Speed Observer of Induction Machine based on Backstepping and Sliding Mode for Low Speed Operation

Marcin Morawiec*, Arkadiusz Lewicki, Filip Wilczyński*****

Abstract – The paper contains design of the speed observer which is based on backstepping and sliding mode approaches. The inputs to the observer are the stator current and voltage vector components. Additionally, such an observer structure is extended to the integrators. The observer stabilizing functions contain the appropriate sliding surfaces which result from Lyapunov function. The rotor angular speed is obtained from the non-adaptive formula with sliding mode mechanism. It allows to improve the robustness of parameters uncertainties and the zero rotor speed working (near to unobservable region). In the sensorless control system, the classical first order sliding-mode controllers are applied with the transformation of the multi-scalar variable. The proposed control system structure can be named full-decoupled due to multi-scalar variables transformation and the feedback control law obtained from Lyapunov theorem. The theoretical derivations are verified in experimental waveforms. The sensorless control system robustness is verified in the experimental investigations by using nominal machine parameters uncertainties method.

Keywords: induction motor drives, variable speed, speed observer, sliding-mode.

NOMENCLATURE

"^"	estimated values,
"~"	error of estimated values,
ω_r	rotor angular speed,
R_r, R_s	rotor and stator resistances,
L_m	mutual-flux inductance,
L_s, L_r	stator and rotor inductances,
T_e	electromagnetic torque,
T_L	load torque,
J	machine torque of inertia,
$i_{s\alpha,\beta}$	stator current vector components,
$\psi_{r\alpha,\beta}$	rotor flux vector components,
$u_{s\alpha,\beta}$	stator voltage vector components,
τ	relative time,
$\hat{\omega}_r$	estimated rotor electrical speed,
$\tilde{\omega}_r$	rotor speed error,
$\tilde{\psi}_{r\alpha,\beta}$	rotor flux vector components error,
$\tilde{i}_{s\alpha,\beta}$	stator current vector components error,
$\hat{Z}_{\alpha,\beta}$	additional observer state variables,
$v_{\alpha,\beta}$	speed observer stabilizing variables,
$c_{\alpha,\beta}$	constant gains,
x_{11}, x_{12}, x_{21}, x_{22}	multi-scalar state variables defined in [19].

1. INTRODUCTION

The sliding-mode approach belongs to the robust control. There are many papers with different control structures containing complicated algorithms of the proposed solutions. These approaches are based on the adaptive mechanism [1-2], backstepping techniques [3-4], fuzzy logic [5-7, 45] with chattering phenomena elimination [5]. Almost all of these cited papers provide robust on machine parameters changes especially [4, 7-9, 38], in which the problem was the main topic. These papers contain different observer structures for example: adaptive [10-12], disturbance observer [13], observers of permanent magnet machine PMSM [14-16] and with the novel sliding surfaces [17-18], backstepping [3-4, 22, 37]. The main inconvenience to put of the sliding mode control into an industrial application is the chattering, which consists in a high-frequency oscillation when the sliding mode takes place. There is a lot of mismatches between the actual object (mathematical model) and the mathematical model developed for the controller design in industrial applications. The control system should guarantee good performance and robust of parameter variations (robust on machine parameters uncertainties). The sliding mode controller consists of continuous linear/non-linear subsystems with discontinuous suitable switching logic. The first order sliding mode (FOSM) was presented in [17-18]. This approach gives possibilities to obtain the convergence of the finite-time error but introduces chattering which is an undesirable effect. The higher order sliding-mode algorithm is one of the solutions to soften the chattering [28]. In [40] the "twisting" algorithm was proposed which reduced the undesirable chattering effect. The supertwisting algorithm (STA) is a second-order sliding-mode approach, and it has been widely used for observation [33-34, 27, 35-37, 43-44], and control [28-29]. The third order sliding mode was presented in [41] to control satellites. The high order sliding mode observer-based backstepping fault-tolerant control for induction motor (IM) was presented in [42].

The sliding observers presented in literature [1-21, 24-29] are based on: adaptive MRAS technique [1], second order sliding mode approach [2, 27, 33-34] and sliding-robust [4, 7-8, 10, 27]. The authors of this paper propose to use the speed observer structure from [22] which is based on backstepping approach. To obtain more robust observer structure than [22], the sliding switching functions are introduced. The speed observer can be extended to the integrator form or filters [21, 3-4, 12]. The stator voltage components are treated as known values and the stator current vector components are measured. The extended observer structure was suitable to use in the sensorless application but was not "permanently" robust on machine parameters uncertainties. The speed observer structure should be a persistence of exciting. For a short period of time to 1-2s, zero rotor speed set, machine loaded of the nominal torque and for machine parameters detuned

* Gdansk University of Technology, Narutowicza 11/12, 80-223 Gdansk, Poland, e-mail: marcin.morawiec@pg.edu.pl
** Gdansk University of Technology, Narutowicza 11/12, 80-223 Gdansk, Poland, e-mail: arkadiusz.lewicki@pg.edu.pl
*** Gdansk University of Technology, Narutowicza 11/12, 80-223 Gdansk, Poland, e-mail: filipwilczynski@pg.edu.pl

test (R_s to about 30% and R_r about 50% or L_m to 10% of nominal values) the estimation errors were smaller than 0.02 p.u. (<2%). As a result, the control system maintained the reference values (reference speed and module of rotor flux vector). After decaying the observer transient states (about 2s of this test) estimation, errors were increased, which resulted in the control system not maintaining the reference values. Stable work of the observer structure is guaranted by backstepping law which is based on Lyapunov stability theory. The observer was still not excited and therefore it was only asymptotically stable - not exponentially. Taking into account this inconvenience with the observer structure [22], the authors of this paper propose to connect the backstepping (continuous) control law to the sliding approach in order to obtain the robust speed observer structure. In literature for example [3-4] authors of these papers propose the sliding – backstepping approach for the control system of machines but not for the observer structure. This is the main contribution of the proposed paper. Extending the observer stabilizing function to the sliding mode approach gives the possibility for permanent exciting of the structure and as a result an exponential decay of estimation errors to zero. The sensorless control system is then more robust on parameter uncertainties. In the control system of an induction machine and in the observer structure, the FOSM is applied. The sliding discontinuous nature contributes to high-frequency oscillations as chattering characteristic but this is desirable in this case. In many different applications, the chattering might be an undesirable effect such as current harmonics and electromagnetic torque pulsation [6-7], but in the speed observer structure it prevents internal observer stationary states (for near to zero or unobservable machine working points). For this working condition, the observer structure should be persistent exciting [24-25], therefore the stator voltage and current vector components should contain the higher harmonics, which are introduced by the sliding approach. The sliding-backstepping approach applied to the observer structure causes that the observer is more robust on disturbances (nominal machine parameter and load torque near to zero changes). These statements are confirmed by experiment results.

