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ABSTRACT

A thermodynamic relation between perturbations of pressure and mass density in the magnetohydrodynamic flow is theoretically studied. Planar magnetohydrodynamic perturbations with the wave vector, which forms a constant angle with the equilibrium magnetic field, are under study. The theory considers thermal conduction of a plasma and the deviation from adiabaticity of a flow due to some kind of heating–cooling function. It also considers nonlinear distortion of a waveform and nonlinear excitation of the entropy mode in the field of intense magnetosonic perturbations. In some conditions, the total density of a plasma enlarges over a cycle of magnetosonic perturbations. These conditions depend on the type of magnetosonic waveform, heating–cooling function, thermal conduction, and equilibrium parameters of a plasma. They depend also on the angle between the wave vector and the magnetic field. The diagrams in the plane of total variation of pressure vs total variation of density indicate the nonlinear phenomena and irreversible processes in a flow. Harmonic perturbation and a bipolar impulse of pressure are considered as magnetosonic exciters of the entropy mode. Exemplary diagrams are plotted and discussed for these particular cases of exciters.

I. INTRODUCTION

Hysteretic behavior reflects the history dependence of physical processes in magnetic materials, mechanics, and engineering. Cycles in the strain–stress diagrams for solids with hysteretic nonlinearity are usually represented by loops.[1,2] They are observed experimentally in micro-inhomogeneous materials as ceramics, rocks, and microcrystalline metals.[3] Irreversible processes that occur during propagation of acoustic waves may also be represented by hysteresis curves in the plane of thermodynamic parameters (total variation of pressure $\Longleftrightarrow$ total variation of density diagrams).[4–6] The energy dissipated per cycle of oscillations (the macroscopic energy is converted into the energy of chaotic motion of molecules) measures the damping properties of a medium such as viscosity, heat conduction, and molecular absorption. Relaxation and mechanical losses are irreversible processes in general described by some integral operator with a kernel reflecting dispersion and frequency-dependent absorption due to the molecular properties of a fluid.[7–10] Among irreversible processes, the nonlinear damping at shock fronts is of key importance. In Newtonian fluid flows, the relation between acoustic pressure and acoustic density includes a term proportional to the partial derivative of acoustic density with respect to time. This term responsible for hysteretic behavior is proportional to the thermal conduction.[8] The mechanical damping does not have an impact on the relation between acoustic pressure and acoustic density but contributes to irreversible heating of a medium.[5,11] The relation contains a nonlinear term that grows in absolute value with the magnitude of acoustic pressure. As usual, quadratic nonlinearity is considered. Attention to the structural features of loops in the phase portraits in the plane of variations of pressure vs variations of density for nonlinear wave processes in fluids has been attracted only recently.[4] Hedberg and Rudenko pointed to physical distinction between various irreversible processes, which accompany nonlinear propagation of intense sound in fluids. They considered loops in the plane of acoustic perturbations for periodic harmonic and saw-tooth sound in a Newtonian fluid flow and in a flow with relaxation and made important conclusions about the frequency-dependent hereditary properties of processes in linear and nonlinear regimes. It has been concluded that hysteresis curves may indicate irreversible and nonlinear processes in a fluid flow.

Open flows, that is, the flows with an external energy supply, are characterized by some heating–cooling function, which incorporates inflow of energy and radiative losses. The heating–cooling function also introduces hysteresis in the thermodynamic diagrams and contributes to the formation of loops.[6] Damping mechanisms in open systems may be overbalanced by the heating–cooling function or enhanced by it. This leads to various scenarios of wave propagation, nonlinear phenomena in a flow, and diagrams in the thermodynamic plane. To conclude about the connection of total perturbations of pressure and density in a medium, one requires knowledge about not only wave perturbations but also irreversible transfer of the wave energy into the energy of the entropy mode. The nonlinear losses of wave energy are followed by an isobaric slow increase in the fluid's temperature and a decrease in its density specifying the entropy mode.[7,8,12] A variation of density in the entropy mode may be positive in acoustically active flows due to some kind of non-equilibrium thermodynamic process or the presence of the heating–cooling function. This leads to unusual nonlinear cooling of a fluid. In spite of smallness of the non-wave perturbations over one period of wave oscillations, they may accumulate over time and take noticeable values. The total variation of density $\Longleftrightarrow$ total variation of pressure diagrams are not fully closed in view of constantly growing absolute value of density perturbation, which specifies the entropy mode. In the magnetohydrodynamic (MHD) flows, the total variation of density is a sum of magnetosonic variation and density perturbation of the entropy mode. The description of hysteresis curves is fairly complex due to the variety of magnetosonic modes (fast or slow) and the impact of the heating–cooling function, the ratio of acoustic and magnetic equilibrium pressures (that is, plasma-β), thermal conduction of a plasma, and an angle between the equilibrium magnetic field and the wave vector. These factors introduce a variety of hysteretic behaviors. The magnitudes of perturbations in the excited entropy mode are proportional to the intensity of the magnetosonic wave. We consider periodic and impulsive magnetosonic exciters in Sec. IV.

The main aim of this study is to attract attention to the history-dependent processes in magnetohydrodynamics to emphasize the importance of links between magnetosonic perturbations and nonlinear excitation of the entropy mode, which reveal hysteresis. The history-dependent processes occur due to some factors reflecting irreversible processes, which may be indicated by means of hysteresis diagrams. In particular, a plasma has different thermodynamic properties before and after passing a magnetosonic wave due to damping mechanisms and heating–cooling function. A kind of magnetosonic wave also has an impact on this difference.

II. INITIAL POINTS AND LINEAR ANALYSIS

The set of MHD equations for perfectly conducting gas includes the continuity equation, momentum equation, energy balance equation, and electrodynamic equations. It considers unspecified inflow of energy into a plasma and radiative losses due to some heating–cooling function $L(p,\rho)$ [this function equals zero in the equilibrium thermodynamic state (p_0, ρ_0)] and thermal conduction of a plasma χ. The classical Braginskii transport theory concludes that thermal conduction parallel to the magnetic field is much more efficient than the perpendicular one, $\chi_\perp \ll \chi_{||}$, and contribution of only parallel compound may be considered with a large margin.[13] We follow conditions and the geometry of a plasma's flow accepted by Chin and Nakariakov *et al.*:[14,15] the wave vector of a planar flow is directed along axis z and forms constant angle θ $(0 \le \theta \le \pi)$ with the straight equilibrium magnetic field $\vec{B}_0$. The y-component of $\vec{B}_0$ equals zero, so as

$$B_{0,x} = B_0 \sin(\theta), \quad B_{0,z} = B_0 \cos(\theta), \quad B_{0,y} = 0.$$

