

## Curve curvature analysis of a grain trajectories in variable honing of cylindrical holes of thin wall cylinder liners as a honing process optimization strategy.

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**Abstract.** The main problem of honing of thin wall cylinder liners is a thermal distortion of honed holes. The higher the value of the temperature of the honed workpiece, the greater its holes deformation. The paper presents a method of reducing the temperature occurring in the honing process as a result of the application of a variable honing kinematics conditions with particular emphasis on the analysis of the effect of the value of the sum of radii of curve curvature on honed workpieces temperature. The lower value of the sum of curve curvatures radius of abrasive grain trajectories function affects the honed workpiece temperature decrease for a longer path length.

The curve curvature radius of the abrasive grain paths function in the variable kinematic condition of honing of cylindrical holes can be used to improve honing of thin wall cylinder liners due to decreasing of amount temperature received in honing process.

**Keywords:** honing, variable kinematic, abrasive grain trajectory

### Introduction

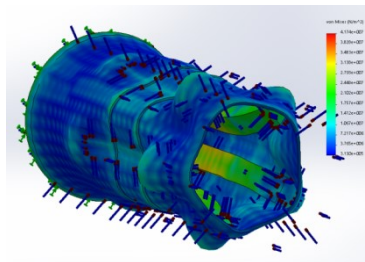
A useful way to represent the curvature of a curve is by plotting curvature against arc length. [...]curvature can be calculated at any point on the curve from the normal and tangent vectors or from the turning angles along the curve [...] for many points on a curve would be time-consuming. It is easier to use a computationally less complex method [1].

Curve curvature  $R$  was calculated in the Mathcad system in many points on received previously in real honed experiment abrasive grain trajectories function of move  $Z(X)$ .

As shown in the literature variable kinematic condition of honing process can improve the cylindricity of honed holes [2, 4, 6]. The way of improving the surface roughness and honing efficiency was described by I. Buj-Corral [3]. The main problem of honing [2, 4, 6] of thin wall cylinder liners is the thermal distortion of honed holes (Fig.1), regardless of how the holes for honing was made [5].

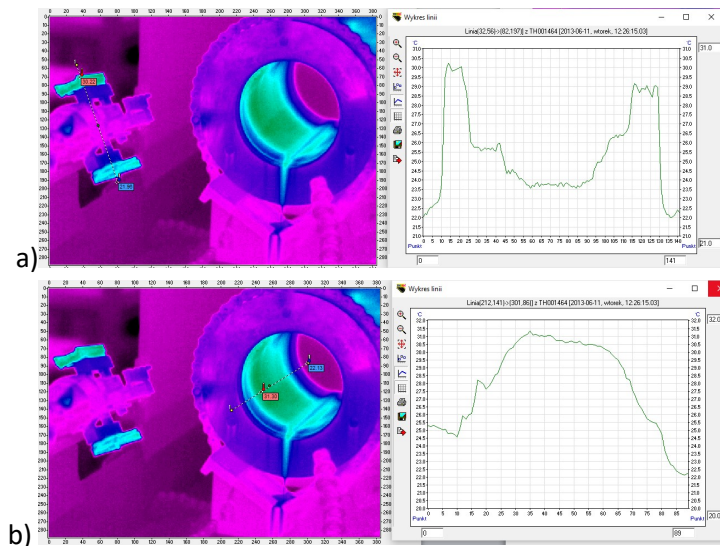
More thematically related and exceptionally interesting literature items are presented by author in his paper [6].

The higher the amount of the workpieces temperature  $t$ , the greater its holes deformation. The paper presents a method of reducing the temperature occurring in the honing process as a result of the application of a variable honing kinematics condition with regards to curve curvature radius  $R$  of abrasive grain trajectory  $Z(X)$ . Honed workpieces holes diameter was  $D = 100\text{mm}$ , length of holes was  $l_h = 250\text{mm}$ . The honing head stroke was  $l_s = 225\text{mm}$  and material of workpieces was St 37-3 N. Research was conducted on the conventional vertical hydraulic honing machine of WMW SZS – 200. Fig. 1 shows the numerical simulation of the deformation of the honed thin wall cylinder liner with different wall thickness. Different workpiece wall thickness has a different amount of cylindricity distortions [2].