2. SPEED OBSERVER OF INDUCTION MACHINE

2.1 THE SPEED OBSERVER BASED ON BACKSTEPPING AND SLIDING MODE APROACH

The standard exponential observer structure can be extended to additional variables that will be treated as the state variables. Such an approach was proposed in [22-23]. These variables are named Z and determined for (αβ) as follows:

$$\hat{Z}_\alpha = \hat{\omega}_r \hat{\psi}_{r\alpha}\,, \tag{1}$$

$$\hat{Z}_\beta = \hat{\omega}_r \hat{\psi}_{r\beta}\,. \tag{2}$$

The rotor angular speed can be obtained from (1)–(2) and has the form [22-23]:

$$\hat{\omega}_r = \frac{\hat{Z}_\alpha \hat{\psi}_{r\alpha} + \hat{Z}_\beta \hat{\psi}_{r\beta}}{\hat{\psi}_{r\alpha}^2 + \hat{\psi}_{r\beta}^2}\,. \tag{3}$$

Since system stay in domain D therefore it is bounded input bounded output state (BIBS) [10-11]. Taking into account the machine model [22], after transformation to (αβ) components, determining the derivative of the Z variables and denoting the estimate values by "^" the speed observer model has the following form:

$$\frac{d\hat{i}_{s\alpha}}{d\tau} = -a_1 \hat{i}_{s\alpha} + a_2 \hat{\psi}_{r\alpha} + a_3 \hat{Z}_\beta + a_4 u_{s\alpha} + v_\alpha\,, \tag{4}$$

$$\frac{d\hat{i}_{s\beta}}{d\tau} = -a_1 \hat{i}_{s\beta} + a_2 \hat{\psi}_{r\beta} - a_3 \hat{Z}_\alpha + a_4 u_{s\beta} + v_\beta\,, \tag{5}$$

$$\frac{d\hat{\psi}_{r\alpha}}{d\tau} = -a_5 \hat{\psi}_{r\alpha} - \hat{Z}_\beta + a_6 i_{s\alpha} + v_{\psi\alpha}\,, \tag{6}$$

$$\frac{d\hat{\psi}_{r\beta}}{d\tau} = -a_5 \hat{\psi}_{r\beta} + \hat{Z}_\alpha + a_6 i_{s\beta} + v_{\psi\beta}\,, \tag{7}$$

$$\frac{d\hat{Z}_\alpha}{d\tau} = \frac{d\hat{\omega}_r}{d\tau}\hat{\psi}_{r\alpha} - \hat{\omega}_r(\hat{Z}_\beta - a_6 i_{s\alpha}) - a_5 \hat{Z}_\alpha + v_{Z\alpha}\,, \tag{8}$$

$$\frac{d\hat{Z}_\beta}{d\tau} = \frac{d\hat{\omega}_r}{d\tau}\hat{\psi}_{r\beta} + \hat{\omega}_r(\hat{Z}_\alpha + a_6 i_{s\beta}) - a_5 \hat{Z}_\beta + v_{Z\beta}\,, \tag{9}$$

where the new correction terms $v_{\alpha,\beta}$, $v_{\psi\alpha,\beta}$, $v_{Z\alpha,\beta}$ and

$$a_1 = \frac{R_s L_r^2 + R_r L_m^2}{L_r w_\sigma},\quad a_2 = \frac{R_r L_m}{L_r w_\sigma},\quad a_3 = \frac{L_m}{w_\sigma},\quad a_4 = \frac{L_r}{w_\sigma}, \tag{10}$$

$$a_5 = \frac{R_r}{L_r},\quad a_6 = \frac{R_r L_m}{L_r},\quad w_\sigma = L_r L_s - L_m^2\,. \tag{11}$$

In the theoretical investigations following assumptions are taken into account: the IM parameters (10)–(11) are known and constant, the stator current vector components $i_{s\alpha}$, $i_{s\beta}$ can be measured, the stator voltage vector components $u_{s\alpha}$, $u_{s\beta}$ are the control variables, rotor flux vector components $\psi_{r\alpha}$, $\psi_{r\beta}$ as well as $i_{s\alpha}$, $i_{s\beta}$ and the rotor angular speed ω_r are estimated by the speed observer structure.

In order to implement the *integrator backstepping* [21] approach the speed observer structure should be extended to the integrators:

$$\frac{d\hat{\xi}_\alpha}{d\tau} = \hat{i}_{s\alpha}\,, \tag{12}$$

$$\frac{d\hat{\xi}_\beta}{d\tau} = \hat{i}_{s\beta}\,. \tag{13}$$

Assuming the strict output-feedback form in which the stator current vector components are only measured and the stator voltage is known, the deviations model has the form:

$$\frac{d\tilde{i}_{s\alpha}}{d\tau} = a_2 \tilde{\psi}_{r\alpha} + a_3 \tilde{Z}_\beta + v_\alpha\,, \tag{14}$$

$$\frac{d\tilde{i}_{s\beta}}{d\tau} = a_2 \tilde{\psi}_{r\beta} - a_3 \tilde{Z}_\alpha + v_\beta\,, \tag{15}$$

$$\frac{d\tilde{\psi}_{r\alpha}}{d\tau} = -a_5 \tilde{\psi}_{r\alpha} - \tilde{Z}_\beta + v_{\psi\alpha}\,, \tag{16}$$

$$\frac{d\tilde{\psi}_{r\beta}}{d\tau} = -a_5 \tilde{\psi}_{r\beta} + \tilde{Z}_\alpha + v_{\psi\beta}\,, \tag{17}$$

$$\frac{d\tilde{Z}_\alpha}{d\tau} = \tilde{P}_\alpha - \hat{\omega}_r \tilde{Z}_\beta - \tilde{\omega}_r(\hat{Z}_\beta - \tilde{Z}_\beta - a_6 i_{s\alpha}) + a_5 \tilde{Z}_\alpha + v_{Z\alpha}\,, \tag{18}$$

$$\frac{d\tilde{Z}_\beta}{d\tau} = \tilde{P}_\beta + \hat{\omega}_r \tilde{Z}_\alpha + \tilde{\omega}_r(\hat{Z}_\alpha - \tilde{Z}_\alpha + a_6 i_{s\beta}) + a_5 \tilde{Z}_\beta + v_{Z\beta}\,, \tag{19}$$

$$\frac{d\tilde{\xi}_\alpha}{d\tau} = \tilde{i}_{s\alpha}\,, \tag{20}$$

$$\frac{d\tilde{\xi}_\beta}{d\tau} = \tilde{i}_{s\beta}\,, \tag{21}$$

where

$$\tilde{P}_\alpha = \frac{d\tilde{\omega}_r}{d\tau}\tilde{\psi}_{r\alpha} + \frac{d\tilde{\omega}_r}{d\tau}\hat{\psi}_{r\alpha}\,, \tag{22}$$

$$\tilde{P}_\beta = \frac{d\tilde{\omega}_r}{d\tau}\tilde{\psi}_{r\beta} + \frac{d\tilde{\omega}_r}{d\tau}\hat{\psi}_{r\beta}\,. \tag{23}$$

The first step in the backstepping procedure is to stabilize the observer structure by the integrators (20)–(21). The stabi lizing function should be chosen to satisfy the Lyapunov the orem. The Lyapunov function is determined:

$$V_1 = \frac{1}{2}(\tilde{\xi}_\alpha^2 + \tilde{\xi}_\beta^2) \ . \qquad (24)$$

