

Research Paper

Non-Linear Interaction of Harmonic Waves in a Quasi-Isentropic Flow of Magnetic Gas

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(received February 24, 2020; accepted April 4, 2020)

The diversity of wave modes in the magnetic gas gives rise to a wide variety of nonlinear phenomena associated with these modes. We focus on the planar fast and slow magnetosound waves in the geometry of a flow where the wave vector forms an arbitrary angle θ with the equilibrium straight magnetic field. Nonlinear distortions of a modulated signal in the magnetic gas are considered and compared to that in unmagnetised gas. The case of acoustical activity of a plasma is included into consideration. The resonant three-wave non-collinear interactions are also discussed. The results depend on the degree of non-adiabaticity of a flow, θ , and plasma- β .

Keywords: non-linear magnetoacoustics; adiabatical instability; acoustic activity; three-wave interaction.

1. Introduction

Violation of the principle of superposition of perturbations in nonlinear acoustics leads to the fact that waves can interact with each other in the course of propagation. If an exciter transmits two planar waves with frequencies ω_1 and ω_2 , there appears a combination frequencies, $m\omega_1 \pm n\omega_2$, where m and n are some integers. In the general case, the mathematical content of a problem is fairly difficult. It is simplified in the case of two close frequencies. In the course of propagation, the combination tone $\Omega = \omega_1 - \omega_2$ enhances. That is of importance in Newtonian flows where low frequencies fade out slower than high ones. The shift of the spectral maximum towards lower frequency may take place starting from some distance from a transducer (RUDENKO, SOLUYAN, 2005). Propagation of a modulated signal with the carrier frequency much larger than that of the modulation reveals similar features.

The nonlinear phenomena in flows different from Newtonian have been of special interest in the last decades. Propagation of perturbations in the open systems is described by similar equations (enriched by the terms originating from heating/cooling of a fluid) in spite of different physical processes in them (OSIPOV, UVAROV, 1992; MOLEVICH, 2001; NAKARIAKOV *et al.*,

2000; LEBLE, PERELOMOVA, 2018). We may list flows with destroyed adiabaticity in gases: with excited degrees of a molecule's freedom, with chemical reactions, and in open plasma. All these flows may be acoustically active under some conditions, that is, sound may enhance in the course of propagation due to some kind of heating-cooling function (FIELD, 1965; PARKER, 1953). In some conditions, the wave perturbations weaken. The Newtonian attenuation always contributes to the losses in momentum and energy, as well as attenuation, due to thermal conduction. Interaction of modes may also occur unusually. This makes the linear and nonlinear features of open flows particular. The most complex case is the flow of magnetic gases. Even in the simplest case of a planar geometry and constant angle between the wave vector and straight magnetic field, the wave parameters (the sound velocity and parameter of nonlinearity) reveal strong dependence on this angle and the plasma- β . There are two Alfvén modes and four modes which rely on compressibility, that is, magnetosound modes (two slow and two fast ones). Attenuation or enhancement of sound also depends on a balance between degree of deviation from adiabaticity due to some heating-cooling function and mechanical and thermal attenuation. The anisotropy of magnetosound speed makes non-collinear interactions of wave modes pos-

sible. That may happen to wave processes in media with dispersion which is a rare case in acoustics of fluids. The wave perturbations may excite also non-wave modes. This excitation occurs unusually in open flows, especially in flows of magnetic gases. Acoustic heating and streaming (that is, excitation of the entropy and vorticity modes) in a plasma has been considered by PERELOMOVA (2016; 2018a; 2018b). They are out of the subject of this study.

