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Modelling of heat transfer in supercritical pressure recuperators

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Abstract: In the paper presented is analysis of convective flow heat transfer at supercritical pressure in channels of heat exchanger working in the thermodynamic cycle. The modelling is based on the division of the flow into three regions, namely the heavy fluid, a two phase flow consisting of the heavy and light fluids and finally the light fluid flow. Modelling is concentrated on the region of simultaneous flow of two fluids divided into the zones with the light and heavy fluids. These agents are considered with averaged thermophysical properties in each region. The surface separating the two zones with respective fluids is assumed to feature the pseudocritical temperature. The problem is solved using a previously developed theoretical model based on considerations of energy dissipation in the flow. The fundamental hypothesis in the model is the fact that heat transfer is considered as being dependent on two contributions

of energy dissipation, one stemming from the shearing pseudo two-phase flow of the heavy and light fluids, whereas the second contribution comes from the energy dissipation due to exchange of mass between the heavy and light fluids. The results of calculations have been compared with some experimental data from literature showing a good consistency.

NOMENCLATURE

a	–	parameter in eq. (15)
A	–	parameter in eq. (16)
b	–	parameter in eq. (24)
C	–	constant
c_p	–	specific heat, J/kgK
d	–	diameter, m
E	–	dissipation energy, W/m^3
f	–	friction factor
g	–	acceleration due to gravity, m/s^2
G	–	mass flux, kg/(m^{2}s)
h	–	specific enthalpy, J/kg
$\dot{m}$	–	mass flow rate, kg/m^3
p	–	pressure, Pa
Pr	–	Prandtl number
q	–	heat flux, W/m^2
r	–	radius, radial coordinate, m
R	–	tube radius, m
Re	–	Reynolds number

T	–	temperature, ºC
u	–	velocity component, m/s
w	–	velocity component, m/s
x	–	quality
z	–	axial coordinate

Greek symbols

α	–	heat transfer coefficient, W/(m^{2}K)
λ	–	thermal conductivity, W/(mK)
ρ	–	density, kg/m^3
μ	–	dynamic viscosity, Pa s
Φ^2	–	two-phase flow multiplier

Subscripts

e	–	pseudo-quality
exp	–	experimental
$grav$	–	gravitational
h	–	heavy fluid
hl	–	transition from heavy to light fluid
i	–	interface
l	–	light fluid
pc	–	pseudo-critical
$PC\text{-}TP$	–	pseudo-critical two-phase flow
TP	–	two-phase

TP-T	–	two-phase friction component
th	–	theoretical
w	–	wall

Superscripts

+	–	non-dimensional
$\bar{}$	–	average value

Keywords: supercritical pressure, heat transfer, modelling, semi-empirical energy dissipation model

Introduction

The topic of supercritical heat transfer for professional power production has been already extensively studied for years in the case of water as well as other working fluids. It is however very difficult to further improve the power conversion efficiency for water as a working fluid, as one of the prohibiting factors is the necessity to increase the live steam temperature before the turbine up to about 700°C or more [2]. Although new superalloys can be used in that range of temperatures and pressures, it is rather not economically viable considering high manufacturing costs [3]. However, a new area of application opens for the case of other than water fluids [1], Sarkar [4]. The possible area of exploration is to convert low-grade heat to electricity or to upgrade it to a higher temperature [5]. A recent study showed that for the EU alone, waste heat potential in industry amounts to 304.13 TWh/year [6]. In that light especially the carbon dioxide supercritical cycles seem to be an interesting option for further scrutiny. The trend into research into supercritical carbon dioxide commenced few decades ago with the concept of supercritical gas reactors, whereas currently the interest is sustained through the opportunity to develop high temperature heat pumps with carbon dioxide as the working fluid or supercritical Organic Rankine Cycles (ORC). Supercritical CO_2 Rankine cycle offers various advantages including simplicity, and highly compact turbomachinery and heat exchangers. Moreover, the CO_2 cycles belong to the environmentally-friendly ones. In order to increase the performance of CO_2 thermodynamic cycles the heat exchangers design improvements are amongst most possible approaches. Significantly less attention has been devoted to hydrocarbons, however in that area some developments are also underway, such as progress in space rockets, advanced hypersonic aircrafts, requirement of an effective cooling techniques. Li et al. [7] presented the heat transfer performance of kerosene in a vertical smooth tube at supercritical pressures and found that heat transfer was enhanced when the fluid temperature was near the critical temperature. Heat transfer performance was better at higher heat fluxes or inlet temperatures, but was worse at higher inlet pressures. On the other hand Zhao et al. [8] investigated thermal cracking of a hydrocarbon fuel at different supercritical pressures. Huang et al [9] compiled an excellent review on the heat transfer of up to date literature at supercritical pressures, presenting the mechanisms detected thus far in supercritical heat transfer. Cai et al [10] presented that supercritical CO_2 fracture is a promising method to improve the efficiency of the shale gas exploration. The heat transfer of supercritical CO_2 plays an important role in both transportation and injection processes. In the light of potential implementation in geothermal systems Hsieh et al [11] performed a systematic study of different fluids at supercritical pressures and volumetric flow rates. In the study, the supercritical pressure of 10MPa constituted an optimal operating condition for CO_2-Enhanced Geothermal Systems (CO_2-EGS) because of the high heat transfer performance at 200°C.