**Fig.1** Numerical simulation of deformation of honed thin wall cylinder liner with different wall thickness (temperature used in numerical simulation from real honing experiment shown on Fig.2)

Fig. 2 shows generated temperatures received in honing process (temperature of workpiece  $t_w=31,3\text{ }^\circ\text{C}$ , ambient temperature  $t_a=20\text{ }^\circ\text{C}$ )



**Fig.2** Temperatures generated in honing process: a) temperature of oilstone  $t_o=21,96\text{ }^\circ\text{C}$ ; b) temperature of honed cylinder liner  $t_w=31,3\text{ }^\circ\text{C}$

## Methodology

Fig. 3 presents the unrolled surface of the honed holes, on which a quadrangular area with its vertices points:  $(0,0)$ ,  $(n, n)$ ,  $(n, -n)$ ,  $(2n, 0)$  was separated. Fig. 3 also shows a straight line  $y = c$  dividing showed area into two halves.

The grain track is shown in a thick line, running in any direction in the axial and radial direction.

The greater angle inclination of the path  $V_1$  with respect to the radial direction axis means the higher axial velocity or the lower amount of honing head rotation, the smaller inclination angle  $V_2$  with respect to the radial direction axis means lower axial velocity or higher amount of rotation of honing head (Fig.3).

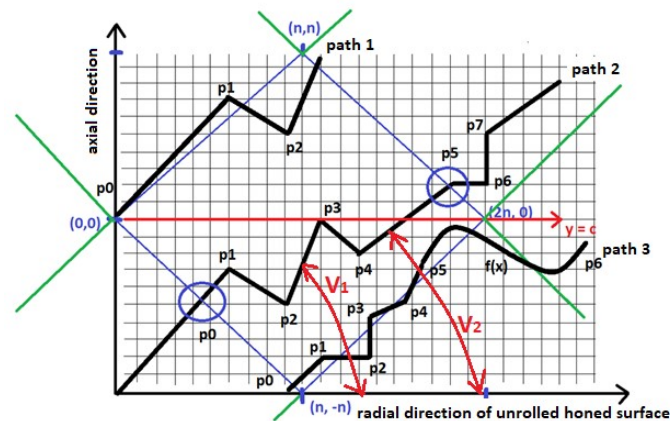


Fig.3 Sample abrasive grain paths on unrolled Surface of honed holes.

Abrasive grain at time  $t = 0$  (Fig. 3) is in point  $(0,0)$ . At times  $t = 0, 1, 2 \dots 2n$  abrasive grain in the honing process performs "jumps". Each "jump" consists in shifting the grain trajectory point by one unit to the right and at the same time by one unit down or up. At time  $t = 2n$ , if the grain were at the point  $(n, n)$ , it would mean that the same number of upward and downward shifts were made. Each broken line connecting the possible position of the points on which the abrasive grains moved at times  $t = 0, 1, 2 \dots, 2n$  is called the trace of the abrasive grain. There are possibilities to find the number of possible traces of the moving grain and the number of traces without common points in the straight line with the equation  $y = c$  ( $c$  - the natural number,  $c < n$ ). All traces of the point are found inside or on the perimeter of the square with the vertices:  $(0,0)$ ,  $(n, n)$ ,  $(2n, 0)$ ,  $(n, -n)$ .

During the abrasive grain straight movement there are  $n$  jumps up and  $n$  jumps down direction. The number of possible traces  $\theta$  is equal to the number of  $n$ -element combinations from the set with  $2n$  elements, i.e. equal to  $\theta = (2n/n)$ .

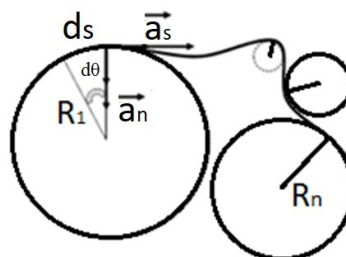
The formula on the curve curvature radius  $R$  of a  $Z(X)$  abrasive grain trajectory move functions is given as (1):

$$R = \frac{\left[ 1 + \left( \frac{d}{dX} Z(X) \right)^2 \right]^{\frac{3}{2}}}{\left| \frac{d^2}{dX^2} Z(X) \right|} \quad (1)$$

The formula on the length of a  $Z(X)$  abrasive grain trajectory move functions from point  $a$  to point  $b$  on circumference's axis  $X$  is given as (2):

$$L = \int_a^b \sqrt{1 + \left[ \frac{dZ}{dX} \right]^2} dX \quad (2)$$

The curvature values of the curve  $Z(X)$  of the abrasive grain trajectory will be presented in the paper as a number equal to the number of revolutions of the honing head occurring for one travel of the honing head in the upper direction during honing with variable kinematic conditions.