2. Magnetosound waves

We start from the set of ideal MHD (magnetohydrodynamic) equations which describe perfectly electrically conducting gas without losses due to mechanical friction and thermal conduction. It includes the continuity equation, momentum equation, energy balance equation, and electrodynamic equations in the differential form (FREIDBERG, 1987; KRALL, TRIVELPIECE, 1973; NAKARIAKOV *et al.*, 2000):

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, \\ \rho \frac{D\mathbf{v}}{Dt} &= -\nabla p + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B}, \\ \frac{Dp}{Dt} - \gamma \frac{p}{\rho} \frac{D\rho}{Dt} &= (\gamma - 1)L(p, \rho), \\ \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}), \\ \nabla \cdot \mathbf{B} &= 0, \end{aligned} \quad (1)$$

where p , ρ , \mathbf{v} are thermodynamic pressure and density of a plasma and particles velocity. The magnetic flux density is denoted by \mathbf{B} , and μ_0 is the permeability of the free space. The third equation in the set (1) refers to an ideal gas with the ratio of specific heats under constant pressure and constant density γ , $\gamma = C_P/C_V$. The fourth equation is the ideal induction equation, and the fifth one is the Maxwell's equation reflecting solenoidal character of \mathbf{B} . $L(p, \rho)$ is the heating-cooling function responsible for non-isentropicity of a flow. NAKARIAKOV *et al.* (2000) reviewed physically meaningful kinds of the heating function in the context of astrophysical applications (heating by Alfvén mode/mode conversion, coronal current dissipation, constant heating per unit mass, heating by cosmic rays and grain photoelectrons, etc). The heating-cooling function accounts also losses due to radiation.

Following NAKARIAKOV *et al.* (2000) and CHIN *et al.* (2010), we assume that the wave vector of a planar flow forms some constant angle θ ($0 \leq \theta \leq \pi$) with the constant straight equilibrium magnetic field \mathbf{B}_0 . The y -component of \mathbf{B}_0 equals zero, so as

$$\begin{aligned} B_{0,x} &= B_0 \sin(\theta), & B_{0,z} &= B_0 \cos(\theta), \\ B_{0,y} &= 0. \end{aligned}$$

In this geometry of a flow, $B_{0,z}$ is constant and there are seven unknown thermodynamic functions in the system (1) and seven modes specifying the flow of infinitely small magnitudes: the entropy non-wave mode and six wave modes including two Alfvén ones. The detailed analysis may be found in (NAKARIAKOV *et al.*, 2000; PERELOMOVA, 2018a; 2018b). Employing a normal mode analysis with all perturbations proportional to $\exp(i\omega t - ikz)$, the dispersion relation of the magnetosound modes is given as

$$\omega = Ck - iCD, \quad (2)$$

where C is the magnetosound speed, one of four possible (corresponding to two fast and two slow modes), including two negative ones, which satisfies the equation

$$C^4 - C^2(c_0^2 + C_A^2) + c_0^2 C_{A,z}^2 = 0, \quad (3)$$

and C_A and c_0

$$C_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}}, \quad c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

designate the Alfvén speed and the acoustic speed in non-magnetised gas in equilibrium,

$$C_{A,z} = C_A \cos(\theta),$$

and

$$D = \frac{C(C^2 - C_A^2)(\gamma - 1)}{2c_0^2(C^4 - c_0^2 C_{A,z}^2)} (c_0^2 L_p + L_\rho).$$

The magnetosound perturbations may enhance if a linear flow is adiabatically unstable (FIELD, 1965; PARKER, 1953) that is, if

$$c_0^2 L_p + L_\rho > 0. \quad (4)$$

We do not consider mechanical and thermal losses which have impact on the energy balance in the flow and may prevent enhancement of wave perturbations even if the above condition is satisfied.