Heat transfer at the conditions above the thermodynamical critical point is different from those conditions present at subcritical pressures. One of the distinct features of heat transfer in that region is that small changes in temperature can lead to large changes in thermo-physical properties of the fluid. Such behavior may subsequently result in heat transfer enhancement or deterioration. Heat transfer deterioration occurs primarily due to severe variation of thermo-physical properties. However, another contributing factor to heat transfer deterioration is attributed to the effects of buoyancy force or flow acceleration. Different empirical correlations for predicting heat transfer deterioration were postulated by several authors, but unfortunately the published results show that the models developed based on the analysis of one working fluid cannot be applied directly to another working fluid, which is a significant deficiency. It ought to be remembered that due to the peculiar distributions of physical properties of the fluid under supercritical conditions, the fluid density, thermal conductivity and specific heat near the wall can be relatively low, which results in worsening of heat transfer between the fluid and the wall. The improved values of heat transfer coefficient with increasing heat flux are due to the enhancement effect, when the fluid temperature dominates over the deterioration effects of local wall temperature. The reversed trend is observed when the effect of deterioration of heat transfer overcomes the enhancement effect. Application of a high heat flux induces a strong buoyancy effect as well as the flow acceleration effect. These two additional influences further complicate the heat transfer. Finally, the effect of pressure on heat transfer is relatively small compared to the effects of mass flux and heat flux.

Scrutiny of the literature on the topic of supercritical heat transfer leads to the conclusion that in general, there can be distinguished three modes of heat transfer to supercritical fluids. One can talk about the classical and augmented heat transfer as well as the heat transfer during boiling crisis, Pioro and Duffey [12], Duffey and Pioro [13]. Classical supercritical heat transfer [14,15] occurs far from the critical point regions with a predictable heat transfer coefficient using modifications of the convective heat transfer correlations. Augmented heat transfer occurs when heat flux is very low, and a local peak value of the heat transfer coefficient exists near the pseudo-critical point [13]. When the heat flux is relatively high and mass flow rate is relatively small, the heat transfer crisis occurs [16]. The heat transfer deterioration (HTD) usually occurs at high heat fluxes and low mass fluxes. There have been a lot of experimental and numerical investigations focusing on heat transfer to water and carbon dioxide at supercritical pressures. Shen et al. [17] experimentally investigated heat transfer to water in downward flow starting from subcritical pressures up to supercritical pressures and found heat transfer enhancement in the pseudo-critical region. Kim et al. [18] compared the heat transfer to supercritical CO_2 in upward and downward flow cases and developed relevant heat transfer correlations. The wall temperature peaked for the vertical upward flow. However, no peak value of the wall temperature was observed for the downward flow cases. Du et al. [19] conducted numerical simulation on the cooling heat transfer of supercritical CO_2 in a horizontal tube. They found that the heat transfer was significantly enhanced due to the buoyancy force, especially near the pseudo-critical point. Jackson [20] presented three physically-based semi-empirical models of turbulent, buoyancy-influenced and acceleration-influenced heat transfer to fluids at supercritical pressure taking into account the strong non-uniformity of physical and transport properties which can be present in such fluids due to the extreme dependence on temperature their properties in the pseudocritical temperature region.

Inspection of the physical property chart distribution suggest that in the region extending from the inlet temperature to the wall temperature with the pseudocritical temperature found in between there practically exist two zones in the fluid with significantly different properties where with the reference to these properties the fluids have been given names of light and heavy ones. Modelling based on that finding has been proposed by Adelt and Mikielewicz [21]. Authors considered a case of pseudo-boiling with a low density, viscosity and thermal conductivity fluid, naming it the light zone, where temperature was ranging from the pseudocritical temperature at the interface between light and heavy zones to the wall temperature. In contrast to this zone, the core of the flow was occupied by the fluid with significantly higher density, viscosity and thermal conductivity and hence was named the "heavy" zone. The interface between these two zones is an isothermal surface having the pseudocritical temperature. The discussed respective regions can be presented in the pressure-temperature coordinates as in Fig. 1.

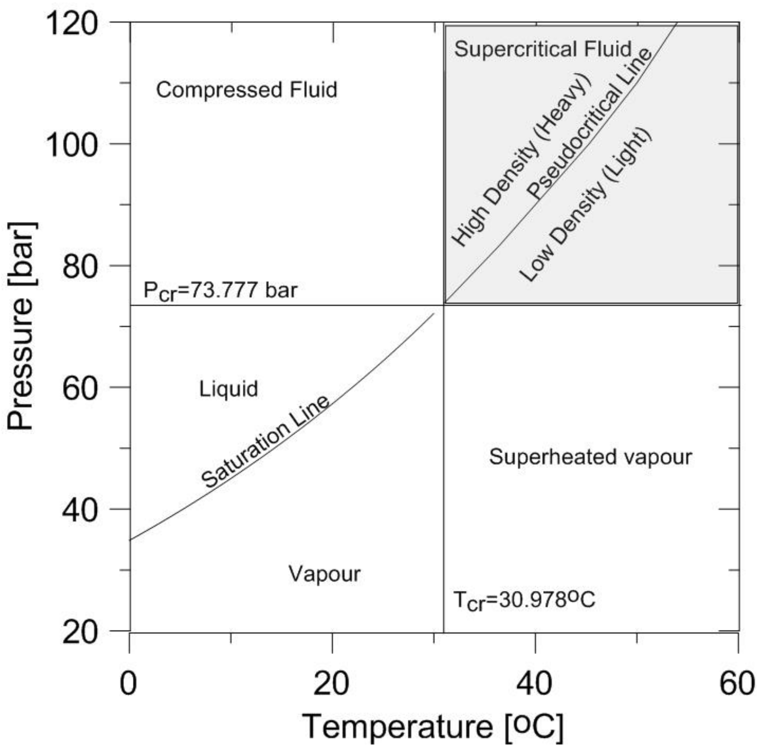


Figure 1: Pressure-temperature diagram for carbon dioxide. Diagram developed on the basis of data from Refprop 9.1 [22].