All perturbations of magnetohydrodynamic variables depend on time t and co-ordinate z. The leading-order system contains quadratically nonlinear terms, and also these ones which are proportional to thermal conduction and derivatives of the heating–cooling function with respect to pressure and density as follows:[16–18]

$$\begin{aligned}
&\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial v'_z}{\partial z} = -\rho' \frac{\partial v'_z}{\partial z} - v'_z \frac{\partial \rho'}{\partial z},\\
&\frac{\partial v'_x}{\partial t} - \frac{B_{0,z}}{\rho_0 \mu_0}\frac{\partial B'_x}{\partial z} = -v'_z \frac{\partial v'_x}{\partial z} - \frac{B_{0,z}}{\rho_0^2 \mu_0}\rho' \frac{\partial B'_x}{\partial z},\\
&\frac{\partial v'_y}{\partial t} - \frac{B_{0,z}}{\rho_0 \mu_0}\frac{\partial B'_y}{\partial z} = -v'_z \frac{\partial v'_y}{\partial z} - \frac{B_{0,z}}{\rho_0^2 \mu_0}\rho' \frac{\partial B'_y}{\partial z},\\
&\frac{\partial v'_z}{\partial t} + \frac{1}{\rho_0}\frac{\partial p'}{\partial z} + \frac{B_{0,x}}{\rho_0 \mu_0}\frac{\partial B'_x}{\partial z}\\
&\quad = \frac{\rho'}{\rho_0^2}\frac{\partial p'}{\partial z} - \frac{B_{0,z}}{\rho_0^2 \mu_0}\rho' \frac{\partial B'_x}{\partial z} - \frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\frac{B'^2_x + B'^2_y}{2\mu_0}\right) - v'_z \frac{\partial v'_z}{\partial z},\\
&\frac{\partial p'}{\partial t} + c_0^2 \rho_0 \frac{\partial v'_z}{\partial z} - (\gamma-1)(L_p p' + L_\rho \rho') - \frac{\chi}{\rho_0 C_P}\frac{\partial^2 \gamma p'}{\partial z^2} + \frac{\chi c_0^2}{\rho_0 C_P}\frac{\partial^2 \rho'}{\partial z^2}\\
&\quad = (\gamma-1)(0.5 L_{pp} p'^2 + 0.5 L_{\rho\rho}\rho'^2 + L_{p\rho} p'\rho') - \gamma p' \frac{\partial v'_z}{\partial z} - v'_z \frac{\partial p'}{\partial z}\\
&\quad - \frac{\chi}{\rho_0^2 C_P}\frac{\partial^2(\gamma p'\rho' - c_0^2 \rho'^2)}{\partial z^2},\\
&\frac{\partial B'_x}{\partial t} + \frac{\partial}{\partial z}\left(B_{0,x} v'_z - B_{0,z} v'_x\right) = -B'_x \frac{\partial v'_z}{\partial z} - v'_z \frac{\partial B'_x}{\partial z},\\
&\frac{\partial B'_y}{\partial t} - \frac{\partial}{\partial z}\left(B_{0,z} v'_y\right) = -B'_y \frac{\partial v'_z}{\partial z} - v'_z \frac{\partial B'_y}{\partial z},
\end{aligned} \tag{1}$$

where p, ρ, $\vec{v}$, $\vec{B}$ denote the hydrostatic pressure and density of a plasma, its velocity, and the magnetic field and μ_0 is the permeability of free space. All thermodynamic quantities are expanded around the equilibrium thermodynamic state as $f(z,t) = f_0 + f'(z,t)$. A plasma is supposed to be static in equilibrium: $\vec{v}_0 = \vec{0}$. The ideal induction equation yields $\frac{\partial B'_z}{\partial t} = 0$, and the Gauss law for magnetism ensures $\frac{\partial B'_z}{\partial z} = 0$. Hence, $B'_z = 0$, and the number of unknowns reduces from eight to seven. The fifth equation incorporates the energy conservation and continuity equations for an ideal gas with adiabatic constant, which equals a ratio of heat capacities under constant pressure and constant volume, $\gamma = \frac{C_P}{C_V}$. The partial derivatives $L_p = \frac{\partial L}{\partial p}$, $L_\rho = \frac{\partial L}{\partial \rho}$, $L_{pp} = \frac{\partial^2 L}{\partial p^2}$, $L_{\rho\rho} = \frac{\partial^2 L}{\partial \rho^2}$, $L_{p\rho} = \frac{\partial^2 L}{\partial p \partial \rho}$ are evaluated at the equilibrium state (p_0, ρ_0). The system describes perturbations of small magnitude, that is, of small Mach numbers. The Mach number M is the ratio of the magnitude of a plasma velocity and the magnetosonic speed. The dispersion relations are established by the linearized equation [Eq. (1)] if one looks for the solution in the form of a sum of planar waves proportional to $\exp(i\omega(k)t - ikz)$, that is, in the integral form

$$f'(z,t) = \int_{-\infty}^{\infty} \tilde{f}(k) \exp(i\omega(k)t - ikz)\, dk,$$

where k designates the wave number. All forthcoming evaluations in this study are leading-order, that is, they contain terms up to the first powers of L_p, L_ρ, L_{pp}, $L_{\rho\rho}$, $L_{p\rho}$, and χ. This concerns also quadratically nonlinear terms. Four dispersion relations are inherent to the magnetosonic modes, which rely on compressibility,

$$\omega = Ck + i\frac{(\gamma-1)(C^2-C_A^2)}{2c_0^2(2C^2-c_0^2-C_A^2)}\left(c_0^2k^2\frac{\chi}{C_P\rho_0}-(c_0^2L_p+L_\rho)\right), \quad (2)$$

where C is one of four magnetosonic speeds satisfying the equation

$$C^4 - C^2(c_0^2+C_A^2)+c_0^2C_{A,z}^2 = 0, \quad (3)$$

and C_A and c_0,

$$C_A = \frac{B_0}{\sqrt{\mu_0\rho_0}}, \quad c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}},$$

designate the Alfvén speed and the acoustic speed in unmagnetized gas in equilibrium, $C_{A,z} = C_A\cos(\theta)$. We consider the weak individual impact of non-adiabaticity associated with the heating–cooling function and thermal conduction on magnetosonic perturbations. This means the weak variation of the MHD wave magnitude in the course of propagation and in fact presupposes introduction of one more small dimensionless parameter, say, λ,

$$\lambda = \frac{(\gamma-1)(C^2-C_A^2)}{2Cc_0^2k(2C^2-c_0^2-C_A^2)}\mathrm{Max}\left(c_0^2k^2\frac{\chi}{C_P\rho_0}, |c_0^2L_p+L_\rho|\right).$$

Smallness of λ depends on θ and plasma-β,

$$\beta = \frac{2}{\gamma}\frac{c_0^2}{C_A^2},$$

and determines the domain of wave numbers of magnetosonic perturbations. The analysis reveals that the ratio $\frac{C^2-C_A^2}{2C^2-c_0^2-C_A^2}$ varies from 0 till 1 for both slow and fast magnetosonic modes. Hence, λ is small with a margin for $\frac{(\gamma-1)\chi k}{2CC_P\rho_0} \ll 1$ and $\frac{\gamma-1}{2Cc_0^2k}|c_0^2L_p+L_\rho| \ll 1$ in the case of both slow and fast magnetosonic modes, and smallness of λ is primarily due to the small impact of thermal conduction and non-adiabaticity introduced by the heating–cooling function. The dispersion relations [Eqs. (2) and (3)] have been established by Chin and Nakariakov *et al.*[14,15] Linear magnetosonic perturbations of wave number k enhance if

$$c_0^2L_p + L_\rho > c_0^2k^2\frac{\chi}{C_P\rho_0}. \quad (4)$$

This is the condition of acoustical activity corrected by contribution of thermal conduction.[19–21] The dispersion relations [Eq. (2)] uniquely determine the links between specific magnetosonic (ms) perturbations in a linear flow,

$$\psi_{lin,ms} = (\rho_{ms} \quad v_{ms,x} \quad v_{ms,y} \quad v_{ms,z} \quad p_{ms} \quad B_{ms,x} \quad B_{ms,y})^T$$