3. Nonlinear distortion of modulated wave and three waves interaction

The evolutionary equation governing longitudinal velocity in individual magnetosound wave has been derived and used by NAKARIAKOV *et al.* (2000), CHIN *et al.* (2010). It takes the form

$$\frac{\partial v_z}{\partial t} + C \frac{\partial v_z}{\partial z} - DCv_z + \varepsilon v_z \frac{\partial v_z}{\partial z} = 0, \quad (5)$$

where ε is responsible for nonlinear distortions,

$$\varepsilon = \frac{3c_0^2 + (\gamma + 1)C_A^2 - (\gamma + 4)C^2}{2(c_0^2 - 2C^2 + C_A^2)}.$$

Equation (5) refers to both slow and fast modes. It is very similar to equations describing perturbations in other open flows which may be acoustically active (OSIPOV, UVAROV, 1992; MOLEVICH, 2001; LEBLE, PERELOMOVA, 2018), but its parameters (sound speed and parameter of nonlinearity) vary with θ and plasma- β ,

$$\beta = \frac{2}{\gamma} \frac{c_0^2}{C_A^2}.$$

For better precision, we consider modes with $C > 0$, that is, slow or fast modes propagating in the positive direction of the axis z . The sign of D coincides with the sign of $c_0^2 L_p + L_\rho$, hence, $D > 0$ is the case of acoustical activity. Equation (5) rearranges into the well known equation for velocity in the Riemann's wave which propagates in the positive direction of axes z with $D = 0$, $C = c_0$, and $\varepsilon = \frac{\gamma+1}{2}$ (LANDAU, LIFSHITZ, 1987; RUDENKO, SOLYAN, 1977). Equation (5) in the new variables (for non-zero D , C)

$$V = v_z \exp(-Dz), \quad Z = \frac{e^{Dz} - 1}{D}, \quad (6)$$

$$\tau = t - z/C$$

may be readily rearranged into the leading order equation:

$$\frac{\partial V}{\partial Z} - \frac{\varepsilon}{C^2} V \frac{\partial V}{\partial \tau} = 0. \quad (7)$$

Note that Z is always positive for non-zero D . Equation (7) is well studied in the nonlinear wave theory (LANDAU, LIFSHITZ, 1987; RUDENKO, SOLYAN, 1977). It may be solved by the method of characteristics.

3.1. Propagation of a modulated wave

Let us consider nonlinear propagation of a modulated at an exciter perturbation, fast or slow,

$$V(Z = 0, \tau) = V_0(1 + m \sin(\Omega\tau)) \sin(\omega\tau)$$

$$= v(z = 0, t)$$

$$= v_0(1 + m \sin(\Omega t)) \sin(\omega t), \quad (8)$$

where $\Omega \ll \omega$, and $m > 0$ is the depth of modulation. An exciter is situated at $Z = 0$. The nonlinear part of the solution to Eq. (5) which oscillates with the frequency Ω , takes the form

$$V_\Omega = \frac{\Omega \varepsilon m V_0^2}{2C^2} Z \cos(\Omega\tau) \quad (9)$$

before formation of a discontinuity (Rudenko, Soluyan, 1977; OSIPOV, UVAROV, 1992), that is, if

$$0 < z < z_{sh} = \ln(1 + z_{sh,0} D) D^{-1},$$

where

$$z_{sh,0} = \frac{C^2}{\varepsilon \omega V_0} = \frac{C^2}{\varepsilon \omega v_0}$$

denotes the distance of shock formation for harmonic at a transducer wave with frequency ω if $D = 0$. We may conclude that z_{sh} is smaller than $z_{sh,0}$ for positive D . This is due to enlargement of sound perturbations and enhancement of nonlinear distortions in acoustically active flows (z_{sh} is larger than $z_{sh,0}$ for negative D). A discontinuity always forms in acoustically active or neutral flows with $D \geq 0$ and, for negative D , in the case $-1 < z_{sh,0} D < 0$ and does not form at all otherwise. After formation of discontinuity, the approximate solution may be found by means of the standard method which eliminates ambiguity of the exact solution to the nonlinear Eq. (5) by establishment of the front so that the momentum per unit mass remains constant at any distance from a transducer. It was found out by RUDENKO, SOLUYAN (1977):

$$V_\Omega = -\frac{\Omega m V_0 \pi}{4\omega} \left(1 - \frac{\pi^2}{2\left(\frac{\pi}{2} + \frac{Z}{z_{sh,0}}\right)^2} \right) \cos(\Omega\tau). \quad (10)$$

