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Free vibration of flexomagnetic nanostructured tubes based on stress-driven nonlocal elasticity theory

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Abstract
A framework for the flexomagneticity influence is here considered extending the studies about this aspect on the small scale actuators. The developed model accommodates and composes linear Lagrangian strains, Euler-Bernoulli beam approach as well as an extended case of Hamilton's principle. The nanostructured tube should subsume and incorporate size effect; however, for the sake of avoiding the staggering costs of experiments, here, via stress-driven nonlocal elasticity theory, the desired influence is captured. A given section is dedicated to reveal the accuracy of the achieved model. In view of solution, the numerical results are generated analytically. We receive the conclusion that in nanoscale tubes the diameter can affect fundamentally the performance of the flexomagnetic effect.

Keywords: Flexomagneticity; Nanotube; Magnetoelasticity; Hamilton's principle

1. Introduction

As a new-discovered material's phenomenon, flexomagneticity absorbs the engineering researchers to study this physical occurrence when materials subject to static and dynamics states. Indeed, flexomagneticity results from strain gradients. This manner can be named as the direct impact of flexomagneticity. In a reverse impact, one can observe the flexo-effect during existence of an outer magnetic field gradient. This effect would not be absolutely and solely in actuators and smart materials, but even can occur in all materials and crystalline structures [1-6].

Since discovering of flexomagneticity effect, a very few publications have been observed on the statics and dynamics responses of small scale actuators and sensors which incorporate the effect [7, 8]. Within these articles, Zhang et al. [7] entirely focused on the effect of flexomagnetic during bending of a nano actuator beam. By means of Euler-Bernoulli beam theory, the static bending equation was formulated. Moreover, the consideration has been carried out by use of surface elasticity. A variety boundary conditions were investigated on the basis of both converse and direct magnetizations. From their result, one can find that the flexomagnetic is a size-dependent material property. On the other hand, Sidhardh and Ray [8] studied the static bending of a piezomagnetic-flexomagnetic Euler-Bernoulli nanosize beam based on the clamped-free ends conditions. Both inverse and direct effects of magnetization were discussed. The surface elasticity aided to examine the size-dependency into the small beam. With a quantitative evaluation, they showed the scale-dependent behavior of flexomagneticity and identified the significance of such the effect into nanostructures even with disregarding the piezomagneticity.

As far as we are aware, no research work is found yet in terms of investigating of natural frequencies of a nano-actuator tube composing the flexomagnetic. We aim to study the flexomagneticity effect on the natural frequencies of a nanostructured tube and intend to evaluate the small scale behavior on the basis of the stress-driven nonlocal model of elasticity. The numerical outcomes pertain to an analytical solution. The magneto-mechanical model is extended by illustrating some drawn graphs during variations in significant and particular criterions.

2. Applied mathematical model

Here a right-handed Cartesian coordinate system is attached to the schematic domain of the flexomagnetic nanotube as presented by Fig. 1. To this, we define L and r, for length and radius of the specimen, respectively.

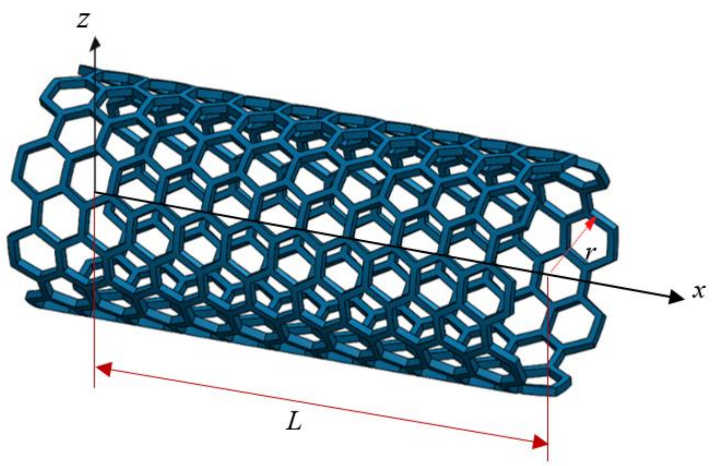

Fig. 1. Pictured geometry of a nanostructured tube presented schematically in the Cartesian coordinate