(T denotes transpose). The links inherent to perturbations in all magnetosonic modes in terms of magnetosonic density perturbation take the form

$$\begin{aligned} v_{ms,x} &= -\frac{C_{A,z}(C^2-c_0^2)}{C_{A,x}C\rho_0}\rho_{ms} + \frac{(\gamma-1)C_{A,z}(C^2-c_0^2)(C^2-2c_0^2-C_A^2)}{2C_{A,x}C^2c_0^2(2C^2-c_0^2-C_A^2)\rho_0} \\ &\quad\times\left((c_0^2L_p+L_\rho)\int\rho_{ms}dz + c_0^2\frac{\chi}{C_P\rho_0}\frac{\partial\rho_{ms}}{\partial z}\right), \\ v_{ms,y} &= 0, \\ v_{ms,z} &= \frac{C}{\rho_0}\rho_{ms} - \frac{(\gamma-1)(C^2-C_A^2)}{2c_0^2(2C^2-c_0^2-C_A^2)\rho_0} \\ &\quad\times\left((c_0^2L_p+L_\rho)\int\rho_{ms}dz + c_0^2\frac{\chi}{C_P\rho_0}\frac{\partial\rho_{ms}}{\partial z}\right), \\ p_{ms} &= c_0^2\rho_{ms} - \frac{(\gamma-1)}{C}\left((c_0^2L_p+L_\rho)\int\rho_{ms}dz + c_0^2\frac{\chi}{C_P\rho_0}\frac{\partial\rho_{ms}}{\partial z}\right), \\ B_{ms,x} &= \frac{(C^2-c_0^2)\mu_0}{B_{0,x}}\rho_{ms} - \frac{(\gamma-1)(C^2-c_0^2)(C^2-c_0^2-C_A^2)\mu_0}{B_{0,x}Cc_0^2(2C^2-c_0^2-C_A^2)} \\ &\quad\times\left((c_0^2L_p+L_\rho)\int\rho_{ms}dz + c_0^2\frac{\chi}{C_P\rho_0}\frac{\partial\rho_{ms}}{\partial z}\right), \\ B_{ms,y} &= 0. \end{aligned} \quad (5)$$

These links may be used for an unequivocal definition of magnetosonic modes on a par with the corresponding dispersion relation. Two Alfvén modes (A) are specified by the dispersion relations

$$\omega_A = \pm C_{A,z}k$$

and links

$$B_{A,y} = \mp\frac{B_0}{C_A}v_{A,y}$$

(all other perturbations are zero), and the perturbations in the entropy mode are determined by the dispersion relation and links as follows:

$$\begin{aligned} \omega_{ent} &= i\frac{(\gamma-1)L_\rho}{c_0^2} + i\frac{\chi}{C_P\rho_0}k^2, \\ v_{ent,z} &= \frac{(\gamma-1)L_\rho}{c_0^2\rho_0}\int\rho_{ent}dz - \frac{\chi}{C_P\rho_0^2}\frac{\partial\rho_{ent}}{\partial z}, \quad v_{ent,x} = \frac{C_{A,x}}{C_{A,z}}v_{z,ent} \end{aligned} \quad (6)$$

(all other perturbations are zero). Vectors of perturbations, corresponding to different modes, are linearly independent. Once all specific links are determined, the projector may be evaluated, which distinguishes the density perturbation in the entropy mode from the vector of total perturbations ψ_{lin}, which represents a sum of all specific ones.[11,22] The projector is a linear operator, which consists of seven elements $m_1, \ldots, m_7$, so as

$$\begin{aligned} P_{ent}\psi_{lin} &= (m_1 \quad m_2 \quad m_3 \quad m_4 \quad m_5 \quad m_6 \quad m_7)\psi_{lin} = \rho_{ent}, \\ \psi_{lin} &= (\rho' \quad v'_x \quad v'_y \quad v'_z \quad p' \quad B'_x \quad B'_y)^T. \end{aligned} \quad (7)$$

The projector's elements take the leading-order form

$$\begin{aligned} m_1 &= 1, \quad m_2 = -\frac{(\gamma-1)C_{A,x}\rho_0}{C_{A,z}c_0^4}\left((c_0^2L_p+L_\rho)\int dz + c_0^2\frac{\chi}{C_P\rho_0}\frac{\partial}{\partial z}\right), \\ m_3 &= 0, \quad m_4 = \frac{C_{A,z}}{C_{A,x}}m_2, \quad m_5 = -\frac{1}{c_0^2}, \quad m_6 = m_7 = 0. \end{aligned}$$

Linear modes, that is, specific perturbations of infinitely small magnitudes, propagate independently on each other. This means that dynamic equations for different specific perturbations do not couple. In particular, application of P_{ent} at the linearized system (1) (which is briefly represented by

$$\frac{\partial}{\partial t}E\psi_{lin} + K\psi_{lin} = 0,$$

with K being the matrix operator containing spatial derivatives and E denoting the unit matrix) readily results in the evolutionary equation for ρ_{ent},

$$P_{ent}\left(\frac{\partial}{\partial t}E\psi_{lin}+K\psi_{lin}\right)=\frac{\partial}{\partial t}\rho_{ent}+\frac{(\gamma-1)L_\rho}{c_0^2}\rho_{ent}-\frac{\chi}{C_P\rho_0}\frac{\partial^2\rho_{ent}}{\partial z^2}=0.$$

The dynamic equation for the specific perturbation in the entropy mode evidently corresponds to the dispersion relation ω_{ent}. The projecting retains perturbations belonging to the entropy mode exclusively and reduces all other specific perturbations in the linear fluid field. The projecting is a powerful tool in studies of weakly nonlinear flow since it allows us to derive dynamic equations for interacting modes with properly distributed nonlinear terms. Going to the studies of nonlinear flows, we still determine modes by specific links. Application of P_{ent} at the system (1) represented by

$$\frac{\partial}{\partial t}E\psi_{lin}+K\psi_{lin}=\phi_{nonl},$$

where ϕ_{nonl} is the nonlinear vector on the right-hand side, yields the dynamic equation for ρ_{ent} with the source $P_{ent}\phi_{nonl}$ containing in general contribution of all modes,

$$P_{ent}\left(\frac{\partial}{\partial t}E\psi_{lin}+K\psi_{lin}\right)=\frac{\partial}{\partial t}\rho_{ent}+\frac{(\gamma-1)L_\rho}{c_0^2}\rho_{ent}-\frac{\chi}{C_P\rho_0}\frac{\partial^2\rho_{ent}}{\partial z^2}$$
$$=P_{ent}\phi_{nonl}.$$

The frames of this study do not allow us to discuss the mathematical content closer. The method has been applied by the author in the description of nonlinear interaction of modes in various fluid flows. The detailed analysis and many examples may be found in Ref. 11 and the related literature. As regards to the MHD flows and excitation of magnetosonic heating, the method has been applied in Refs. 16, 17, and 23