The theorem will be fulfilled if the integrator stabilizing function $\sigma_{\alpha,\beta}$:

$$\sigma_\alpha = -c_\alpha \tilde{\xi}_\alpha \ , \qquad (25)$$

$$\sigma_\beta = -c_\alpha \tilde{\xi}_\beta \ . \qquad (26)$$

The second step in the backstepping procedure is to introduce the deviation variables $z_{\alpha,\beta}$ which are the differences between $\tilde{i}_{s\alpha}$, $\tilde{i}_{s\beta}$ and the introduced $\sigma_{\alpha,\beta}$:

$$z_\alpha = \tilde{i}_{s\alpha} + c_\alpha \tilde{\xi}_\alpha \ , \qquad (27)$$

$$z_\beta = \tilde{i}_{s\beta} + c_\alpha \tilde{\xi}_\beta \ . \qquad (28)$$

Using (27)–(28) the integrators (20)–(21) take the form:

$$\frac{d\tilde{\xi}_\alpha}{d\tau} = z_\alpha - c_\alpha \tilde{\xi}_\alpha \ , \qquad (29)$$

$$\frac{d\tilde{\xi}_\beta}{d\tau} = z_\beta - c_\alpha \tilde{\xi}_\beta \ , \qquad (30)$$

and the characteristic for backstepping method – back-step th rough the integrator is achieved.
In the next steps of the backstepping procedure the derivative of $z_{\alpha,\beta}$ should be determined. Calculation of the (27)–(28) derivatives gives:

$$\dot{z}_\alpha = a_2 \tilde{\psi}_{r\alpha} + a_3 \tilde{Z}_\beta + v_\alpha + c_\alpha \tilde{i}_{s\alpha} \ , \qquad (31)$$

$$\dot{z}_\beta = a_2 \tilde{\psi}_{r\beta} - a_3 \tilde{Z}_\alpha + v_\beta + c_\alpha \tilde{i}_{s\beta} \ . \qquad (32)$$

The Lyapunov function is determined for the dynamics of the $\xi_{\alpha,\beta}$, $z_{\alpha,\beta}$ and $Z_{\alpha,\beta}$ variables and for the rotor flux vector components. Therefore, the Lyapunov function is chosen as follows:

$$V_2 = \frac{1}{2}(\tilde{\xi}_\alpha^2 + \tilde{\xi}_\beta^2 + z_\alpha^2 + z_\beta^2 + \tilde{\psi}_{r\alpha}^2 + \tilde{\psi}_{r\beta}^2 + \tilde{Z}_\alpha^2 + \tilde{Z}_\beta^2) \ . \qquad (33)$$

Derivative of (33) must be negatively determined. This condition is satisfied if the following stabilizing functions have the form:

$$v_\alpha = -a_2 \tilde{\psi}_{r\alpha} - c_\alpha \tilde{i}_{s\alpha} - c_\beta z_\alpha - \tilde{\xi}_\alpha \ , \qquad (34)$$

$$v_\beta = -a_2 \tilde{\psi}_{r\beta} - c_\alpha \tilde{i}_{s\beta} - c_\beta z_\beta - \tilde{\xi}_\beta \ , \qquad (35)$$

$$v_{\psi\alpha} = \tilde{Z}_\beta \ , \qquad (36)$$

$$v_{\psi\beta} = -\tilde{Z}_\alpha \ , \qquad (37)$$

$$v_{Z\alpha} = k_{Z1}\left(a_5 v_{\psi\beta} + a_3 z_\beta\right) - \tilde{P}_\alpha \ , \qquad (38)$$

$$v_{Z\beta} = k_{Z2}\left(a_5 v_{\psi\alpha} - a_3 z_\alpha\right) - \tilde{P}_\beta \ , \qquad (39)$$

where the estimation errors are defined:

$$\tilde{i}_{s\alpha,\beta} = \hat{i}_{s\alpha,\beta} - i_{s\alpha,\beta} \ , \qquad (40)$$

$$\tilde{\omega}_r = \hat{\omega}_r - \omega_r \ , \qquad (41)$$

$$\tilde{\psi}_{r\alpha,\beta} = \hat{\psi}_{r\alpha,\beta} - \psi_{r\alpha,\beta} \ , \qquad (42)$$

$$\tilde{Z}_\alpha = \hat{Z}_\alpha - Z_\alpha \ , \qquad (43)$$

$$\tilde{Z}_\beta = \hat{Z}_\beta - Z_\beta \ , \qquad (44)$$

c_α, $c_\beta > 0$ and $k_z > 0$ are the gains introduced to (38)–(39).

Derivative of the rotor angular speed which occurs in (8)–(9) can be obtained by the adaptive mechanism.

Directly from Lyapunov theorem and taking into account (33) extended to $\frac{1}{2\gamma}\tilde{\omega}_r^2$,the following form of the rotor speed derivative can beobtained:

$$\frac{d\tilde{\omega}_r}{d\tau} = \gamma_1\left(\tilde{Z}_\alpha(\hat{Z}_\beta - a_6 \hat{i}_{s\alpha}) - \tilde{Z}_\beta(\hat{Z}_\alpha + a_6 \hat{i}_{s\beta})\right) , \qquad (45)$$

where $\gamma_1 > 0$.

To obtain the sliding properties the switching surfaces can be introduced:

$$\mathbf{s} = \begin{bmatrix} s_{\xi\alpha} \\ s_{\xi\beta} \\ s_{Z\alpha} \\ s_{Z\beta} \end{bmatrix} = \begin{bmatrix} \hat{\xi}_\alpha - \xi_\alpha \\ \hat{\xi}_\beta - \xi_\beta \\ \hat{Z}_\alpha - \hat{\omega}_r \hat{\psi}_{r\alpha} \\ \hat{Z}_\beta - \hat{\omega}_r \hat{\psi}_{r\beta} \end{bmatrix} . \qquad (46)$$

The sliding control law is introduced by the discontinuous function [1-3, 28-30, 35]:

$$\mathbf{v} = \mathbf{v}_a sign(\mathbf{s}) \ . \qquad (47)$$

Taken into account the above control law in the stabilizing function of observer structure (34)–(39), one obtains:

$$v_\alpha = -a_2 \tilde{\psi}_{r\alpha} - c_\beta z_\alpha - \tilde{\xi}_\alpha - c_{\alpha 1} sign(s_{\tilde{\xi}\alpha}) \ , \qquad (48)$$

$$v_\beta = -a_2 \tilde{\psi}_{r\beta} - c_\beta z_\beta - \tilde{\xi}_\beta - c_{\alpha 2} sign(s_{\tilde{\xi}\beta}) \ , \qquad (49)$$

$$v_{\psi\alpha} = k_{\psi 1} sign(s_{z\beta}) \ , \qquad (50)$$

$$v_{\psi\beta} = -k_{\psi 2} sign(s_{z\alpha}) \ , \qquad (51)$$

$$v_{Z\alpha} = k_{Z1}\left(-a_5 k_{\psi 1} sign(s_{z\alpha}) + a_3 z_\beta\right) - \tilde{P}_\alpha \ , \qquad (52)$$

$$v_{Z\beta} = k_{Z2}\left(a_5 k_{\psi 2} sign(s_{z\beta}) - a_3 z_\alpha\right) - \tilde{P}_\beta \ , \qquad (53)$$

where values of $\tilde{\psi}_{r\alpha,\beta}$ can be obtained from (16)–(17) and $\tilde{P}_{\alpha,\beta}$ from (22)–(23) (using (45)).

Deviation variables $z_{\alpha,\beta}$, defined in (27)–(28), can be determined:

$$z_\alpha = \tilde{i}_{s\alpha} + c_{\alpha 1} sign(s_{\tilde{\xi}\alpha}) \ , \qquad (54)$$

$$z_\beta = \tilde{i}_{s\beta} + c_{\alpha 2} sign(s_{\tilde{\xi}\alpha}) \ . \qquad (55)$$

The speed observer equations are discretized using Euler's method and presented in the Appendix section.