Assuming that the nanostructured tube contains flexomagneticity influence give the constitutive equations as [7, 8]

$$\sigma_{xx} = C_{11}\varepsilon_{xx} - q_{31}H_z \tag{1}$$

$$\xi_{xxz} = g_{31}\eta_{xxz} - f_{31}H_z \tag{2}$$

$$B_z = a_{33}H_z + q_{31}\varepsilon_{xx} + f_{31}\eta_{xxz} \tag{3}$$

in which g_{31} illustrates the influence of the sixth-order gradient elasticity tensor, H_z and B_z exhibit the component of magnetic field and the magnetic flux, respectively, ξ_{xxz} is the component of the higher-order hyper stress tensor and is an induction of coverse flexomagnetic effect, q_{31} depicts the component of the third-order piezomagnetic tensor, a_{33} represents the component of the second-order magnetic permeability tensor, f_{31} denotes the component of the fourth-order flexomagnetic coefficients tensor, σ_{xx} is the axial stress, C_{11} is the elastic modulus, ε_{xx} and η_{xxz} are the axial elastic strain and its gradients.

To have a movement for each node of body of the applied model after deformation, the Euler-Bernoulli hypothesis is used as [9-11]

$$u_1(x,z,t) = u(x,t) - z\frac{\partial w(x,t)}{\partial x} \tag{4a}$$

$$u_3(x,z,t) = w(x,t) \tag{4b}$$

in which the general movements along x and z directions are shown by u_i (i=1,3) and the movements of the middle plane of the thickness alone the aforementioned directions there have been used as u and w, respectively. More importantly, we employ z to dedicate the thickness coordinate.

Axial strain and the related gradient by means of linear Lagrangian strains as well as Eq. (4), are attained as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \tag{5a}$$

$$\eta_{xxz} = \frac{\partial \varepsilon_{xx}}{\partial z} = -\frac{\partial^2 w}{\partial x^2} \tag{5b}$$

To yield the equation which governs the domain subjected to vibrational state, the Hamiltonian can be extended and nominated as

$$\delta\int_{t_1}^{t_2}\left(\Pi_K - \Pi_U + \Pi_W\right) = 0 \tag{6}$$

for which the total internal strain energy, work of external forces and the kinetic energy are introduced by Π_U, Π_W and Π_K.

To determine the total strain energy, one should collect the strain energy by mechanics and the magnetic strain energy which lead to

$$\delta\Pi_U = \int_V \left(\sigma_{xx}\delta\varepsilon_{xx} + \xi_{xxz}\delta\eta_{xxz} - B_z\delta H_z\right)dV \tag{7}$$

In a magnetic-mechanical coupling problem, the resultants of stress can be defined as

$$N_x = \int_{-h/2}^{h/2} \sigma_{xx} dz \tag{8}$$

$$M_x = \int_{-h/2}^{h/2} \sigma_{xx} z dz \tag{9}$$

$$T_{xxz} = \int_{-h/2}^{h/2} \xi_{xxz} dz \tag{10}$$

To write a relation between the transverse component of the magnetic field and magnetic potential, one can show

$$H_z + \frac{\partial \Psi}{\partial z} = 0 \tag{11}$$

Here we assume a closed circuit state for the modeled system giving the boundary conditions for the magnetic potential as

$$\Psi\left(+\frac{h}{2}\right) = \psi,\ \Psi\left(-\frac{h}{2}\right) = 0 \tag{12a-b}$$

To determine the magnetic potential which is externally applied on the model as a result of the existence magnetic field, we symbolize ψ .