III. EXCITATION OF THE ENTROPY MODE IN THE FIELD OF INTENSE MAGNETOSONIC PERTURBATIONS

For definiteness, we consider one dominant magnetosonic mode (fast or slow) propagating in the positive direction of axis z, that is, with $C > 0$. The dominance means that magnitudes of perturbations in all other wave and non-wave modes are comparatively small. Hence, only magnetosonic terms are considered in $P_{ent}\phi_{nonl}$ among all varieties of nonlinear ones. We focus on the nonlinear excitation of the entropy mode since it is the only mode contributing in the variation of density, which may accumulate in time. In the studies of nonlinear phenomena, the linear magnetosonic links should be corrected by nonlinear terms. The leading-order links are as follows:

$$\psi_{nonl,ms}=\psi_{lin,ms}+\begin{pmatrix}0 & A_1c_0\frac{\rho_{ms}^2}{\rho_0^2} & 0 & A_2c_0\frac{\rho_{ms}^2}{\rho_0^2} & A_3c_0^2\frac{\rho_{ms}^2}{\rho_0} & A_4B_0\frac{\rho_{ms}^2}{\rho_0^2} & 0\end{pmatrix}^T, \tag{8}$$

where

$$A_1=\frac{C_{A,z}(C^2-c_0^2)^2(3C^6-3C^2c_0^2C_{A,z}^2+C^4(c_0^2(\gamma-2)-C_{A,z}^2)-c_0^2C_{A,z}^4(\gamma-3))}{4c_0C^5C_{A,x}^3(2C^2-c_0^2-C_A^2)},$$
$$A_2=\frac{C(c_0^2+C^2(\gamma-4)-C_A^2(\gamma-3))}{4c_0(2C^2-c_0^2-C_A^2)},$$
$$A_3=\frac{\gamma-1}{2},$$
$$A_4=\frac{(C^2-c_0^2)^2(c_0^2+C_A^2-C^2)(c_0^2+C_A^2(\gamma-1)-\gamma C^2)}{2c_0^2C_AC_{A,x}^3(2C^2-c_0^2-C_A^2)}.$$

They have been obtained in terms of $v_{ms,z}$ in Ref. 17. Quadratically nonlinear terms correct the linear links, making the dominant magnetosonic mode isentropic in a flow without thermal conduction and zero L, that is, these terms are of order $M^2\lambda^0$. In the absence of the magnetic field, $C = c_0$, $C_A = 0$, and the relations are readily transformed as

$$\begin{aligned} v_{s,x}&=v_{s,y}=0,\\ v_{s,z}&=\frac{c_0}{\rho_0}\rho_s-\frac{(\gamma-1)}{2\rho_0}\frac{\chi}{C_P\rho_0}\frac{\partial\rho_s}{\partial z}-\frac{(\gamma-1)(c_0^2L_p+L_\rho)}{c_0^2\rho_0}\int\rho_s dz+\frac{(\gamma-3)c_0}{4\rho_0^2}\rho_s^2,\\ p_s&=c_0^2\rho_s-c_0(\gamma-1)\frac{\chi}{C_P\rho_0}\frac{\partial\rho_s}{\partial z}-\frac{(\gamma-1)(c_0^2L_p+L_\rho)}{c_0}\int\rho_s dz+\frac{(\gamma-1)c_0^2}{2\rho_0}\rho_s^2,\\ B_{s,x}&=B_{s,y}=0, \end{aligned} \tag{9}$$

which correspond to the sound (s) planar wave. The nonlinear terms support isentropicity of the Riemann wave, and the linear terms in the links reflect the dispersion relation.[8,24] The link between p_s and ρ_s generalizes the equation of state in the Riemann wave by including terms associated with the thermal conduction and heating–cooling function. It may be considered as a "constitutive equation" instead of "an equation of state," which implies an instantaneous relation of variables.[4] It has been used by Soluyan and Khokhlov in studies on nonlinear acoustics in 1961.[25] An energy loss that does not associate with L results in a term proportional to the derivative of an acoustic density with respect to time and thermal conduction. This term was considered erroneously as proportional to the total attenuation (including mechanical one due to shear and bulk viscosity) in Ref. 4.

The variations of pressure associate only with the magnetosonic wave, but the total variation of density consists of parts belonging to the magnetosonic wave and to the entropy mode, ρ_{ent}. By means of projecting, we are able to derive dynamic equations accounting for weak nonlinear interaction of finite-magnitude modes. Particularly, application of P_{ent} at the system (1) taking into account links (6) and (8) distinguishes the dynamic equation for density perturbation in the entropy mode, yielding nonlinear terms properly attributed to specific equations.[17,18] Among all nonlinear terms that form a driving "source" of the entropy mode, we consider only these belongings to the dominant magnetosonic wave. A variation of density in the entropy mode is governed by the leading-order dynamic equation

$$\frac{\partial \rho_{ent}}{\partial t} + \frac{(\gamma-1)L_\rho}{c_0^2}\rho_{ent} - \frac{\chi}{C_p\rho_0}\frac{\partial^2 \rho_{ent}}{\partial z^2} = Q_L + Q_\chi = Q_{ms}, \tag{10}$$

where the magnetosonic source Q_{ms} is a sum of two parts, Q_L and Q_χ,

$$Q_L = Q_{L,1}\rho_{ms}^2 + Q_{L,2}\frac{\partial \rho_{ms}}{\partial t}\int \rho_{ms}(z,t)dt, \tag{11}$$

$$Q_\chi = Q_{\chi,1}\left(\left(\frac{\partial \rho_{ms}}{\partial t}\right)^2 + \rho_{ms}\frac{\partial^2 \rho_{ms}}{\partial t^2}\right) + Q_{\chi,2}\rho_{ms}\frac{\partial^2 \rho_{ms}}{\partial t^2}, \tag{12}$$

and

$$\begin{aligned}
Q_{L,1} = &-\frac{(\gamma-1)}{4\rho_0 c_0^2(C^4 - c_0^2C_{A,z}^2)}\Bigg[-\frac{(c_0^4C_{A,z} + C^4(C_{A,x}+C_{A,z}) - 2C^2c_0^2C_{A,z})((4+\gamma)C^2 - (\gamma+1)C_A^2 - 3c_0^2)}{C_{A,x}} \\
&\times(c_0^2L_p + L_\rho) + 0.5c_0^2(-2c_0^4(c_0^2L_p + L_\rho) + 4C^4((5-2\gamma)L_\rho + 2c_0^4\rho_0L_{pp} + 2\rho_0L_{\rho\rho} + 3c_0^2L_p + 4c_0^2\rho_0L_{p\rho}) \\
&-2C^2(2c_0^6\rho_0L_{pp} + c_0^4((2\gamma-1)L_p + 2C_A^2\rho_0L_{pp} + 4\rho_0L_{p\rho}) + 2C_A^2((4-2\gamma)L_\rho + \rho_0L_{\rho\rho}) + c_0^2(L_\rho + 2\rho_0L_{\rho\rho} + C_A^2(2(3-\gamma)L_p + 4\rho_0L_{p\rho})))\Bigg]
\end{aligned}$$

$$Q_{L,2} = \frac{(\gamma-1)C^2(c_0^2L_p + L_\rho)}{2\rho_0c_0^2(C^4 - c_0^2C_{A,z}^2)}\left[(2\gamma c_0^2 + (\gamma+1)C_A^2 - (3\gamma+1)C^2)\right],$$

