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Investigations of the Methods of Time Delay Measurement of Stochastic Signals Using Cross-correlation with the Hilbert Transform

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***Abstract*— The article presents the results of simulation studies of four methods of time delay estimation for random signals using cross-correlation with the Hilbert Transform. Selected models of mutually delayed stochastic signals were used in the simulations, corresponding to the signals obtained from scintillation detectors in radioisotope measurements of liquid-gas two-phase flow. Standard deviations of the values of the individual functions were designated and compared, along with standard deviations of time delay estimates determined on their basis. The obtained results were compared with the results for classic cross-correlation function (CCF). It was found that for the analysed range of the signal-to-noise ratio (SNR): 0.2 ≤ SNR ≤ 5, the lowest values of standard deviation of time delay estimates were obtained for the CCFHT function (cross-correlation with the Hilbert Transform of the delayed signal).**

Keywords — Time delay measurement, random signals, cross-correlation, Hilbert Transform.

I. INTRODUCTION

In areas such as radiolocation, acoustics, medical diagnostics, seismology, and two-phase flow measurement estimation of time delay is a very important issue. This theme is widely reported on in subject literature, among others in works [1-5]. In order to determine the time delay for random signals obtained from two or more sources, statistical methods are used [6-13]. The most well known classic methods, used primarily for Gaussian stationary signals, are: the cross-correlation function and the cross-spectral density phase [1-3, 6, 11-13].

A modification of the cross-correlation method may be substituting the measuring signals with Hilbert Transforms (HT) of these signals, or the so-called analytical signals obtained using that transform [14-20].

The present paper describes the possibilities of using the Hilbert Transform in measurements of time delay for random signals by cross-correlation. It presents examplary results of the simulation of the four methods for models of signals corresponding to real signals obtained in radioisotope studies of liquid-gas two-phase flow. The obtained standard deviations of the estimations of the studied functions and the transport time delay were compared with the corresponding results obtained using the classic cross-correlation.

II. APPLICATION OF THE HILBERT TRANSFORM IN CROSS-CORRELATION MEASUREMENTS OF TIME DELAY

The cross-correlation function $R_{xy}(\tau)$ of two ergodic signals $x(t)$ and $y(t)$ equals [6]:

$$R_{xy}(\tau) = E[(x(t)y(t+\tau)] \tag{1}$$

where $E[\cdot]$ denotes the expected value, while τ denotes time delay.

CCF reaches the maximum value for $\tau = \tau_0$, so transportation time delay may be obtained as the argument of the maximum of this function:

$$\tau_0 = \arg\{\max R_{xy}(\tau)\} = \arg\{R_{xy}(\tau_0)\} \tag{2}$$

The Hilbert Transform of a real signal $x(t)$ produces a real signal $\widetilde{x}(t)$ in accordance with the definition [12, 14]:

$$\widetilde{x}(t) = H[x(t)] = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(u)}{(t-u)}du \tag{3}$$

The Hilbert Transform can be applied to implement the so-called analytic signal:

$$\underline{x(t)} = x(t) + jH[x(t)] = x(t) + j\widetilde{x}(t) \tag{4}$$

The module of an analytic signal:

$$\left|\underline{x(t)}\right| = \sqrt{x^2(t) + \widetilde{x}^2(t)} \tag{5}$$

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is called the envelope of signal $x(t)$.

Literature presents several possible applications of HT and the analytic signal in time delay measurements of random signals using cross-correlation [12, 14-20].

Article [20] proposed to use a delayed signal $\tilde{y}(t)$ instead of signal $y(t)$ to calculate the cross-correlation function. Thus obtained, the CCFHT function:

$$R_{x\tilde{y}}(\tau) = \tilde{R}_{xy}(\tau) = E[(x(t)\tilde{y}(t+\tau)] \tag{6}$$

is zero for $\tau = \tau_0$. The location of the maximum CCF (1) corresponds in this case to the search for an argument for which function (6) passes through zero, which is easier to achieve.

Work [14] contains an analysis of the envelope of CCF, described by the relation:

$$O_{1xy}(\tau) = \sqrt{R_{xy}^2(\tau) + \tilde{R}_{xy}^2(\tau)} \tag{7}$$

Since for $\tau = \tau_0$ $\tilde{R}_{xy}(\tau_0) = R_{x\tilde{y}}(\tau_0) = 0$, therefore $O_{1xy}(\tau_0) = R_{xy}(\tau_0)$ and the maximums of CCF and $O_{1xy}(\tau_0)$ overlap. The properties of the envelope function (7) were discussed in [14, 17].

