Expedited Optimization of Antenna Input Characteristics with Adaptive Broyden Updates

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Structured Abstract

Purpose

A technique for accelerated design optimization of antenna input characteristics is developed and comprehensively validated using real-world wideband antenna structures. Comparative study using a conventional trust-region algorithm is provided. Investigations of the effects of the algorithm control parameters are also carried out.

Design/methodology/approach

A methodology is introduced that is based on replacing finite differentiation (FD) by a combination of FD and selectively used Broyden updating formula for antenna response Jacobian estimations. The updating formula is utilized for directions that are sufficiently well aligned with the design relocation that occurred in the most recent algorithm iteration. This allows for significant reduction of the number of full-wave

electromagnetic simulations necessary for the algorithm to converge, hence leads to the reduction of the overall design cost.

Findings

Incorporation of the updating formulas into the Jacobian estimation process in a selective manner considerably reduces the computational cost of the optimization process without compromising the design quality. The algorithm proposed in the paper can be used to speed up direct optimization of the antenna structures as well as surrogate-assisted procedures involving variable-fidelity models.

Research limitations/implications

The presented study sets a direction for further studies on accelerating procedures for local optimization of antenna structures. Further investigations on the effects of the control parameters on the algorithm performance are necessary along with the development of means to automate the algorithm setup for the particular antenna structure, especially from the point of view of the search space dimensionality.

Originality/value

The proposed algorithm proved useful for reduced-cost optimization of antennas and has been demonstrated to outperform conventional algorithms. To our knowledge, this is one of the first attempts to address the problem in this manner. In particular, it goes beyond traditional approaches, especially by combining various sensitivity estimation update measures in an adaptive way.

Abstract

Simulation-driven adjustment of geometry and/or material parameters is a necessary step in the design of contemporary antenna structures. Due to their topological complexity, other means, such as supervised parameter sweeping, does not usually lead to satisfactory results. On the other hand, rigorous numerical optimization is computationally expensive due to a high cost of underlying full-wave electromagnetic (EM) analyses, otherwise required to assess antenna performance in a reliable manner. Design closure normally requires a local search, often carried out by means of gradient-based procedures. In this work, accelerated trust-region gradient-search algorithm is proposed for expedited optimization of antenna structures. In our approach, finite differentiation conventionally used to estimate the antenna response Jacobian is replaced, for selected variables, by a rank-one Broyden updating formula. The selection of variables is governed by the alignment between the direction of the recent design relocation and the coordinate system axes. Operation and performance of the algorithm is demonstrated using a set of benchmark wideband antennas. Comprehensive numerical validation indicates significant computational savings of up to 70 percent that can be achieved without compromising the design quality in a significant manner.

1. Introduction

Antennas are fundamental components of wireless communications systems with wide applications in telecommunication, remote sensing, radars, biomedicine, and many others (Balanis, 2008; Elliot, 2003; Hansen, 2009; Volakis, 2007). Contemporary antenna engineering heavily relies on full-wave electromagnetic (EM) simulation tools. In the past, EM analysis was mostly used for design verification. Nowadays, EM-driven design has become the standard and the necessity. The primary reason is that due to topological complexity of antenna systems, design-ready theoretical models (analytical or equivalent network ones) are no longer available. Also, antenna structures have to be analyzed together with their immediate environment such as connectors, housing/radomes, installation fixtures, feeding structures, or other radiating components (Koziel and Ogurtsov, 2014; Nair and Jha, 2014). Enlarging computational domains as well as utilization of highly-graded meshes for electrically large structures (e.g., antenna array apertures (Koziel and Ogurtsov, 2014; Mailloux, 2005)) increases the computational cost of the antenna simulation. Another factor are various geometrical modifications utilized to improve the antenna performance, e.g., to permit size reduction (Nosrati and Tavassolian, 2017; Zhou and Cheung, 2017), to increase element isolation in MIMO (multiple-input multiple-output) structures (Iqbal *et al.*, 2018; Zhang and Pedersen, 2016), or to enable additional functionality such as band notches (Sarkar *et al.*, 2014), results in increasing the number of parameters that have to be adjusted.