$$\begin{aligned}
Q_{\chi,1} = &\frac{(\gamma-1)\chi}{2C_p\rho_0^2c_0^2C_{A,x}^3C^2(C^4 - c_0^2C_{A,z}^2)}\Big[(C^6(c_0^2C_{A,x} + 2(C_{A,x}^2C_{A,z} + C_{A,x}C_A^2 - C_{A,z}^3)) - C^4c_0^2(2c_0^2C_{A,x} + 2C_{A,x}^2C_{A,z} + 7C_{A,x}C_{A,z}^2 - 6C_{A,z}^3 \\
&+ 4(\gamma-1)C_{A,x}^3) + C^2c_0^2(c_0^4C_{A,x} + c_0^2(8C_{A,x}C_{A,z}^2 - 6C_{A,z}^3 + (\gamma-1)C_{A,x}^3) + 2C_{A,x}^2C_{A,z}^2(C_{A,x}(\gamma-2) - \gamma C_{A,z})) \\
&+c_0^4(C_{A,z}^2(c_0^2(2C_{A,z} - 3C_{A,x}) + C_{A,x}^2((\gamma-1)C_{A,x} + 2\gamma C_{A,z})))\Big],
\end{aligned}$$

$$Q_{\chi,2} = \frac{(\gamma-1)^2\chi}{C_PC^2\rho_0^2}.$$

The reason for the separation of two sources Q_L and Q_χ on the right-hand side of Eq. (10) is in association of different terms with various non-adiabatic effects caused by the heating–cooling function and thermal conduction. The dynamic equation for the variation of density in the entropy mode without account of thermal conduction and the corresponding source Q_L have been derived by the author in Ref. 17. Taking in mind that for the wave mode $\frac{\partial}{\partial z} \approx -\frac{1}{C}\frac{\partial}{\partial t}$, the total pressure perturbation relates to the total density perturbation in the leading order as

$$\begin{aligned}
p' = p_{ms} &= c_0^2(\rho' - \rho_{ent}) + \frac{(\gamma-1)c_0^2}{2\rho_0}\rho_{ms}^2 - \frac{\gamma-1}{C}(c_0^2L_p + L_\rho) \\
&\times\int \rho_{ms}dz - \frac{(\gamma-1)c_0^2}{C}\frac{\chi}{C_P\rho_0}\frac{\partial \rho_{ms}}{\partial z} \\
&\approx c_0^2(\rho' - \rho_{ent}) + \frac{(\gamma-1)c_0^2}{2\rho_0}\rho'^2 + (\gamma-1)(c_0^2L_p + L_\rho) \\
&\times\int \rho' dt + \frac{(\gamma-1)c_0^2}{C^2}\frac{\chi}{C_P\rho_0}\frac{\partial \rho'}{\partial t}
\end{aligned} \tag{13}$$

and respectively,

$$\begin{aligned}
\rho' = \rho_{ent} &+ \frac{p'}{c_0^2} - \frac{(\gamma-1)}{2\rho_0c_0^4}p'^2 - \frac{(\gamma-1)}{c_0^4}(c_0^2L_p + L_\rho)\int p'dt \\
&- \frac{(\gamma-1)}{C^2c_0^2}\frac{\chi}{C_P\rho_0}\frac{\partial p'}{\partial t}.
\end{aligned} \tag{14}$$

There are terms of different orders on the right-hand side of Eq. (13). The leading-order term is $c_0^2\rho_{ms}$, where ρ_{ms} is a difference of ρ' $[O(M)]$ and ρ_{ent}, which is governed by Eq. (10) with the magnetosonic source $[O(M^2\lambda)]$. The next terms in Eq. (13) are of comparative order, that is, $\frac{(\gamma-1)c_0^2}{2\rho_0}\rho'^2$ $[O(M^2)]$, $(\gamma-1)(c_0^2L_p + L_\rho)\int \rho' dt$ $[O(M\lambda)]$, and $\frac{(\gamma-1)c_0^2}{C^2}\frac{\chi}{C_P\rho_0}\frac{\partial \rho'}{\partial t}$ $[O(M\lambda)]$. In these terms, we replace ρ_{ms} by the total ρ' in view of smallness of damping/enhancement due the heating–cooling function, thermal conduction, and nonlinear effects. The sources (11) and (12) in the unmagnetized case ($C = c_0$, $C_A = 0$) take the forms

$$Q_L = \frac{(\gamma-1)L_T}{2\rho_0^2C_P}\left[\frac{3(\gamma-1)}{2}\rho_s^2 - (\gamma+1)\frac{\partial \rho_s}{\partial t}\int \rho_s dt\right], \tag{15}$$

$$Q_\chi = -\frac{(\gamma-1)\chi}{2\rho_0^2 c_0^2 C_P}\left[(3\gamma-5)\left(\frac{\partial\rho_s}{\partial t}\right)^2 + (\gamma-3)\rho_s\frac{\partial^2\rho_s}{\partial t^2}\right]. \quad (16)$$

We will treat L as a function of temperature T, so as

$$L_p = \frac{\gamma L_T}{C_P(\gamma-1)\rho_0}, \quad L_\rho = -\frac{c_0^2 L_T}{C_P(\gamma-1)\rho_0},$$
$$L_{pp} = \frac{\gamma^2 L_{TT}}{C_P^2(\gamma-1)^2\rho_0^2}, \quad L_{\rho\rho} = \frac{c_0^2(2C_P(\gamma-1)L_T + c_0^2 L_{TT})}{C_P^2(\gamma-1)^2\rho_0^2},$$
$$L_{p\rho} = -\frac{C_P(\gamma-1)\gamma L_T + c_0^2\gamma L_{TT}}{C_P^2(\gamma-1)^2\rho_0^2},$$

and discard the second derivative of L with respect to T, L_{TT}. The condition (4) rearranges as

$$L_T > L_{T,0} = \frac{\omega^2\chi}{c_0^2}.$$

Equation (15) matches the result of Ref. 26 for weak magnetic strength and nearly periodic sound, and Eq. (16) agrees with conclusions of Refs. 22 and 24.

IV. HYSTERESIS CURVES FOR PERIODIC AND IMPULSIVE EXCITERS

In general, ρ_{ent} satisfies Eq. (10), which is difficult for analytical solution. In order to simplify evaluations, we suppose that the variation of density in the entropy mode may be evaluated by integrating the magnetosonic source with respect to time ($\rho_{ent} = 0$ at $t = 0$),

$$\rho_{ent} = \int_0^t Q_{ms}dt.$$

This approach imposes that the impact of thermal conduction and heating–cooling function is considered on the right-hand side (that is, in the magnetosonic source) but is discarded on the linear left-hand side of equation.

