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Research Paper

Nonlinear Interaction of Magnetoacoustic Modes in a Quasi-Isentropic Plasma Flow

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The nonlinear interaction of magnetoacoustic waves in a plasma is analytically studied. A plasma is an open system. It is affected by the straight constant equilibrium magnetic flux density forming constant angle with the wave vector which varies from 0 till π. The nonlinear instantaneous equation which describes excitation of secondary wave modes in the field of intense magnetoacoustic perturbations is derived by use of projecting. There is a diversity of nonlinear interactions of waves in view of variety of wave modes, which may be slow or fast and may propagate in different directions. The excitation is analysed in the physically meaningful cases, that is: harmonic and impulsive exciter, oppositely or accordingly directed dominant and secondary wave modes.

Keywords: non-linear magnetohydrodynamics; adiabatical instability.

1. Introduction

Magnetohydrodynamic (MHD) perturbations indicate physical processes in plasma, geometry of its flow and equilibrium state. They have a key role in transport of energy and momentum at large distances. In the case of open flows, the radiative losses and inflow of energy in plasma may be described by a generic heating-cooling function (Vesecky *et al.*, 1979; Dahlburg, Mariska, 1988; Ibáñez, Parravano, 1994). It is one of the factors of non-adiabaticity of a flow which comes to a balance with other mechanisms such as mechanical damping, thermal conduction, and electrical resistivity. The heating-cooling function may crucially affect the wave processes and nonlinear phenomena in a plasma's flow, in particular, formation of discontinuity in a wave form and nonlinear excitation of the entropy mode by intense waves (Chin *et al.*, 2010; Perelomova, 2018a; 2018b; 2019a). Wave processes in a plasma are very similar to processes in other open flows (Osipov, Uvarov, 1992; Molevich, 2001) but they are much more diverse. Wave perturbations of small magnitudes enhance in the course of propagation in acoustically active flows if they are not suppressed by irreversible processes such as mechanical viscosity and thermal conduction or some kind of thermodynamic relaxation (Field, 1965; Parker, 1953). For moderate magnitudes of perturbations, the nonlinear distortion of a wave form, nonlinear attenuation at the shock fronts, and nonlinear transfer of momentum and energy between modes go into play. MHD perturbations stand out among the rest of wave processes in open flows due to their complexity in view of coexistence of slow, fast sound modes, and the Alfvén modes. Wave processes in a plasma depend strongly on the direction and intensity of the magnetic field and demand much more compound matematical description. The nonlinear evolution of individual wave modes in a plasma was paid attention to with regard to simple analytical and complex cases which involve numerical simulations (Anderson, 1953; Ponomarev, 1961; Sharma *et al.*, 1987). Nakariakov *et al.* (2000) analysed the linear speed, parameter of nonlinearity, and damping of planar MHD wave perturbations depending on the equilibrium parameters of a plasma, the heating-cooling function, the equilibrium magnetic flux density, and angle which it forms with the wave vector (see also (Chin *et al.*, 2010)). The authors have derived the dynamic equation governing wave perturbations in a weakly nonlinear open flow of a thermoconducting plasma and made some important conclusions concerning propagation of waves, and, in particular, autowaves formation. The conclusions concern the dynamics of individual magnetoacoustic waves excluding its nonlinear interaction with other modes.

Perturbations of infinitely-small magnitude evolute independently, but finite-magnitude perturbations do interact. The reason for that, apart from nonlinearity, is deviation from adiabaticity due to the heating-cooling function or some kind of an irreversible thermodynamic process. These two factors lead to excitation of variety of modes in the flow, to scattering of waves on other waves or on thermal inhomogeneities and vortex bulk streams which in turn may represent secondary modes enhancing in the wave field (Brodin *et al.*, 2006; Zavershinsky, Molevich, 2014; Lyubchyk, Voitenko, 2014; Perelomova, 2016a; 2016b; Leble, Perelomova, 2018). Mathematical description of modes' interactions is much more difficult than description of nonlinear evolution of an individual mode. As usual one wave mode is treated as dominant. The distortion of the dominant wave occurs due to its nonlinear self-interaction. Nonlinear transfer of momentum and energy between modes, which constantly takes place, leads to the nonlinear enhancement of the secondary modes, weakening of the dominant mode, and invalidity of its individual dynamic equation starting from some moment of evolution.