The aforementioned factors make simulation-driven design of antennas a challenging process. The major bottleneck is a high computational cost of the design optimization, often prohibitive when using conventional methods (both local (Bekasiewicz and Koziel, 2016; Nocedal and Wright, 2006), and global, typically population-based metaheuristics (Al-Azza *et al.*, 2016; Chamaani *et al.*, 2011; Chiu and Chen, 2015; Goudos *et al.*, 2011; Lalbakhsh *et al.*, 2017; Soltani *et al.*, 2018)). Another issue is the necessity of handling multiple performance figures pertinent to impedance matching, directivity, gain, axial ratio, etc. Reduction of the computational overhead can be achieved by various means, among others, adjoint sensitivities (Ghassemi, *et al.*, 2013; Koziel and Bekasiewicz, 2016a), or, more and more popular, surrogate-based optimization (SBO) techniques (Jacobs, 2016; Koziel, 2015; Koziel and Ogurtsov, 2014). SBO procedures exploit a fast auxiliary representation of the structure under design (referred to as a surrogate model) which may be a data-driven model (de Villiers *et al.*, 2017) or a physics-based one (Koziel and Bekasiewicz, 2016b). In the case of antennas, the latter is typically the only option when higher-dimensional parameters spaces (beyond a few dimensions) need to be considered. Physics-based antenna surrogates are normally constructed from an underlying low-fidelity model which is obtained through coarse-discretization EM simulations (Koziel and Bekasiewicz, 2016b). A consequence is that the surrogate model optimization step of the SBO procedure still requires multiple EM simulations (at the coarse-mesh level).

No matter what particular design closure approach is utilized (e.g., direct optimization or iterative prediction-correction SBO schemes (Al-Azza *et al.*, 2016; Bekasiewicz and Koziel, 2016; Chamanni *et al.*, 2011; Chiu and Chen, 2015; Ghassemi, *et al.*, 2013; Goudos *et al.*, 2011; Koziel and Bekasiewicz, 2016b; Koziel and Ogurtsov, 2014; Lalbakhsh *et al.*, 2017; Soltani *et al.*, 2018)), it is the reduction of the number of EM simulations that is a key to improve the overall efficiency of the EM-driven design process. In this paper, an accelerated trust-region (TR) gradient search algorithm for a reduced-cost antenna optimization is introduced. The proposed technique involves reduced-cost updates of the antenna response Jacobian involving a Broyden formula (Koziel, *et al.*, 2010), adaptively adjusted based on the alignment between the recent design relocation direction and the coordinate system axes. The performance of the procedure is demonstrated using a representative set of benchmark wideband antenna examples optimized for best matching (a typical antenna design task). Reliability of the algorithm is validated through multiple optimization runs from random initial designs. The average computational savings with respect to the conventional TR algorithm are as high as seventy percent with acceptable degradation of the final design quality.

2. Antenna Design Optimization Using Selective Broyden Updates

In this section, we recall the formulation of the antenna design closure problem, outline the conventional trust-region algorithm, as well as describe the proposed accelerated procedure based on selective utilization of Broyden updating formula. Comprehensive numerical validation is provided in Section 3.

2.1. Design Closure of Antenna Structures

Design closure refers to the last stage of the design process where the antenna topology has already been selected and fixed, and the task is to adjust the values of geometry parameters (on some occasions, also material parameters) so as to improve the structure performance in a given sense. Usually, design closure requires a local search that is carried out by means of gradient-based procedures. In order to assess the quality of the design, a properly defined performance measure is required. Here, we employ a scalar cost function, as it permits utilization of single-objective search routines. The task is formulated as a nonlinear continuous minimization problem of the form (Koziel and Ogurtsov, 2014)

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} U(\boldsymbol{R}(\boldsymbol{x})) \tag{1}$$

in which $\boldsymbol{x}$ is a vector of independent antenna parameters, $\boldsymbol{R}(\boldsymbol{x})$ is a response of the EM-simulation antenna model, whereas U is an objective function encoding given performance specifications. The antenna response is typically a vector-valued function representing relevant characteristics such as reflection coefficient, gain, axial ratio (all being functions of frequency f) (Koziel and Ogurtsov, 2014). In the paper, the response $\boldsymbol{R}(\boldsymbol{x})$ refers to an input reflection characteristics versus frequency f, i.e., $\boldsymbol{R}(\boldsymbol{x}) = S_{11}(\boldsymbol{x},f)$.

The definition of U depends on a particular choice of performance figures to be handled. Here, perhaps the most common problem of improving antenna in-band reflection is considered, i.e., minimization of the maximum in-band reflection $|S_{11}(\boldsymbol{x},f)|$, which is also referred to as the optimization for best matching. The objective function U is defined as

$$U(\boldsymbol{R}(\boldsymbol{x})) = \max\{f \in F : |S_{11}(\boldsymbol{x}, f)|\} \tag{2}$$

where $|S_{11}(\boldsymbol{x},f)|$ is the modulus of the reflection coefficient (here shown with an explicit dependence on frequency), whereas F is the frequency range of interest (e.g., 3.1 GHz to 10.6 GHz in case of ultra-wideband antennas). The presented formulation of the design closure problem in a minimax sense is one of the most popular ones.