In the next step, it will be shown that the proposed observer (4) -(9), (29)-(30) with the stabilization functions (48)–(53), can exponentially converge vectors values $\hat{\mathbf{i}}_s$ to $\mathbf{i}_s$, $\hat{\boldsymbol{\psi}}_r$ to $\boldsymbol{\psi}_r$ and $\hat{\mathbf{Z}}$ to $\mathbf{Z}$. The Lyapunov function can be selected as follows:

$$V = V_2 + \frac{1}{2\gamma}\tilde{\omega}_r^2 \ . \qquad (56)$$

Taking into account the theorems about practical stability presented in [10-11] and the derivative of (56), the speed observer will be practically stable if the observer gains are taken into account [10-11]:

$$k_{\psi 1} = \max\left\{-\tilde{Z}_\beta + k_{\psi 1}\,\mathrm{sgn}(s_{z\beta})\right\} + \delta_{\psi 1} \ , \qquad (57)$$

$$k_{\psi 2} = \max\left\{\tilde{Z}_\alpha - k_{\psi 2}\,\mathrm{sgn}(s_{z\alpha})\right\} + \delta_{\psi 2} \ , \qquad (58)$$

$$c_{\alpha 1} = \max\left\{-\tilde{i}_{s\alpha} - c_{\alpha 1} sign(s_{\tilde{\xi}\alpha})\right\} + \delta_{\tilde{\xi}\alpha} \ , \qquad (59)$$

$$c_{\alpha 2} = \max\left\{-\tilde{i}_{s\beta} - c_{\alpha 2} sign(s_{\tilde{\xi}\beta})\right\} + \delta_{\tilde{\xi}\beta} \ , \qquad (60)$$

$$k_{z1} = \max\left\{\frac{a_6}{a_5}\tilde{\omega}_r i_{s\alpha} - k_{z1}\,\mathrm{sgn}(s_{z\alpha})\right\} + \delta_{kz1} \ , \qquad (61)$$

$$k_{z2} = \max\left\{\frac{a_6}{a_5}\tilde{\omega}_r i_{s\beta} + k_{z2}\,\mathrm{sgn}(s_{z\beta})\right\} + \delta_{kz2} \ , \qquad (62)$$

with $\delta_{\psi 1}$, $\delta_{\psi 2}$, $\delta_{\tilde{\xi}\alpha}$, $\delta_{\tilde{\xi}\beta}$, δ_{kz1}, δ_{kz2} > 0 and for $\tilde{i}_{s\alpha},_\beta \le \varepsilon_1$, $\tilde{Z}_\alpha,_\beta \le \varepsilon_2$, $\tilde{\omega}_r \le \varepsilon_3$, $\varepsilon_1,_2,_3 << 1$ are sufficient small reals, $c_\alpha,_\beta > 0$, the derivative (56) has the following form:

$$\dot{V} = \begin{cases} -c_\alpha(\tilde{\xi}_\alpha^2 + \tilde{\xi}_\beta^2) - a_5(\tilde{\psi}_{r\alpha}^2 + \tilde{\psi}_{r\beta}^2) - a_5(\tilde{Z}_\alpha^2 + \tilde{Z}_\beta^2) + \\ -c_\beta \delta_{\tilde{\xi}\alpha}|z_\alpha| - c_\beta \delta_{\tilde{\xi}\beta}|z_\beta| - \delta_{\psi 1}|\tilde{\psi}_{r\alpha}| - \delta_{\psi 2}|\tilde{\psi}_{r\beta}| + \\ -\delta_{kz1}|\tilde{Z}_\alpha| - \delta_{kz2}|\tilde{Z}_\beta| \end{cases} \le -\mu\sqrt{V} \ , \qquad (63)$$

where

$\mu = \min(\sqrt{2}\delta_{\tilde{\xi}\alpha}, \sqrt{2}\delta_{\tilde{\xi}\beta}, \sqrt{2}\delta_{\psi 1}, \sqrt{2}\delta_{\psi 2}, \sqrt{2}\delta_{kz1}, \sqrt{2}\delta_{kz2})$.

The above condition implies the convergence of vectors values $\hat{\mathbf{i}}_s$ to $\mathbf{i}_s$, $\hat{\boldsymbol{\psi}}_r$ to $\boldsymbol{\psi}_r$ and $\hat{\mathbf{Z}}$ to $\mathbf{Z}$ in finite time, noted as t_2. Consequently, for $\hat{\psi}_{r\alpha}^2+\hat{\psi}_{r\beta}^2>0$, the estimated rotor angular speed from (3), converges exponentially to ω_r in finite time $t>t_3>t_2$.

The Lyapunov function for the whole speed observer of IM operating range leads to conservatism of stabilizing function design. This problem was presented in [46]. Authors of that paper proposed minimization of function cost and based on this, feedback controller tuning gains. In this paper, to minimize the Lyapunov function conservatism, the authors propose the minimization of the estimation errors (63) and appropriate selection of observer gains as well as the minimization to zero of the vector product, defined by:

$$\hat{Z}_\alpha\hat{\psi}_{r\beta}-\hat{Z}_\beta\hat{\psi}_{r\alpha}\approx 0\,. \tag{64}$$

The dependence (64) is near to zero for the nominal parameters of IM and in the stationary state (constant rotor speed command).

The rotor speed value should be estimated using the following dependence

$$\hat{\omega}_r=\frac{\hat{Z}_\alpha\hat{\psi}_{r\alpha}+\hat{Z}_\beta\hat{\psi}_{r\beta}+C_f\left(\hat{Z}_\alpha\hat{\psi}_{r\beta}-\hat{Z}_\beta\hat{\psi}_{r\alpha}\right)}{\hat{\psi}_{r\alpha}^2+\hat{\psi}_{r\beta}^2}, \tag{65}$$

where

$$C_f=k_\omega\,\mathrm{sgn}\left(s_\omega\right)=\begin{cases}1 & s_\omega<0\\ -1 & s_\omega\geq 0\end{cases},$$

where $k_\omega>0$ and the switching surface is defined as follows

$$s_\omega=\hat{Z}_\alpha\hat{\psi}_{r\alpha}+\hat{Z}_\beta\hat{\psi}_{r\beta}\,.$$

3. FIRST ORDER SLIDING MODE CONTROL OF INDUCTION MACHINE

The sliding mode control of an induction machine with the multi-scalar transformation was presented in [19, 39]. In this paper the control structure was optimized. The IM state of variables $\mathbf{M}_i=[i_{s\alpha},i_{s\beta},\psi_{r\alpha},\psi_{r\beta},\omega_r]^T$ can be transformed to the multi-scalar form. The multi-scalar transformation was introduced in [19]. The multi-scalar variables take the form:

$$\begin{bmatrix}\hat{x}_{11}\\ \hat{x}_{12}\\ \hat{x}_{21}\\ \hat{x}_{22}\end{bmatrix}=\begin{bmatrix}\hat{\omega}_r\\ \hat{\psi}_{r\alpha}\hat{i}_{s\beta}-\hat{\psi}_{r\beta}\hat{i}_{s\alpha}\\ \hat{\psi}_{r\alpha}^2+\hat{\psi}_{r\beta}^2\\ \hat{\psi}_{r\alpha}\hat{i}_{s\alpha}+\hat{\psi}_{r\beta}\hat{i}_{s\beta}\end{bmatrix}. \tag{66}$$

Using the IM model (1)–(2) after transformation to the ($\alpha\beta$) reference frame components, the differential equations of the multi-scalar variables (66) have the following form:

$$\frac{dx_{11}}{d\tau}=\frac{L_m}{JL_r}x_{12}-\frac{1}{J}T_L\,, \tag{67}$$

$$\frac{dx_{12}}{d\tau}=-a_1x_{12}-x_{11}(x_{22}+a_3x_{21})+a_4u_1\,, \tag{68}$$

$$\frac{dx_{21}}{d\tau}=-2a_5x_{21}+2a_6x_{22}\,, \tag{69}$$

$$\frac{dx_{22}}{d\tau}=-a_1x_{22}+x_{11}x_{22}+a_2x_{21}+a_6\frac{x_{12}^2+x_{22}^2}{x_{21}}+a_4u_2\,, \tag{70}$$

where a_i, i=1…6 are defined in (10)–(11) and

$$u_1=-\psi_{r\beta}u_{s\alpha}+\psi_{r\alpha}u_{s\beta}\,, \tag{71}$$

$$u_2=\psi_{r\alpha}u_{s\alpha}+\psi_{r\beta}u_{s\beta}\,, \tag{72}$$

where the control variables of the IM are $\mathbf{u}=[u_{s\alpha},u_{s\beta}]^T$.

