}_M$  and motor load coefficient  $\bar{M}_M$ . The area of changes of these coefficients is limited by the inequalities:  $0 \leq \bar{\omega}_M < \bar{\omega}_{Mmax}$  and  $0 \leq \bar{M}_M < \bar{M}_{Mmax}$ .

The goal of hydrostatic system operation is to meet power requirements of the driven device or machine in the entire range of its working area. It is assumed that the machine can work at each point of the working area, hence the information about its energy efficiency in newly created conditions is

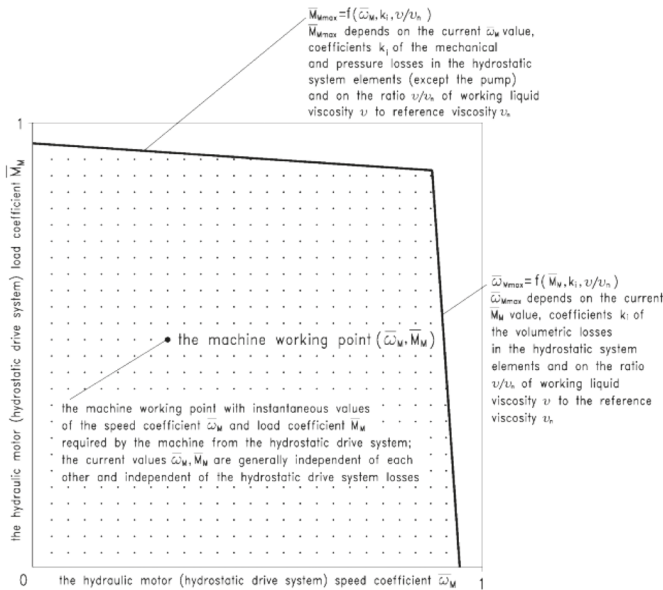


Fig. 1. The range of drive motor speed coefficient  $\bar{\omega}_M$  and motor load coefficient  $\bar{M}_M$  ( $0 \leq \bar{\omega}_M < \bar{\omega}_{Mmax}$ ,  $0 \leq \bar{M}_M < \bar{M}_{Mmax}$ ) in a hydrostatic drive system [9]

important. In practice, however, this information is not available for a machine or system user.

To find the range of the working area of the hydrostatic system driving a machine, we should determine the maximal values of the motor speed coefficient  $\bar{\omega}_{Mmax}$  and motor load coefficient  $\bar{M}_{Mmax}$ . The value of  $\bar{\omega}_{Mmax} = f(\bar{M}_M, k_i, \frac{v}{v_n})$  depends on the current value of the motor load coefficient  $\bar{M}_M$ , the volumetric loss coefficients  $k_i$  in hydrostatic system elements, and the current ratio  $v/v_n$  between the working medium viscosity  $v$  and the reference viscosity  $v_n$ . In turn, the value of  $\bar{M}_{Mmax} = f(\bar{\omega}_M, k_i, \frac{v}{v_n})$  depends on the current value of the motor speed coefficient  $\bar{\omega}_M$ , the mechanical and pressure loss coefficients  $k_i$  in hydrostatic system elements (excluding pump), and the current ratio  $v/v_n$  between the working medium viscosity  $v$  and the reference viscosity  $v_n$ .

To perform the energy balance in the displacement machine (pump or hydraulic motor) used in a hydrostatic drive system, we should add the energy loss rate to the useful (output) power, (see Fig. 2), as it is the output power parameters which are decisive for individual energy loss rates [8].

It is noteworthy that the maximal values of the motor speed coefficient  $\bar{\omega}_{Mmax}$  and the motor load coefficient  $\bar{M}_{Mmax}$ , which define the boundaries of the system's working area, depend on the parameters of the displacement machines used in the system. They also depend on the control structure applied for controlling the hydraulic motor speed, and on losses generated in the remaining elements of the system.

### METHODS TO DETERMINE $k_i$ COEFFICIENTS AND "A" EXPONENTS DESCRIBING THE EFFECT OF INDIVIDUAL PARAMETERS ON ENERGY LOSSES

According to the theory by Z. Paszota [9], the basis for describing losses and energy efficiency of a displacement machine (pump, rotating hydraulic motor) is the diagram of power increase in the drive system in the direction opposite to the power flow direction, (Fig. 2).

The presented formulas make it possible to calculate the value of each individual loss generated in a rotating displacement machine. These formulas contain selected energy loss coefficients  $k_i$ , and "a" exponents that describe the effect of individual parameters on energy losses.

### MODEL OF VOLUMETRIC LOSS IN PUMP AND HYDRAULIC MOTOR

The mathematical model of volumetric loss in pump which takes into account the dependence of the volumetric