2.2. Reference Algorithm

The reference algorithm is a trust-region (TR)-embedded gradient search procedure (Conn *et al.*, 2009). The TR algorithm is a convenient way of solving (1) for objective functions and constraints evaluated through EM analysis, which usually exhibit certain level of numerical noise. The algorithm generates a series $\boldsymbol{x}^{(i)}$, i = 0, 1, …, of approximations to the solution $\boldsymbol{x}^*$ of (1) as

$$\boldsymbol{x}^{(i+1)} = \arg \min_{\boldsymbol{x};\, -\boldsymbol{d}^{(i)} \le \boldsymbol{x}-\boldsymbol{x}^{(i)} \le \boldsymbol{d}^{(i)}} U(\boldsymbol{L}^{(i)}(\boldsymbol{x})) \tag{3}$$

where $\boldsymbol{L}^{(i)}(\boldsymbol{x}) = \boldsymbol{R}(\boldsymbol{x}^{(i)}) + \boldsymbol{J}_{\boldsymbol{R}}(\boldsymbol{x}^{(i)})\cdot(\boldsymbol{x} - \boldsymbol{x}^{(i)})$ is a linear approximation (first-order Taylor expansion) of $\boldsymbol{R}$ at $\boldsymbol{x}^{(i)}$. Unless adjoint sensitivities (Ghassemi *et al.*, 2013) are available, the Jacobian $\boldsymbol{J}_{\boldsymbol{R}}$ is evaluated using the finite differentiation (FD), which incurs n additional EM analyses (n being the number of the antenna parameters) per algorithm iteration. In order to account for (often significantly) different ranges of parameters (e.g., fractions of millimeters for gaps versus tens of millimeters for substrate/ground plane dimensions), variables can be scaled (Ghassemi *et al.*, 2013). Here, a different approach is taken, i.e., a hypercube-like search region (cf. (3)) is used, rather than an Euclidean ball $||\boldsymbol{x} - \boldsymbol{x}^{(i)}|| \le d^{(i)}$, normally utilized by TR algorithms (Conn *et al.*, 2009). The components of the size vector $\boldsymbol{d}^{(i)}$ are proportional to the parameter ranges.

2.3. Accelerated TR Algorithm with Selective Broyden Updates

In the proposed accelerated algorithm, estimation of the Jacobian through the finite differentiation is—in some cases—replaced by the update using a rank-one Broyden formula (cf. Koziel *et al.*, 2010)

$$\boldsymbol{J}_{\boldsymbol{R}}^{(i+1)} = \boldsymbol{J}_{\boldsymbol{R}}^{(i)} + \frac{\left(\boldsymbol{f}^{(i+1)} - \boldsymbol{J}_{\boldsymbol{R}}^{(i)} \cdot \boldsymbol{h}^{(i+1)}\right)\cdot \boldsymbol{h}^{(i+1)T}}{\boldsymbol{h}^{(i+1)T}\boldsymbol{h}^{(i+1)}}, \quad i = 0, 1, \ldots \tag{4}$$

where $\boldsymbol{f}^{(i+1)} = \boldsymbol{R}(\boldsymbol{x}^{(i+1)}) - \boldsymbol{R}(\boldsymbol{x}^{(i)})$, and $\boldsymbol{h}^{(i+1)} = \boldsymbol{x}^{(i+1)} - \boldsymbol{x}^{(i)}$. The above formula improves the current Jacobian estimate along the direction of $\boldsymbol{h}^{(i+1)}$. Here, in the first iteration, the initial estimate of the Jacobian is obtained through full finite differentiation. In the subsequent iterations, however, the finite differentiation is substituted by the above formula only for selected parameters. The sole usage of the Broyden update normally produces poor results because the i-th estimate of the Jacobian $\boldsymbol{J_R}^{(i)}$ (obtained upon i updates) only contains information about $\boldsymbol{J_R}$ in the subspace spanned by vectors $\boldsymbol{h}^{(1)}$, $\boldsymbol{h}^{(2)}$, …, $\boldsymbol{h}^{(i)}$. In particular, to get the Jacobian estimate for all directions, at least n iterations are required. This issue is alleviated by means of the proposed technique of the selecting the directions, for which the Broyden update is performed. Operation of the proposed expedited algorithm is explained in the form of a flow diagram shown in Fig. 1.