A mathematical combination of Eqs. (3), (7), (11) and (12), we can obtain the magnetic potential along the thickness and the magnetic field as below [7, 8]

$$\Psi = -\frac{q_{31}}{2a_{33}}\left(z^2 - \frac{h^2}{4}\right)\frac{\partial^2 w}{\partial x^2} + \frac{\psi}{h}\left(z + \frac{h}{2}\right) \tag{13}$$

$$H_z = z\frac{q_{31}}{a_{33}}\frac{\partial^2 w}{\partial x^2} - \frac{\psi}{h} \tag{14}$$

Therefore, one can insert Eqs. (13) and (14) into Eqs. (1)-(3) to harvest the magnetic induction and stress also higher-order moment stress component as follows

$$\sigma_{xx} = C_{11}\frac{\partial u}{\partial x} - z\left(C_{11} + \frac{q_{31}^2}{a_{33}}\right)\frac{\partial^2 w}{\partial x^2} + \frac{q_{31}\psi}{h} \tag{15}$$

$$\xi_{xxz} = -\left(g_{31} + \frac{q_{31}f_{31}z}{a_{33}}\right)\frac{\partial^2 w}{\partial x^2} + \frac{f_{31}\psi}{h} \tag{16}$$

$$B_z = -f_{31}\frac{\partial^2 w}{\partial x^2} - \frac{a_{33}\psi}{h} \tag{17}$$

Thus, the magnetic-mechanical stress resultants can be developed as

$$M_x = -I_z\left(C_{11} + \frac{q_{31}^2}{a_{33}}\right)\frac{\partial^2 w}{\partial x^2} \tag{18}$$

$$T_{xxz} = -g_{31} h \frac{\partial^2 w}{\partial x^2} + f_{31} \psi \tag{19}$$

where the general form of the area moment of inertia is as $I_z = \int_A z^2 dA$.

The general form of kinetic energy is displayed below

$$\Pi_K = \frac{1}{2} \int \int_A \rho(z) \left[\left(\frac{\partial u_1}{\partial t} \right)^2 + \left(\frac{\partial u_3}{\partial t} \right)^2 \right] dA dz \tag{20}$$

The first variation of kinetic energy leads to

$$\delta \Pi_K = \int_A \left[I_2 \frac{\partial^4 w}{\partial x^2 \partial t^2} \delta w - I_0 \frac{\partial^2 w}{\partial t^2} \delta w \right] dA \tag{21}$$

where the mass moment of inertias are

$$I_0, I_2 = \int_{-h/2}^{h/2} \rho(z) \left(1, z^2\right) dz$$

We consider the general case of established work by external forces as

$$\Pi_W = \frac{1}{2} \int_0^L N_x^0 \left(\frac{\partial w}{\partial x} \right)^2 dx \tag{22}$$

which its first variational case will be

$$\delta \Pi_W = \int_0^L N_x^0 \left(\frac{\partial \delta w}{\partial x} \frac{\partial w}{\partial x} \right) dx \tag{23}$$

in which N_x^0 depicts the axial load. In this paper, we investigate the axial magnetic force as in-plane axial resultant. To this,

$$N_x^0 = \psi q_{31} \tag{24}$$

Eventually, based on the above formulation the governing equation which gives the natural frequencies of the flexomagnetic nanotube can be taken as

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 T_{xxz}}{\partial x^2} + N_x^0 \frac{\partial^2 w}{\partial x^2} = I_0 \frac{\partial^2 w}{\partial t^2} - I_2 \frac{\partial^4 w}{\partial x^2 \partial t^2} \tag{25}$$

Here, we employ the stress-driven nonlocal elasticity model (NDM). It stands here differentially as below [12-13]

$$\frac{\partial^2 \chi(x)}{\partial x^2} - \frac{1}{L_c^2} \chi(x) = -\frac{1}{D L_c^2} M(x) \tag{26}$$

in which L_c shows a nonlocal characteristic length. And for $\chi(x)$ we have

$$\chi(x) = \frac{\partial^2 w}{\partial x^2} \tag{27}$$

Consequently,

$$D \left(L_c^2 \frac{\partial^6 w}{\partial x^6} - \frac{\partial^4 w}{\partial x^4} \right) = B \frac{\partial^4 w}{\partial x^4} + N_x^0 \frac{\partial^2 w}{\partial x^2} - I_0 \frac{\partial^2 w}{\partial t^2} + I_2 \frac{\partial^4 w}{\partial x^2 \partial t^2} \tag{28}$$

where $B = -g_{31}h$, and $D = I_z\left(C_{11} + \frac{q_{31}^2}{a_{33}}\right)$.

It is required to solve the above characteristic equation to obtain the natural frequencies of the flexomagnetic nanotube.