Hence, the solution to Eq. (5) with the boundary condition (8), sounds as

$$v_\Omega = \frac{\Omega m v_0}{2D\omega z_{sh,0}} \exp(Dz) (\exp(Dz) - 1) \cos(\Omega\tau) \quad (11)$$

before formation of discontinuity, and

$$v_\Omega = -\frac{\Omega m v_0 \pi}{4\omega} \exp(Dz) \left(1 - \frac{\pi^2}{a^*} \right) \cos(\Omega\tau), \quad (12)$$

where

$$a^* = 2 \left(\frac{\pi}{2} + \frac{(\exp(Dz) - 1)}{D z_{sh,0}} \right)^2$$

after formation of discontinuity The dimensional amplitude of the velocity $\frac{\omega v_{\Omega,A}}{m v_0 \Omega}$ of the wave with frequency Ω , is shown in Fig. 1 for different $D z_{sh,0}$. Whereas dynamics of the longitudinal velocity in the magnetosound wave if $D \neq 0$ is described by Eq. (7) with variables (6), the neutral case $D = 0$ corresponds to the same equation but with variables

$$V = v_z, \quad Z = z, \quad \tau = t - z/C.$$

On Fig. 1, the plot in the bottom row, right panel represents dimensional magnitude of velocity $\frac{\omega v_{\Omega,A}}{m v_0 \Omega} |D| z_{sh,0}$ in the case of negative D when discontinuity does not form at all.

Some mismatches in junction of curves before and after formation of discontinuity are caused by approximate solution (10) which considers a triangular profile as modulated wave propagates in order to simplify analysis (RUDENKO, SOLUYAN, 1977). For a triangular profile of signal at an exciter, the shock formation distance equals $\frac{\pi}{2} z_{sh,0}$ if $D = 0$.

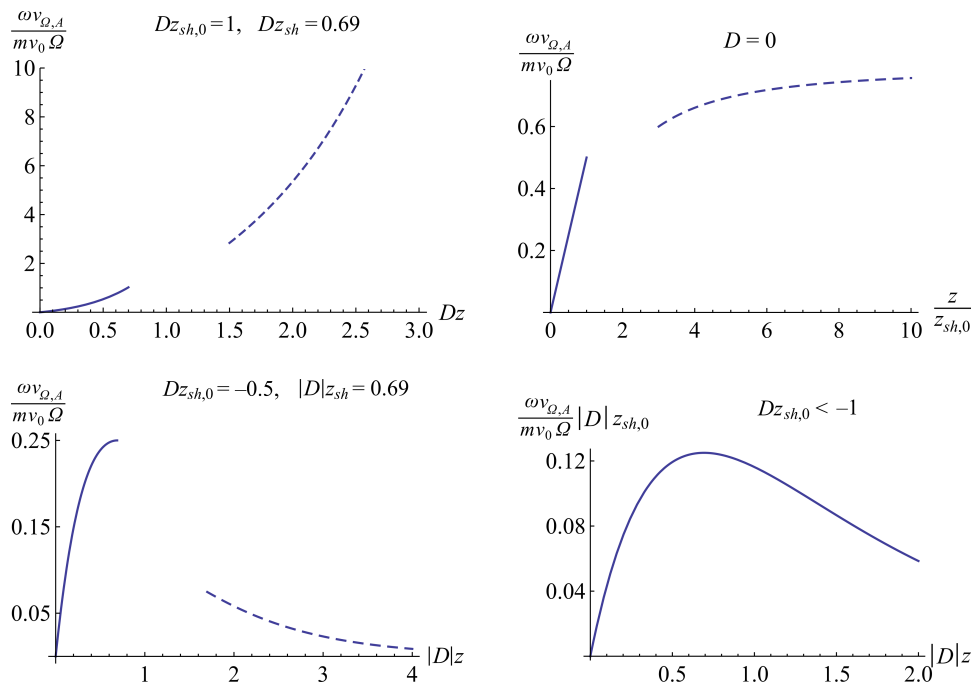


Fig. 1. Dimensionless amplitude of velocity in the modulation wave as a function of a dimensionless distance from a transducer. The solid lines correspond to distances before formation of discontinuities, and the dotted lines correspond to distances after formation of discontinuities. The plot in the bottom row, right panel is the case of $Dz_{sh,0} < -1$ when a discontinuity does not form at all.