**Fig.4** The curvature of honed oil paths:  $R = ds/d\theta$  - the curve curvature radius;  $ds/d\theta$  - increase of the grain path move relative to the angle increment,  $a_n$  - normal acceleration,  $a_s$  - tangential acceleration

The formula on the speed of abrasive grain movement in motion with acceleration  $a = 0$  (3):

$$V = \frac{L}{t} \quad (3)$$

where:  $L$  - path length [m]  
 $t$  - time [s]

The average speed of grain movement in motion with acceleration  $a \neq 0$  is given by the formula (4):

$$V_{av} = \frac{L_2 - L_1}{t_2 - t_1} = \frac{dL}{dt} \quad (4)$$

where:  $L_2$  - end path length,  
 $L_1$  - beginning path length,  $t_2$  - end time,  $t_1$  - beginning time

Total acceleration  $a$  is the acceleration of the tangential and normal acceleration (5):

$$a = \sqrt{a_n^2 + a_s^2} \quad (5)$$

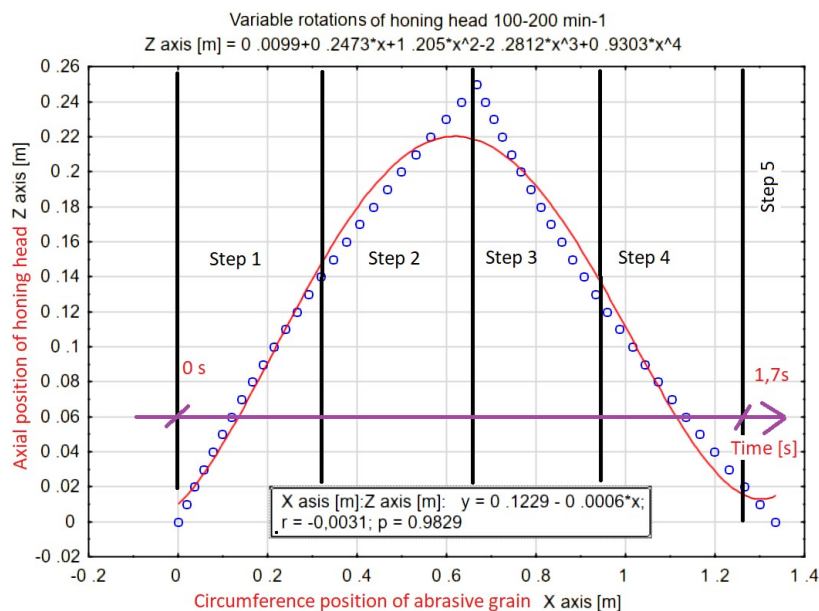
Tangent acceleration  $\overline{a_s}$  is characterized by the rate of change of the numerical value of the speed of abrasive grain movement (6):

$$\overline{a_s} = \frac{dV}{dt} \quad (6)$$

Normal acceleration  $\overline{a_n}$  is characterized by the speed of change of grain speed move direction (7):