The nonlinear self-interaction and nonlinear excitation of the secondary magnetoacoustic modes in the field of some intense magnetoacoustic wave are the subject of this study. We make use of the initial points and geometry of a flow following Nakariakov *et al.* (2000) (see also (Chin *et al.*, 2010)) and consider quadratically nonlinear terms which are of major importance in the weakly nonlinear flows. They condition corrections in the links between specific magnetoacoustic perturbations in the dominant mode making it isentropic in the leading order (they are responsible for the self-interaction), and nonlinear terms which are in charge of interaction of modes. The quadratically nonlinear "forces" in equations describing interactions between modes may be evaluated by the method of projecting. The method was used by the author in the studies of nonlinear interactions of different modes in a wide variety of fluid flows (Leble, Perelomova, 2018). It is fruitful in investigations of a plasma flow and has been applied in evaluations of heating/cooling excited by magnetoacoustic perturbations (Perelomova, 2016a; 2016b; 2018a; 2018b).

The key issue is derivation of the system of coupling equations for the interacting modes. The projecting method allows to derive a system of instantaneous coupling dynamic equations describing perturbations in all specific modes with properly distributed nonlinear terms. There are no restrictions on the comparative magnitudes of perturbations in interacting modes, and the nonlinear "forces" in general contain perturbations belonging to all modes. The system may be considerably simplified if one mode (the magnetoacoustic one in this study) is dominant. The excitation of the secondary wave perturbations by the dominant wave demands resolution of two problems: description of nonlinear distortions of the dominant perturbations and solution of dynamic equations for the secondary perturbations. Acoustical activity of a plasma has the crucial impact on propagation of the dominant wave and corresponding nonlinear phenomena. It may take place only due to some kind of the heating-cooling function. We do not consider mechanical and thermal losses in a plasma and its finite electrical conductivity. These effects introduce additional attenuation which is well studied and contributes both to distortion of the dominant magnetoacoustic mode and coupling of interacting modes making it stronger. In this study, we derive and analyse the instantaneous dynamic equation for excitation of the secondary magnetoacoustic mode by the (other) dominant magnetoacoustic mode (they both may be fast or slow). The excited perturbations contain parts which propagate with the speed of the dominant mode and their own linear speed (Perelomova, 2019b). The results are discussed in some physically meaningful cases of wave perturbations (periodic and impulsive) and the generic heating-cooling function. The impact of plasma's boundaries is not considered. This study expands the previous investigations of the author concerning nonlinear interaction of modes in a plasma flow.

2. Modes in the linear MHD flow

We consider perturbations in an ideal open plasma's flow and remind the conservation system which contains the continuity equation, the momentum equation, the energy balance equation, and completing electrodynamic equations (Krall, Trivelpiece, 1973; Callen, 2003):

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, \\
\rho \frac{D\mathbf{v}}{Dt} &= -\nabla p + \mu_0 (\nabla \times \mathbf{B}) \times \mathbf{B}, \\
\frac{Dp}{Dt} - \gamma \frac{p}{\rho}\frac{D\rho}{Dt} &= (\gamma - 1) L(p, \rho), \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}), \\
\nabla \cdot \mathbf{B} &= 0,
\end{aligned} \tag{1}$$

where p, ρ, $\mathbf{v}$, $\mathbf{B}$, are the pressure and density of a plasma, its velocity, the magnetic flux density, and μ_0 is the magnetic permeability of the free space. $L(p, \rho)$ designates some generic heating-cooling function which may lead to deviation of adiabaticity of wave perturbations in a plasma (Nakariakov *et al.*, 2000; Chin *et al.*, 2010). The third equation in the set (1) relates to an ideal gas, where γ is the ratio of specific heats under constant pressure and constant volume, $\gamma = C_P/C_V$. The planar geometry of a flow is the same as in the studies (Nakariakov *et al.*, 2000; Chin *et al.*, 2010). The equilibrium magnetic density flux $\mathbf{B}_0$ forms a constant angle θ $(0 \le \theta \le \pi)$ with the positive direction of axis z, so that

$$B_{0,x} = B_0 \sin(\theta), \qquad B_{0,z} = B_0 \cos(\theta),$$

and $B_{0,y} = 0$. Axis z points the direction of a planar wave propagation. All quantities are expanded in the vicinity of the equilibrium state, and perturbations are functions of z and t, so as $f(z,t) = f_0 + f'(z,t)$. The conclusion from the last equation is that $B'_z = 0$ and $B_{0,z}$ is constant. Hence the number of unknown functions (and modes) reduces to seven. We consider initially static plasma with zero equilibrium velocity $\mathbf{v}_0 = 0$ and constant unperturbed thermodynamic parameters. The leading-order equations containing quadratic nonlinear terms follow from Eqs (1):