3.2. Non-collinear interaction of harmonic waves

In this section, we consider three frequency interaction between harmonic waves: pump wave (ω_3), signal wave (ω_1), and differential frequency wave (ω_2),

$$\omega_1 + \omega_2 = \omega_3. \tag{13}$$

For effective nonlinear interaction, they should be in the phase synchronism (RUDENKO, SOLUYAN, 1977),

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3, \tag{14}$$

that is, modules of wave vectors k_1, k_2, k_3 , and $\theta_1, \theta_2, \theta_3$ must satisfy the system

$$k_1 \cos(\theta_1) + k_2 \cos(\theta_2) = k_3 \cos(\theta_3),$$

$$k_1 \sin(\theta_1) + k_2 \sin(\theta_2) = k_3 \sin(\theta_3),$$

$$C_1 k_1 + C_2 k_2 = C_3 k_3.$$

The general description is fairly difficult in view of a complex form of $C(\theta, c_0, C_A)$ and different species of magnetosound waves, fast or slow ones. The possibility of non-collinear three wave interaction is due to dependence of C on θ .

For example, let us consider the first mode correspondent to the wave vector parallel to the magnetic field, $\theta_1 = 0$, and the third mode correspondent to the wave vector perpendicular to the magnetic field, $\theta_3 = \frac{\pi}{2}$

(Fig. 2, top row, left panel). This geometry leads to equation

$$C_1 \cos(\theta_2) - C_2 = -C_3 \sin(\theta_2) = -\sqrt{c_0^2 + C_A^2} \sin(\theta_2),$$

which determines θ_2 in dependence to the ratio $\frac{C_A}{c_0}$. $C_1 = c_0$, if $c_0 \geq C_A$, and $C_1 = C_A$, if $c_0 < C_A$. The effective angle θ_2 is the same if the second mode is fast or slow. It is shown in Fig. 2 (top row, right panel). There are two solutions. In the limit zero or infinite $\frac{C_A}{c_0}$, they both tend to $\frac{3\pi}{4}$. There are also degenerative solutions $\theta_2 = 0, \theta_2 = \pi$, and $\theta_2 = \frac{\pi}{2}$ corresponding to zero or infinitely large k_2 and k_3 . The next geometry of interaction is shown in Fig. 2 in the bottom row, left panel. It corresponds to equation

$$C_1 \cos(\theta_3) + C_2 \sin(\theta_3) = C_3.$$

The solutions θ_3 in fact equals $\pi - \theta_2$ from the first example. As well as θ_2 does not depend on fast or slow C_2 , θ_3 does not depend on fast or slow C_3 , and θ_3 tends to $\frac{\pi}{4}$ as $\frac{C_A}{c_0}$ tends to infinity. The limiting values ($\theta_2 = \pi$ and $\theta_3 = 0$) correspond to the cases of zero k_3 and zero k_2 . The top and middle rows of Fig. 3 show geometry and effective angle θ_1 for $k_1 = k_2$ and symmetric wave vectors which produce perturbations with perpendicular to the direction of magnetic field wave vector \mathbf{k}_3 . The cases of both interacting waves are slow or fast, are degenerative and yield a single