$$\overline{a_n} = \frac{V^2}{R} \quad (7)$$

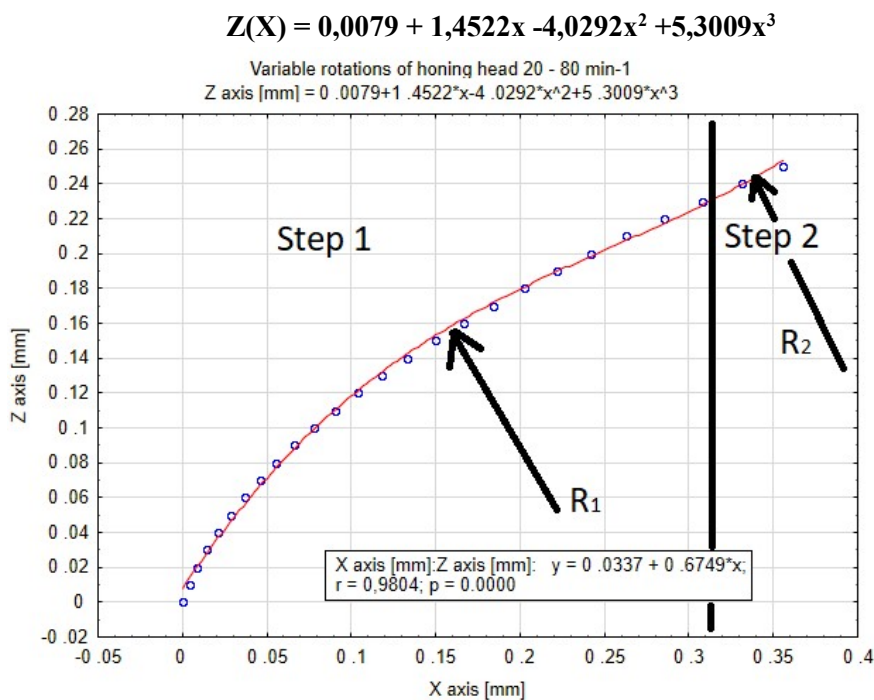
Variable kinematic of the honing process is the best way of honing of cylindrical holes especially with a thin wall of honed parts. Fig. 5 is showing the abrasive grain trajectory received during honing with variable rotations  $100-200 \text{ min}^{-1}$  of honing head in up and down direction. The graph showed on Fig.5 presents the trajectory of the abrasive grain on the developed surface of the honed hole for one complete honing cycle lasting  $t = 1,7 \text{ s}$  and containing 5 revolutions of honing head. This way of presenting of honed grain path trajectories help to receive the function  $Z(X)$  of a single abrasive grain trajectory in honing with the variable kinematic condition.



**Fig.5** Variable rotations of honing head  $100-200 \text{ min}^{-1}$ , up and down move, 5 steps (5 turns of honing head) S1, S2...S5 per 1 machining cycle lasting 1,7s

The graph shown on Fig. 5 presents the function of the fourth degree polynomial, which is not able to properly adjust to the trajectory of the abrasive grain moving up and down during variable honing condition, for this reason the next graphs will present fragments of the abrasive grain trajectory described using the function of the third degree polynomial moving during the honing only upwards. The number of calculated steps depends on the number of rotations of honing head for each graph.

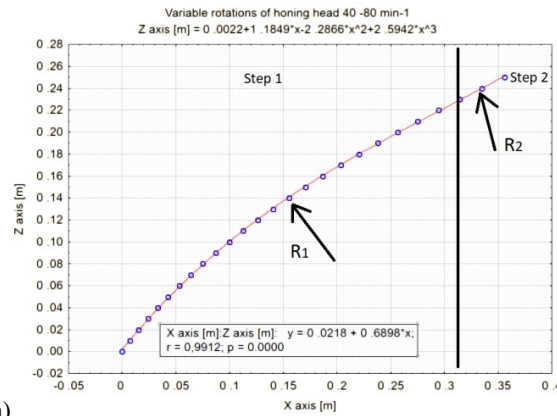
Fig. 6 – Fig .11 presents the number of radiuses of the curve curvature  $R$  of honing abrasive grain trajectories curves  $Z(X)$  received in variable kinematic condition with different parameters of the honing process.



- a)  
b)  $R_1 = 0,504$  m,  $R_2 = 0,589$  m ; c)  $L_1 = 0,392$  m,  $L_2 = 0,048$  m,  $L_1 + L_2 = 0,44$  m,  
d)  $a_{n1} = 0,29$  m/s<sup>2</sup>  $a_{n2} = 2,109$  m/s<sup>2</sup>

**Fig.6** Variable rotations of honing head 20-80 min<sup>-1</sup>, up move direction of honing head; temperature increase of honed workpiece in 1 minute of honing is 3.025°C; grain trajectory length on the circumference length  $X = 0.356$  m, total acceleration  $a = 0,37$  m/s<sup>2</sup>; b) curve curvature radius in the middle of each honing cycle step; c) path length in each honing steps, d) normal acceleration amount in each honing step