$$\begin{aligned}
\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial v_z}{\partial z} &= -\rho' \frac{\partial v_z}{\partial z} - v \frac{\partial \rho'}{\partial z},\\
\frac{\partial v_x}{\partial t} - \frac{B_{0,z}}{\rho_0\mu_0}\frac{\partial B_x}{\partial z} &= -v_z \frac{\partial v_x}{\partial z} - \frac{B_{0,z}}{\rho_0^2\mu_0}\rho' \frac{\partial B_x}{\partial z},\\
\frac{\partial v_y}{\partial t} - \frac{B_{0,z}}{\rho_0\mu_0}\frac{\partial B_y}{\partial z} &= -v_z \frac{\partial v_y}{\partial z} - \frac{B_{0,z}}{\rho_0^2\mu_0}\rho' \frac{\partial B_y}{\partial z},\\
\frac{\partial v_z}{\partial t} + \frac{1}{\rho_0}\frac{\partial p'}{\partial z} + \frac{B_{0,x}}{\rho_0\mu_0}\frac{\partial B_x}{\partial z} &= \frac{\rho'}{\rho_0^2}\frac{\partial p'}{\partial z} + \frac{B_{0,x}}{\rho_0^2\mu_0}\rho' \frac{\partial B_x}{\partial z}\\
&\quad - \frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\frac{B_x^2 + B_y^2}{2\mu_0}\right) - v_z \frac{\partial v_z}{\partial z}, \qquad (2)
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial p'}{\partial t} + \gamma p_0 \frac{\partial v_z}{\partial x} - (\gamma - 1)(L_p p' + L_\rho \rho') =\\
&(\gamma-1)(0.5 L_{pp} p'^2 + 0.5 L_{\rho\rho}\rho'^2 + L_{p\rho} p'\rho') - \gamma p' \frac{\partial v_z}{\partial z} - v_z \frac{\partial p'}{\partial z},
\end{aligned}$$

$$\begin{aligned}
\frac{\partial B_x}{\partial t} + \frac{\partial}{\partial z}\left(B_{0,x} v_z - B_{0,z} v_x\right) &= -B_x \frac{\partial v_z}{\partial z} - v_z \frac{\partial B_x}{\partial z},\\
\frac{\partial B_y}{\partial t} - \frac{\partial}{\partial z}\left(B_{0,z} v_y\right) &= -B_y \frac{\partial v_z}{\partial z} - v_z \frac{\partial B_y}{\partial z},
\end{aligned}$$

where

$$L_p = \frac{\partial L}{\partial p}, \qquad L_\rho = \frac{\partial L}{\partial \rho}, \qquad L_{pp} = \frac{\partial^2 L}{\partial p^2},$$

$$L_{\rho\rho} = \frac{\partial^2 L}{\partial \rho^2}, \qquad L_{p\rho} = \frac{\partial^2 L}{\partial p \partial \rho}$$

are partial derivatives of the heating-cooling function $L(p, \rho)$ evaluated at the equilibrium state (p_0, ρ_0) (Perelomova, 2018a; 2018b). The system (2) is an initial point for evaluations that follow which will be undertaken with accuracy up to the first powers of the first derivatives of L with respect to its variables. That imposes smallness of the heating-cooling function impact on the wave processes which remains weakly deviating from isentropic.

The preliminary conclusions may be deduced from the linearised version (2) which describes a flow of infinitely-small magnitudes. We consider any disturbance as a sum of planar waves proportional to $\exp(i\omega(k_z)t - ik_z z)$, where k_z is the wave number, so that

$$f'(z,t) = \int_{-\infty}^{\infty} \widetilde{f}(k_z) \exp(i\omega(k_z)t - ik_z z)\,\mathrm{d}k_z.$$

The dispersion relations follow from the solvability of the linearised version of Eqs (2) (we mean non-zero solutions):

$$\begin{aligned}
\omega_j &= C_j k_z + i \frac{(\gamma - 1)(C_j^2 - C_A^2)}{2c_0^2(c_0^2 + C_A^2 - 2C_j^2)}(c_0^2 L_p + L_\rho),\\
\omega_{5,6} &= \pm C_{A,z} k_z, \qquad \omega_7 = \frac{i(\gamma - 1)L_\rho}{c_0^2},
\end{aligned} \qquad (3)$$

where C_j is the magnetoacoustic speed $(j = 1, ..., 4)$, one of the roots of the equation

$$C_j^4 - C_j^2(c_0^2 + C_A^2) + c_0^2 C_{A,z}^2 = 0, \qquad (4)$$

where

$$C_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}}, \qquad c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