The sliding surfaces are chosen as:

$$s_{x11}=\lambda_{x11}\sigma_{x11}+\dot{\sigma}_{x11}\,, \tag{73}$$

$$s_{x21}=\lambda_{x21}\sigma_{x21}+\dot{\sigma}_{x21}\,, \tag{74}$$

where

$$\sigma_{x11}=x_{11}^*-x_{11}\,, \tag{75}$$

$$\sigma_{x21}=x_{21}^*-x_{21}\,. \tag{76}$$

By using the sliding mode control theory presented in [1-3, 28] it yields gains α_1,α_2 such that $s_{x11}\dot{s}_{x11}\leq-\eta_1\left|s_{x11}\right|$ and $s_{x21}\dot{s}_{x21}\leq-\eta_2\left|s_{x21}\right|$ where

$$\dot{\mathbf{s}}=\boldsymbol{\varphi}+\boldsymbol{\Psi}_r\mathbf{cu}\,, \tag{77}$$

$$\mathbf{u}=\Psi_r^{-1}\mathbf{c}^{-1}\left[-\boldsymbol{\varphi}\ +\mathbf{u}_d\right], \tag{78}$$

$$\dot{\mathbf{s}}=\left[s_{x11}\quad s_{x21}\right]^T, \tag{79}$$

$$\boldsymbol{\varphi}_N=\begin{bmatrix}\lambda_{x11}(\dot{x}_{11}^*-f_1)+\ddot{x}_{11}^*+f_2\\ \lambda_{x21}(\dot{x}_{21}^*-f_3)+\ddot{x}_{21}^*+2a_5f_3+f_4\end{bmatrix}, \tag{80}$$

$$\mathbf{c}_N=\begin{bmatrix}-a_7a_4 & 0\\ 0 & -2a_6a_4\end{bmatrix}, \tag{81}$$

$$\mathbf{u}_d=\begin{bmatrix}-\alpha_1 sign(s_{x11})\\ -\alpha_2 sign(s_{x21})\end{bmatrix}, \tag{82}$$

$$\boldsymbol{\Psi}_r=\begin{bmatrix}-\psi_{r\beta} & \psi_{r\alpha}\\ \psi_{r\alpha} & \psi_{r\beta}\end{bmatrix}, \tag{83}$$

and

$$\begin{aligned}&f_1=a_7x_{12}-\frac{1}{J}T_L,\\ &f_2=a_1a_7x_{12}+a_7x_{11}(x_{22}+a_3x_{21}),\\ &f_3=-2a_5x_{21}+2a_6x_{22},\\ &f_4=2a_6a_1x_{22}-2a_6x_{11}x_{22}-2a_6a_2x_{21}-2a_6a_6\frac{x_{12}^2+x_{22}^2}{x_{21}},\end{aligned} \tag{84}$$

for $\lambda_{x11},\lambda_{x21}>0$ and $\gamma_1>0$, $\gamma=\gamma_1\lambda_{x11}/J$, $\mathbf{c}_N$ and Ψ_r^{-1} matrixes are invertible and $\det(\boldsymbol{\Psi}_r)\neq 0$.

The control structure scheme is presented in Fig. 1.

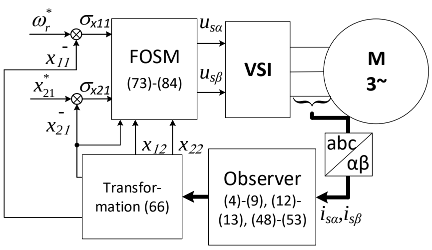

Fig. 1. Control structure scheme with the Observer and FOSM blocks

4. EXPERIMENTAL RESULTS

The experimental tests were prepared in the 5.5 kW IM supplied by the voltage source inverter. The transistors impulse frequency was f_k=3.3 kHz. The control system was implemented in an interface with a *DSP Sharc* ADSP21363 floating point signal processor and Altera Cyclone 2 FPGA. All the waveforms have been registrated by a measurement console prepared at Gdańsk University of Technology. In this console the frequency of the measurements can be changed (it is limited to 2500 sample points per one measurement and 5 channels by flash memory). The induction machine parameters are presented in Table 1.

The experimental investigation has been divided into three scenarios: 1) Machine starts up and reverse to 1.0 p.u. (with nominal IM parameters condition) – Fig. 2; 2) Very low or zero of rotor speed tests with a load torque injections – Fig. 3 and 4a; 3) Parameters uncertainties tests – Fig. 4b, 5. In the figures presented in this section the following state of variables are shown: x_{11}, x_{12}, x_{21}, x_{22} – multi-scalar variables defined in (66), i_m

- stator current vector module, $\tilde{\omega}_r$ - rotor speed error defined in (41), $u_{s\alpha}$ – stator voltage component (the control variable).

Fig. 2a presents the waveforms while the IM starts-up from 0.1 to nominal speed 1.0 p.u. Fig. 2b presents IM revers from -1.0 to 1.0 p.u. IM is not loaded in these tests. The square of rotor flux command is about 0.92 p.u. In both cases the electromagnetic torque responses very quickly to the rotor speed command. The electromagnetic and electromechanic subsystems are full decoupled which proves a good quality of proposed sensoress control system. In Fig. 3a machine revers from 0.005 to -0.005 p.u. is presented (about 7.1 rpm and IM is not loaded). The accuracy of the speed encoder has influence on the speed error transient. Nevertheless, the rotor speed error of estimation is smaller than 0.02 p.u. and near to zero speed, the control system is stable what is the most important advantage of the proposed sliding mode control system. It results from the persistence of excitation of the observer system through the sliding controllers and it affects the robustness of the control system. In Fig. 3b, after 1.5 s IM is loaded to about 0.55 p.u. and after 4.7 s the rotor speed command is equal to zero. After these changes, the sensorless control system is still stable.

Fig. 4a presents the waveforms while the zero rotor speed is set and after 1 s the load torque of about 0.85 p.u. is injected. The drive works stable even when zero rotor speed and disturbance in the form of the load torque are applied. These testes confirm that the control system is robust on load torque changes even when zero rotor speed was set. In Fig. 4b the parameters uncertainties test is shown. In fig. 4b the rotor resistance is changed to about 130% (R_r=2.28R_{rN}). In the IM is loaded to about 0.6 p.u. These tests show that the sensorless control system is stable but the estimation errors occurred.