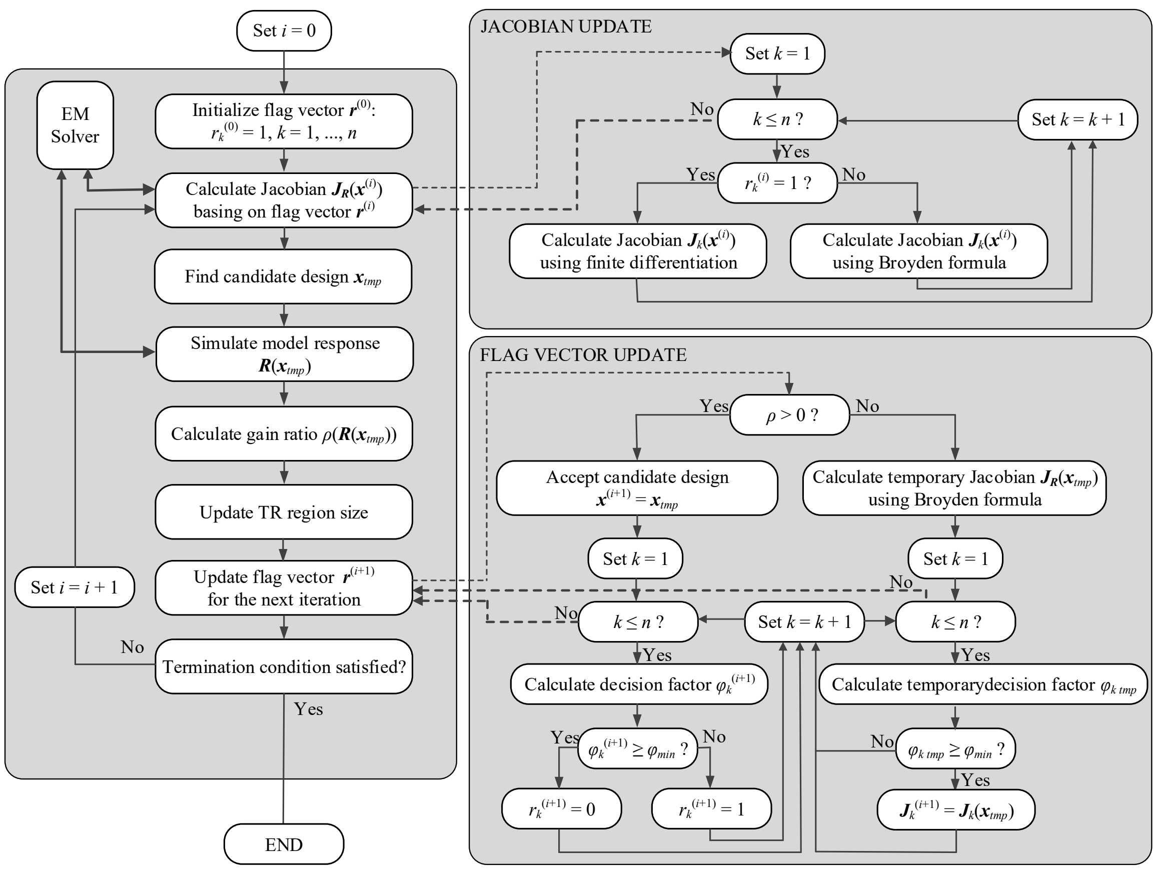


Fig. 1. Flow diagram the proposed adaptive Broyden update algorithm.

The left panel of Fig. 1 shows the essential stages of the algorithm. The following notation is used: $\boldsymbol{e}^{(k)}$ represent the standard basis vectors, i.e., $\boldsymbol{e}^{(k)} = [0 \ldots 0\ 1\ 0 \ldots 0]^T$ with 1 on the k-th position. In addition, the k-th column of $\boldsymbol{J_R}$ (meaning the antenna response sensitivities w.r.t. the k-th parameter) is denoted by $\boldsymbol{J}_k = \partial\boldsymbol{R}/\partial x_k$. Furthermore, $\boldsymbol{r}^{(i)}$ stands for the binary flag vector; its nonzero elements $r_k^{(i)}$ refer to the Jacobian components $\boldsymbol{J}_k(\boldsymbol{x})$ that are to be obtained in the (i+1)th iteration through the finite differentiation. Otherwise, $\boldsymbol{J}_k(\boldsymbol{x})$ is updated with the use of the Broyden formula (4) (see the top-right panel of Fig. 1). In the first iteration, all entries of the flag vector $\boldsymbol{r}^{(0)}$ are set to 1. Consequently, the entire Jacobian $\boldsymbol{J_R}(\boldsymbol{x}^{(0)})$ is calculated using the finite differentiation for the initial parameter vector $\boldsymbol{x}^{(0)}$, as mentioned above. In subsequent iterations, the update of the column of the Jacobian $\boldsymbol{J}_k$, for a given index k, is performed accordingly to the flag vector $\boldsymbol{r}^{(i)}$, calculated in the previous iteration.