designate the Alfvén speed and the acoustic speed in unmagnetised gas in equilibrium, and

$$C_{A,z} = C_A \cos\theta.$$

The first four dispersion relations determine two slow and two fast magnetoacoustic modes of different direction of propagation. The relations ω_5, ω_6 specify the Alfvén waves of different direction of propagation and ω_7 specifies the non-wave entropy mode. They are out of attention in this study but contribute to the projecting operators. We consider the small impact of the heating-cooling function on a magnetoacoustic mode during the wave period:

$$|C_j| k_z \gg \left| \frac{(\gamma - 1)}{c_0^2}(c_0^2 L_p + L_\rho) \right|. \qquad (5)$$

The dispersion relations Eqs (3) and Eq. (4) have been established by Nakariakov *et al.* (2000) (see also (Chin *et al.*, 2010)). The conditions of acoustic (isentropic) instability are common in all non-adiabatic flows (not only in the presence of a magnetic field) and sounds as (Field, 1965; Parker, 1953):

$$c_0^2 L_p + L_\rho > 0. \tag{6}$$

Every dispersion relation determines the links of perturbations in any individual mode. The magnetoacoustic branches are established by the linear links ($j = 1, ..., 4$) (Perelomova, 2018a):

$$\psi_{lin,j} = \begin{pmatrix} \rho' \\ v_x \\ v_y \\ v_z \\ p' \\ B_x \\ B_y \end{pmatrix}_j = \begin{pmatrix} A_1^* \\ A_2^* \\ 0 \\ 1 \\ A_3^* \\ A_4^* \\ 0 \end{pmatrix} v_{z,j}, \tag{7}$$

where

$$A_1^* = \frac{\rho_0}{C_j} + \frac{\rho_0(\gamma-1)(C^2 - C_A^2)}{2c_0^2(C_j^4 - c_0^2 C_{A,z}^2)}(c_0^2 L_p + L_\rho)\int \mathrm{d}z,$$

$$A_2^* = \frac{C_{A,z}}{C_{A,x}}\left(\frac{c_0^2}{C_j^2} - 1\right) - \frac{(\gamma-1)C_{A,z}(C_j^2 - c_0^2)}{C_{A,x}C_j(C_j^4 - c_0^2 C_{A,z}^2)}(c_0^2 L_p + L_\rho)\int \mathrm{d}z,$$

$$A_3^* = \frac{c_0^2\rho_0}{C_j} - \frac{(\gamma-1)(3C_j^2 - 2c_0^2 - C_A^2)}{2(C_j^4 - c_0^2 C_{A,z}^2)} \cdot (c_0^2 L_p + L_\rho)\int \mathrm{d}z,$$

$$A_4^* = \frac{(C_j^2 - c_0^2)B_0}{C_j C_A C_{A,x}} - \frac{(\gamma-1)(C_j^2 - c_0^2)(C_j^2 - 2c_0^2 - C_A^2)B_0}{2c_0^2 C_A C_{A,x}(C_j^4 - c_0^2 C_{A,z}^2)} \cdot (c_0^2 L_p + L_\rho)\int \mathrm{d}z.$$

The integrals in the links reflect the impact of non-adiabaticity of flow due to L. The projecting rows may be established which distinguish an excess density in the individual magnetoacoustic mode,

$$P_{ms,j}\begin{pmatrix} \rho' & v_x & v_y & v_z & p' & B_x & B_y \end{pmatrix}^{\mathrm{T}} = \rho_j, \qquad j = 1, ..., 4. \tag{8}$$

They follow from the system of algebraic Eqs (8) in view of that the total perturbations are sums of specific ones:

$$v_x = \sum_{j=j}^{7} v_{x,j}, \quad v_y = \sum_{j=1}^{7} v_{y,j}, \quad v_z = \sum_{j=1}^{7} v_{z,j},$$

$$B_x = \sum_{j=1}^{7} B_{x,j}, \quad B_y = \sum_{j=1}^{7} B_{y,j},$$

$$p' = \sum_{j=1}^{7} p_j, \quad \rho' = \sum_{j=1}^{7} \rho_j$$

and links determined by ψ_j, ($j = 1, ..., 7$). The first four projectors take the form (Perelomova, 2018a; 2018b):

$$P_{ms,j} = \begin{pmatrix} A_1^{**} \\ A_2^{**} \\ 0 \\ A_3^{**} \\ A_4^{**} \\ A_5^{**} \\ 0, \end{pmatrix}^{\mathrm{T}}, \tag{9}$$