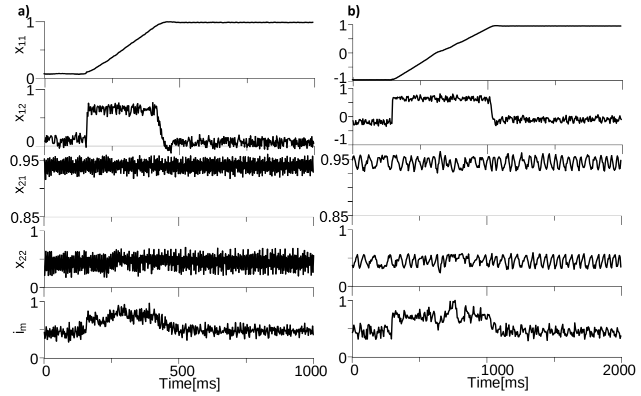

Fig. 2. a) Machine is starting up to 1.0 p.u., b) Machine is reversing from -1.0 to 1.0 p.u.

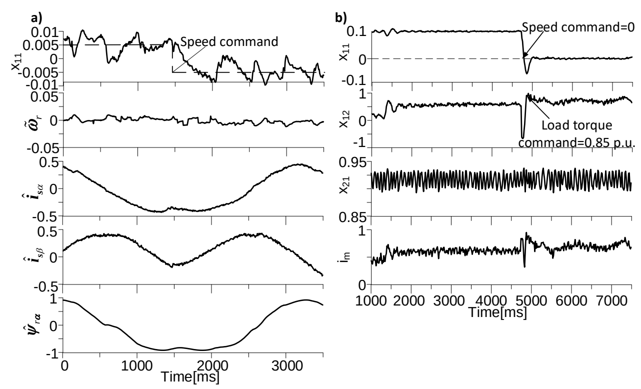

Fig. 3. a) Very low speed machine reverse from 0.005 to -0.005 p.u. (~7.2 rpm) without load torque command, b) after 4.7 s zero speed command and 0.85 p.u. load torque are applied

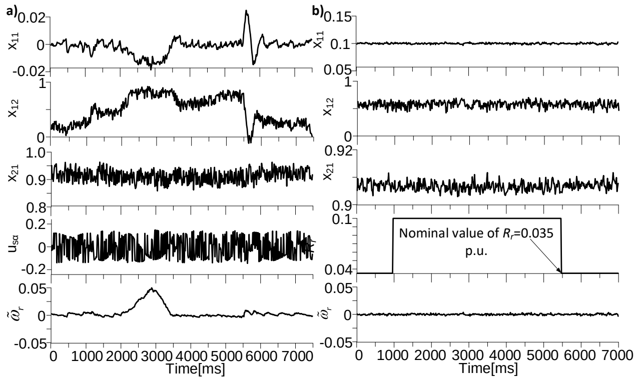

Fig. 4. a) Zero rotor speed and load torque injection about 0.85 p.u., b) Load torque about 0.6 p.u. and after 1.0 s rotor resistance is changed to about 185% of nominal (R_r=2.85R_{rN})

In Fig. 5a the stator resistance R_s is changed by about 185% of nominal value. The reference speed is 0.1 p.u. The load torque is about 0.45 p.u. In Fig. 5b the rotor speed command is 0.005 p.u. (7.2 rpm) and IM is loaded to about 0.6 p.u. The main inductance, stator and rotor inductances respectively are changed by about 5 %.

Experimental results presented in this section confirm the advantageus properties of the sensorless control system with sliding mode actions than classical PI controllers structure [22, 23]. The control system is robust on nominal parameter uncertainties. While the IM parameters are detuned, the sensorless control system is stable but the properties are not sufficient (the estimation errors occur). However, the proposed speed observer structure with backstepping and sliding mode action has good properties when the rotor speed is very small (near to zero) and the load torque is injected. In this case, the IM working points can achieve the unstable (unobservable) region [8, 10-11]. Therefore, the modified formula for the rotor speed estimation is proposed (65) which allows to obtain the stable electrical drive working even if the zero speed and almost nominal load torque are applied (Fig. 3a, 4a).

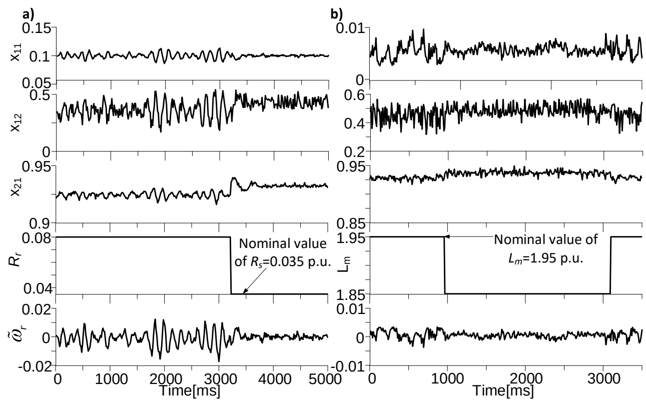

Fig. 5. a) After 3 s the stator resistance is changed from (R_s=2.28R_{sN}) to nominal value, b) After 1.0 s the main inductance value is changed from 1.95 to 1.85 p.u. (5%),

4. CONCLUSIONS

Presented results of experimental tests show that the sensorless control system is robust on machine parameters detuned up to above 100% of nominal value and under load torque injection (of course within the accepted scope of investigations). Obtained results can be compared to the sensorless control system with the PI controllers and the backstepping speed observer [22] but without the sliding mode features. Backstepping-sliding approach causes that the speed observer structure has more oscillation nature than the linear form [22] due to chattering. However, it allows better robustness of whole control system, especially near to zero speed tests, nominal machine parameters variation and the control system perturbation (load torque injection). This results from the precipitation of the observer structure from the stationary state and permanent of excitation condition through the high frequency switching. The speed observer structure is stable even if permanent zero speed and almost nominal load torque are set. Increasing the speed observer robustness does not eliminate unobservable machine working points in which the estimation process is not established [10]. The subject presented in this paper can be further developed in the direction of introducing the higher order sliding-mode actions to Observer-Controller structure (to chattering effect reduction). These problems still remain and will be studied in future papers.

APPENDIX

The discrete form of the proposed speed observer is as follows:

$$\hat{i}_{s\alpha}(k)=\hat{i}_{s\alpha}(k-1)+T_s\begin{pmatrix}-a_1 i_{s\alpha}(k-1)+a_2\hat{\psi}_{r\alpha}(k-1)+a_3\hat{Z}_\beta(k-1)+\\+a_4u_{s\alpha}(k-1)+v_\alpha(k-1)\end{pmatrix}, \qquad (85)$$

$$\hat{i}_{s\beta}(k)=\hat{i}_{s\beta}(k-1)+T_s\begin{pmatrix}-a_1 i_{s\beta}(k-1)+a_2\hat{\psi}_{r\beta}(k-1)-a_3\hat{Z}_\alpha(k-1)+\\+a_4u_{s\beta}(k-1)+v_\beta(k-1)\end{pmatrix}, \qquad (86)$$

$$\hat{\psi}_{r\alpha}(k)=\hat{\psi}_{r\alpha}(k-1)+T_s\begin{pmatrix}-a_5\hat{\psi}_{r\alpha}(k-1)-\hat{Z}_\beta(k-1)+a_6 i_{s\alpha}(k-1)+\\+v_{\psi\alpha}(k-1)\end{pmatrix}, \qquad (87)$$

$$\hat{\psi}_{r\beta}(k)=\hat{\psi}_{r\beta}(k-1)+T_s\begin{pmatrix}-a_5\hat{\psi}_{r\beta}(k-1)+\hat{Z}_\alpha(k-1)+a_6 i_{s\beta}(k-1)+\\+v_{\psi\beta}(k-1)\end{pmatrix}, \qquad (88)$$