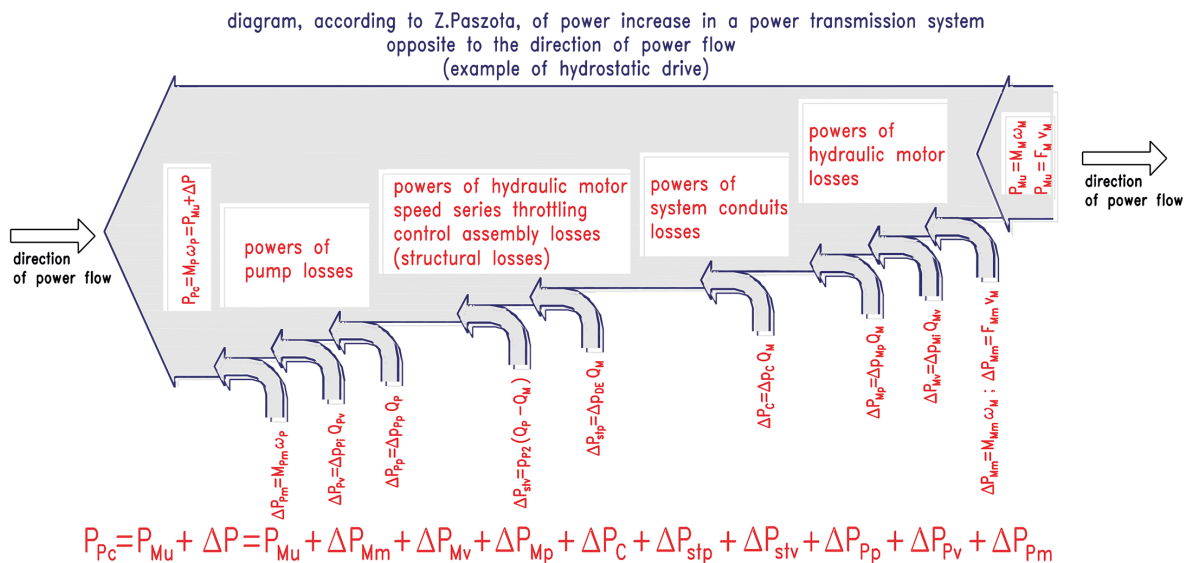


Fig. 2. Diagram of power increase in the hydrostatic drive system, in the direction opposite to that of power flow [9]

loss rate  $Q_{pv}$  in the pump on the induced pressure increase  $\Delta p_{pi}$  in pump's working chambers and on the ratio  $v/v_n$  between oil viscosities has the form [9]:

$$Q_{pv} = k_1 \cdot q_{pt} \cdot n_p \left| \frac{q_{pt}}{\Delta p_{pi} = p_n} \right| \cdot \left( \frac{\Delta p_{pi}}{p_n} \right)^{a_{pv}} \cdot \left( \frac{v}{v_n} \right)^{a_{vv}} \quad (1)$$

where  $k_1$  is the coefficient of the volumetric loss  $Q_{pv}$  generated during one revolution of constant or variable delivery pump shaft for the induced pressure increase  $\Delta p_{pi}$  in the pump equal to the nominal pressure  $p_n$  of the hydraulic system:  $\Delta p_{pi} = p_n$ , and for the reference viscosity  $v_n$  of the working medium. The value of the coefficient  $k_1$  is referenced to the theoretical working volume  $q_{pt}$  of the pump:

$$k_1 = \frac{Q_{pv} \left| \frac{q_{pt}}{\Delta p_{pi} = p_n} \right| \cdot 1}{n_p \left| \frac{q_{pt}}{\Delta p_{pi} = p_n} \right| \cdot v_n} \quad (2)$$

Here,  $q_{pt}$  is the theoretical working volume of the constant delivery pump, and  $n_p$  is the rotational speed of the pump, determined for its theoretical working volume and the temperature corresponding to the reference viscosity  $v_n$ , and for the induced pressure increase  $\Delta p_{pi}$  in pump's working chamber equal to the nominal pressure  $p_n$  of the hydraulic system:  $\Delta p_{pi} = p_n$ .

$(\Delta p_{pi} = p_n)^{a_{pv}}$  represents the effect of the induced pressure increase  $\Delta p_{pi}$  in pump's working chambers on the total amount of losses in the rotating displacement machine. The value of the exponent  $a_{pv}$  varies from  $a_{pv} = 1$  for laminar flow to  $a_{pv} \approx 0,57$  for flow with not fully developed turbulence, and  $a_{pv} = 0,5$  for fully developed turbulent flow. The exact value of the exponent  $a_{pv}$  should be determined experimentally for each pump type.

$v$  is the kinematic viscosity of the working medium (hydraulic oil) for which the volumetric loss rate  $Q_{pv}$  at pump inlet is calculated, and  $v_n$  is the reference kinematic viscosity of the working medium (hydraulic oil), equal to  $v_n = 35 \text{ mm}^2\text{s}^{-1}$ , determined for the atmospheric pressure, i.e. for  $p = 0$ .

$(v/v_n)^{a_{vv}}$  represents the effect of medium viscosity  $v$  on the total amount of volumetric loss in the rotating displacement machine. The value of the exponent  $a_{vv}$  varies from  $a_{vv} = -1$  for laminar leakage flows to  $a_{vv} = -0,14$  for leakage flows with not fully developed turbulence, and  $a_{vv} = 0$  for fully developed turbulent leakage flows. The exact value of the exponent  $a_{vv}$  should be determined experimentally for each displacement pump type.

The volumetric loss rate  $Q_{Mv}$  in working chambers of the hydraulic motor are given by the following mathematical simulation model [9]:

$$Q_{Mv} = k_9 Q_{pt} \left( \frac{\Delta p_{Mi}}{p_n} \right)^{a_{pv}} \left( \frac{n_M}{n_{Mt}} \right)^{a_{nv}} \left( \frac{v}{v_n} \right)^{a_{vv}} \quad (3)$$

where is the coefficient of the volumetric loss  $Q_{Mv}$  determined for: the induced pressure drop  $\Delta p_{Mi}$  equal to the nominal pressure  $p_n$  of the hydraulic system:  $\Delta p_{Mi} = p_n$ , the reference viscosity  $v_n$  of the working medium, and the theoretical

rotational speed  $n_{Mt}$  of motor shaft. The value of the coefficient  $k_9$  is referenced to the theoretical pump delivery  $Q_{pt}$ :

$$k_9 = \frac{Q_{Mv} | q_{Mi} ; \Delta p_{Mi} = p_n ; n_{Mt} ; v_n}{Q_{pt}} \quad (4)$$

Here,  $(\Delta p_{Mi} = p_n)^{a_{pv}}$  represents the effect of the induced pressure drop  $\Delta p_{Mi}$  in motor's working chambers on the total amount of volumetric loss in the rotating displacement machine. The value of the exponent  $a_{pv}$  varies from  $a_{pv} = 1$  for laminar flow to  $a_{pv} \approx 0,57$  for flow with not fully developed turbulence, and  $a_{pv} = 0,5$  for fully developed turbulent flow. The exact value of the exponent  $a_{pv}$  should be determined experimentally for each motor type.