In the present paper the principles of energy dissipation budget, which proved to be successful in the modelling of flow boiling process under subcritical conditions such as saturated flow boiling and further subcooled and the mist flow (Mikielewicz [23], Mikielewicz and Mikielewicz [24-27], was applied to modelling of heat transfer in heat exchanger channels under supercritical conditions. Considered is the case where the pseudocritical temperature is found between the fluid and wall temperatures. Under such conditions two-cases can be distinguished, namely the pseudo-boiling and pseudo-condensation, however in the paper only the first case is considered. The results of calculations have been compared with some experimental correlations from literature showing a good consistency.

Theoretical considerations

The versatile semi-empirical model for calculations of flow boiling due to Mikielewicz [23] and its final version for calculations of saturated subcritical flow boiling due to Mikielewicz and Mikielewicz [24-27] has been hitherto tested for an extensive number of experimental data for numerous fluids such as water, synthetic refrigerants (R134a, R11, R12, R404, R407, R410, R152a and other) and natural refrigerants (NH_3, R290, R600a, R601, CO_2) as well as new perspective fluids (SES36, R245fa, R1234yf, R1233ze) and returned satisfactory results both in conventional size channels as well as minichannels (Mikielewicz and Jakubowska [28-30], Mikielewicz et al. [31].

The success in modelling of the complex phenomenon of flow boiling encouraged authors to develop a model for calculation of supercritical heat transfer basing on the similar hypothesis, namely that the energy dissipation in the supercritical two-phase flow can be also modelled due to constituent contributions, that is the shearing convective flow and the dissipation related to the generation of the light phase fluid. The procedure for establishing such model is described in the further. The schematic of the flow under scrutiny is presented in Fig. 2 for the case of pseudo-boiling. In the figure flow separation into the heavy and light fluid zones is delineated by the contour corresponding to the pseudo-critical temperature. In the case where the heat flux is removed from the channel then the opposite situation is found, i.e. the light zone is found in the core, and the heavy zone is at the wall. Such case is called pseudo-condensation.

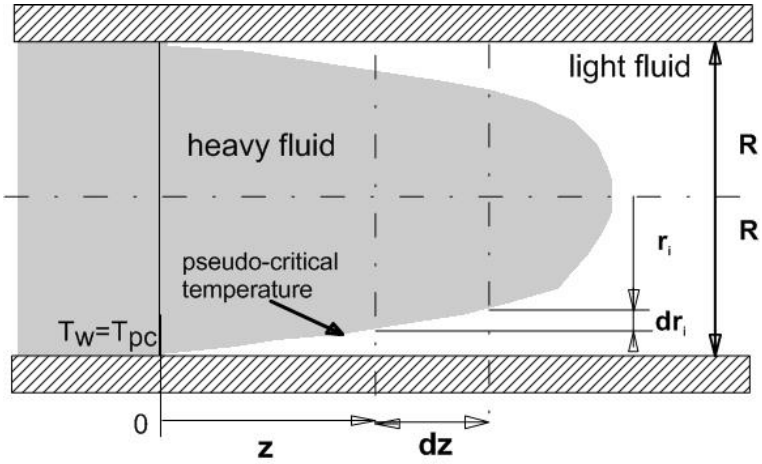


Figure 2: Schematic of heat transfer in a channel at supercritical pressure.

In the analysis of pseudo-boiling we assume that:

1. Phases in the core and wall regions are flowing with the same average velocities.
2. Due to heat transfer (evaporation of the heavy-phase in case of pseudo-boiling) the volume of the core is reducing its size.
3. Core flow and annular flow surrounding it can be modelled as the respective homogenous two-phase flows.
4. Distribution of shear stresses in the light phase and heavy phase are non-linear, which directly results from the model as the buoyancy effects are included.

As mentioned earlier and what can be noticed from the schematic presented in Fig. 2, prior to reaching the pseudocritical temperature by the heavy fluid, in the case of pseudo-boiling, we are dealing with the flow of single phase fluid until the wall temperature will reach the pseudo-critical temperature. That occurs at the location where z=0, as indicated in the figure. Until that location is reached the heat transfer calculations should be carried out using the single phase correlations, such as for example the one due to Dittus-Boelter [33]. Next, the split of the flow is observed into the light and heavy fluids. That is the region, where the model presented below is applicable. Once the bulk of fluid reaches the pseudocritical temperature the flow becomes again a single phase one and the correlations such as the one due to Dittus-Boelter [33] can be used for determination of the heat transfer using the properties of the light fluid.

The fundamental hypothesis applied in the considered model is the fact that heat transfer in the two phase flow in the channel can be considered as a sum of two contributions constituting the total energy dissipation in the flow, E_{PC-TP}. In the case of supercritical pseudo-boiling heat transfer the heavy (in the core) and light (at the wall) zones are distinguished. The energy dissipation can be defined by the dissipation due to the convective shearing two-phase flow of a mixture of light and heavy phases, E_{TP-T}, and energy dissipation due to generation of the light phase, E_{hl}, constituting in such way the total energy dissipation in the flow:

$$E_{PC-TP} = E_{TP-T} + E_{hl} \qquad (1)$$

In case of pseudo-condensation the situation is opposite, i.e. the light zone is located in the core region, whereas the heavy one is found at the wall. Phase transition takes place from the light to the heavy phase.