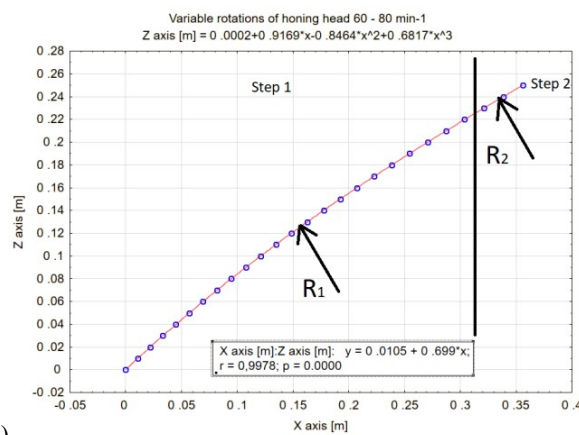
$$Z(X) = 0,0022 + 1,1849x - 2,2866x^2 + 2,5942x^3$$



- b)  $R_1 = 0,807$  m,  $R_2 = 0,765$  m; c)  $L_1 = 0,391$  m,  $L_2 = 0,047$  m,  $L_1 + L_2 = 0,438$  m,  
d)  $a_{n1} = 0,193$  m/s<sup>2</sup>,  $a_{n2} = 1,095$  m/s<sup>2</sup>

**Fig.7** Variable rotations of honing head 40-80 min<sup>-1</sup>, up move direction of honing head; temperature increase of honed workpiece in 1 minute of honing is 5.03°C; grain trajectory length on the circumference length  $X = 0,356$  m, total acceleration  $a = 0,246$  m/s<sup>2</sup>; b) curve curvature radius in each honing step; c) path length in each honing, d) normal acceleration amount in each honing step

$$Z(X) = 0,0002 + 0,9169x - 0,8464x^2 + 0,6817x^3$$

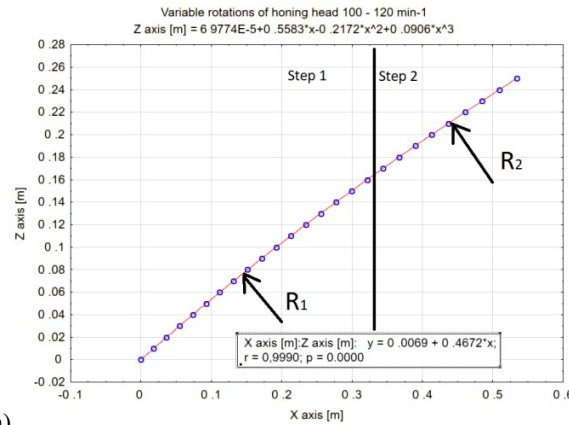


- b)  $R_1 = 1,735$  m,  $R_2 = 1,476$  m, c)  $L_1 = 0,388$  m,  $L_2 = 0,048$  m,  $L_1 + L_2 = 0,436$  m,  
d)  $a_{n1} = 0,095$  m/s<sup>2</sup>,  $a_{n2} = 0,177$  m/s<sup>2</sup>

**Fig.8** Variable rotations of honing head 60-80 min<sup>-1</sup>, up move direction of honing head; temperature increase of honed workpiece in 1 minute of honing is 5.75°C; grain trajectory length on the circumference length  $X = 0,3560474$  m, total acceleration  $a = 0,123$  m/s<sup>2</sup>  
b) curve curvature radius in each honing step; c) path length in each honing, d) normal acceleration amount in each honing step



$$Z(X) = 0,69774 + 0,5583x - 0,2172x^2 + 0,0906x^3$$

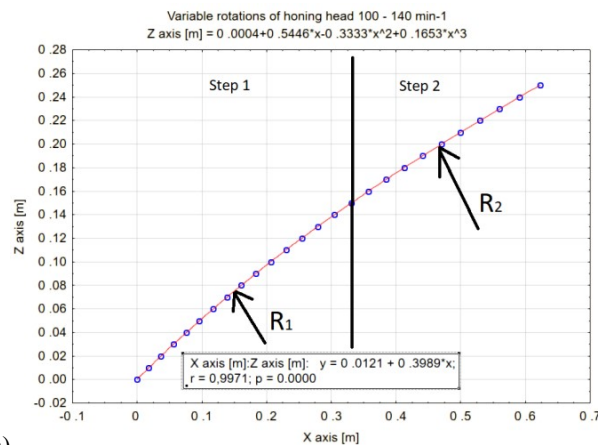


a)