$(n_M / n_{Mt})^{a_{nv}}$  represents the effect of the rotational speed  $n_M$  of the motor shaft on the volumetric loss rate  $Q_{Mv}$ . The exact value of the exponent  $a_{nv}$  should be determined experimentally for each type of hydraulic motor.

$(v/v_n)^{a_{vv}}$  represents the effect of medium viscosity  $v$  on the total amount of volumetric loss in the rotating displacement machine. The value of the exponent  $a_{vv}$  varies from  $a_{vv} = -1$  for laminar leakage flows to  $a_{vv} \approx -0,14$  for flows with not fully developed turbulence, and  $a_{vv} = 0$  for fully developed turbulent flows. The exact value of the exponent  $a_{vv}$  should be determined experimentally for each motor type.

The models (2.1) and (2.3) of volumetric loss in a displacement machine make it possible to determine very accurately the dependence of the rates  $Q_{pv}$  or  $Q_{Mv}$  in the displacement machine on the induced pressure increase  $\Delta p_{pi}$  or pressure drop  $\Delta p_{Mi}$  in machine's working chambers and on oil viscosity  $v$ , as well as on the rotational speed  $n_M$  of motor shaft in the case of hydraulic motor. The models utilise the coefficients  $k_1$  (in pump) and  $k_9$  (in hydraulic motor) of volumetric loss rates generated at the induced pressure increase  $\Delta p_{pi}$  or pressure drop  $\Delta p_{Mi}$  in chambers equal to the nominal pressure  $p_n$  of the hydrostatic system in which the pump is installed:  $\Delta p_{pi} = p_n$ , or in which the motor is installed:  $\Delta p_{Mi} = p_n$ . The obtained loss values are referenced to the theoretical working volume  $q_{pt}$  of the pump, or, in the case of hydraulic motor, to the theoretical delivery  $Q_{pt}$  of the pump.

## MODEL OF PRESSURE LOSS IN PUMP AND HYDRAULIC MOTOR

The mathematical simulation formula which determines the pressure loss  $\Delta p_{pp}$  in pump channels for varying viscosity  $v$  has the form [9]:

$$\Delta p_{pp} = k_3 p_n \left( \frac{Q_p}{Q_{pt}} \right)^{a_{qp}} \left( \frac{v}{v_n} \right)^{a_{vp}} \quad (5)$$

where  $k_3$  is the coefficient of the pressure loss  $\Delta p_{pp}$  in pump's inner channels and distributor (if exists). This coefficient is determined for the pump delivery  $Q_p$  equal to its theoretical delivery  $Q_{pt}$  and for the reference viscosity  $v_n$  of the working medium. The value of the coefficient  $k_3$  is referenced to the nominal pressure  $p_n$  of the system:

$$k_3 = \frac{\Delta p_{pp}|_{Q_p=Q_{pt}}}{p_n}, \quad (6)$$

Here,  $(Q_p / Q_{pt})^{a_{Qp}}$  represents the effect of the flow rate  $Q_p$  of the liquid medium (pump delivery  $Q_p$ ) in the channels on the pressure loss  $\Delta p_{pp}$  in the pump. The value of the exponent  $a_{Qp}$  varies from  $a_{Qp} = 1$  for laminar flow to  $a_{Qp} \approx 1,75$  for flow with not fully developed turbulence, and to  $a_{Qp} = 2$  for fully developed turbulent flow. The exact value of the exponent  $a_{Qp}$  should be determined experimentally for each type of displacement pump.

$(\nu/\nu_n)^{a_{vp}}$  is the effect of medium viscosity  $\nu$  on the total amount of pressure loss in the rotating displacement machine. The value of the exponent  $a_{vp}$  varies from  $a_{vp} = 1$  for laminar flow to  $a_{vp} \approx 0,25$  for flow with not fully developed turbulence, and to  $a_{vp} = 0$  for fully developed turbulent flow. The exact value of the exponent  $a_{vp}$  should be determined experimentally for each type of displacement pump.

The model of pressure loss in hydraulic motor has the form [9]:

$$\Delta p_{Mp} = k_8 p_n \left( \frac{Q_M}{Q_{pt}} \right)^{a_{Qp}} \left( \frac{\nu}{\nu_n} \right)^{a_{vp}}, \quad (7)$$

with the coefficient:

$$k_8 = \frac{\Delta p_{Mp}|_{Q_M=Q_{pt}, \nu_n}}{p_n}, \quad (8)$$

determining the pressure loss  $\Delta p_{Mp}$  generated in inner channels and distributor of the hydraulic motor at absorbing capacity  $Q_M$  of the motor equal to the theoretical delivery  $Q_{pt}$  of the pump. The value of the coefficient  $k_8$  is referenced to the nominal pressure  $p_n$  of the system in which the hydraulic system is installed.