$$\hat{Z}_\alpha(k)=\hat{Z}_\alpha(k-1)+T_s\begin{pmatrix}-\hat{\omega}_r(k-1)(\hat{Z}_\beta(k-1)-a_6 i_{s\alpha}(k-1))+\\-a_5\hat{Z}_\alpha(k-1)+v_{Z\alpha}(k-1)\end{pmatrix}, \qquad (89)$$

$$\hat{Z}_\beta(k)=\hat{Z}_\beta(k-1)+T_s\begin{pmatrix}\hat{\omega}_r(k-1)(\hat{Z}_\alpha(k-1)+a_6 i_{s\beta}(k-1))+\\-a_5\hat{Z}_\beta(k-1)+v_{Z\beta}(k-1)\end{pmatrix}, \qquad (90)$$

where

$\tilde{\xi}_\alpha(k)=\tilde{\xi}_\alpha(k-1)+T_s\tilde{i}_{s\alpha}(k-1)$,

$\tilde{\xi}_\beta(k)=\tilde{\xi}_\beta(k-1)+T_s\tilde{i}_{s\beta}(k-1)$,

$v_\alpha(k-1)=-a_2\tilde{\psi}_{r\alpha}(k-1)-c_\beta z_\alpha(k-1)-\tilde{\xi}_\alpha(k-1)-c_{\alpha1}sign\left(s_{\tilde{\xi}\alpha}(k-1)\right)$,

$v_\beta(k-1)=-a_2\tilde{\psi}_{r\beta}(k-1)-c_\beta z_\beta(k-1)-\tilde{\xi}_\beta(k-1)-c_{\alpha2}sign\left(s_{\tilde{\xi}\beta}(k-1)\right)$,

$v_{\psi\alpha}(k-1)=k_{\psi1}sign(s_{z\beta}(k-1))$,

$v_{\psi\beta}(k-1)=-k_{\psi2}sign\left(s_{z\alpha}(k-1)\right)$,

$v_{Z\alpha}(k-1)=k_{Z1}\left(-a_5k_{\psi1}sign\left(s_{z\alpha}(k-1)\right)+a_3z_\beta s_{z\alpha}(k-1)\right)-\tilde{P}_\alpha(k-1)$,

$v_{Z\beta}(k-1)=k_{Z2}\left(a_5k_{\psi2}sign\left(s_{z\beta}(k-1)\right)-a_3z_\alpha(k-1)\right)-\tilde{P}_\beta(k-1)$,

T_s is discretization time, T_s= 0.00015 s.

Table 1. Induction machine parameters

Name	SI	Per unit/base
P_n	5.5 kW	7.6 kW
U_n	400 V	U_b=400 V
I_n	10.8 A	I_b=18.9 V
n	1420 rpm	1.0 p.u.
R_s	0.98 Ω	0.046 p.u.
R_r	0.96 Ω	0.046 p.u
L_m	0.142 H	1.95 p.u.
L_s, L_r	0.148 H	2.05 p.u.

References

[1] R. Abd. Rahman, L. He, N. Sepehri, “Design and experimental study of a dynamical adaptive backstepping–sliding mode control scheme for position tracking and regulating of a low-cost pneumatic cylinder”, Inter. J. of robust and non. Cont., Vol. 26, Issue 4, 10 March 2016, Pages: 853–875, DOI: 10.1002/rnc.3341.

[2] M. Taleb, F. Plestan and B. Bououlid, An adaptive solution for robust control based on integral high-order sliding mode concept, Inter. J. of Robust and Non. Control, Vol. 25, Issue 8, 25 May 2015, Pages: 1201–1213, DOI: 10.1002/rnc.3135.

[3] R. Trabelsi, A. Khedher, M. F. Mimouni, F. M’Sahli, “Backstepping control for an induction motor using an adaptive sliding rotor-flux observer”. *Electric Power Systems Research*, Vol. 93, pp. 1–1, Dec. 2012.

[4] S. Drid, M. Tadjine, M. S. Nait-Said, “Robust backstepping vector control for the doubly fed induction motor”, *IET Control Theory & Applications*, 2007, Vol.: 1, Issue: 4, pp: 861-868, DOI: 10.1049/iet-cta:20060053.

[5] Bin Lin, Xiaoyu Su, Xiaohang Li, “Fuzzy Sliding Mode Control for Active Suspension System with Proportional Differential Sliding Mode Observer”, Asian Journal of Control, 2018.

[6] F. Barrero, A. Gonzalez, A. Torralba, E. Galvan, and L. G. Franquelo, “Speed control of induction motors using a novel fuzzy sliding-mode structure,” *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 3, pp. 375–383, Jun. 2002.

[7] Z. Jinhui, S. Peng, and X. Yuanqing, “Robust adaptive sliding-mode control for fuzzy systems with mismatched uncertainties,” *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 4, pp. 700–711, Aug. 2010.

[8] M. Comanescu, “Design and Implementation of a Highly Robust Sensorless Sliding Mode Observer for the Flux Magnitude of the Induction Motor”, *IEEE Trans. on Energy Conv.*, vol.: 31, Issue: 2, pp: 649 - 657, DOI: 10.1109/TEC.2016.2516951.

[9] A. Saim, W. Haoping, T. Yang: Robust Adaptive Fractional-Order Terminal Sliding Mode Control for Lower-Limb Exoskeleton, Asian Journal of Control. 28 November 2018.

[10] M. Ghanes, Z. Gang, “On Sensorless Induction Motor Drives: Sliding-Mode Observer and Output Feedback Controller”. *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3404-3413, Sep. 2009.

[11] D. Traore, F. Plestan, A. Glumineau; J. de Leon, “Sensorless Induction Motor: High-Order Sliding-Mode Controller and Adaptive Interconnected Observer”, *IEEE Trans. on Ind. Electron.*, Vol. 55, Issue 11, 2008.

[12] A. Benchaib, A. Rachid, E. Audrezet, M. Tadjine, “Real time sliding mode observer and control of an induction motor”. *IEEE Trans. Ind. Electron.*, vol.46, no. 1, pp.128–138, Feb.1999.

[13] Mou Chen, Shao-dong Chen and Qing-xian Wu, “Sliding mode disturbance observer-based adaptive control for uncertain MIMO nonlinear systems with dead-zone”, Inter. Journal of Adaptive Con. and signal processing, 3 Nov. 2016

[14] X. Zhang, Z. Li, “Sliding-Mode Observer-Based Mechanical Parameter Estimation for Permanent Magnet Synchronous Motor”, *IEEE Trans. On Power Electr.*, Vol. 31, No 8, August 2016.

[15] Y. Zhao, Z. Zhang, W. Qiao, L. Wu, “An Extended Flux Model-Based Rotor Position Estimator for Sensorless Control of Salient-Pole Permanent-Magnet Synchronous Machines”, *IEEE Trans. On Power Electr.*, vol. 30, No 8, 2015.

[16] G. Corradini, S. Ippoliti, D. Longhi, Marchei G. Orlando, “A Quasi-Sliding Mode Observer-based Controller for PMSM Drives”, Asian Journal of Control, Vol. 15, Issue 2, March 2013, Pages: 380–390, M.L., DOI: 10.1002/asjc.555

[17] V. I. Utkin, J. G. Guldner, and J.Shi, *Sliding Mode Control in Electromechanical Systems.* ISBN 9781420065602. CRC Press 2009.

[18] K. D. Young, V. I. Utkin, “A control engineer’s guide to sliding mode control”, *IEEE Trans. on Control Sys. Tech*., vol. 7, no. 3, may 1999.