If two analytic signals (the original and the delayed one) are used to determine CCF, the obtained complex CCF will take the following form [12, 14]:

$$\begin{aligned} \underline{R}_{xy}(\tau) &= E[(\underline{x}^*(t)\underline{y}(t+\tau)] = \\ &= R_{xy}(\tau) + R_{\tilde{x}\tilde{y}}(\tau) + j\left[R_{x\tilde{y}}(\tau) + R_{\tilde{x}y}(\tau)\right] \end{aligned} \tag{8}$$

Seeing as $R_{\tilde{x}\tilde{y}}(\tau) = R_{xy}(\tau)$ and $R_{\tilde{x}y}(\tau) = -R_{x\tilde{y}}(\tau) = \tilde{R}_{xy}(\tau)$ [14], the modulus of relation (8) produces the function:

$$O_{2xy}(\tau) = \left|\underline{R}_{xy}(\tau)\right| = 2\sqrt{R_{xy}^2(\tau) + \tilde{R}_{xy}^2(\tau)} = 2O_{1xy}(\tau) \tag{9}$$

For $\tau = \tau_0$ the arguments of function maximums (1), (7) and (9) overlap.

Article [12] provided a discussion concerning the use in cross-correlation analysis signals defined as follows: $x_1(t) = [x(t)]^2 + [H\{x(t)\}]^2$, $y_1(t) = [y(t)]^2 + [H\{y(t)\}]^2$. The obtained real CCF takes the following form [11]:

$$R_{x1y1}(\tau) = 4\sigma_x^2\sigma_y^2 + 4\left[R_{xy}^2(\tau) + \tilde{R}_{xy}^2(\tau)\right] \tag{10}$$

If one applies centred signals in calculating the correlation [10], one obtains a function expressed thus:

$$O_{3xy}(\tau) = 4\left[R_{xy}^2(\tau) + \tilde{R}_{xy}^2(\tau)\right] = 4O_{1xy}^2(\tau) \tag{11}$$

III. Simulation Studies

A. Models of Signals

When considering the estimation of time delay, the relation between signals $x(t)$ and $y(t)$ obtained from two sensors is most often expressed using the following formulae [12]:

$$x(t) = s(t) + m(t) \tag{12a}$$

$$y(t) = c \cdot s(t - \tau_0) + n(t) \tag{12b}$$

where: $s(t)$ – stationary low-band random signal with normal probability distribution $N(0, \sigma_s)$, c – constant coefficient (most often $c = 1$); τ_0 – transportation time delay; $m(t)$, $n(t)$ – stationary white noises with Gaussian distributions $N(0,\sigma_m)$, and $N(0,\sigma_n)$ uncorrelated with signal $s(t)$ or with each other. With the above presumptions for models of signals [12], the following relations are true:

$$\sigma_x^2 = R_{xx}(0) = \sigma_s^2 + \sigma_m^2 \tag{13a}$$

$$\sigma_y^2 = R_{yy}(0) = c^2\sigma_s^2 + \sigma_n^2 \tag{13b}$$

where σ_x and σ_y denote standard deviations of signals $x(t)$ and $y(t)$ respectively, while $R_{xx}(\,)$ and $R_{yy}(\,)$ are their autocorrelation functions.

For low-band noises with limited frequency band B, power spectral density of the signal $s(t)$ is:

$$S_{ss}(f) = \begin{cases} K/2 & |f| \le B \\ 0 & other\ f \end{cases} \tag{14}$$

and the autocorrelation function is expressed by the equation:

$$R_{ss}(\tau) = KB\left(\frac{\sin 2\pi B\tau}{2\pi B\tau}\right) \tag{15}$$

Depending on the considered issue, in models [12] the following values are adopted: $m(t) \neq n(t) \neq 0$ or $m(t) = n(t) \neq 0$ or $m(t) = 0$ and $n(t) \neq 0$. In practice, the last two cases are most commonly applied.

B. Examples of Results

In order to compare the properties of functions (6), (7), (9) and (11) in relation to the classic CCF (1), simulations were carried out using LabVIEW software. Discrete stochastic signals $x(n)$ and $y(n)$ were generated, where: $n = t/\Delta t$, Δt - sampling interval, corresponding to the models (12). Signal $s(n)$ was formed with white noise using digital low-pass filtering, interferences $m(n)$ and $n(n)$ were Gaussian white noises uncorrelated with signals $x(n)$ and $y(n)$. The signal parameters were selected in such a way as to obtain CCF graphs similar to the radioisotope measurements of the liquid-gas flow [3, 21].