3. Solution of the equation

The methodology here comprises the analytical solution as [14]

$$w(x) = \sum_{m=1}^{\infty} \sin\left(\frac{m\pi}{L}x\right)\exp(i\omega_n t) \tag{29}$$

The above-mentioned series can satisfy the conditions for pined-pined beams.

To compute and present the numerical values of the natural frequencies of the flexomagnetic nanostructured tube, we apply Eq. (28) on Eq. (29). Finally, the characteristic equation of frequency would be

$$\omega_n = \sqrt{\left(-DL_c^2\alpha_m^6 - (D+B)\alpha_m^4 - \psi q_{31}\right)/\left(I_0 + I_2\alpha_m^2\right)} \tag{30}$$

where $\alpha_m = m\pi/L$.

4. Results' discussions

4.1. Results' validation

This section associates a comparison for the present formulation. The examination of formulation is based on the ignoring piezo-flexomagnetic features. Table 1 is prepared to estimate natural frequencies in dimensionless quantities with respect to [15] in which one can observe the evaluations for stress-driven nonlocal integral model (SDM) and strain gradient theory (SGT). As it is clear, slight differences are seen between NDM and SDM when the characteristic parameter (λ) is sufficiently small. However, increasing this dimensionless characteristic parameter results in further conflicts. Nevertheless, it is so far easier to use the NDM vis-à-vis the SDM and results can be acceptable.

$\Omega = \omega_n L^2\sqrt{\rho A/C_{11}I_z}$, $\lambda = \frac{L_c}{L}$, E=30×10^6, υ= 0.3, h=1, L/h=10, ρ=1, SS.

Table 1. Evaluation of natural frequencies of a nanobeam

λ	[15] SDM	[15] SGT	[Present] NDM
0	9.82927	9.82927	9.82927
0.01	9.83402	9.83392	9.83412
0.02	9.84787	9.8471	9.84865
0.03	9.87022	9.86761	9.87282
0.04	9.90042	9.89427	9.90657
0.05	9.93783	9.9259	9.94979

λ	[15]		[Present]
	SDM	SGT	NDM
0.06	9.98183	9.9614	10.0024
0.07	10.0318	9.9997	10.0641
0.08	10.0871	10.0398	10.1349
0.09	10.1472	10.081	10.2146
0.1	10.2115	10.1223	10.3029

4.2. Computational model

This section devotes some tabulated results for natural frequencies of the nanotube in the presence and absence of the flexomagneticity impact. To do this, the Table 2 aids us [7, 8]. Additionally, the results are shown for a non-dimensional manner of natural frequency as $\Omega = \omega_n L^2 \sqrt{\rho A / C_{11} I_z}$.

Table 2. Material specifications of an assumed piezo-actuator nanotube

C_{11}=286e9 N/m^2 f_{31}=10^{-10} N/A q_{31}=580.3 $N/A.m$ a_{33}=1.57×10^{-4} N/A^2
L=15d, d=1 nm, h=0.34 nm

We initially evaluate the effect of length scale parameter variations in accordance with the Table 3. The nanotube is assumed in two states. The former has been investigated with regard to the effects of flexomagnetic and the latter without considering the effect and merely under piezomagnetic conditions. It is important to note that the flexomagnetic effect makes the natural frequencies smaller. It is also worth mentioning that the larger the values of L_c, the higher the natural frequencies. It can be observed that while the value of L_c is set to be zero in contrast to the when its value is at 4, give further difference for natural frequencies of the mentioned tubes. It can be stated that this decreasing behavior in the difference of results of both cases can be because the length scale parameter increases the strength of the tubes and as far as the flexomagnetic effect makes the material more flexible, hence, in higher values of the length scale the influence of flexomagneticity is slighter. More significantly, as the variation of the length scale parameter creates differences between results of a piezomagnetic nanotube against a flexo-piezomagnetic one, this behavior can confirm that the flexomagneticity is a size-dependent phenomenon similar to the flexoelectricity [16-19].

Tables 4 and 5 give the numerical values of natural frequencies for the both aforementioned cases of nanotubes in variations of diameter and length of the tubes.