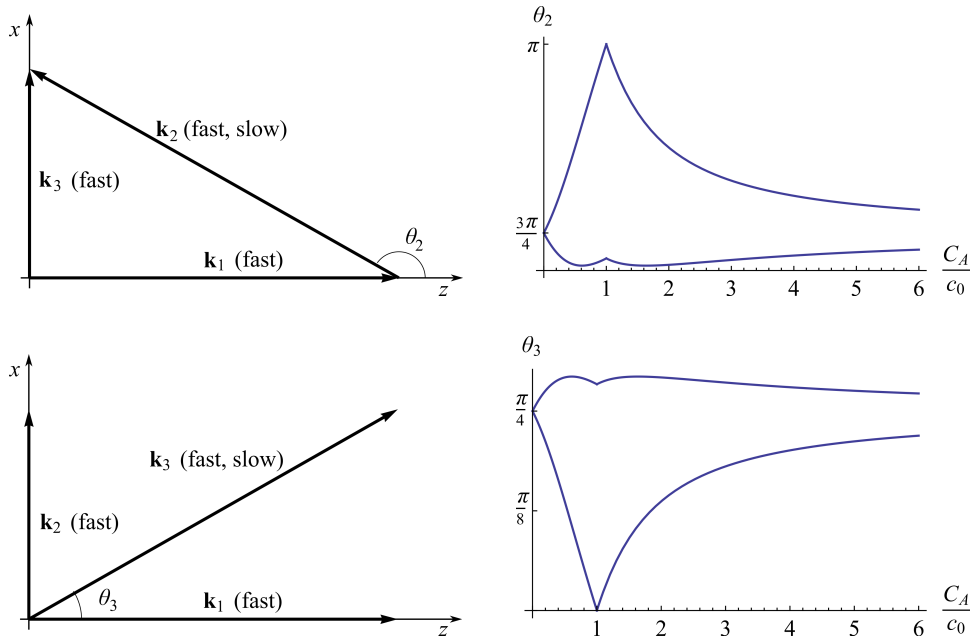


Fig. 2. Geometry of three wave interaction (left panels). The angles θ_2 , θ_3 ensure effective interaction (right panels).