Discrete estimator of the CCF was calculated from:

$$\hat{R}_{xy}(l) = \frac{1}{N-l} \sum_{n=0}^{N-l-1} x(n) y(n+l) \qquad (16)$$

where: N – number of samples, $l = \tau/\Delta t$.

This estimator was used in determining the CCF and CCFHT for appropriate signals as well as O_{1xy}, O_{2xy} and O_{3xy} in accordance with the relations provided in the second chapter. In the simulations that were carried out, it was assumed that $\sigma_m = \sigma_n$, so the signal to noise ratio (SNR) was equal to: $\mathrm{SNR} = (\sigma_s/\sigma_m)^2 = (\sigma_s/\sigma_n)^2$.

Figure 1 shows the graphs of functions: CCF, CCFHT, O_{1xy}, O_{2xy} and O_{3xy} for models of signals (12) and parameters: relative band $B\Delta t = 0.025$, $N = 100{,}000$, $l_0 = \tau_0/\Delta t = 100$, $c = 1$, $\sigma_s = 1$, and $\sigma_m = \sigma_n = 0.45$ (SNR = 5).

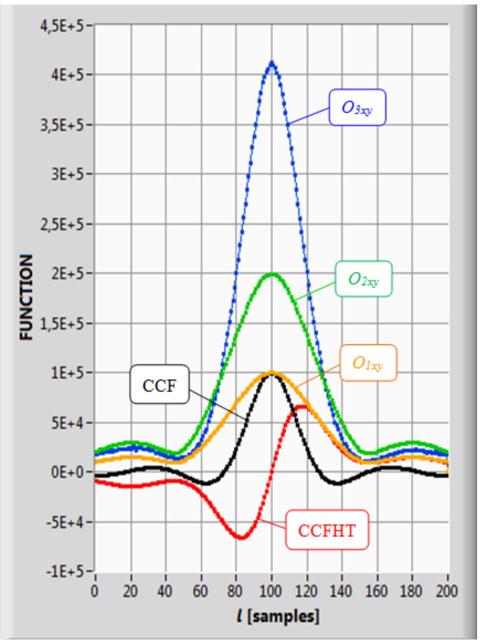


Fig. 1. CCF, CCFHT, O_{1xy}, O_{2xy} and O_{3xy} for models of stochastic signals (12) and parameters: $N = 100{,}000$, $l_0 = 100$, $c = 1$, $SNR = 5$.

Increasing the values of σ_m and σ_n reduces SNR, which causes distortion of the graphs of the analysed functions. An examples for SNR= 0.5 and SNR= 0.2 are shown in Figure 2.

In the first stage of research, for given SNR values experimental standard deviations of the value of the individual functions with neighbourhood $l = l_0$ were designated from the formula:

$$\hat{\sigma}\left[\hat{f}(l)\right] = \left\{ \frac{1}{M} \sum_{i=1}^{M} \left[\frac{\hat{f}(l)_i - \overline{\hat{f}(l)}}{\hat{f}(l)_{i_{\max}}} \right]^2 \right\}^{1/2} \qquad (17)$$

where f denotes the considered function (CCF, CCFHT, O_{1xy}, O_{2xy} or O_{3xy}), and M – the number of repetitions of the experiment.

The number of repetitions in the study was adopted as $M = 10^4$. Next, values of the quotient $k_{\sigma f}(l)$ were calculated:

$$k_{\sigma f}(l) = \frac{\hat{\sigma}\left[\hat{f}(l)\right]}{\hat{\sigma}\left[CCF(l)\right]} \qquad (18)$$