After all columns $\boldsymbol{J}_k$, $k = 1, \ldots, n$, are updated, the candidate design $\boldsymbol{x}_{tmp}$ is obtained by solving (3). Next, the gain ratio $\rho = (U(\boldsymbol{R}(\boldsymbol{x}_{tmp}) - U(\boldsymbol{R}(\boldsymbol{x}^{(i)})/(\boldsymbol{L}^{(i)}(\boldsymbol{x}_{tmp}) - \boldsymbol{L}^{(i)}(\boldsymbol{x}^{(i)}))$ is calculated and used to adjust the TR region size using the standard rules (Conn *et al.*, 2009). Subsequently, the flag vector update procedure is carried out, as shown in the bottom-right panel of Fig. 1. If the iteration was successful (i.e., $\rho > 0$), the candidate design is accepted and the decision factors $\varphi_k^{(i+1)}$ for the next iteration are calculated as a normalized design change in the k-th direction: $\varphi_k^{(i+1)} = |\boldsymbol{h}^{(i+1)T}\boldsymbol{e}^{(k)}|/||\boldsymbol{h}^{(i+1)}||$. The factors affect the Jacobian update in the next iteration as they are used to construct the flag vector $\boldsymbol{r}^{(i+1)}$. For a given index k, if the factor $\varphi_k^{(i+1)}$ exceeds the user specified threshold $\varphi_{\min}$, the corresponding element of the flag vector $r_k^{(i+1)}$ is set to 0, otherwise it is set to 1.

The alignment threshold $0 \leq \varphi_{\min} \leq 1$ is a control parameter of the algorithm. The higher the threshold, the more rigorous condition for using the Broyden formula gets. In the case of $\varphi_k = 1$, the two vectors (i.e. the direction of the recent design relocation $\boldsymbol{h}^{(i+1)}$ and the k-th base vector $\boldsymbol{e}^{(k)}$) are co-linear; whereas, for $\varphi_k = 0$, the vectors are orthogonal. From the point of view of the computational savings, lower threshold values are desirable. At the same time, the final design quality may be compromised to a certain extent. In the next section, these trade-offs are illustrated by appropriate numerical experiments.

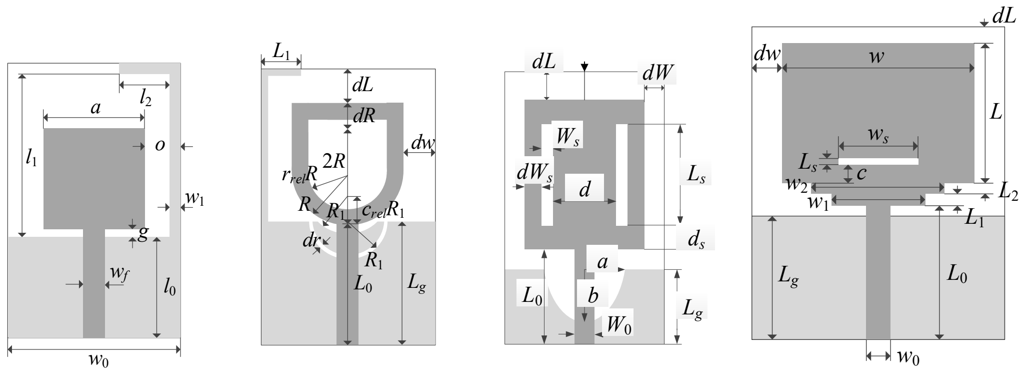
In the bottom-right panel of Fig. 1, the case of the rejected iteration is also shown. In that situation, the temporary Jacobian $\boldsymbol{J_R}(\boldsymbol{x}_{tmp})$ is calculated exclusively based on the Broyden formula (4) with the use of the rejected candidate step $\boldsymbol{x}_{tmp}$ instead of $\boldsymbol{x}^{(i+1)}$. Subsequently, the decision factors φ_k^i are calculated again for each parameter. The temporary Jacobian columns $\boldsymbol{J}_k(\boldsymbol{x}_{tmp})$ substitute the Jacobian columns $\boldsymbol{J}_k$ calculated in the previous iteration for those parameters, for which the factors are beyond the threshold $\varphi_{\min}$. The motivation behind it is that the information content included in the rejected iteration may better guide the next iteration of the optimization process (Nocedal and Wright, 2006).