The virtue of the model postulated in [23] is that energy dissipation under steady state conditions in the two-phase flow can be approximated as energy dissipation in the laminar sublayer, which is eventually defined as the power lost in the control volume. The flow power is defined as a product of a shearing force and the flow velocity of the mixture. Following relevant substitutions the resulting dissipation energy definition yields:

$$E = \frac{\tau^2}{\mu} \qquad (2)$$

Substituting the respective expressions for the terms describing dissipation energies into (1) a geometrical summation between respective shear stresses in the flow is obtained:

$$\tau^2_{PC-TP} = \tau^2_{TP-T} + \tau^2_{hl} \tag{3}$$

Substituting the definition of respective shear stresses into (3) and performing necessary reductions a geometrical relation between respective friction factors is obtained:

$$f^2_{PC-TP} = f^2_{TP-T} + f^2_{hl} \tag{4}$$

Making use of the known analogy between the exchange of momentum and heat we can generalize the above result to extend it over to the case of heat transfer coefficient to yield its value of complex heat transfer in terms of simpler modes of heat transfer, namely the heat transfer coefficient in for the convective pseudocritical two-phase flow and heat transfer coefficient resulting from the conversion of the heavy fluid to the light phase for the pseudo-boiling case:

$$\alpha^2_{PC-TP} = \alpha^2_{TP-T} + \alpha^2_{hl} \tag{5}$$

The way to evaluate α_{TP-T} and α_{hl} will be presented in the further.

It ought to be borne in mind that the case of pseudo-condensation is easily to be derived from the presented below considerations.

Determination of the envelope of heavy fluid

In the view of the necessity of knowing the region where the light and heavy fluids are existent there is a need to establish the envelope of heavy fluid. In order to establish that the mass and energy conservation equations in the heavy and light fluids will be considered.

Mass flow rate in the supercritical flow consists of the mass flow rate of the heavy phase in the core, $\dot{m}_h$, and the mass flow rate of light phase between the core and the wall, $\dot{m}_l$, i.e.:

$$\dot{m}_{TP} = \dot{m}_h + \dot{m}_l \tag{6}$$

Rate of heat supplied to the heavy core phase goes to its conversion into the light phase (at the same time the radius of the core reduces its size). The energy balance on the element of the flow in the core, where heat flux is supplied from the wall (pseudo-boiling), and transport of heat into the light phase around core yields:

$$d\dot{m}_h h_{hl} = 2\pi r_i q_i \mathrm{d}z \quad \text{where heat transfer at interface is} \quad q_i = q_w \frac{r_i}{R} \tag{7}$$

Mass flow rate of the heavy phase in the core reads:

$$\dot{m}_h = \pi r_i^2 w_{TP} \rho_h \tag{8}$$

From equation (7) the elementary change of the heavy fluid mass flow rate reads:

$$\mathrm{d}\dot{m}_h = \frac{2\pi r_i q_w}{h_{hl}} \frac{r_i}{R} \mathrm{d}z \tag{9}$$

where: $h_{hl} = c_{p_h} T_b - c_{p_l} \frac{T_{pc}+T_w}{2}$ denotes the pseudo latent heat of the phase change.

Hence the elementary change of the mass flow rate of the heavy phase stemming from (8) due to heat absorption in the core reads:

$$\mathrm{d}\dot{m}_h = [2\pi\rho_h w_{TP} r_i(\mathrm{d}r_i) + \pi\rho_h r_i^2 \mathrm{d}w_{TP}] \tag{10}$$

Introducing (10) into (9) enables to obtain the relation for the envelope of the heavy fluid, i.e. the change of core radius with the distance from the beginning of two-phase flow:

$$\frac{\mathrm{d}r_i}{\mathrm{d}z} = \frac{-\frac{2q_w}{\rho_h h_{hl} R}}{2\frac{w_{TP}}{r_i} + \frac{\mathrm{d}w_{TP}}{\mathrm{d}r_i}} \tag{11}$$

Equation (11) requires for its solution information about the velocity distribution. That implies consideration of the mass balance of the heavy phase exchanging mass between the core and the light phase flowing close to the wall.

In order to obtain the two-phase flow velocity the balance of mass flow in annular region for the light phase flow rate can be formulated as:

$$\dot{m}_l = \pi\rho_l w_{TP}(R^2 - r_i^2) \tag{12}$$

The two-phase velocity $w_{TP}(r_i)$ due to heat transfer on the infinitesimal distance dz of the light phase is generated from the heavy phase in the core flow (10).

Differentiating equation (12) with respect to r_i and w_{TP} we obtain:

$$\mathrm{d}\dot{m}_l = -2\pi\rho_l w_{TP} r_i(-\mathrm{d}r_i) + \pi\rho_l(R^2 - r_i^2)\mathrm{d}w_{TP} \tag{13}$$

Rearranging and combining (10) and (13) we arrive at:

$$\frac{\mathrm{d}w_{TP}}{\mathrm{d}r_i} = \frac{-2\left(\frac{\rho_h}{\rho_l}-1\right)w_{TP}r_i}{\left(R^2-r_i^2\right)+r_i^2\frac{\rho_h}{\rho_l}} \tag{14}$$

Separating variables in (14) we obtain the solution assuming that at the beginning of our region under scrutiny of the process of heat and mass transfer we are dealing with the heavy phase only flowing in the channel, i.e. for: $r_i=R \rightarrow w_{TP}=w_0$.

$$w_{TP} = \frac{(a+1)w_0}{1+a\frac{r_i^2}{R^2}} \tag{15}$$

where: $w_0 = \frac{\dot{m}}{\pi R^2 \rho_h}$, $a = \frac{\rho_h}{\rho_l} - 1$, $r^+ = \frac{r}{R}$.