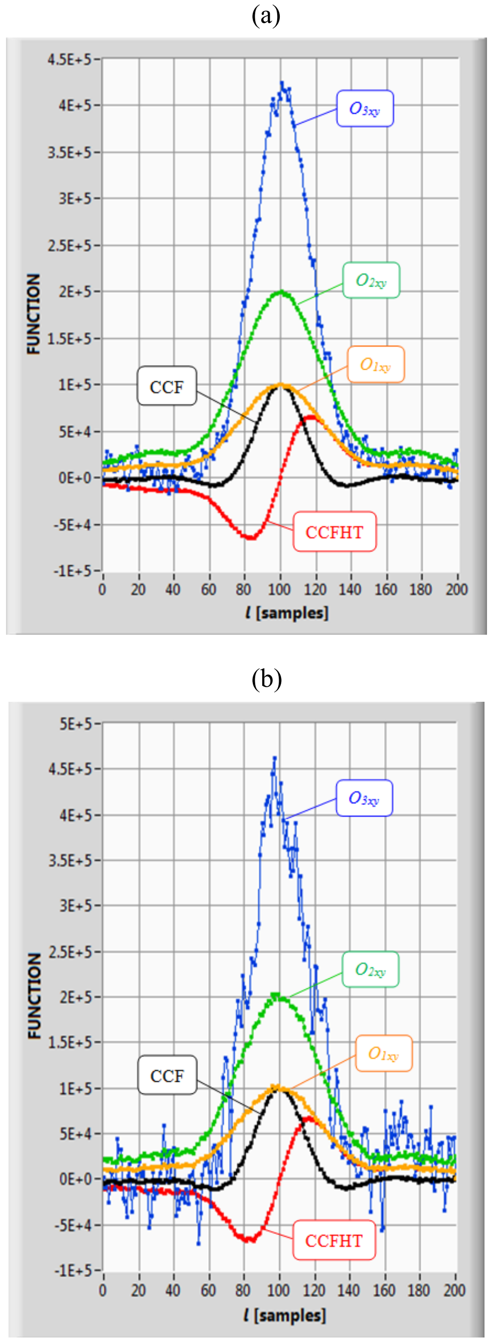


Fig. 2. CCF, CCFHT, O_{1xy}, O_{2xy} and O_{3xy} for models of stochastic signals (12) and parameters: $N = 100{,}000$, $l_0 = 100$, $c = 1$; (a) $SNR = 0.5$, (b) $SNR = 0.2$.

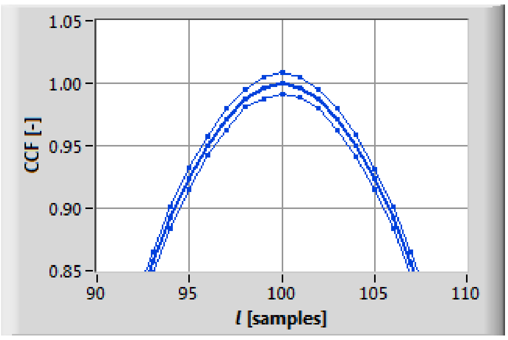
The experiments were carried out for $N = 100{,}000$ and several SNR values. The results for $l = l_0 = 100$ are summarized in Table I. The lowest values of $k_{\sigma f}(l_0)$ were obtained, respectively, for functions: CCF, O_{1xy}, O_{2xy}, CCFHT and O_{3xy}.

TABLE I. VALUES OF COEFFICIENT $k_{\sigma f}(l_0)$ OBTAINED FOR l_0 =100

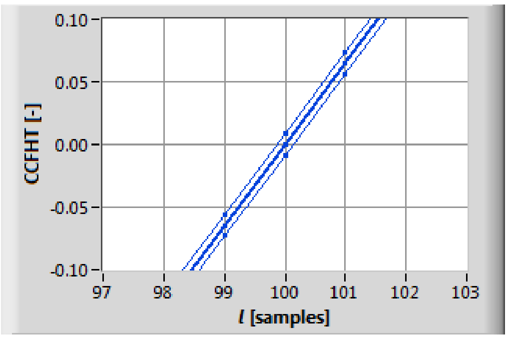
Parameter / Function	$k_{\sigma f}(l_0)$			
	SNR=5	SNR=1	SNR=0.5	SNR=0.2
CCF	1.00	1.00	1.00	1.00
CCFHT	1.24	1.45	1.54	1.44
O_{1xy}	1.00	1.00	1.00	1.00
O_{2xy}	1.01	1.00	1.00	1.00
O_{3xy}	14.64	6.95	5.92	6.92

Figure 3 shows examples of graphs of normalised functions: CCF, CCFHT, O_{1xy}, O_{2xy}, and O_{3xy} in neighborhood $l = l_0$ with a marked interval of one relative standard deviation (17). The values of each function were normalized in relation to their maximum values. The adopted values of parameters are: N = 100,000, l_0 = 100, c = 1 and SNR= 0.5.