- b)  $R_1 = 3,989$  m,  $R_2 = 3,793$  m; c)  $L_1 = 0,351$  m,  $L_2 = 0,239$  m,  $L_1 + L_2 = 0,59$  m,  
d)  $a_{n1} = 0,086$  m/s<sup>2</sup>,  $a_{n2} = 0,199$  m/s<sup>2</sup>

**Fig.9** Variable rotations of honing head 100-120 min<sup>-1</sup>, up move direction of honing head a); temperature increase of honed workpiece in 1 minute of honing is 5.216°C; grain trajectory length on the circumference length  $X = 0.534$  m; total acceleration  $a = 0,123$  m/s<sup>2</sup>; b) curve curvature radius in each honing step; c) path length in each honing, d) normal acceleration amount in each honing step

$$Z(X) = 0,0004 + 0,5446x - 0,3333x^2 + 0,1653x^3$$

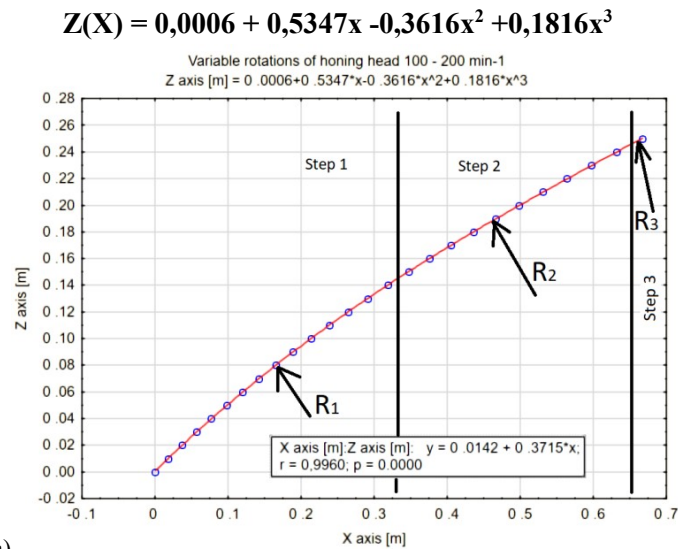


a)

- b)  $R_1 = 2,588$  m,  $R_2 = 2,578$  m c)  $L_1 = 0,346$  m,  $L_2 = 0,332$  m,  $L_1 + L_2 = 0,678$  m,  
d)  $a_{n1} = 0,582$  m/s<sup>2</sup>,  $a_{n2} = 1,647$  m/s<sup>2</sup>

**Fig.10** Variable rotations of honing head 100-140 min<sup>-1</sup>, up move of honing head a); temperature increase of honed workpiece in 1 minute of honing is 7,524°C; grain trajectory length on the circumference length  $X = 0.62308301$  m; total acceleration  $a = 0,246$  m/s<sup>2</sup>; b) curve curvature radius in each honing step; c) path length in each honing, d) normal acceleration amount in each honing step

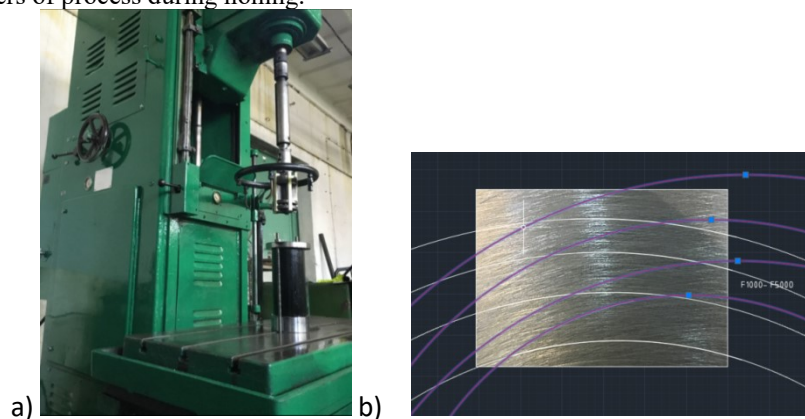




b)  $R_1 = 2,348$  m,  $R_2 = 2,348$  m,  $R_3 = 2,042$  m; c)  $L_1 = 0,343$  m,  $L_2 = 0,33$  m,  $L_3 = 0,041$  m,  $L_1 + L_2 + L_3 = 0,714$  m; d)  $a_{n1} = 0,7$  m/s<sup>2</sup>,  $a_{n2} = 1,928$  m/s<sup>2</sup>,  $a_{n3} = 3,32$  m/s<sup>2</sup>