Introducing (15) into (11) we can find variation of the interface radius with respect to the distance z counting from the beginning of the process of developing the light phase region:

$$\frac{\mathrm{d}r_i}{\mathrm{d}z} = \frac{-A\ r_i}{2\frac{w_{TP}}{r_i}+\frac{\mathrm{d}w_{TP}}{\mathrm{d}r_i}} = \frac{-A\ r_i\ \left[a\left(\frac{r_i}{R}\right)^2+1\right]^2}{w_0(a+1)} \tag{16}$$

where $A = \frac{q_w}{\rho_h h_{lh} R}$

Substitution of the velocity distribution (15) and its derivative into (16) leads to the differential equation which has an analytical solution with respect to the core radius r(z):

$$z = \frac{w_0(a+1)}{2\ A}\left\{\ln\left(\frac{a\left(\frac{r_i}{R}\right)^2+1}{a\left(\frac{r_i}{R}\right)^2}\right) - \frac{1}{\left[a\left(\frac{r_i}{R}\right)^2+1\right]}\right\} + C \tag{17}$$

The constant C can be evaluated from the initial condition, that for $z^+=0$ radius of the core is equal to $r^+=1$. Hence the constant C is:

$$C = -\frac{w_0(a+1)}{2\ A}\left\{\ln\left(\frac{a+1}{a}\right) - \frac{1}{[a+1]}\right\} \tag{18}$$

Relation (18) combined with (17) allows to calculate the radius of interface between two phase core and annular vapour region with respect to distance z and in final form reads:

$$z = \frac{w_0(a+1)}{2\ A}\left\{\ln\left(\frac{a\left(\frac{r_i}{R}\right)^2+1}{\left(\frac{r_i}{R}\right)^2(1+a)}\right) - \frac{1}{\left[a\left(\frac{r_i}{R}\right)^2+1\right]} + \frac{1}{a+1}\right\} \tag{19}$$

Relation (17) allows to present resultant values of heat transfer coefficient against z coordinate.

The pseudo-quality x_e of the two pseudo phases can be defined which is understood as a ratio of the mass flowrate of light fluid to the total mass flowrate, which reads:

$$x_e = \frac{\dot{m}_l}{\dot{m}} = \frac{\pi\left(R^2-r_i^2\right)\rho_l}{\pi\left(R^2-r_i^2\right)\rho_l+\pi r_i^2\rho_h} \tag{20}$$

Determination of the shear stress distribution in the flow of two pseudo phases

In the further the analysis will be performed to establish the shear stress distribution in the two-phase flow of the light fluid with account of buoyancy force. In the arbitrary cross-section of the flow the momentum equation for the pseudocritical two-phase mixture is firstly scrutinized:

$$\rho\bar{u}\frac{\mathrm{d}\bar{u}}{\mathrm{d}z} = -\frac{\mathrm{d}p}{\mathrm{d}z} - \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\tau) - \rho g \tag{21}$$

Integrating that equation in the radial direction leads to the expression:

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\rho\bar{u}\frac{\mathrm{d}\bar{u}}{\mathrm{d}z} - \frac{2\tau_w}{R} - \bar{\rho}g \tag{22}$$

Substituting relation (22) into (21) we obtain the following:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\tau) = \frac{2\tau_w}{R} + (\bar{\rho} - \rho)g \tag{23}$$

The density appearing in equation (23) refers to the heavy liquid in case of $r<r_{pc}$, and the light fluid for $r_{pc}<r<R$, hence:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\tau) = \begin{cases} \frac{2\tau_w}{R} + (\bar{\rho} - \rho_l)g = b_l & \text{for} \quad r_{pc} < r < R \\ \frac{2\tau_w}{R} + (\bar{\rho} - \rho_h)g = b_h & \text{for} \quad r < r_{pc} \end{cases} \tag{24}$$

Solving equation (24) returns the general solution for the shear stress, applicable both in the light and heavy fluid zones:

$$\tau_l = \frac{1}{2}b_l r + \frac{C_l}{r} \tag{25a}$$

$$\tau_h = \frac{1}{2}b_h r + \frac{C_h}{r} \tag{25b}$$

The term $b_{l,h}$ present in equation (25) is defined in equation (24). Equation (25) can be solved at the following boundary conditions:

$$r = 0 \quad \rightarrow \quad \tau_h = 0 \tag{26}$$

$$r = R \quad \rightarrow \quad \tau_l = \tau_{w,l} \tag{27}$$

$$r = r_{pc} \quad \rightarrow \quad \tau_l = \tau_{pc} \tag{28}$$

In equation (28) the term τ_{pc} denotes the shear stress at the pseudo-critical temperature. Application of the boundary condition described by equation (26) leads to determination of the value of constant C_h=0.

With the view to determine the constant C_l let's examine the boundary condition given by equations (27) and (28). Hence:

$$\tau_l = \frac{1}{2} b_l r_{pc} + \frac{C_l}{r_{pc}} = \tau_{pc} \tag{29}$$

Value of the constant C_l:

$$C_l = \tau_{pc} r_{pc} - \frac{1}{2} b_l r_{pc}^2 = \tau_{pc} r_{pc} - \frac{1}{2}\left[\frac{2\tau_{w,l}}{R} + (\bar{\rho} - \rho_l) g\right] r_{pc}^2 \tag{30}$$

In result the shear stress in the light phase flow reads:

$$\tau_l = \frac{\tau_{w,l}}{R}\left(r - \frac{r_{pc}^2}{r}\right) + \frac{1}{2} g\left[(\bar{\rho} - \rho_l) r - (\bar{\rho} - \rho_l)\frac{r_{pc}^2}{r}\right] + \frac{\tau_{w,h}}{R}\frac{r_{pc}^2}{r} + \frac{1}{2} g(\bar{\rho} - \rho_h)\frac{r_{pc}^2}{r} \tag{31}$$

Utilising the boundary condition (27) we can derive the relation describing the wall value of the shear stress for two phases in function of the heavy fluid shear stress and the influence of buoyancy force:

$$\tau_{w,l} = \tau_{w,h} + gR\left[(\bar{\rho} - \rho_l)\left(\frac{R}{r_{pc}}\right)^2 - (\rho_h - \rho_l)\right] \tag{32}$$

That value is to be used in the present novel model of energy dissipation. Summarizing, the convective shear stress in the pseudocritical two-phase flow consists of two contributions, the shearing part of the two-phase mixture and the influence of buoyancy, i.e.:

$$\tau_{TP-T} = \tau_h + \tau_{grav} \tag{33}$$

The shear stress component due to the influence of gravity yields:

$$\tau_{grav} = gR\left[(\bar{\rho} - \rho_l)\left(\frac{R}{r_{pc}}\right)^2 - (\rho_h - \rho_l)\right] \tag{34}$$

The pseudocritical radius is to be obtained from the relation provided by equation (19).