$(Q_M / Q_{pt})^{a_{Qp}}$  represents the effect of the absorbing capacity  $Q_M$  of the liquid medium in the channels on the pressure loss  $\Delta p_{Mp}$  in the rotating hydraulic motor. The value of the exponent  $a_{Qp}$  varies from  $a_{Qp} = 1$  for laminar flow to  $a_{Qp} \approx 1,75$  for flow with not fully developed turbulence, and to  $a_{Qp} = 2$  for fully developed turbulent flow. The exact value of the exponent  $a_{Qp}$  should be determined experimentally for each motor type.

$(\nu/\nu_n)^{a_{vp}}$  represents the effect of medium viscosity  $\nu$  on the total amount of pressure loss in the rotating displacement machine. The exponent  $a_{vp}$  varies from  $a_{vp} = 1$  for laminar flow to  $a_{vp} \approx 0,25$  for flow with not fully developed turbulence, and to  $a_{vp} = 0$  for fully developed turbulent flow. The exact value of the exponent  $a_{vp}$  should be determined experimentally for each motor type.

The type of the medium flow developed in displacement machine channels is mainly affected by the local flow resistance (flow direction and cross section disturbances) which depends on structural design and dimensions of the machine. The created pressure loss models (2.5) and (2.7) can be applied in analysing an individual machine design to determine experimentally the exact values of the exponent  $a_{vp}$  describing the effect of viscosity, and the exponent  $a_{Qp}$  describing the effect of flow rate on the pressure loss in machine channels. The models utilise the pressure loss coefficients  $k_3$  (in pump) and  $k_8$  (in hydraulic

motor) determined for the flow rate  $Q_p$  or  $Q_M$  in the channel equal to the nominal pump delivery  $Q_{pt}$ , and referenced to the nominal pressure  $p_n$  of the system in which the pump is installed.

## MODEL OF MECHANICAL LOSS IN PUMP AND HYDRAULIC MOTOR

The mathematical model describing the mechanical loss moment  $M_{Pm}$  in the pump with geometric (variable) delivery  $q_{pgv}$  ( $q_{pgv} = b_p q_{pt}$ ) per shaft revolution has the form:

$$\begin{aligned} M_{Pm|\Delta p_{pi}, b_p, \nu} &= (k_{4.1.1} + k_{4.1.2} b_p) M_{Pt} \left( \frac{\nu}{\nu_n} \right)^{a_{vm}} + k_{4.2} M_{Pt} b_p \frac{\Delta p_{pi}}{p_n} = \\ &= \left[ (k_{4.1.1} + k_{4.1.2} b_p) \left( \frac{\nu}{\nu_n} \right)^{a_{vm}} + k_{4.2} b_p \frac{\Delta p_{pi}}{p_n} \right] M_{Pt} = \\ &= \left[ (k_{4.1.1} + k_{4.1.2} b_p) \left( \frac{\nu}{\nu_n} \right)^{a_{vm}} + k_{4.2} b_p \frac{\Delta p_{pi}}{p_n} \right] \frac{q_{pt} p_n}{2\pi}, \quad (9) \end{aligned}$$

Here, the coefficient  $k_{4.1.1}$  expresses the ratio of the mechanical loss moment  $M_{Pm|\Delta p_{pi}=0, b_p=0, \nu_n}$  of the unloaded pump, determined for the delivery  $q_{pgv}$  per pump shaft revolution (pump delivery change coefficient  $b_p$ ) equal to zero:  $q_{pgv} = 0$  ( $b_p = 0$ ) and the working medium reference viscosity  $\nu_n$ , to the theoretical moment  $M_{Pt}$  of the pump:

$$k_{4.1.1} = \frac{M_{Pm|\Delta p_{pi}=0, b_p=0, \nu_n}}{M_{Pt}} = \frac{M_{Pm|\Delta p_{pi}=0, b_p=0, \nu_n}}{\frac{q_{pt} p_n}{2\pi}}, \quad (10)$$

The coefficient  $k_{4.1.2}$  expresses the ratio of the difference between the mechanical loss moment  $M_{Pm|\Delta p_{pi}=0, b_p=1, \nu_n}$  in the unloaded pump, determined for the pump delivery  $q_{pgv}$  per pump shaft revolution (pump delivery change coefficient  $b_p$ ) equal to the theoretical working delivery  $q_{pt}$  of the pump,  $q_{pgv} = q_{pt}$  ( $b_p = 1$ ) and the mechanical loss moment  $M_{Pm|\Delta p_{pi}=0, b_p=0, \nu_n}$  in the unloaded pump, determined for the pump delivery  $q_{pgv}$  per pump shaft revolution equal to zero:  $q_{pgv} = 0$  ( $b_p = 0$ ) and the working medium reference viscosity  $\nu_n$ , to the theoretical moment  $M_{Pt}$  of the pump:

$$\begin{aligned} k_{4.1.2} &= \frac{M_{Pm|\Delta p_{pi}=0, b_p=1, \nu_n} - M_{Pm|\Delta p_{pi}=0, b_p=0, \nu_n}}{M_{Pt}} = \\ &= \frac{M_{Pm|\Delta p_{pi}=0, b_p=1, \nu_n} - M_{Pm|\Delta p_{pi}=0, b_p=0, \nu_n}}{\frac{q_{pt} p_n}{2\pi}}, \quad (11) \end{aligned}$$