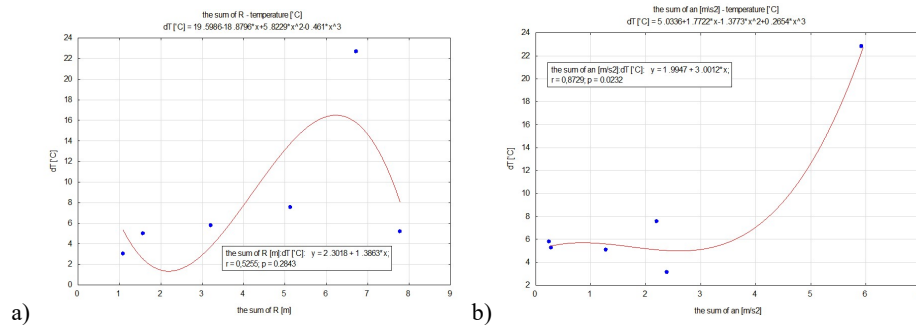
**Fig.11** Variable rotations of honing head 100-200 min<sup>-1</sup>, up move of honing head: a) grain trajectories, b) curve curvature radius in 5 full rotations in one full honing cycle; temperature increase in 1 minute of honing: 22.701°C; grain trajectory length on the circumference length  $X = 0.667588$  m; total acceleration  $a = 0,616$  m/s<sup>2</sup>; b) curve curvature radius in each honing step; c) path length in each honing, d) normal acceleration amount in each honing step

Fig. 12 presents a research stand enables a smooth possibilities to change of all parameters of process during honing.



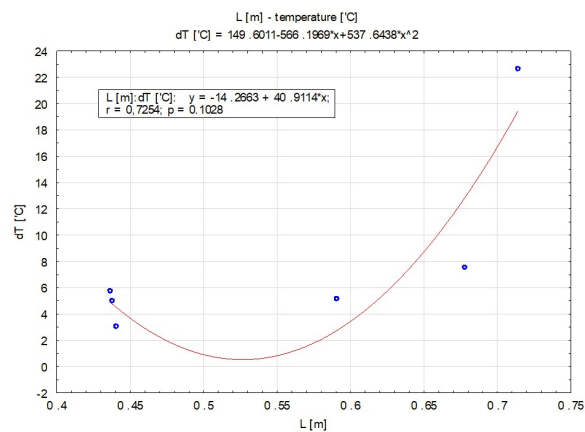
**Fig.12** Research stand: a) conventional hydraulic honing machines WMW SZS-200; b) sample photo of honed surface with grain trajectories received in variable kinematic conditions of honing process

Fig. 13 presents the influence of the sum of curve curvatures radius  $\Sigma R$  of the function's  $Z(X)$  on temperatures of honed workpiece  $t$  increase a) and the influence of the normal acceleration value  $a_n$  on the honed workpieces temperatures  $t$  b). The amount of sum of curve curvatures radius  $\Sigma R$  of abrasive grain trajectory path  $Z(X)$  length 0,438 m and the sum of the amount of normal acceleration  $a_n$  2,4 m/s<sup>2</sup> had the lowest impact on honed workpieces temperature.



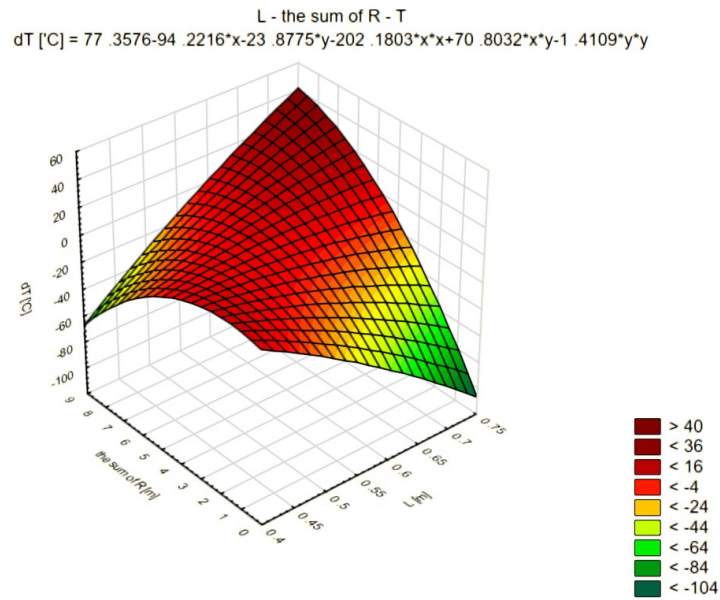
**Fig.13** The influence of the sum of curve curvatures radius  $\Sigma R$  on honed workpiece's temperatures increase  $t$  °C a); the influence of the amount of normal acceleration  $a_n$  on honed workpiece's temperatures  $t$  increase °C b).