Determination of shear stresses and heat transfer coefficient

The hypothesis of energy dissipation in the flow results in the necessity of definition of shear stresses referenced to the wall, which will be done next. Our earlier analysis in this paper enabled determination of shear stresses resulting from the convective shearing flow of a pseudo two-phase flow mixture. That annular flow of the light phase is additionally influenced by the presence of buoyancy force. In the considered problem there is also another contribution of shear stress in action, i.e. due to conversion of heavy phase into the light phase. Exchange of the mass between heavy and light phases modifies momentum and this can be expressed by the change of shear stress on the interface. In such case the two-phase mixture accelerates and the shear stresses increase with respect to the heavy phase flow only. That stress occurs at the interface between the core and the developing light phase, τ_{hl}. Hence, the presence of conversion of the heavy phase to the light phase creates additional to the convective light phase flow shear stresses and hence the additional source of energy dissipation.

Application of the principle of momentum conservation to the single-phase pipe flow without the effect of buoyancy returns a linear distribution of shear stress. The analysis presented below enables determination of the action of shear stress at the interface between heavy and light fluids transformed to the wall, as that is the requirement of the energy dissipation hypothesis balance. The procedure for such extrapolation is presented below. Sketch of the problem is shown in Fig. 3.

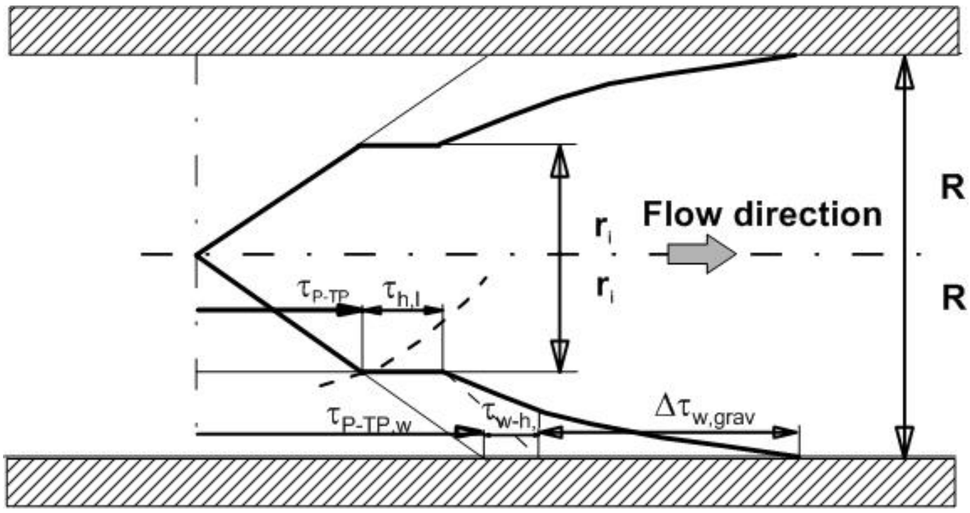


Figure 3: Schematic of distribution of shear stress in equivalent supercritical pressure two phase flow.

The buoyancy-free shear stress term resulting from the shearing flow of the fluid can be determined from the following expression:

$$\tau_{TP-T} = f_{TP}\frac{\rho_{TP}^2 w_{TP}^2}{2} = \Phi^2 f_h \ \frac{\rho_{TP}^2 w_{TP}^2}{2} \tag{35}$$

where the two-phase friction factor is a product of the two-phase flow multiplier and the single phase friction factor, $f_{TP} = \Phi^2 f_h$.

As in the model considering the dissipation we require a value of the friction factor, hence the resulting friction factor due to action of gravity yields:

$$f_{grav} = \frac{2\tau_{grav}\rho_{TP}}{G^2} \tag{36}$$

It is apparent that at $r=r_{pc}$ in case of no buoyancy the shear stress between the light and heavy fluids is the same, i.e. $\tau_h=\tau_l$. There is however a process of generation of the light phase fluid, which renders that there are in action different values of τ_h and τ_l. Hence the value of the resulting shear stress due to that process can be denoted as $\tau_{h,l}$. That term which is in action at the interface between the heavy and light phases needs to be transformed to the wall value:

$$\tau_{h,l-w} = \tau_{h,l}\frac{r}{R} \tag{37}$$

Energy dissipation in the laminar sublayer close to the wall according to the hypothesis of energy dissipation (1) is expressed as a ratio of the square of shear stress to the dynamic viscosity. Taking advantage of relation (3) we can derive from the relation describing the hypothesis of energy dissipation the relation between overall shear stresses in the flow and the shear stress of the mixture, $\tau_{TP-T,w}$, and the additional stress stemming from the transformation of heavy fluid into the light fluid, $\tau_{h,l-w}$:

$$\tau_{PC-TP,w}^2 = \tau_{TP-T,w}^2 + \tau_{h,l-w}^2 = \tau_{TP-T,w}^2 + \left(\frac{r_i}{R}\right)^2 \tau_{h,l}^2 \tag{38}$$

It is assumed that the transformation of heavy fluid into the light fluid, $\tau_{h,l}$ can be modelled as a difference in shear stresses between respective phases. Expressing every shear stress present in eq. (38) in terms of the respective definitions finally results in the relation between respective friction factors, which assumes the form:

$$f_{PC-TP}^2 = f_{TP-T,w}^2 + \left(\frac{r_i}{R}\right)^2 \left[f_h \ - f_l \ \right]^2 \tag{39}$$

where the two-phase friction factor is a product of the two-phase flow multiplier and the single phase friction factor, i.e. $f_{TP-T,w} = \Phi^2 f_h$.