A. Harmonic magnetosonic pressure

We consider harmonic magnetosonic dimensionless pressure at a transducer in the form

$$P = \frac{p'}{Mc_0^2\rho_0} = \sin(\omega t), \quad (17)$$

where ω is the magnetosonic frequency. The leading-order form of the total dimensionless variation of density in accordance with Eq. (14) is described by

$$R = \frac{\rho'}{M\rho_0} = \sin(\omega t) + \frac{(\gamma-1)}{\omega C_P\rho_0}\left(L_T - \frac{\omega^2}{C^2}\chi\right)\cos(\omega t) - \frac{1}{2}M(\gamma-1)\sin^2(\omega t) + R_{ent}, \quad (18)$$

where

$$R_{ent} = \frac{\rho_{ent}}{M\rho_0} = M\rho_0\frac{Q_{L,1} - Q_{L,2} - Q_{\chi,2}\omega^2}{2\omega}\omega t - M\rho_0\frac{Q_{L,1} + Q_{L,2} - (2Q_{\chi,1} + Q_{\chi,2})\omega^2}{4\omega}\sin(2\omega t). \quad (19)$$

Equations (17)–(19) determine the dependence of the total density perturbation ρ' on the total pressure perturbation p' in the parametric form. One may conclude that the term

$$\frac{(\gamma-1)}{\omega C_P\rho_0}\left(L_T - \frac{\omega^2}{C^2}\chi\right)\cos(\omega t) \quad (20)$$

is responsible for the width of a hysteresis curve in the plane of magnetosonic pressure vs magnetosonic density [it is nonzero if the factor by $\cos(\omega t)$ is non-zero], that is, $P \Longleftrightarrow R$ diagram not taking into account R_{ent}. If $L_T > \frac{\omega^2}{C^2}\chi$, a hysteresis curve in the magnetosonic perturbation diagram has the clockwise direction, and if $L_T < \frac{\omega^2}{C^2}\chi$, it has the counterclockwise direction. The term

$$\frac{1}{2}M(\gamma-1)\sin^2(\omega t) \quad (21)$$

is responsible for the nonlinear distortion of a curve (quadratic nonlinearity deforms an elliptic diagram into a crescent with downcast ends). The term

$$M\rho_0\frac{Q_{L,1} + Q_{L,2} - (2Q_{\chi,1} + Q_{\chi,2})\omega^2}{4\omega}\sin(2\omega t) \quad (22)$$

with non-zero factor by $\sin(2\omega t)$ may lead to formation of intersections in a hysteretic curve. This term reflects the contribution of the entropy mode, as well as the term

$$M\rho_0\frac{Q_{L,1} - Q_{L,2} - Q_{\chi,2}\omega^2}{2\omega}\omega t. \quad (23)$$

This is the only term that may be non-zero on the average over the magnetosonic period. It is responsible for slow accumulation of positive or negative variations of density due to excitation of the entropy mode. It could unlimitedly enlarge in absolute value if it were not opposed by the growing nonlinear interaction with other modes. When the variation of density in the entropy mode becomes comparable with density perturbation in the dominant mode, Eq. (10) is no longer valid. The inflow of energy balances losses due to thermal conduction over a period if

$$L_T = L_{T,th},$$

which ensures $Q_{L,1} - Q_{L,2} - Q_{\chi,2}\omega^2 = 0$. Figure 1 shows the ratio $\frac{L_{T,th}}{L_{T,0}}$ for the fast magnetosonic mode as a function of β at different θ values. In the cases $\theta = 0$ and $\theta = \pi$, $\beta < \frac{2}{\gamma}$ corresponds to $C = C_A$ and ratio 0 ($\beta > \frac{2}{\gamma}$ corresponds to $C = c_0$ and ratio 2). In all evaluations, $\gamma = \frac{5}{3}$.

So, the threshold values for acoustical activity, $L_{T,0}$, and that for an anomalous decrease in the background temperature associated with the entropy mode, $L_{T,th}$, are different. This is due to the distribution of incoming/outcoming energy between the wave and non-wave modes. The acoustical activity is the linear effect, but excitation of the entropy mode in the field of intense MHD waves is the nonlinear phenomenon in essence. Acoustical activity makes the energy associated with the magnetosonic mode enlarge until suppressed by damping and/or nonlinear transfer of energy into the entropy mode and other wave modes. This part of energy "goes away" with the running wave.

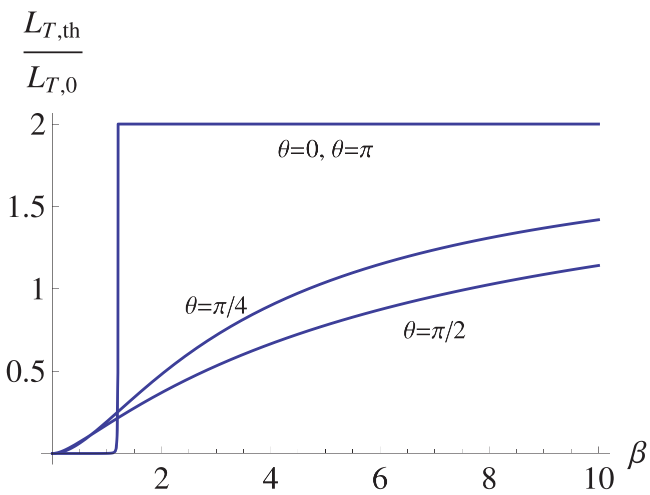

FIG. 1. The ratio $\frac{L_{T,th}}{L_{T,0}}$ for the fast magnetosonic mode as a function of β at different θ values.

The part of energy, which accumulates in the space domain, associates with the entropy mode. The sketchy plots of two cycles of hysteresis curves for different ratios of $L_{T,th}$ and $L_{T,0}$ are shown in Fig. 2. The starting time of excitation is $t = 0$.

The counterclockwise direction in the last plot in the low row is due to a constant increase in the density perturbation associated with the entropy mode, R_{ent}. The total density perturbation decreases or increases after passing of a wave period in dependence on energy balance. The variation in the internal energy of a medium during the magnetosonic period is determined by dynamics of the entropy mode. The relative variation of the internal energy U over the period may be positive or negative. It equals

$$\frac{\omega}{2\pi}\frac{\delta U}{U_0} = -\left\langle \frac{1}{\rho_0}\frac{\partial \rho_e}{\partial t}\right\rangle = -M\rho_0(Q_{L,1} - Q_{L,2} - Q_{\chi,2}\omega^2), \qquad (24)$$

where square brackets denote average over the exciter's period and U_0 is an unperturbed internal energy of a plasma.

B. A bipolar impulse

The next kind of an exciter is bipolar at a transducer,

$$P = \sqrt{2e}\,\omega t \exp\left(-(\omega t)^2\right), \qquad (25)$$

where ω denotes the characteristic inverse duration of an impulse (the multiplier $\sqrt{2e}$ ensures unit maximum P). The leading-order form of the total dimensionless density perturbation in accordance with Eq. (13) equals