The coefficient  $k_{4.2}$  expresses the ratio of the difference between the mechanical loss moment  $M_{Pm|\Delta p_{pi}=p_n, b_p=1, \nu_n}$  in the pump loaded with nominal pressure and the mechanical loss moment  $M_{Pm|\Delta p_{pi}=0, b_p=1, \nu_n}$  in the unloaded pump at the working medium reference viscosity  $\nu_n$ , to the theoretical moment  $M_{Pt}$  of the pump:

$$\begin{aligned}
k_{4.2} &= \frac{\Delta M_{Pm|\Delta p_{pi}, b_p, v_n}}{M_{Pi}} = \frac{\Delta M_{Pm|\Delta p_{pi}, b_p, v_n}}{b_p q_{Pt} \Delta p_{Pi}} = \frac{\Delta M_{Pm|\Delta p_{pi}, b_p=1, v_n}}{q_{Pt} \Delta p_{Pi}} = \\
&= \frac{M_{Pm|\Delta p_{pi}=p_n, b_p=1, v_n} - M_{Pm|\Delta p_{pi}=0, b_p=1, v_n}}{\frac{q_{Pt} P_n}{2\Pi}} = \\
&= \frac{M_{Pm|\Delta p_{pi}=p_n, b_p=1, v_n} - M_{Pm|\Delta p_{pi}=0, b_p=1, v_n}}{M_{Pt}}, \quad (12)
\end{aligned}$$

Here,  $(v/v_n)^{a_{vm}}$  represents the effect of medium viscosity  $v$  on the mechanical loss moment in the displacement piston machine with medium filling the casing. The value of the exponent  $a_{vm}$  should be determined experimentally for each type of displacement pump.

$\Delta p_{Pi}/p_n$  represents the effect of the induced pressure increase  $\Delta p_{Pi}$  in pump's working chambers on the total amount of mechanical loss in the rotating displacement machine. According to [9], the assumed value of the exponent  $a_{pm}$  is 1.

The mathematical model of the mechanical loss moment  $M_{Mm}$  in the rotating hydraulic motor with geometrical (variable) absorbing capacity  $q_{Mgv}$  per shaft revolution has the form [9]:

$$\begin{aligned}
M_{Mm|M_M, n_M, b_M, v} &= \left( k_{7.1.1} + k_{7.1.2} \frac{n_M}{n_{Mt}} b_M \right) M_{Mt} \left( \frac{v}{v_n} \right)^{a_{vm}} + k_{7.2} M_M = \\
&\left( k_{7.1.1} + k_{7.1.2} \frac{n_M}{n_{Mt}} b_M \right) \frac{q_{Mt} P_n}{2\pi} \left( \frac{v}{v_n} \right)^{a_{vm}} + k_{7.2} M_M, \quad (13)
\end{aligned}$$

where the coefficient  $k_{7.1.1}$  expresses the ratio of the mechanical loss moment  $M_{Mm}$  in unloaded motor ( $M_M = 0$ ), determined for the motor shaft rotational speed equal to zero:  $n_M = 0$ , theoretical capacity per motor shaft revolution equal to  $q_{Mt}$  (coefficient  $b_M$  of absorbing capacity change per motor revolution:  $b_M = 1$ ), and working medium viscosity equal to  $v_n$ , to the theoretical moment  $M_{Mt}$  of the motor:

$$k_{7.1.1} = \frac{M_{Mm|M_M=0, n_M=0, b_M=1, v_n}}{M_{Mt}}, \quad (14)$$

The coefficient  $k_{7.1.2}$  expresses the ratio of the difference – in unloaded motor:  $M_M = 0$ , at the theoretical capacity  $q_{Mt}$  per motor shaft revolution ( $b_M = 1$ ) and the working medium reference viscosity  $v_n$  – between the mechanical loss moment  $M_{Mm|M_M=0, n_M=n_{Mt}, b_M=1, v_n}$  determined for the theoretical motor shaft rotational speed  $n_{Mt}$  and the mechanical loss moment  $M_{Mm|M_M=0, n_M=0, b_M=1, v_n}$  determined for the motor shaft rotational speed  $n_M$  equal to zero ( $n_M = 0$ ), to the theoretical moment  $M_{Mt}$  of the motor:

$$k_{7.1.2} = \frac{M_{Mm|M_M=0, n_M=n_{Mt}, b_M=1, v_n} - M_{Mm|M_M=0, n_M=0, b_M=1, v_n}}{M_{Mt}}, \quad (15)$$

The coefficient  $k_{7.2}$  expresses the ratio of the difference – at the theoretical capacity  $q_{Mt}$  per motor shaft revolution

( $b_M = 1$ ), the theoretical motor shaft rotational speed  $n_{Mt}$ , and the working medium reference viscosity  $v_n$  – between the mechanical loss moment  $M_{Mm|M_M=M_{Mt}, n_M=n_{Mt}, b_M=1, v_n}$  in the motor loaded with the theoretical moment  $M_{Mt}$  and the mechanical loss moment  $M_{Mm|M_M=0, n_M=n_{Mt}, b_M=1, v_n}$  in the unloaded motor to the theoretical moment  $M_{Mt}$  of the motor:

$$k_{7.2} = \frac{M_{Mm|M_M=M_{Mt}, n_M=n_{Mt}, b_M=1, v_n} - M_{Mm|M_M=0, n_M=n_{Mt}, b_M=1, v_n}}{M_{Mt}}, \quad (16)$$

The ratio  $n_M/n_{Mt}$  represents the effect of the rotational speed on the total amount of mechanical loss in the rotating hydraulic motor. According to [9], the assumed value of the exponent  $a_{nm}$  is 1.

$(v/v_n)^{a_{vm}}$  represents the effect of medium viscosity  $v$  on the mechanical loss moment in the displacement piston motor with medium filling the casing. The exact value of the exponent  $a_{vm}$  should be determined for each type of rotating hydraulic motor.