According to the Colburn analogy between heat and momentum transfer we can derive the equivalent heat transfer coefficient for the supercritical conditions heat transfer:

$$\alpha_{PC-TP}^2 = \alpha_{TP-T,w}^2 + \left(\frac{r_i}{R}\right)^2 \left[\alpha_h\ (r_{pc}) - \alpha_l\ \ (r_{pc})\right]^2 \tag{40}$$

The two-phase flow heat transfer coefficient, $\alpha_{TP-T,w}$, present in (40) can be modelled in terms of the product of the two phase flow multiplier and single phase heat transfer coefficient and the term related to gravity:

$$\alpha_{TP-T} = \left[1 + \frac{f_{grav}}{f_h}\frac{\overline{\rho}}{\rho_h}\right] (\Phi^2)^{0.76} \tag{41}$$

Authors own studies established the exponent n at the two-phase flow multiplier to be equal 0.76, and in case of laminar flow to be equal 2, [23].

As a reference heat transfer coefficient, we will assume the single phase heat transfer coefficient evaluated at the pseudocritical temperature, α_{pc}.

$$\frac{\alpha_{PC-TP}}{\alpha_{pc}(R)} = \sqrt{\left[1 + \frac{f_{grav}}{f_h}\frac{\overline{\rho}}{\rho_h}\right] (\Phi^2)^{0.76} + \frac{\left[\alpha_h\ (r_i) - \alpha_l\ \ (r_i)\right]^2}{\alpha_{pc}^2(R)} \left(\frac{r_i}{R}\right)^2} \tag{42}$$

The core flow is treated as homogenous mixture, hence the heat transfer coefficient for the heavy fluid is evaluated from the following expressions:

$$\alpha_h = 0.023 Re_h^{0.8} Pr_h^{0.4} \frac{\lambda_h}{2R} \tag{43}$$

where $Re_h = \frac{2\ G\ R}{\mu_h}$

In case of light phase flow:

$$\alpha_l(r) = 0.023Re_l^{0.8}Pr_l^{0.4}\frac{\lambda_l}{2R} \tag{44}$$

where $Re_l(r_i) = \frac{2\ \ G\ \ R}{\mu_l}$

$$\alpha_{pc}(R) = 0.023Re_{pc}^{0.8}Pr_{pc}^{0.4}\frac{\lambda_{pc}}{2R} \tag{45}$$

where $Re_{pc} = \frac{2\ \ G\ \ R}{\mu_{pc}}$

Validation procedure - comparison with experimental data

Sample verification examples of the model presented above have been presented in the next using the experimental data due to Zahlan et al. [32]. In case of presentation of supercritical experiments, the data are presented in the form of distributions of wall temperature and heat transfer coefficients in terms of quality or bulk enthalpy. In order to use the presented above model the following data reduction procedure has to be applied. First the distance z_1 from the beginning of the zone where the light zone fluid is being formed must be established. That distance is evaluated from the energy balance equation, which reads:

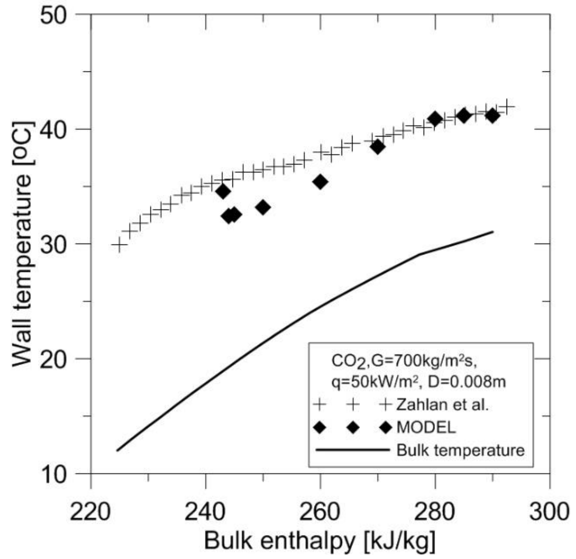
$$\dot{m}(h_i - h_0) = q_w 2\pi R z_1 \tag{46}$$

Expression (46) enables determination of the value of z_1, which is a value of the distance from the beginning of the validity zone of the model, i.e. location where wall temperature is equal to pseudocritical temperature T_w=T_{pc}. At that location the value of heat transfer coefficient, α_0, must be established for the case as the entire flow is of the heavy phase. The value of such heat transfer coefficient should be determined from the known single phase correlations, such as for example due to Dittus and Boelter [33]. Subsequently from the definition of the heat transfer coefficient the bulk temperature at that location can be obtained, i.e.:

$$\alpha_0 = \frac{q_w}{T_w - T_0} \tag{47}$$

At that location the wall temperature is equal to the pseudocritical temperature. Having the values of z_1 we can determine from the formula (19) the corresponding radius which distinguishes the light and heavy zone regions. We have to remember that the presented here model is suitable only for the case where the wall temperature is greater than the pseudocritical temperature. Only in such situation the developed equations for the calculation of heat transfer coefficient can be used.

Sample results of calculations of boiling have been presented in Figures 4 to 7 for pseudo-boiling in supercritical conditions. The values of critical parameters for carbon dioxide read: p_{cr}=73.770 bar, T_{cr}=30.978^{o}C. In comparisons the experimental data due to Zahlan et al [32] have been used for two different heat fluxes, namely q_w=50 and 70 kW/m^2, mass velocity G=700kg/(m^{2}s) and tube diameter of 8mm. Distributions of the wall temperature as well as heat transfer coefficient have been presented. The consistency between experiment and the proposed model seems to be remarkably good in case of smaller values of the applied heat flux. The discrepancy increases when the heat flux is raised, however qualitatively the agreement still seems to be satisfactory.