where

$$A_1^{**} = -\frac{(\gamma-1)C_j(C_j^2 - C_A^2)}{2c_0^2(C_j^4 - c_0^2 C_{A,z}^2)} L_\rho \int \mathrm{d}z,$$

$$A_2^{**} = -\frac{C_{A,x}C_{A,z}C_j\rho_0}{2(C_j^4 - c_0^2 C_{A,z}^2)} - \left(c_0^2 L_p + L_\rho\right) \cdot \frac{(\gamma-1)C_{A,x}C_{A,z}(2C_j^6 - 3C_j^4 C_{A,z}^2 + c_0^2 C_{A,z}^4)\rho_0}{2(C_j^4 - c_0^2 C_{A,z}^2)^3}\int \mathrm{d}z,$$

$$A_3^{**} = \frac{C_j(C_j^2 - C_{A,z}^2)\rho_0}{2(C_j^4 - c_0^2 C_{A,z}^2)} + \left(c_0^2 L_p + L_\rho\right) \cdot \frac{(\gamma-1)(C_j^2 - C_{A,z}^2)(B_1^*)}{2(C^4 - c_0^2 C_{A,z}^2)^3}\int \mathrm{d}z,$$

$$B_1^* = C_j^6 + c_0^2 C_{A,z}^2 C_j^2 - 3C_j^4 C_{A,z}^2 + c_0^2 C_{A,z}^4,$$

$$A_4^{**} = \frac{C_j^2 - C_{A,z}^2}{2(C_j^4 - c_0^2 C_{A,z}^2)} + L_\rho \frac{(\gamma-1)(C_j^2 - C_{A,z}^2)(B_2^*)}{4C_j(C_j^4 - c_0^2 C_{A,z}^2)^3}\int \mathrm{d}z + L_p \frac{(\gamma-1)(C_j^2 - C_{A,z}^2)C_j^3(B_3^*)}{4(C_j^4 - c_0^2 C_{A,z}^2)^3}\int \mathrm{d}z,$$

$$B_2^* = 3C_j^6 + c_0^2 C_j^2 C_{A,z}^2 - 7C_j^4 C_{A,z}^2 + 3c_0^2 C_{A,z}^4,$$

$$B_3^* = 2c_0^4 + 2C_j^4 - 5C_j^2 C_A^2 + C_A^4 - 3c_0^2(C_j^2 - C_A^2),$$

$$A_5^{**} = \frac{C_j^2 C_{A,x} C_A^2 \rho_0}{2B_0 C_A(C_j^4 - c_0^2 C_{A,z}^2)} + \left(c_0^2 L_p + L_\rho\right) \cdot \frac{(\gamma-1)C_j C_{A,x} C_A (B_4^*)\rho_0}{4B_0(C_j^4 - c_0^2 C_{A,z}^2)^3}\int \mathrm{d}z,$$

$$B_4^* = 3C_j^6 + c_0^2 C_j^2 C_{A,z}^2 - 5C_j^4 C_{A,z}^2 + c_0^2 C_{A,z}^4.$$

Projectors into Alfvén specific velocity and perturbation in density in the entropy mode, are

$$P_A = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \mp\frac{C_A}{2B_0} \end{pmatrix},$$

$$P_{\text{ent}} = \begin{pmatrix} 1 \\ -\frac{(\gamma-1)C_{A,x}\rho_0}{C_{A,z}c_0^4}(c_0^2 L_p + L_\rho)\int \mathrm{d}z \\ 0 \\ -\frac{(\gamma-1)\rho_0}{c_0^4}(c_0^2 L_p + L_\rho)\int \mathrm{d}z \\ -\frac{1}{c_0^2} \\ 0 \\ 0 \end{pmatrix}^{\mathrm{T}}. \tag{10}$$

The projectors satisfy usual properties of projecting operators. In particular,

$$\sum_{j=1}^{4} P_{ms,j} + P_{\text{ent}} = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0).$$