Fig. 14 presents the influence of the length of abrasive grain trajectories  $L$  on workpiece temperature  $t$ . The length of the abrasive grain path  $L$  from 0,53 m had the lowest impact on honed workpieces temperature  $t$ .



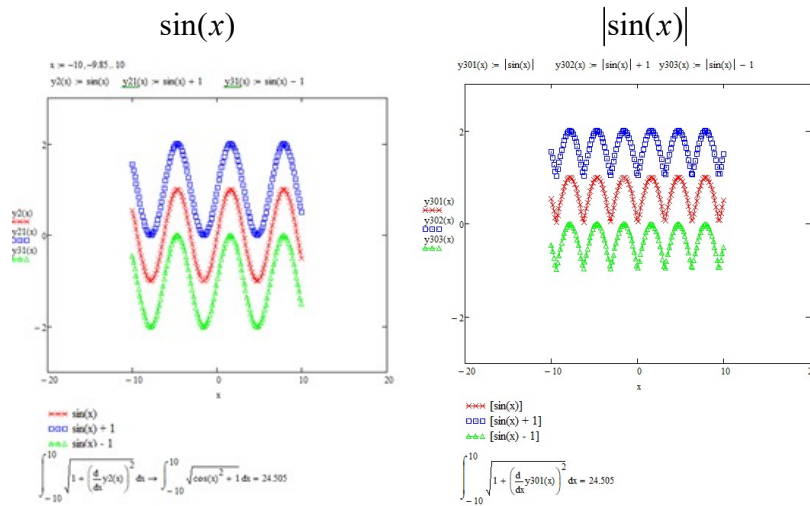
**Fig.14** The influence of the length of abrasive grain trajectories  $L$  on honed workpiece's temperatures  $t$  °C.

Fig. 15 presents the influence of the sum of curve curvatures radius  $\Sigma R$  and abrasive grain path length  $L$  on honed workpiece temperature  $t$ . The lower  $\Sigma R$  amount affects the honed workpiece temperature  $t$  decrease for a longer path length  $L$ .



**Fig.15** The influence of the sum of curve curvatures radius  $\Sigma R$  and abrasive grain path length  $L$  on honed workpiece's temperature  $t$  °C.

The fig. 16 shows an example of curves with the same path length and with different amplitudes, frequency and curvatures. The choice of the desired shape of the trajectory can be supported by the analysis of the radii of curvature of abrasive grain moves function  $Z(X)$ .



**Fig.16** An example of abrasive grain trajectories  $Z(X)$  curves, with the same path length  $\hat{x} \in \langle -10, 10 \rangle$   $L = 24,505$  and with different amplitudes, frequency and curvatures.

## Conclusions

The lower value of the sum of curve curvatures radius  $\Sigma R$ , of abrasive grain trajectories  $Z(X)$  function, affects the honed workpiece temperature  $t$  decrease for a longer path length  $L$ .

The curve curvature radius  $R$  of the abrasive grain paths function  $Z(X)$ , in the variable kinematic condition of honing of cylindrical holes, can be used to improve honing of thin wall cylinder liners, due to decreasing of amount temperature received in honing process.

Detailed knowledge of the impact of the curvature of the abrasive grain path on the machining process can bring many benefits in the development of methods of honing of cylindrical holes, especially in the case of thin-walled workpieces.

Further research should make it possible to verify the legitimacy and benefits of introducing into the control of modern CNC honing machines the possibility of programming of the trajectory of the abrasive grain, using a function describing the motion of the honing head.

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