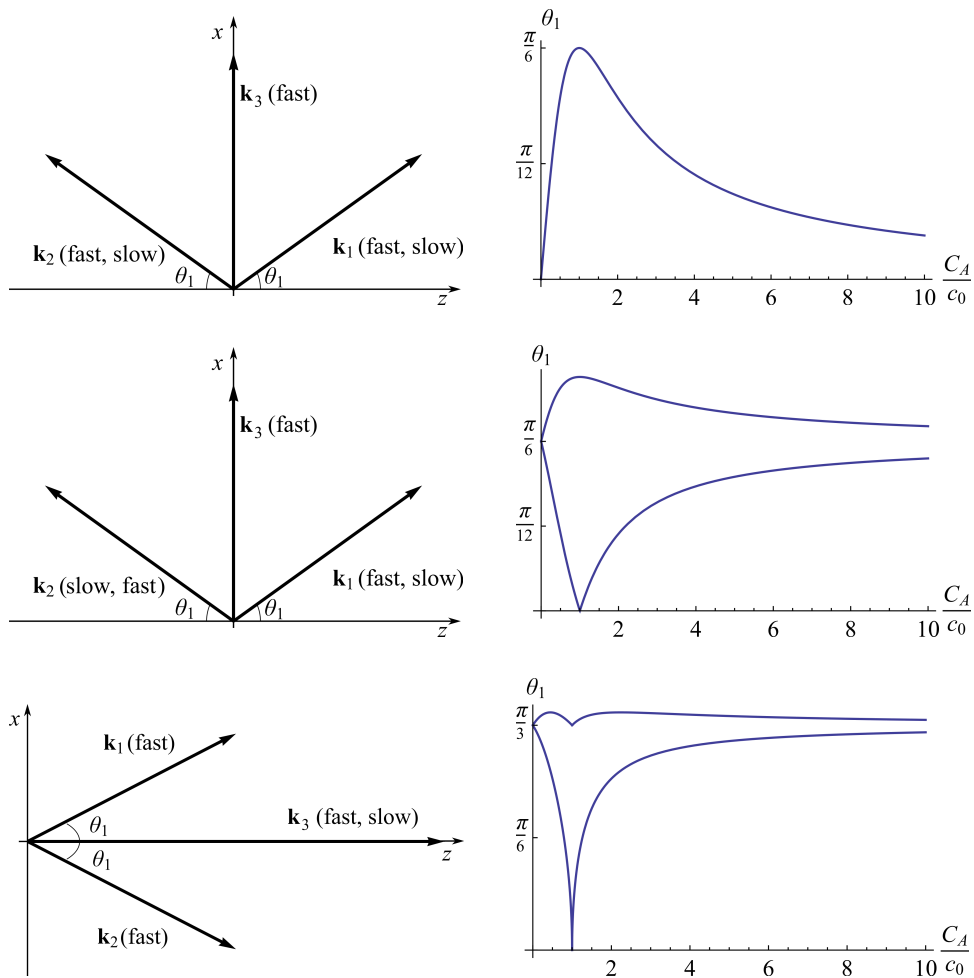


Fig. 3. Geometry of three wave interaction (left panels). The interacting waves with wave vectors \mathbf{k}_1 , \mathbf{k}_2 may be simultaneously fast or slow (top row) or simultaneously different (middle row). The angle θ_1 provides effective interaction (right panels).

solution (top row, right panel at Fig. 3). The equation establishing θ_1 sounds as

$$C_1 + C_2 = 2C_3 \sin(\theta_1).$$

The left panel in the bottom row of Fig. 3 represents the geometry of interaction with summary vector \mathbf{k}_3 parallel to the magnetic field. The plot in the bottom row, right panel shows the solution θ_1 satisfying the equation

$$C_1 + C_2 = 2C_3 \cos(\theta_1)$$

for both fast first and second waves. The solution takes the same form if either of these modes is fast, and the other is slow. In the case of two slow modes, there are only degenerate solutions $0, \frac{\pi}{2}, \pi$.

All types of non-collinear interactions are described by Eqs (13) and (14). There are mathematical difficulties in establishing solutions to them (that is, angles between the wave vectors and axis z in dependence to $\frac{C_A}{c_0}$) in the general case.

4. Concluding remarks

This study continues the author's investigations concerning nonlinear flows of magnetic gases. The first important results on this way starting from the choice of geometry, derivation of nonlinear dynamic equation, and analysis of some of its approximate solutions were done by NAKARIAKOV *et al.* (2000), CHIN *et al.* (2010). PERELOMOVA (2019) has derived exact solutions in the case of saw-tooth at an exciter periodic or impulsive perturbations. Also, interaction of wave and non-wave entropy mode, that is, excitation of acoustic heating, has been discussed in detail (PERELOMOVA, 2018a; 2018b).

In this study, we consider nonlinear propagation of modulated perturbations and three wave interactions in a magnetic gas. The magnitude of the low-frequency oscillations in the modulated wave behaves differently in the cases $z_{sh,0}D \leq -1$, $-1 < z_{sh,0}D \leq 0$, $D = 0$, and $D > 0$ (D is responsible for deviation from adiabaticity of the wave processes due to some kind of the heating-cooling function). This entails a difference in the forms of modulated waves before and after formation of a discontinuity, as well as a difference in the distances of formation of discontinuity, z_{sh} and $z_{sh,0}$. In particular, the magnitude of the modulated wave grows accordingly to Eq. (12) as $\exp(Dz)$ at large distances from a transducer, $\frac{\exp(Dz)-1}{Dz_{sh,0}} \gg 1$ in acoustically active flow (that is, if $D > 0$). When $D < 0$, there are two special cases: $-1 < z_{sh,0}D \leq 0$ in which discontinuity forms, and $z_{sh,0}D \leq -1$, in which discontinuity does not form at all. In both cases, magnitude of the modulated signal tends to zero at large z as $\exp(Dz)$. In the case when discontinuity does not form, magnitude of the modulated signal v_0 achieves maximum $\frac{\Omega m v_0}{8\omega|D|z_{sh,0}}$