3. Verification Examples

The algorithm of Section 2.3 has been comprehensively validated using four UWB (ultra-wideband) antennas shown in Fig. 2. Antenna I (Koziel and Bekasiewicz, 2016c) is implemented on the Taconic RF-35 substrate (dielectric permittivity $\varepsilon_r = 3.5$, and height $h = 0.762$ mm). It is a standard rectangular monopole described by seven parameters $\boldsymbol{x} = [l_0\ g\ a\ l_1\ l_2\ w_1\ o]^T$ (all dimensions in mm). Other design parameters for Antenna I are: w_0

$= 2o + a$, along with the feeding line width $w_f = 1.7$ mm which ensures 50 ohm input impedance. Antenna II (Alsath and Kanagasabai, 2015) is also implemented on the same substrate and its independent geometry parameters are $\boldsymbol{x} = [L_0\ dR\ R\ r_{rel}\ dL\ dw\ Lg\ L_1\ R_1\ dr\ c_{rel}]^T$. Antenna III is based on the structure of (Haq *et al.*, 2017) and it is implemented on the FR4 substrate ($\varepsilon_r = 4.3$, $h = 1.55$ mm). The geometry parameters are $\boldsymbol{x} = [L_g\ L_0\ L_s\ W_s\ d\ dL\ d_s\ dW_s\ dW\ a\ b]^T$. Finally, Antenna IV (Suryawanshi and Singh, 2014) is implemented on the RO4350 substrate ($\varepsilon_r = 3.48$, $h = 0.762$ mm) with the following design variables $\boldsymbol{x} = [L_0\ L_1\ L_2\ L\ dL\ L_g\ w_1\ w_2\ w\ dw\ L_s\ w_s\ c]^T$. The parameters of all antennas from Fig. 2 are summarized in Table I, which contains also the parameter values for the representative algorithm runs presented in Fig. 3 ($\varphi_{\min} = 0.025$). The computational models for all antennas are implemented in CST Microwave Studio and evaluated using its transient solver. The EM models incorporate the SMA connectors.

To assess the algorithm robustness, ten random starting points have been used for each antenna to collect statistics of the algorithm performance. The optimization cost expressed in terms of the number of EM simulations along with the objective function values, obtained using the proposed algorithm and the conventional TR algorithm, are presented in Table II. Moreover, in Table III, complementary results are included: percentage-wise cost savings w.r.t. the reference algorithm, degradation of objective function value w.r.t. the reference algorithm as well as the standard deviation of the objective function across the initial design set, being a measure of the result repeatability. Figure 3 shows the initial and optimized antenna responses for the representative algorithm runs ($\varphi_{\min} = 0.025$): $S_{11}(\boldsymbol{x}_0,f)|$ for the initial design $\boldsymbol{x}_0$ and $|S_{11}(\boldsymbol{x}_{opt},f)|$ for the optimized design $\boldsymbol{x}_{opt}$, respectively. The design specification –10dB, which is met for all antennas, is also included in Fig. 3.



(a) (b) (c) (d)

Fig. 2. Benchmark antenna structures: (a) Antenna I, (b) Antenna II, (c) Antenna III, (d) Antenna IV. Ground plane marked using light gray shade.

Table. I. Optimal Geometry Parameter Values of the Antennas from Fig. 2. for the Representative Algorithm Runs of Fig. 3.

Antenna	Geometry parameters [mm]												
I	l_0	g	a	l_1	l_2	w_1	o						
	23.27	19.85	10.72	6.00	5.10	0.97	2.40						
II	L_0	dR	R	r_{rel}	dL	dw	Lg	L_1	R_1	dr	c_{rel}		
	11.16	0.06	6.63	0.12	0.40	7.36	10.82	3.93	2.04	0.49	0.80		
III	L_g	L_0	L_s	W_s	d	dL	d_s	dW_s	dW	a	b		
	9.65	13.83	8.69	0.39	3.85	7.21	1.33	0.72	3.82	0.33	0.50		
IV	L_0	L_1	L_2	L	dL	L_g	w_1	w_2	w	dw	L_s	w_s	c
	12.16	1.63	2.00	14.79	4.46	11.42	0.69	0.60	20.20	6.08	0.08	0.19	0.10

The following values of the threshold $\varphi_{\min}$ were considered: 0 (corresponding to Broyden-only algorithm with no FD calculations apart from the initial Jacobian estimation), 0.01, 0.025, 0.05, 0.1, 0.2 and 0.3. The results confirm the expected dependence between the applied threshold value and both the achieved design quality and the number of simulations necessary to obtain the solution.