[19] Z. Krzeminski, “Nonlinear control of induction motor”. *Proc. of the 10th IFAC World Congress*, Munich 1987.

[20] H. Abu-Rub, A. Iqbal, J. Guzinski: *"High Performance Control of AC Drives with Matlab / Simulink Models"*. John Wiley & Sons (2012).

[21] M. Krsti´c, I. Kanellakopoulos, P. Kokotovi´c, "*Nonlinear and Adaptive Control Design*", Wiley-Interscience Publication 1995.

[22] M. Morawiec, "Z type Observer Backstepping For Induction Machines*", IEEE Trans. on Ind. Electron.*, Vol. 62, Issue: 4, pp. 2090-2103, 2015.

[23] Z. Krzeminski, "Observer of induction motor speed based on exact disturbance model", *Power Electr. and Motion Control Conf., 13th EPE-PEMC*, pp. 2294-2299, 2008.

[24] R. Marino, P. Tomei, C. M. Verrelli, *Induction motor control design*, Springer, London, 2010.

[25] R. Marino, S. Peresada, and P. Valigi, "Adaptive input-output linearizing control of induction motors," *Automatic Control, IEEE Trans. on*, vol. 38, no. 2, pp. 208–221, Feb 1993

[26] A. Sabanovic and D.B.Izosimov, "Application of sliding modes to induction motor control," *IEEE Trans. on Ind. Appl.*, vol. 1A-17, pp. 41–49, 1981.

[27] S. Solvar, V. Le, M. Ghanes, J.-P. Barbot, and G. Santomenna, "Sensorless second order sliding mode observer for induction motor," in *Proc. IEEE CCA*, Yokohama, Japan, 2010, pp. 1933–1938.

[28] S. Gennaro, J. Rivera Dominguez, M. A. Meza, "Sensorless High Order Sliding Mode Control of Induction Motors With Core Loss," *IEEE Trans. Ind. Electron.*, vol. 61, no. 6, pp. 2678–2689, June 2014.

[29] O. Barambones, A.J. Garrido and F.J. Maseda, "Integral sliding mode controller for induction motor based on field oriented control theory", *IET Control Theory & Applications*, vol. 1, no. 3, pp. 786-794, May. 2007.

[30] A. Bartoszewicz, "Discrete-time quasi-sliding-mode control strategies", *IEEE Trans. on Ind. Electron.*, 45, 4, 1998, pp. 633-637.

[31] A. Proca, A. Keyhani, V. Utkin, and J.Miller, "Discrete time sliding mode, continuous time sliding mode and vector control of induction motors," *Int. J. Control*, vol. 75, no. 12, pp. 901–909, 2002.

[32] J. Rivera Dominguez, "Discrete-Time Modeling and Control of Induction Motors by Means of Variational Integrators and Sliding Modes—Part II: Control Design", *IEEE Trans. on Ind. Electron.*, Vol. 62, No 10, pp. 6183-6193, 2015.

[33] Lihang Zhao, Jin Huang, He Liu, Bingnan Li, Wubin Kong: "Second-Order Sliding-Mode Observer With Online Parameter Identification for Sensorless Induction Motor Drives", *IEEE Trans. on Ind. Electron.*, Vol. 64, Issue: 10, No 10, pp. 5280 - 5289, 2014.

[34] J. A. Moreno and M. Osorio, "Strict Lyapunov functions for the super-twisting algorithm," *IEEE Trans. Autom. Control.*, vol. 57, no. 4, pp. 1035–1040, Apr. 2012.

[35] C. L. Baratieri, H. Pinheiro, "New variable gain super-twisting sliding mode observer for sensorless vector control of nonsinusoidal back-EMF PMSM", *Control Engineering Practice*, vol. 52, pp. 59-69, 2016.

[36] S. K. Kommuri, M. Defoort, H. R. Kaimi, K. C. Veluvolu, "A robust observer-base sensor fault-tolerant control for PMSM in electric vehicles", *IEEE Trans. on Ind. Elect.*, vol. 63, no. 12, 2016.

[37] M. A. Hamida, A. Glumineau, "High-order sliding mode observer and integral backstepping control for sensorless IPMS motor", *International Journal of Control*, vol. 87, no. 10, pp. 2176-2193, 2014.

[38] O. Naifar, G. Boukettaya, A. Ouali, "Robust software sensor with online estimation of stator resistance applied to WECS using IM", *Int J Adv Manuf Technol* (2016) 84: 885. doi:10.1007/s00170-015-7753-3.

[39] M. Morawiec, A. Lewicki, Z. Krzemiński (2008). Sliding mode multiscalar control of induction motor, International Review of Electrical Engineering-IREE nr 3, strony 892 - 900, ISSN: 1827-6660.

[40] A. Levant. Sliding order and sliding accuracy in Sliding Mode Control. International Journal of Control, 58(6):1247–1263, 1993.

[41] Zhang Z, Ye D, Xiao B,Sun Z. Third-order sliding mode fault-tolerantcontrol for satellites based on iterative learningobserver.Asian J Control. 2019; 21:43–51.

[42] Guezmil A, Berriri H, Pusca R, Sakly A, Romary R, Mimouni MF. High order sliding mode observer-based backstepping fault-tolerant control for induction motor. Asian J Control. 2019; 21:33–42.

[43] Shaofang W., Jianwu Z., Benben Ch. A Robust Backstepping Sensorless Control for Interior Permanent Magnet Synchronous Motor Using a Super-Twisting Based Torque Observer, Vol. 21, No. 1, pp. 302–311, Asian J Control 2019.

[44] Wei Sh., Honglei H., Jiehao W. Robust Backstepping Sliding Mode Controller Investigation for a Port Plate Position Servo System Based on an Extended States Observer, Vol. 21, No. 1, pp. 302–311, Asian J Control 2019.

[45] Bin L., Xiaoyu S., Xiaohang L. Fuzzy Sliding Mode Control for Active Suspension System with Proportional Differential Sliding Mode Observer. Vol. 21, No. 1, pp. 264–276, Asian J Control 2019.

[46] Takashi Sh., Takehiro K. Gain-Scheduled Control under Common Lyapunov Functions: Conservatism Revisited, 2005 American Control Conference June 8-10, 2005. Portland, OR, USA.

Marcin Morawiec received Ph.D. and D. Sc. degrees in electrical drives from the Faculty of Electrical Engineering, Gdansk University of Technology, Gdansk, Poland in 2007 and 2016, respectively. He is author of over 70 articles and 2 monographs and 2 chapters in books, 1 Polish patent and 5 patent applications. His main scientific activities are concentrated on multi-scalar models, nonlinear control of any electrical machines, sensorless control, nonlinear control, backstepping control, adaptive observer backstepping and sliding mode control.

Arkadiusz Lewicki received the Ph.D. and D.Sc degrees in electrical drives from the Faculty of Electrical Engineering, Gdansk University of Technology, Gdansk, Poland in 2003 and 2013, respectively. He is currently with the Institute of Automatic Control of Electric Drives, Gdansk University of Technology. His scientific activities are concentrated on microprocessor control of converters, multilevel converters, pulse width modulation techniques, and nonlinear control of drives.

Filip Wilczyński received the M.Sc. degree in electrical engineering from Faculty of Electrical Engineering, Gdansk University of Technology, Gdansk, Poland in 2016. He is currently a PhD student in the Institute of Automatic Control of Electric Drives and Energy Conversion. His research interest include nonlinear control of electric drives, multiphase machines, sensorless control and current source inverters.