The mathematical model (2.9) describing the mechanical loss in pump makes it possible to determine precisely the dependence of the mechanical loss moment  $M_{Pm}$  on: the induced pressure increase  $\Delta p_{Pi}$  in pump's working chambers, the pump delivery  $q_{Pi}$  per pump shaft revolution, and the oil viscosity  $v$ . The model utilises the mechanical loss coefficients  $k_{4.1}$ ,  $k_{4.1.1}$ ,  $k_{4.1.2}$ , and  $k_{4.2}$ , which provides opportunities for performing simulation tests.

The mathematical model (2.13) describing the mechanical loss moment  $M_{Mm}$  in rotating hydraulic motor makes it possible to determine precisely the dependence of this moment on: the motor shaft rotational speed  $n_M$ , the oil viscosity  $v$ , and the capacity  $q_{Mt}$  per motor shaft revolution. The model utilises the mechanical loss coefficients  $k_{7.1.1}$ ,  $k_{7.1.2}$ , and  $k_{7.2}$ , which provides opportunities for performing simulation tests.

## EXAMINED PUMPS AND MOTORS

The examined pumps and hydraulic motors are shown in Figs. 3 ÷ 7.

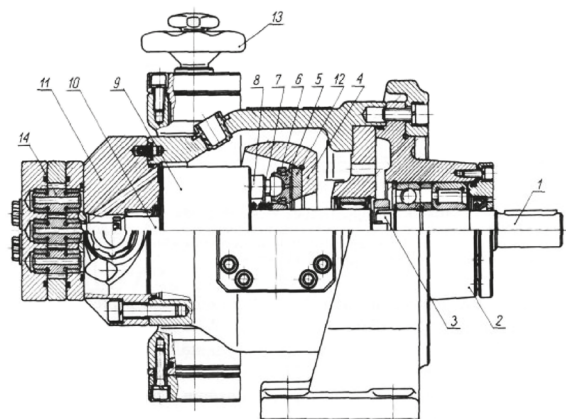


Fig. 3. PTO22-25 variable displacement pump, PZL-HYDRAL [11]; 1 – drive shaft, 2 – attachment drive, 3 – clutch, 4 – swivel disc, 5 – sliding foot, 6 – separator, 7 – spring, 8 – plunger, 9 – cylinder block, 10 – main shaft, 11 – cover, 12 – sliding disc, 13 – control unit

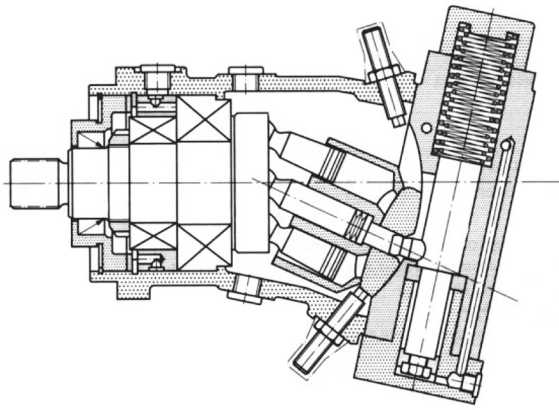


Fig. 4. Variable displacement piston pump (A7V.58R.DR.1.R.P.F.00 by HYDROMATIK) [3]

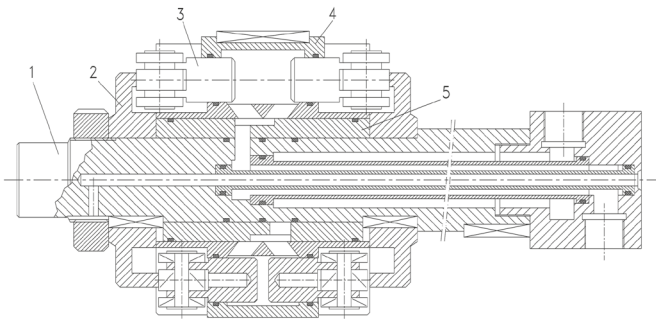


Fig. 5. Structure diagram of a low-speed hydraulic high-torque motor type SWSB with axial arrangement of pistons, HYDROSTER [8]; 1 – drive shaft; 2 – cam equipped with humps; 3 – plunger equipped with bearings; 4 – rotary cylinder block; 5 – distributor bushing

## COMPARING ENERGY LOSS COEFFICIENTS IN PISTON PUMPS

**Table 1** collates the energy loss coefficients  $k_i$  obtained for the examined piston pumps as a result of applying a new perspective to loss and efficiency assessment in rotating displacement machines used in hydrostatic drive and control systems [2, 3, 4, 10]. These coefficients were calculated neglecting the compressibility of the working medium. Constructional solutions of the examined pumps are shown in Figs. 3 and 4.

The coefficient  $k_i$  of the volumetric loss rate  $Q_{pv}$  provides the information about the scale of leakage flows taking place in the displacement machine. The volumetric efficiency  $\eta_{pv}$  can be calculated by subtracting the obtained value of coefficient  $k_i$  from 1. Hence, the volumetric efficiency of the PTOZ2-25 pump was 94,4%. At the same time, the efficiency of the A7V.58R.DR.1.R.P.F.00 pump with  $q_{pt} = 58,227 \text{ cm}^3$  was equal to 93,5%, and increased to 94,3% when its delivery was decreased to  $q_{pt} = 32,20 \text{ cm}^3$ .