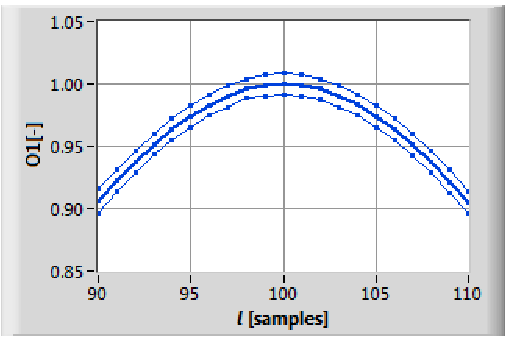
(a)



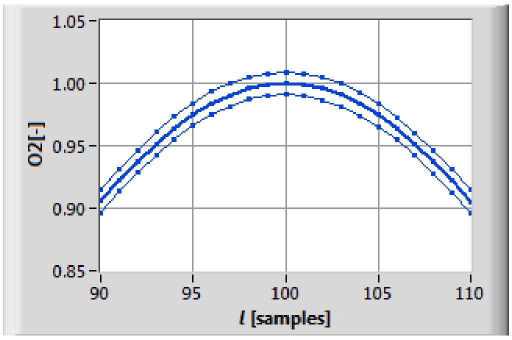
(b)



(c)



(d)



(e)

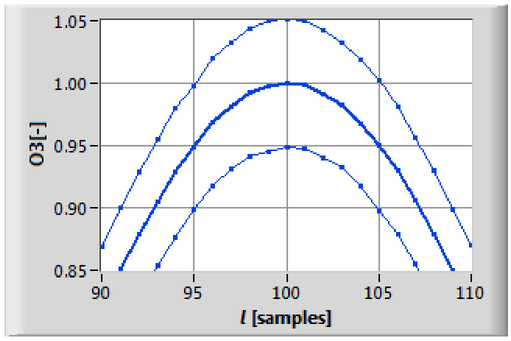


Fig. 3. Normalized functions in neighbourhood $l = l_0$ with one interval of relative standard deviation marked: (a) CCF, (b) CCFHT, (c) O_{1xy}, (d) O_{2xy}, (e) O_{3xy}.

During the main phase of the study, the values of experimental standard deviations of the time delay $\hat{\sigma}(\hat{\tau}_0)_f$ and the coefficient $k_{\sigma\tau_0}$ for each function and the same analysis parameters were calculated from the following relations:

$$\hat{\sigma}(\hat{\tau}_0)_f = \left[\frac{1}{M}\sum_{i=1}^{M}\left(\hat{\tau}_{0f_i} - \overline{\hat{\tau}_{0f}}\right)^2\right]^{1/2} \tag{19}$$

$$k_{\sigma\tau_0} = \frac{\hat{\sigma}(\hat{\tau}_0)_f}{\hat{\sigma}(\hat{\tau}_0)_{CCF}} \tag{20}$$

In order to designate time delay estimates $\hat{\tau}_0$, parabolic approximation was used for CCF, O_{1xy}, O_{2xy} and O_{3xy} and linear approximation was used for CCFHT in neighborhood l_0. The obtained results are summarized in Table II.

TABLE II. VALUES OF COEFFICIENT $k_{\sigma\tau_0}$ OBTAINED IN EXPERIMENT

Parameter / Function	$k_{\sigma\tau_0}$			
	SNR=5	SNR=1	SNR=0.5	SNR=0.2
CCF	1.00	1.00	1.00	1.00
CCFHT	0.36	0.16	0.15	0.15
O_{1xy}	3.70	1.93	1.85	1.73
O_{2xy}	3.74	1.93	1.84	1.72
O_{3xy}	4.98	2.65	2.86	3.64

Based on the results in Table II, it can be concluded that for all the SNR values shown, the smallest values of coefficient $k_{\sigma\tau_0}$ are obtained for the CCFHT function.

IV. Summary and Conclusion

The article summarizes the four applications of the Hilbert Transform in time delay measurements of stochastic signals by cross-correlation described in literature. The results were provided for simulation studies of functions CCF, CCFH, O_{1xy}, O_{2xy} and O_{3xy} for models of mutually delayed random signals, corresponding to the signals from the scintillation probes in measurements of two-phase flows using radioisotopes. Standard deviations of the values of the individual functions were designated and compared, along with standard deviations of time delay estimates determined on their basis. It was found that the lowest values of the standard deviation of function value for $l = l_0$ in the examined area of signal-to-noise ratio are (in order): CCF, O_{1xy}, O_{2xy} CCFHT, and O_{3xy}. In the case of estimating transportation time delay, which was the main objective of the study, the lowest values of the standard deviation for time delay estimates were obtained for CCFHT throughout the entire analysed range of SNR: $0.2 \leq$ SNR ≤ 5.

The findings of the studies presented in the article were used in works relating to the application of the CCFHT method in radioisotope flow measurements of liquid-gas mixtures and liquid-solid particles in pipelines [22, 23].

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