3. Nonlinear interaction of magnetoacoustic modes

3.1. Nonlinear dynamics of an individual dominant magnetoacoustic wave. Excitation of multiple harmonics

We will consider one magnetoacoustic mode, say, ordered by $j = 1$, fast or slow, as dominant. That means that magnitudes of its perturbations are much bigger than those of other modes, at least over some temporal and spatial domains. In order to properly take into account nonlinear quadratic effects associated with the dominant mode, it should be corrected by inclusion of terms making it isentropic in the leading order, without an impact of nonadiabaticity which is introduced by the heating-cooling function. The corrected links have been obtained by Perelomova (2018b):

$$\psi_1 = \psi_{lin,1} + \begin{pmatrix} A_1^{***} \\ A_2^{***} \\ 0 \\ 0 \\ A_3^{***} \\ A_4^{***} \\ 0 \end{pmatrix} v_{z,1}^2, \tag{11}$$

where

$$A_1^{***} = \frac{(c_0^2 + C_1^2(\gamma-4) - C_A^2(\gamma-3))\rho_0}{4C_1^2(c_0^2 + C_A^2 - 2C_1^2)},$$

$$A_2^{***} = \frac{c_0^2(C_1^2-c_0^2)(c_0^4 - C_1^2(2c_0^2+(\gamma-1)C_A^2)+\gamma C_1^4)}{2C_1^5(C_1^2 - C_A^2)(c_0^2 + C_A^2 - 2C_1^2)}\frac{C_{A,z}}{C_{A,x}},$$

$$A_3^{***} = \frac{c_0^2(c_0^4 - 3\gamma C_1^4 + C_1^2(2c_0^2(\gamma-1) + C_A^2(\gamma+1))\rho_0}{4C_1^4(c_0^2 + C_A^2 - 2C_1^2)},$$

$$A_4^{***} = \frac{B_0(C_1^2 - c_0^2)(B_5^*)}{4C_1^2(C_1^4 - c_0^2 C_{A,z}^2)C_A C_{A,x}^3},$$

$$B_5^* = C_1^6 - C_1^2 c_0^4 - C_1^4(3C_{A,z}^2 + c_0^2(\gamma-3)) + c_0^2 C_{A,z}^2(C_{A,z}^2(\gamma+1) - c_0^2).$$

The dynamic equation for excess density in dominant magnetoacoustic wave may be obtained by application of projector $P_{ms,1}$ at the system (2). It may be readily rearranged in the terms of longitudinal velocity $v_{z,1}$ making use of links between ρ_1 and $v_{z,1}$ given by Eq. (7). All linear terms corresponding to other wave and non-wave modes are reduced in the linear part of the resulting equation, and only terms belonging to the first mode are kept among the variety of quadratic nonlinear terms. The resulting equation describes the nonlinear self-interaction of the dominant first magnetoacoustic mode:

$$\frac{\partial v_{z,1}}{\partial t} + C_1\frac{\partial v_{z,1}}{\partial z} + D_1 C_1 v_{z,1} + \varepsilon_1 v_{z,1}\frac{\partial v_{z,1}}{\partial z} = 0, \tag{12}$$

with

$$D_1 = \frac{C_1(C_1^2 - C_A^2)}{2(C^4 - c_0^2 C_{A,z}^2)}(c_0^2 L_p + L_\rho),$$

$$\varepsilon_1 = -\frac{C_1^2(3c_0^2 + C_A^2(\gamma+1) - C_1^2(\gamma+4))}{2(C^4 - c_0^2 C_{A,z}^2)}.$$

In the absence of magnetic field and external inflow of energy, Eq. (12) coincides with the well known equation for velocity in the progressive planar Riemann's wave. This is the case $D_1 = 0$, $C_1 = c_0$, $\varepsilon_1 = \frac{\gamma+1}{2}$ (Landau, Lifshitz, 1987; Rudenko, Soluyan, 1977). Equation (12) may be readily rearranged into the leading-order pure nonlinear equation, if $D_1 \neq 0$:

$$\frac{\partial V_1}{\partial Z} - \frac{\varepsilon_1}{C_1^2}V_1\frac{\partial V_1}{\partial \tau} = 0, \tag{13}$$

by means of new variables

$$V_1 = v_{z,1}\exp(D_1 z), \qquad Z = \frac{1 - e^{-D_1 z}}{D_1},$$

$$\tau = t - z/C_1.$$

Equation (13) may be solved by the method of characteristics. Note that Z is always positive for non-zero D_1. If $D_1 = 0$, $V_1 = v_{z,1}$, $Z = z$. The exact solution to Eq. (13), which is sinusoidal at $z = 0$ with the frequency ω and amplitude v_0, is well known (Landau, Lifshitz, 1987; Rudenko, Soluyan, 1977). The average over period kinetic energies of multiple harmonics $n\omega$ (per unit mass of a plasma) equal

$$E_n = \exp(2D_1 z)\left(\frac{2J_n(nK_1^{-1}(\exp(D_1 z)-1))}{nK_1^{-1}(\exp(D_1 z)-1)}\right)^2 \frac{v_0^2}{2}, \qquad n = 1,2,...,D_1 \neq 0, \tag{14}$$