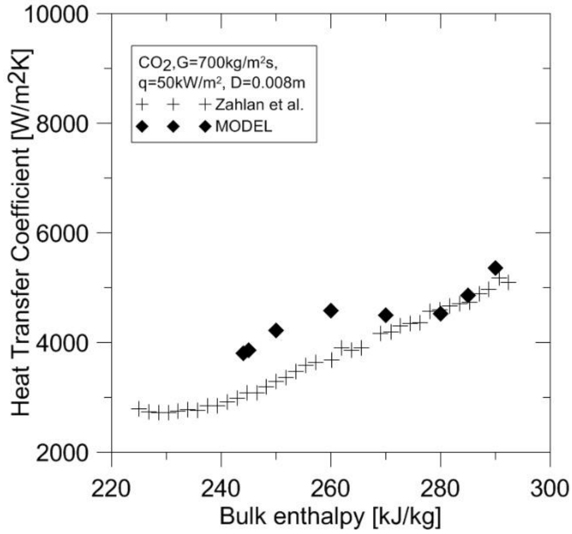
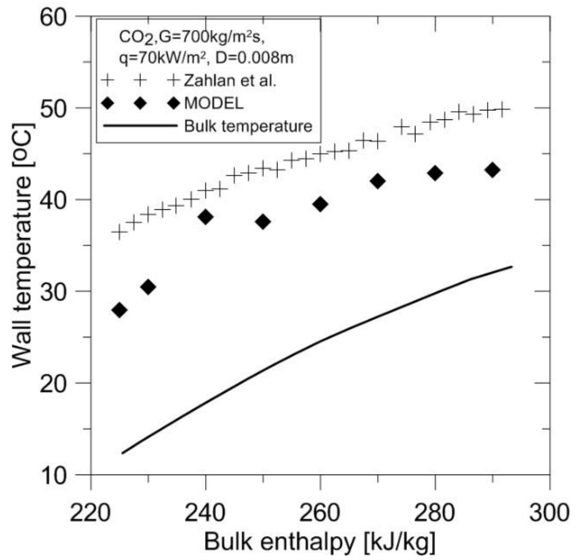


Fig. 4. Comparisons in terms of wall temperature with data due to Zahlan et al. [32], G=700kg/m^{2}s, d=8mm, q_w=50kW/m^2, p_r=1.13, CO_2

Fig. 5. Comparisons in terms of heat transfer coefficient with data due to Zahlan et al. [32], G=700kg/m^{2}s, d=8mm, q_w=50kW/m^2, p_r=1.13, CO_2



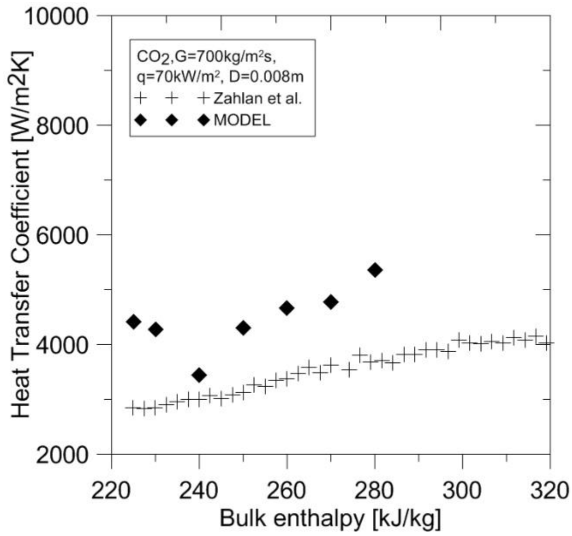


Fig. 6. Comparisons in terms of wall temperature with data due to Zahlan et al. [32], G=700kg/m^{2}s, d=8mm, q_w=70kW/m^2, p_r=1.13, CO_2

Fig. 7. Comparisons in terms of heat transfer coefficient with data due to Zahlan et al. [32], G=700kg/m^{2}s, d=8mm, q_w=70kW/m^2, p_r=1.13, CO_2

Using the model presented above it is also possible to model pseudo-condensation case in supercritical flow. An appropriate experiment has been accomplished by Yang [34]. The experiment was carried out in an 9.4mm tube with R404A as a test fluid. Only one experimental point has been presented, which could be used for the present comparisons. The experiment was reported at mass velocity G=401.6kg/m^{2}s, bulk temperature T_b=81.85^{o}C and wall temperature of Tw=65^{o}C. For these conditions the experimental value of heat transfer obtained was $\alpha_{exp} = 2235W/m^2K$, wheras calculations using the model returned $\alpha_{th} = 2753W/m^2K$.

Conclusions

In the paper presented is analysis of the convective flow heat transfer at supercritical pressure in channels of heat exchangers working in a thermodynamic cycle. The modelling is based on the division of the supercritical flow into three regions, namely the heavy fluid, a two phase flow consisting of the heavy and light fluids and finally the light fluid flow. Presented in the paper modelling is concentrated on the region of simultaneous flow of two fluids divided into the zones with the light and heavy fluids.

Basing on the concept of dissipation of energy in flow a semi-empirical model for supercritical heat transfer in a vertical channel has been developed, which is of a general character. The postulated model can be used for estimation of heat transfer coefficients for different fluids, even these which have not yet to be investigated experimentally from the heat transfer point of view. Validity of the model has been proved for the case of carbon dioxide flow under supercritical conditions in a channel.

The presented approach is heuristic. Further research is required into determination of the shear stress distribution at the interface between light and heavy phases, which seems to be one of the important factors in the analysis. Generally, the conclusion can be drawn that complex supercritical heat transfer phenomenon is possible to be described with the aid of simpler modes of heat transfer, which has been presented in the present paper.

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