The coefficient  $k_3$  of pressure loss  $\Delta p_{pp}$  provides the information about the scale of losses generated in machine's inner channels and distributor. These losses are mainly the effect of local pressure losses resulting from direction and velocity changes of the flowing medium. The pressure efficiency  $\eta_{pp}$  can be calculated by subtracting the obtained value of coefficient  $k_3$  from 1. The pressure efficiency of the examined pumps was relatively high and ranged about  $\sim 99,9\%$ .

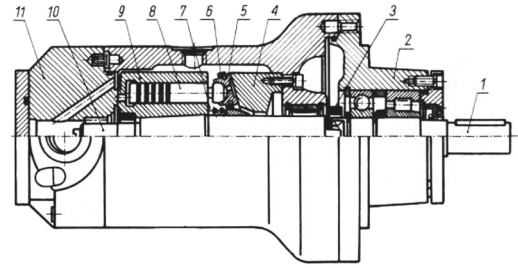


Fig. 6. Hydraulic motor PTO2-16 with constant rotational speed, PZL-HYDRAL [11]: 1 – drive shaft, 2 – drive starter, 3 – clutch, 4 – tread slope, 5 – sliding foot, 6 – separator assembly, 7 – spring, 8 – piston, 9 – rotor, 10 – main shaft, 11 – cover

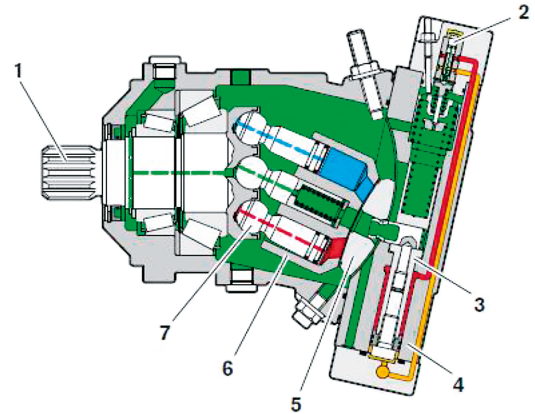


Fig. 7. A6VM variable displacement axial piston hydraulic motor by Bosch Rexroth [12]: 1 – drive shaft, 2 – control piston, 3 – control piston stroke, 4 – body in which the control piston is positioned, 5 – divider, 6 – cylinder block, 7 – piston

The coefficient  $k_{4,1}$  of the mechanical loss moment  $M_{Mm}$  (being the sum of coefficients  $k_{4,1}$  and  $k_{4,2}$ ) provides the information about losses caused by friction between construction elements (bearings, for instance) and between the liquid medium filling the crank chamber and the cylinder block (in piston machines). The largest losses were recorded in the PTOZ2-25 pump.

The coefficient  $k_{4,2}$  of the mechanical loss moment  $M_{Mm}$  provides the information about the scale of the mechanical loss moment increase  $\Delta M_{Pm|\Delta p_{pi}, b, v}$  in the pump, as a result of loading its structural elements with the induced moment  $M_{pi}$  generated by the induced pressure increase  $\Delta p_{Mi}$  in pump's working chambers. The largest losses were recorded in the PTOZ2-25 pump.

The mechanical efficiency  $\eta_{pm}$  of the pump can be calculated by subtracting the obtained sum of coefficients  $k_{4,i}$  from 1. Hence, the mechanical efficiency of the PTOZ2-25 pump was 87,7%, while that of the A7V.58R.DR.1.R.P.F.00 pump with  $q_{pt} = 58,227 \text{ cm}^3$  was 96,4%, and of the same pump with  $q_{pt} = 32,20 \text{ cm}^3$  was 94,6%.

## COMPARING ENERGY LOSS COEFFICIENTS IN PISTON MOTORS

**Table 2** collates the energy loss coefficients  $k_i$  obtained for the examined piston motors as a result of applying a new perspective to loss and efficiency assessment in rotating

Tab. 1. Energy loss coefficients  $k_i$  in displacement piston pumps [2÷4, 10]

		PTO2-25	A7V*	A7V**
	$v_n$ [mm <sup>2</sup> s <sup>-1</sup> ]	35	35	35
	$q_{Pt}$ [m <sup>3</sup> ]	$16,50 \cdot 10^{-6}$	$32,20 \cdot 10^{-6}$	$58,23 \cdot 10^{-6}$
	$n_{Pn}$ [s <sup>-1</sup> ]	16,67	25	25
	$p_n$ [MPa]	16	16	32
	$P_{Pc}$ [kW]	4,40	12,88	23,10
$Q_{Pv}$	$k_{l=}$	0,056	0,057	0,065
	$a_{pv=}$	-	-	0,97
	$a_{vv=}$	-	-	-0,30
	$k_{3=}$	0,001	0,002	0,001
$\Delta p_{Pp}$	$k_{3=}$	0,001	0,002	0,001
	$a_{Qp=}$	1,59	-	1,76
	$a_{vp=}$	-	-	0,26
$M_{Pm}$	$k_{4,1=}$	0,076	0,039	0,021
	$k_{4,1,1=}$	0,063	-	0,014
	$k_{4,1,2=}$	0,013	-	0,007
	$k_{4,2=}$	0,047	0,015	0,015
	$a_{vm=}$	-	-	0,316

\* – coefficients  $k_i$  and exponents “a” determined for nominal operating parameters of the system in which the A7V.58.DR.1.R.P.F.00 pump was used

\*\* – coefficients  $k_i$  and exponents “a” determined for nominal parameters of the A7V.58.DR.1.R.P.F.00 pump

displacement machines used in hydrostatic drive and control systems [1, 2, 4]. Constructional solutions of the examined motors are shown in Figs. 5 ÷ 7.