$$E_n = \left(\frac{2J_n(nz/z_{sh,0})}{nz/z_{sh,0}}\right)^2 \frac{v_0^2}{2}, \quad n = 1,2,...,D_1 = 0,$$

where $K_1 = \frac{D_1 C_1^2}{\omega\varepsilon_1 V_0}$. This is valid before formation of a discontinuity (Osipov, Uvarov, 1992; Rudenko, Soluyan, 1977), that is, if

$$0 \le z < z_{sh} = \ln(1+K_1)D_1^{-1}, \qquad D_1 \neq 0,$$

$$0 \le z < z_{sh,0} = \frac{C_1^2}{\omega\varepsilon V_0}, \qquad D_1 = 0.$$

Before formation of discontinuity, the kinetic energy per unit mass varies with a distance from an exciter as

$$\sum_{n=1}^{\infty} E_n = \exp(2D_1 z)\frac{v_0^2}{2}$$

and remains constant if $D_1 = 0$. A discontinuity always forms in acoustically active flows with $D_1 > 0$ at the distance z_{sh} (and $z_{sh,0}$ if $D_1 = 0$), and does not form at all if $K_1 \le -1$. The average kinetic energies per unit mass of multiple harmonics after formation of discontinuity, at $z > \pi z_{sh}/2$ (this is the case $K_1 > -1$ and negligible curvature in the sloping parts of waveform), equal

$$E_n = \exp(2D_1 z)\left(\frac{2}{n(1+K_1^{-1}(\exp(D_1 z)-1))}\right)^2 \frac{v_0^2}{2}, \qquad n = 1,2,...,D_1 \neq 0, \tag{15}$$

and, at $z > \pi z_{sh,0}/2$,

$$E_n = \left(\frac{2}{n(1+\frac{z}{z_{sh,0}})}\right)^2 \frac{v_0^2}{2}, \quad n = 1,2,...,D_1 = 0.$$

At these distances, the total kinetic energy per unit mass equals

$$\sum_{n=1}^{\infty} E_n = \exp(2D_1 z)\frac{2\pi^2}{3(1+K_1^{-1}(\exp(D_1 z)-1))^2}\frac{v_0^2}{2}, \qquad D_1 \neq 0, \tag{16}$$

$$\sum_{n=1}^{\infty} E_n = \frac{2\pi^2}{3(1+\frac{z}{z_{sh,0}})^2}\frac{v_0^2}{2}, \qquad D_1 = 0.$$

The kinetic energy per unit mass gets smaller in the course of propagation in the neutral case due to nonlinear attenuation at the shock front. The domain between z_{sh} and $\pi z_{sh}/2$ ($z_{sh,0}$ and $\pi z_{sh,0}/2$) is difficult for analytical description. Figure 1 shows average kinetic energies per unit mass of three first harmonics as functions of distance from an exciter before and after formation of discontinuity in accordance to Eqs (14) and (15) at different K_1. $K_1 = 1$ is the case of equilibrium between inflow of energy and nonlinear attenuation at the discontinuity after its formation. The stationary saw-tooth wave forms with an amplitude independent from the distance from a transducer.

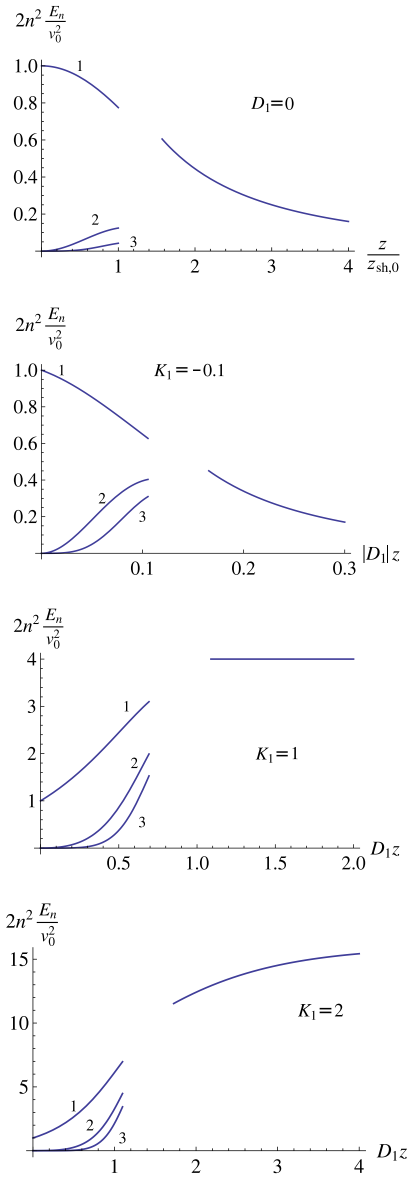


Fig. 1. Average kinetic energies per unit mass of multiple harmonics before and after formation of discontinuity. Three first harmonics are ordered as 1, 2, 3. After formation of discontinuity, curves for all harmonics cover.