Again here the size dependency behavior of flexomagneticity can be seen. The increase in the diameter leads to decrease of the discrepancy between response of the two tubes. Howbeit it is noteworthy that the reducing effect in Table 3 is further remarkable than the Table 2. In addition, it seems that the diminishing effect as a result of enlarging diameter is more noticeable than the lessening effect of the length scale parameter as the previous Table. Accordingly, it is important to say that the diameter plays as a crucial factor to study size-dependent response of nanotubes possessing flexomagneticity.

In Table 6 the natural frequencies of both cases of nanotubes are tabulated in order to exhibit whether the magnetic field affects a flexo-piezomagnetic nanotube more than a piezomagnetic one or not. As can be observed, there is no highlight difference among the two tubes, although a very little difference can be seen. The meaning of difference is here about difference between results of two cases when the magnetic potential is chosen as minimum against when it is selected as maximum in the Table. As a matter of fact, it can be said that the magnetic field has approximately identical influence on the two tubes. Furthermore, it is substantial that the values of the external potential are insignificant, but their effect is major. In fact, it is concluded that the effect of outer magnetic potential on the natural frequencies of a nanoscale actuator tube having piezo-flexomagnetic influences is momentous.

Table 3. Dimensionless natural frequencies in variations of the length scale parameter (Ψ=1 μA)

L_c *(nm)*	Piezomagnetic nanotube with considering flexomagneticity	Piezomagnetic nanotube
0	14.4114	14.4303
0.25	14.4307	14.4496
0.5	14.4886	14.5074
0.75	14.5845	14.6032
1	14.7177	14.7362
1.25	14.8872	14.9055
1.5	15.0918	15.1099
1.75	15.3301	15.3479
2	15.6006	15.6180
2.5	16.2312	16.2480
3	16.9702	16.9863
3.5	17.8041	17.8194
4	18.7202	18.7347

Table 4. Dimensionless natural frequencies in variations of the diameter (L_c=0.5 *nm*, Ψ=1 μA)

d (nm)	Piezomagnetic nanotube with considering flexomagneticity	Piezomagnetic nanotube
0.7	10.6768	10.7023
1	14.4886	14.5074
1.2	16.6303	16.6467
1.5	19.5766	19.5905
2	24.2887	24.2999
2.5	29.2463	29.2556
3	34.8146	34.8224
3.5	41.2301	41.2367
4	48.6422	48.6478

Table 5. Dimensionless natural frequencies in variations of the Length (L_c=0.5 *nm*, Ψ=1 μA)

L/d	Piezomagnetic nanotube with considering flexomagneticity	Piezomagnetic nanotube
10	14.4471	14.4660
15	14.4886	14.5074
20	14.8129	14.8313
25	15.5244	15.5420
30	16.7575	16.7737
35	18.6175	18.6321
40	21.1582	21.1710
45	24.3866	24.3977
50	28.2822	28.2919

Table 6. Dimensionless natural frequencies in variations of the magnetic potential (L_c=0.5 *nm*)

Ψ *(μA)*	Piezomagnetic nanotube with considering flexomagneticity	Piezomagnetic nanotube
-2	13.9794	13.9989
-1	14.1512	14.1704
0	14.3209	14.3399
1	14.4886	14.5074
2	14.6544	14.6730
3	14.8183	14.8367
4	14.9805	14.9987
5	15.1409	15.1589
6	15.2996	15.3174
7	15.4567	15.4743
8	15.6122	15.6297

9	15.7662	15.7835
10	15.9187	15.9358

5. Conclusions

In this paper, we successfully combined the flexomagnetic effect with elasticity relations to consider this impact on the natural frequencies of a nanotube. We further considered the nanosize effects based on the stress-driven nonlocal elasticity model. The extended Hamiltonian demonstrated governing equation in a magnetic-mechanical coupling. We verified our results regarded to a nanotube and correspond well to the open literature. In an analytical framework, we established some tabulated results to show the flexomagnetic effect. Based on our numerical exercises, it was found that the variation of diameter is more notable to show the effect of flexomagneticity. And the lesser the diameter, the larger the flexomagnetic effect. Likewise, the smaller the length of the tube, the greater the flexomagneticity effect. It can confirm that the flexomagneticity is a size-dependent feature of materials, and its impact is more considerable in nanoscale.

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