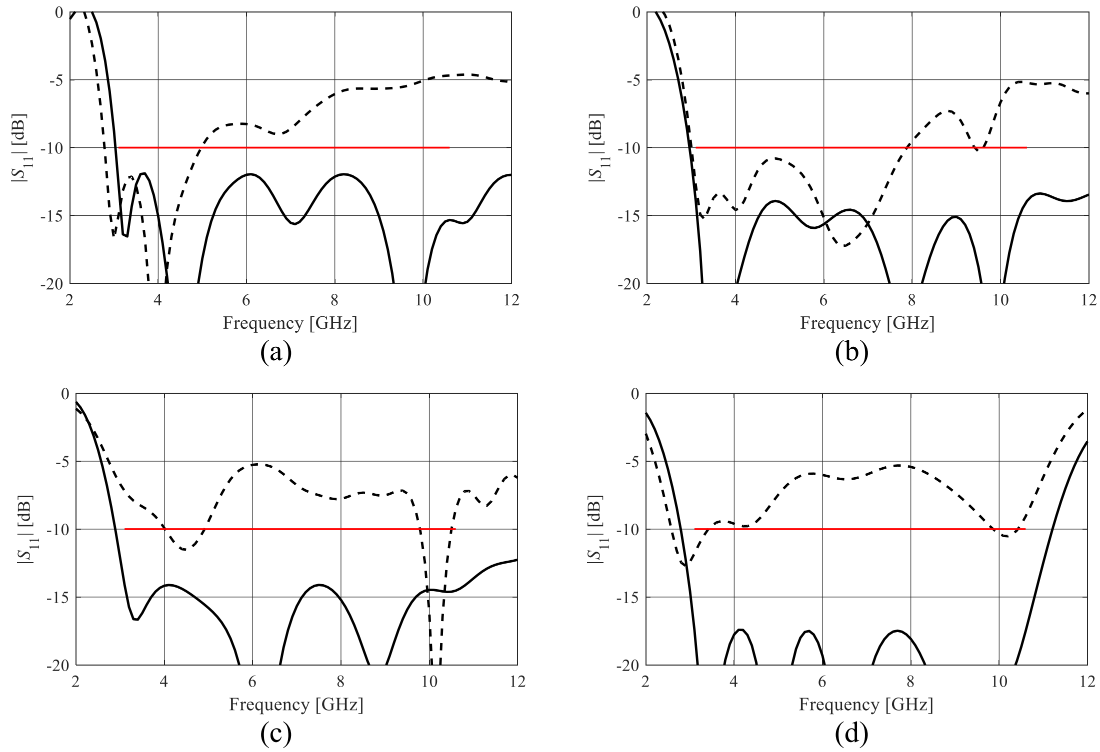


Fig. 3. Reflection characteristics for the representative algorithm runs ($\varphi_{\min}$ = 0.025): (a) Antenna I, (b) Antenna II, (c) Antenna III, (d) Antenna IV. Horizontal lines mark the design specifications; (- - -) initial design, (—) optimized design.

Table. II. Optimization Results for Antennas I through IV

Algorithm		Antenna							
		I		II		III		IV	
		Cost*	max$\lvert S_{11}\rvert$#	Cost*	max$\lvert S_{11}\rvert$#	Cost*	max$\lvert S_{11}\rvert$#	Cost*	max$\lvert S_{11}\rvert$#
Reference		97.6	–11.9	111.2	–14.9	111.0	–13.9	139.7	–17.6
This work	0	25.5	–10.4	26.5	–13.5	26.5	–10.8	34.3	–13.4
	0.01	27.5	–10.7	30.7	–13.6	31.7	–11.0	34.4	–13.7
	0.025	31.5	–11.0	37.5	–13.9	36.1	–11.2	44.0	–14.0
	0.05	36.4	–11.1	47.9	–14.0	43.1	–11.3	51.8	–14.1
	0.1	36.8	–11.2	58.4	–13.7	63.7	–11.6	65.5	–14.7
	0.2	53.0	–10.7	75.9	–14.3	80.0	–11.9	89.2	–15.1
	0.3	63.0	–11.6	89.3	–14.2	91.0	–12.0	124.8	–17.2

* Number of EM simulations averaged over 10 algorithm runs (random initial points).
# Objective function values (maximum in-band reflection in dB).