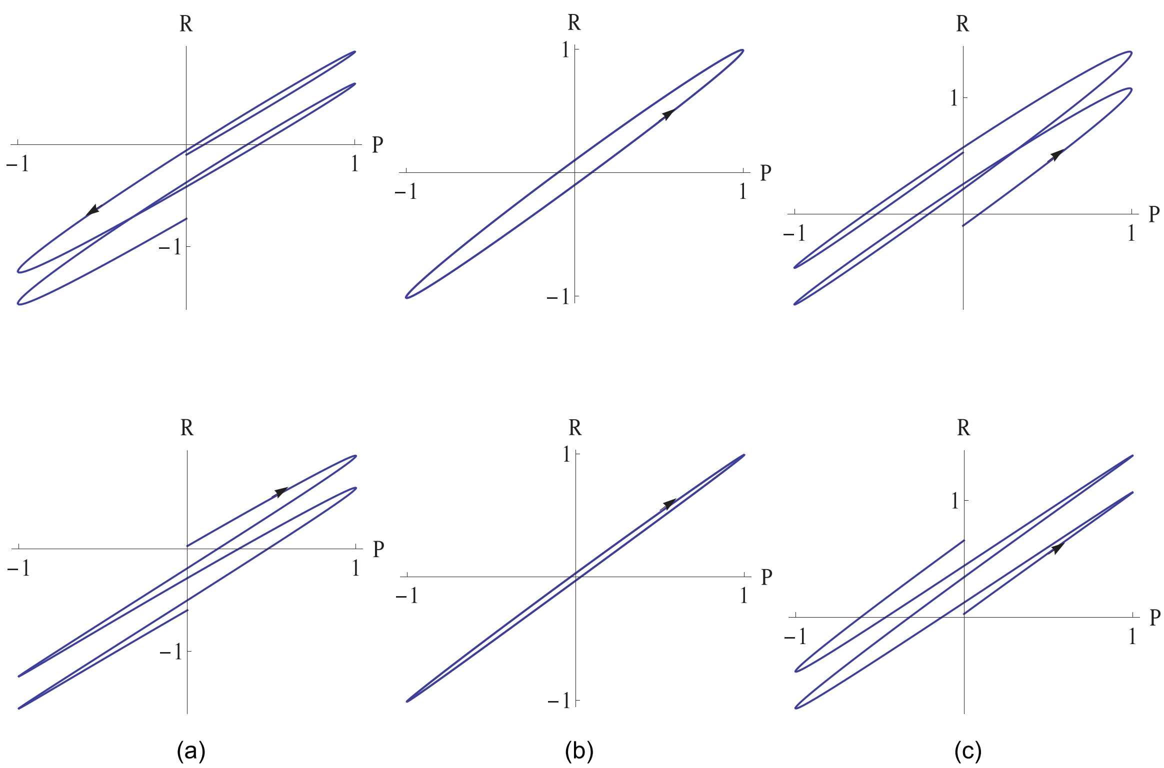

FIG. 2. Dependence of the dimensionless total density perturbation on the dimensionless total pressure perturbation in the MHD flow: case of the periodic pressure at a transducer and $L_T < L_{T,th}$ (a), $L_T = L_{T,th}$ (b), and $L_T > L_{T,th}$ (c). The upper row corresponds to $L_T < \frac{\omega^2\chi}{C^2}$, and the lower row corresponds to $L_T > \frac{\omega^2\chi}{C^2}$. Arrows indicate the direction of evolution. The curves show two cycles of the harmonic excitation.

$$R = \frac{\rho'}{M\rho_0} = \sqrt{2e}\Big[\omega t \exp\left(-(\omega t)^2\right) + \frac{(\gamma-1)L_T}{2\omega C_P \rho_0}\exp\left(-(\omega t)^2\right) - \frac{(\gamma-1)\omega}{C^2}\frac{\chi}{C_p\rho_0}\exp\left(-(\omega t)^2\right)\left(\omega t - 2(\omega t)^3\right) - \frac{\sqrt{2e}}{2}M(\gamma-1)\exp\left(-2(\omega t)^2\right)(\omega t)^2\Big] + R_{ent}$$

and

$$R_{ent} = -M\rho_0 \frac{\exp\left(1-(\omega t)^2\right)}{2\omega}\big(\omega t\left((Q_{L,1}+Q_{L,2}) + \omega^2(-4Q_{\chi,1} - 3Q_{\chi,2} + 4\omega^2 t^2(2Q_{\chi,1}+Q_{\chi,2})\right)\big) + eM\rho_0\frac{\sqrt{\pi}(Q_{L,1}-Q_{L,2}-3\omega^2 Q_{\chi,2})}{4\sqrt{2}\omega}\left(Erf\left(\sqrt{2}\omega t\right)+1\right). \quad (26)$$

The threshold $L_{T,th}$ is determined by equality $Q_{L,1} - Q_{L,2} - 3\omega^2 Q_{\chi,2} = 0$ and hence is three times larger than the threshold value for the harmonic signal. Figure 3 shows exemplary $P \Longleftrightarrow R$ diagrams as an impulse develops from $-\infty$ till ∞ in time.

The total relative increase/decrease in the internal energy due to the balance of incoming energy and increase/decrease in the magnetosonic energy is given by the following formula:

$$\frac{\delta U}{U_0} = -\frac{1}{\rho_0}\int_{-\infty}^{\infty} Q_{ms}dt = -eM\rho_0\sqrt{\frac{\pi}{2}}\frac{(Q_{L,1}-Q_{L,2}-3\omega^2 Q_{\chi,2})}{2\omega}. \quad (27)$$

V. CONCLUDING REMARKS

The subject of study is the relation of total variation of density and total variation of pressure taking into account excitation of the entropy mode in the field of intense magnetosonic perturbations. The pictorial rendition of these relations is a diagram that reflects the hysteretic character of irreversible thermodynamic processes. The main results of this study are Eq. (13) [Eq. (14)] along with the evolutionary equation for ρ_{ent} given by Eq. (10). They determine diagrams in the plane of total perturbation of pressure vs total perturbation of density in a magnetohydrodynamic flow, $p' \Longleftrightarrow \rho'$. Equation (13) makes use of the links between dominant magnetosonic pressure and magnetosonic density and considers nonlinear excitation of the entropy mode. After passing a wave, a medium does not return back to the equilibrium, and the parameters behavior in both the wave and a medium is irreversible. As usual, the temperature of a medium gets larger (and its density smaller) due to transfer of the wave energy into that of the chaotic motion of the molecules in nonlinear flows with attenuation. Some kinds of the heating–cooling function may overbalance damping and lead to a cooling of a plasma. This is reflected in the hysteresis diagrams.

Equation (10) is simplified by discarding terms $\frac{(\gamma+1)L_\rho}{c_0^2}\rho_{ent}$, $\frac{\chi}{C_p\rho_0}\frac{\partial^2\rho_{ent}}{\partial z^2}$ in the first approximation, so as the solution is a simple integral of Q_{ms} over time. The impact of these disregarded terms may decrease or increase (this is the case of thermal instability $L_\rho < 0$ and comparatively small thermal conduction) the magnitude of ρ_{ent} evaluated by simple integration. For more precise evaluations, Eq. (10) with a magnetosonic source should be solved. The variation of density in the entropy mode ρ_{ent} is the only term that may accumulate and enlarge in absolute value with time until suppressed by nonlinear transfer of energy between modes. When the absolute value of ρ_{ent} approaches a magnetosonic density, other modes also enhance, the dominant mode weaken, and Eqs. (10) and (13) are no longer valid. The terms on the right-hand side of Eq. (13) $(\gamma-1)(c_0^2 L_p + L_\rho)\int \rho' dt$ and $\frac{(\gamma-1)c_0^2}{C^2}\frac{\chi}{C_P\rho_0}\frac{\partial\rho'}{\partial t}$, as well as ρ_{ent}, also reflect hysteretic behavior, that is, the dependence of the current thermodynamic state on the history. The first term that associates with the heating–cooling function includes an integral operator. The exemplary exciters considered in this study give non-zero variation in magnetosonic density over one period or after an impulse passing due to this term. In some cases of exciters, the non-zero integral results in non-zero magnetosonic density. In particular, this happens to mono-polar impulses. In the case of the Gaussian exciter,

$$p_{ms} = M\rho_0 c_0^2 \exp\left(-(\omega\tau)^2\right),$$

where ω is the characteristic duration of an impulse, and a magnetosonic density after an impulse passes equals

$$\rho_{ms} = -M\rho_0\frac{2\sqrt{\pi}(\gamma-1)(c_0^2 L_p + L_\rho)}{c_0^2\omega}. \quad (28)$$