The coefficient  $k_{7,1}$  of the mechanical loss moment  $M_{Mm}$  (being the sum of coefficients  $k_{7,1,1}$  and  $k_{7,1,2}$ ) provides the information about losses caused by friction between construction elements (bearings, for instance) and between the liquid medium filling the crank chamber and the cylinder block (in piston machines). The largest losses were recorded in the PTO2-16 motor.

The coefficient  $k_{7,2}$  of the mechanical loss moment  $M_{Mm}$  provides the information about the scale of mechanical loss moment increase  $\Delta M_{Mm}$  in the motor as a result of the increase in motor shaft loading. The largest losses were recorded in the SWSB-63 motor.

The mechanical efficiency  $\eta_{Mm}$  of the hydraulic motor can be calculated by subtracting the obtained sum of coefficients  $k_{7,1}$  from 1. Hence, the mechanical efficiency of the SWSB-63 motor was 96,8%, while that of the PTO2-16 motor was 94,0% and of the A6VM160 motor was 96,2%.

The coefficient  $k_8$  of the pressure loss  $\Delta p_{Pp}$  provides the information about the scale of losses generated in machine's inner channels and distributor. These losses are mainly the effect of local pressure losses resulting from direction and velocity changes of the flowing medium. The pressure

Tab. 2. Energy loss coefficients  $k_i$  in displacement piston motors [1, 2, 4]

		SWSB-63	PTO2-16	A6VM
	$v_n$ [mm <sup>2</sup> s <sup>-1</sup> ]	26	35	22
	$q_{Mt}$ [m <sup>3</sup> ]	$639 \cdot 10^{-6}$	$12,74 \cdot 10^{-6}$	$160 \cdot 10^{-6}$
	$n_{Mn}$ [s <sup>-1</sup> ]	2,67	21,54	50
	$p_n$ [MPa]	6,2	16	40
	$P_{Mc}$ [kW]	10,6	4,39	320
$M_{Mm}$	$k_{7,1=}$	0,032	0,060	0,038
	$k_{7,1,1=}$	0,018	0,035	0,002
	$k_{7,1,2=}$	0,014	0,025	0,036
	$a_{vm=}$	0,15	-	-
	$k_{7,2=}$	0,076	0,033	0,036
$\Delta p_{Mp}$	$k_{8=}$	0,039	0,001	0,002
	$a_{Qp=}$	1,76	1,87	1,76
	$a_{vp=}$	0,25	-	-
$Q_{Mv}$	$k_{9=}$	0,053	0,047	0,054
	$a_{pv=}$	1,62	0,92	-
	$a_{vv=}$	-0,62	-	-
	$a_{nv=}$	0,12	-	-

efficiency  $\eta_{Mv}$  can be calculated by subtracting the obtained value of coefficient  $k_8$  from 1. The highest pressure efficiency, equal to 99,0%, was recorded for the PTO2-16 motor.

The coefficient  $k_9$  of the volumetric loss rate  $Q_{Mv}$  provides the information about the scale of leakage flows in the displacement machine. The volumetric efficiency  $\eta_{Mv}$  can be calculated by subtracting the obtained value of coefficient  $k_9$  from 1. The lowest volumetric efficiency, approximately equal to 95,5%, was recorded for A6VM and SWSB-63 motors.

## CONCLUSIONS

The presented research is the next step in scientific development of the author involved in creating a library of energy loss coefficients  $k_i$ . This work discusses its practical implementation for assessing the quality of the examined machines.

Each energy loss coefficient  $k_i$  describes a relative value of individual loss in a hydrostatic system element (pump, hydraulic motor, but also in a conduit and the motor speed throttle control unit).

The  $k_i$  coefficients provide the opportunity for assessing proportions and scale of losses, as well as the resulting (volumetric, pressure, and/or mechanical) energy efficiency of an element, being the result of losses generated at the reference viscosity and nominal operating pressure  $p_n$  of the system in which the element is installed.

The information on exact values of the energy loss coefficients  $k_i$  for individual loss types provides opportunities for determining losses and (volumetric, pressure, mechanical, and total) energy efficiency of an element working in the drive system, as well as the total efficiency of the system with a given speed control structure as a function of speed coefficient  $\bar{\omega}_M$  and load coefficient  $\bar{M}_M$  of the hydraulic motor referred to in [9].

Tables 1 and 2 collate the obtained values of  $k_i$  coefficients and such exponents as:  $a_{Qp}$ ,  $a_{vp}$ ,  $a_{pv}$ ,  $a_{vv}$ ,  $a_{vm}$ ,  $a_{nv}$  which describe the effect of such parameters as: flow rate, pressure, rotational speed, and viscosity on individual losses. The values of these coefficients were obtained using the power sum method described in [4, 6], among other tools.

The values of such exponents as:  $a_{Qp}$ ,  $a_{vp}$ ,  $a_{pv}$ ,  $a_{vv}$  provide the information about the type of flow taking place in the examined machine.

The exact value of the exponent  $a_{vm}$  could only be calculated for two machines. It was equal to  $a_{vm} = 0,316$  for the A7V.58.DR.1.R.P.F.00 pump, and to  $a_{vm} = 0,148$  for the SWSB-63 motor.

The article compares the values of energy loss coefficients  $k_i$  for the A7V.58.DR.1.R.P.F.00 pump examined at its nominal parameters [3] and in the conditions of nominal operation of the entire system in which it was installed [10]. The obtained  $k_i$  values are different, as the pump working in conditions which differ from its nominal parameters generates higher energy loss coefficients  $k_i$  and, consequently, has lower efficiency.

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