Table. III. Optimization Results for Antennas I through IV: Computational Savings, Design Quality Degradation and Results Repeatability

Algorithm		Antenna											
		I			II			III			IV		
		Cost savings* [%]	Δ max |S11|# [dB]	std (max $\|S_{11}\|$)# [dB]	Cost savings* [%]	Δ max |S11|# [dB]	std (max $\|S_{11}\|$)# [dB]	Cost savings* [%]	Δ max |S11|# [dB]	std (max $\|S_{11}\|$)# [dB]	Cost savings* [%]	Δ max |S11|# [dB]	std (max $\|S_{11}\|$)# [dB]
Reference		–	–	0.4	–	–	0.6	–	–	1.0	–	–	2.5
This work	0	70.4	1.5	1.8	76.2	1.4	1.7	76.1	3.1	2.6	75.4	4.2	4.9
	0.01	71.8	1.2	1.2	72.4	1.3	1.2	71.4	2.9	2.4	75.4	3.9	4.8
	0.025	67.7	0.9	1.6	66.3	1.0	1.3	67.5	2.7	2.2	68.5	3.6	4.5
	0.05	62.7	0.8	1.7	56.9	0.9	0.9	61.2	2.6	2.2	62.9	3.5	3.8
	0.1	62.3	0.7	1.1	47.5	1.2	1.1	42.6	2.3	1.9	53.1	2.9	3.5
	0.2	45.7	1.2	0.7	31.7	0.6	0.9	27.9	2.2	1.9	36.1	2.5	3.2
	0.3	35.5	0.3	0.6	19.7	0.7	0.8	18.0	1.9	1.2	10.7	0.4	2.9

* Percentage-wise cost savings w.r.t. the reference algorithm.
# Degradation of objective function value w.r.t. the reference algorithm.
$ Standard deviation of the objective function in dB across 10 algorithm runs (random initial points).

Smaller threshold values are preferred in terms of ensuring better computational savings. The reason is that the bigger the threshold value, the stricter the condition for using the Broyden formula becomes. Hence, the Jacobian update is performed more often using the finite differentiation than with the simplified technique and the number of simulations increases. For all antennas, the threshold value $\varphi_{\min} = 0$ delivers the lowest quality of the solution which is beyond acceptance for essentially all considered antennas. It should be emphasized that this case was merely included to indicate that resignation from occasional FD updates is not a practical option for a reliable optimization procedure. As the threshold value $\varphi_{\min}$ increases, the design quality improves, however the cost savings decline significantly (to as low as 10.7% for Antenna IV), which is a not practical solution either.

Here, as a measure of the results repeatability, a standard deviation of the objective function over the set of ten algorithm runs is employed (see Table III). Clearly, the reference algorithm (full Jacobian update) yields the lowest standard deviation, although it can also be observed that the value grows with the increasing dimensionality of the parameter space. This effect can be explained by the fact that the optimized designs, obtained in particular algorithm runs, likely correspond to different local optima, the number of which grows with the design space dimension. At the same time, the algorithms considered here are local optimization routines so that finding a globally optimal design is neither possible nor sought for in this paper. Both the average objective function degradation and the standard deviation values are more or less monotonic with respect to φ_{min}. The observed fluctuations are primarily a result of relatively large variances of determining these figures (due to a small sample set of ten algorithm runs). The analysis of the results of Tables II and III allows us to draw certain conclusions concerning the recommended values of the threshold φ_{min}. These locate between 0.025 and 0.1 and somehow depend on the parameter space dimensionality: larger for Antennas I and II and smaller for Antennas III and IV. The values below 0.025 are associated with considerably degraded design quality, therefore they are not recommended. Within the range from 0.025 to 0.1, the proposed algorithm delivers notable average reduction of the optimization cost over the considered benchmark set exceeding 70%: from around 71 percent (for Antenna III) to around 75 percent (for Antenna IV). However, the degradation of the design quality is satisfactory (around 1 dB on average, cf. Table III) for Antennas I and II. It should also be emphasized that apparently large standard deviation values for Antennas III and IV should be viewed in the context of already large deviations obtained with the reference algorithm. As explained above, this effect has nothing to do with the performance of the optimization algorithm but with the complexity of the functional landscape to be handled. At the same time, the aforementioned dependence of the recommended threshold values on the number of antenna variables indicates the need for further algorithm enhancement that would aim at making its control parameters dimensionality independent.

4. Conclusion

An expedited trust-region gradient-search algorithm with adaptive Broyden updates for antenna design optimization was presented. The proposed algorithm allows for significant reduction of the number of EM simulations required in the optimization process by replacing the time consuming calculation of the part of the Jacobian through the finite differentiation with the rank-one updating formula. The updates are controlled by the relative change of the design variable vector between iterations, as well as the alignment between the design relocation vector and coordinate system basic vectors. The efficiency of the approach is demonstrated by comprehensive numerical experiments carried out for several wideband antenna structures and multiple starting points. Considerable computational speedup has been observed of 70 percent on average across the benchmark set for the optimum value of the Broyden acceptance threshold. At the same time, for most cases the design quality does not deteriorate considerably. The future work will aim at further reduction of the computational cost by making the algorithm control parameters adjusted based on the convergence status of the optimization process, search space dimensionality, as well as the response Jacobian changes along the optimization path.

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