This is a constant quantity in contrast to the non-wave density perturbation in the entropy mode, ρ_{ent}. In some parts of the curve $\rho'(p')$,

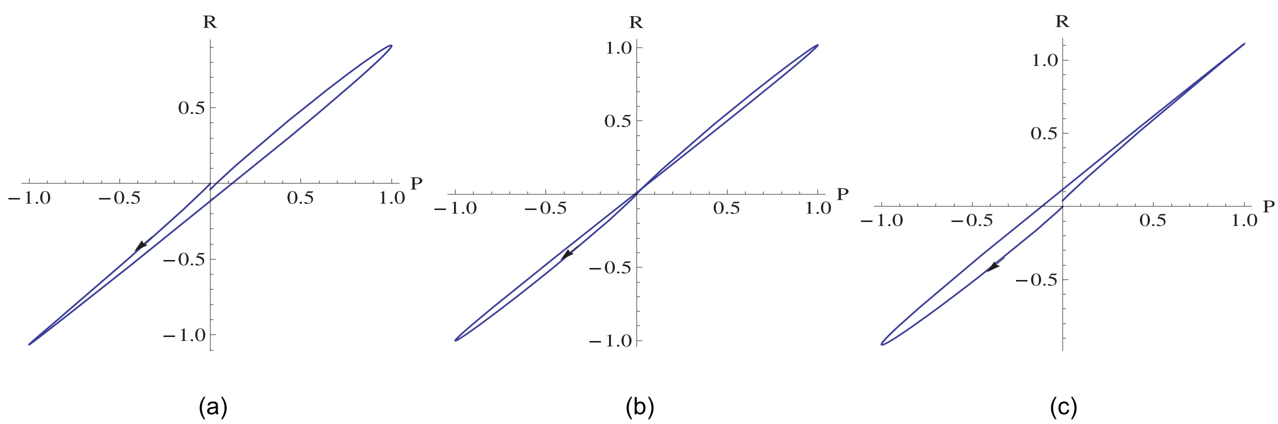

FIG. 3. $P \Longleftrightarrow R$ diagrams: case of the bipolar pressure impulse at a transducer and $L_T < L_{T,th}$ (a), $L_T = L_{T,th}$ (b), and $L_T > L_{T,th}$ (c).

density may temporarily additionally grow due to the term $\frac{(\gamma-1)}{C^2c_0^2}\frac{\chi}{C_P\rho_0}\frac{\partial p'}{\partial t}$ in Eq. (14). This may happen to the domains where pressure decreases with time. This term is zero on average for periodic exciters and tends to zero if $t \to \pm\infty$ for impulsive exciters. That is why the ultimate variation in magnetosonic density is determined exclusively by the integral term, which in turn is determined by the heating–cooling function. The total density perturbation includes as a part ρ_{ent}, which depends on the magnetosonic source in accordance with Eq. (10) and may take positive and negative values. Involving into consideration mechanical viscosity would contribute to Eq. (10), enriching the source right-hand term with a part associated with mechanical damping, Q_{mech}, which is always negative: $Q = Q_L + Q_\chi + Q_{mech}$. Magnetosonic links (13) and (14) remain unchanged since mechanical losses do not contribute to the pressure–density links for wave perturbations.

The linear links (not to mention nonlinear corrections to them) between specific perturbations are undeservedly underestimated. The integral link between perturbations of pressure and density in the wave motion is inherent to flows with relaxation of thermodynamic parameters.[7,8,10,27] It appears in flows of spatially inhomogeneous in equilibrium media like the atmospheric gas affected by the gravitational force.[11] Generally, an integral link contains a kernel, which is frequency-dependent. The links determine the dynamic equations for wave and non-wave perturbations in all frequency ranges, including weakly nonlinear ones accounting for the interaction of different modes.[5,28,29] This is of especial importance in the case of impulsive exciters with the broad frequency spectrum. Wave processes reveal frequency-dependent behavior due to the impact of the heating–cooling function and thermal conduction. In this study, we consider high-frequency magnetosonic perturbations weakly damping/amplified over wavelength and make use the series expansion in the first powers of L_p, L_ρ, and χ. The dispersion relations, and in particular the real part of magnetosonic frequency, will experience modification in the general, not asymptotic case.[30,31] Coverage of the entire frequency range would have an impact on links of wave perturbations and the source of the entropy mode as it happens to flows of gases with various relaxation processes.[5,28,29,32] The links between total perturbations may indicate thermodynamic processes in a medium (characteristic frequency of magnetosonic perturbations and their magnitude), its equilibrium state, and parameters responsible for the deviation from adiabaticity of a flow. The diagrams may be useful in reconstruction of dispersive and viscous properties of a medium in the general case as well.[4,8] Plotted for different frequencies of wave perturbations, these pictorial images make possible to evaluate the time scales at which the dispersive properties of the wave are most pronounced. The analogous problems in optics are usually solved by laser spectroscopy.[33] In the context of this study, the only term in $p_{ms} \Longleftrightarrow \rho_{ms}$ relation, which depends on the magnetosonic speed C, that is, on an angle between the magnetic field and the wave vector θ and plasma-β, is $\frac{(\gamma-1)c_0^2}{c_0^2C^2}\frac{\chi}{C_P\rho_0}\frac{\partial p'}{\partial t}$. The diagrams may be used in order to determine these parameters. The links between thermodynamic perturbations in the wave mode, which may be referred to as "the constitutive equation" or "the polarization relation" rather than an equation of state, are useful in many wave problems not only belonging to acoustics. Khokhlov used a similar equation referring to nonlinear electromagnetic waves.[34] The mathematical tool for evaluations in this study is projecting, which is actually a way for linear combination of conservation equations in order to eliminate all linear non-specific terms.

An efficiency of the irreversible heating of a medium depends on the magnitude and shape of an exciting magnetosonic pressure, as well as $p' \Longleftrightarrow \rho'$ diagrams. It is also determined by $Q_{L,1}$, $Q_{L,2}$, $Q_{\chi,1}$, $Q_{\chi,2}$, which in turn depend on plasma-β and θ [Eq. (10)]. This causes studies of magnetohydrodynamic flows especially difficult even in a simple geometry of a flow. For a medium with known thermodynamic properties, the diagrams may be useful in reconstruction of the remote magnetosonic source. The type of excitation may be selected in order to produce the lowest or highest heating at the same intensity of the initial perturbations. We do not consider exciters with discontinuities in this study. The saw-tooth wave forms in acoustically active flows if mechanical damping and thermal conduction are comparatively small,[18] and it may appear at sudden excitation of a medium. The magnetosonic wave with discontinuity experiences nonlinear damping.[7,8] The peculiarities of nonlinear damping at the fronts compared to Newtonian attenuation and difference in the hysteresis curves have been discussed by Hedberg and Rudenko.[4]

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study, which is a purely theoretical one.

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