at $z = |D|^{-1} \ln 2$ (Fig. 1, bottom row, right panel). It varies in accordance to Eq. (11). In the case of $D = 0$, magnitude of the modulated signal grows linearly with the distance from a transducer before formation of discontinuity. In this case, the magnitude tends to $\frac{\pi \Omega m v_0}{4\omega}$ as $\frac{z}{z_{sh,0}}$ tends to infinity (Fig. 1, top row, right panel).

In synchronous interactions, accumulation of nonlinear interactions occurs. As a result, the energy of a dominant wave can be completely converted into the energy of initially weak waves of different frequencies. This phenomenon is well known in radio engineering and nonlinear optics and has much in common with the wave interaction in acoustics. Equations (13) and (14) are responsible for the parametric interaction in quadratically nonlinear medium. Parametric phenomena can be clearly interpreted in quantum language as processes of splitting of high-frequency phonons of a pump wave $\hbar\omega_3$ into two phonons of lower frequencies $\hbar\omega_1$ and $\hbar\omega_2$. If we evaluate the products of the Planck constant \hbar , Eq. (13) and the condition of synchronism (14), these equations can be interpreted as the laws of conservation of energy and quasimomentum during an elementary three phonon interaction. Parametric phenomena in radio engineering and nonlinear optics are usually treated by the spectral method (KHARKEVICH, 1965; LANDAU *et al.*, 1984). This method is very convenient in MHD applications, because presence of a strong dispersion makes interaction between only a few waves possible. The spectral methods in acoustics are used much less frequently in view of weak dispersion. In the magnetic gas, the anisotropy in propagation of planar waves, that is, dependence of sound speed on the angle between the wave vector and magnetic field, causes the possibility of non-collinear interactions.

The starting point is the conservation system of PDE in ideal MHD equations. Ideal magnetohydrodynamics refers to the single fluid model dealing with macroscopic equilibrium quantities and equal temperatures of electrons and ions. It approximates well the majority of astrophysical gases which are weakly coupled plasmas. In particular, the results may be useful in coronal seismology, that is, in remote observations of wave perturbations in the solar corona and conclusions about plasma's properties and heating/cooling in it. The solar corona consists of loops which extend up to 700 000 km and have radii between 1000 and 10 000 km. Hence, the model of a plasma column affected by straight magnetic field considered in the present paper is a good approximation for coronal loops. The results may be used in the flows of laboratory plasmas. We do not consider effects connected with mechanical viscosity and thermal conduction of a plasma. The impact of thermal conduction on the magnetosound wave propagation has been considered by NAKARIAKOV *et al.* (2000). The damping mechanisms alter conditions of acoustical activity and have

impact on all nonlinear phenomena in the course of wave propagation. In the case of small Reynolds numbers, that is, strong damping compares to nonlinearity, $Dz_{sh,0} < -1$, magnitude of the modulated wave enlarges at small distances from the exciter, then passes through a maximum, and attenuates at the large distances slower than the carrier wave. This is the case of plot in Fig. 1 (bottom row, right panel). We may expect that Newtonian attenuation and thermal conduction may prevent acoustical activity and growth of the magnitude of the modulated wave, if $D > 0$. That happens if

$$DC^3 < \frac{b(\Omega - \omega)^2}{\rho_0},$$

where b designates the total attenuation, including mechanical and the one due to thermal conduction (RUDENKO, SOLUYAN, 1977). Observed dissipation of slow magnetosound modes is difficult to explain by linear damping mechanisms (KRISHNA PRASAD *et al.*, 2014). The application of nonlinear theory seems promising because it considers nonlinear damping at discontinuities and nonlinear transfer of energy between wave and non-wave modes.

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