3.2. Interaction of different magnetoacoustic modes

The dynamic equation which describes excitation of excess density in magnetoacoustic mode ordered as second, is the result of application of $P_{ms,2}$ (Eqs (9)) at the system (2) which eliminates all "foreign" terms but retains the terms referring to ρ_2 on the left:

$$\frac{\partial \rho_2}{\partial t} + C_2 \frac{\partial \rho_2}{\partial z} - D_2 C_2 \rho_2 = F_1. \tag{17}$$

Among nonlinear terms forming the magnetoacoustic force, only terms belonging to the dominant first mode, are kept. The leading order term on the right of Eq. (17), is

$$\frac{(C_1 + C_2)(C_1^2 + C_2^2 - c_0^2 - C_A^2)\varepsilon_1}{2C_1^2(c_0^2 + C_A^2 - 2C_2^2)} v_{z,1} \frac{\partial v_{z,1}}{\partial z}.$$

If $C_1 = C_2$ ($\varepsilon_1 = \varepsilon_2$, $v_{z,1} = v_{z,2}$), it equals

$$-\frac{\varepsilon_2 \rho_0}{C_2} v_{z,2} \frac{\partial v_{z,2}}{\partial z}$$

and reflects the self-interaction of the second mode which has been discussed in Subsec. 3.1. For any $C_2 \neq C_1$, it equals zero. In this case, the quadratic nonlinear terms belonging to the first mode proportional to the first or second derivatives of L with respect to its arguments, form the magnetoacoustic force F_1. The impact of second order derivatives of L with respect to its arguments is ignored. In general, the magnetoacoustic force takes the form

$$F_1 = \alpha_1 v_{z,1}^2 + \alpha_2 \frac{\partial v_{z,1}}{\partial z} \int v_{z,1}\, \mathrm{d}z, \tag{18}$$

where α_1, α_2 depend on the equilibrium parameters of a plasma and θ. The parts of coefficients α_1, α_2 proportional to L_ρ and L_p are shown in Table 1 ($C_2 = -C_1$) and Table 2 ($C_2 = \frac{c_0 C_{A,z}}{C_1}$) in the Appendix. The solution to Eq. (17) satisfying zero initial conditions, sounds as

$$\rho_2(z,t) = \exp(C_2 D_2 t) \int_0^t \exp(-C_2 D_2 \tau) F_1 \cdot (z - C_2(t-\tau), \tau)\, \mathrm{d}\tau. \tag{19}$$

In the leading order, the magnetoacoustic force F_1 is a function of $z - C_1 t$, which yields

$$\rho_2(z,t) = \exp(C_2 D_2 t) \int_0^t \exp(-C_2 D_2 \tau) F_1 \cdot (z - C_2 t + \tau(C_2 - C_1))\, \mathrm{d}\tau. \tag{20}$$

4. Excitation of the secondary wave mode by some kinds of exciters

4.1. Periodic exciter

In particular, for the harmonic exciter

$$v_{z,1} = v_0 \sin(\omega(t - z/C_1)), \tag{21}$$

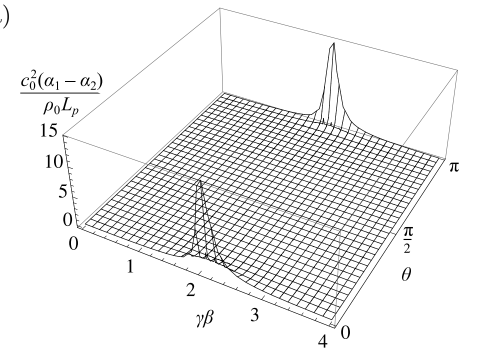
the leading-order solution to (17) takes the form

$$\rho_2 = v_0^2 \frac{(\alpha_1 - \alpha_2)}{2} t + v_0^2 C_1 \frac{(\alpha_1 + \alpha_2)\sin(2\omega(z - C_1 t)/C_1)}{4(C_1 - C_2)\omega} - \frac{(\alpha_1 - \alpha_2)\sin(2\omega(z - C_2 t)/C_1)}{4(C_1 - C_2)\omega}. \tag{22}$$

At enough large times, the sign of ρ_2 is evidently determined by the sign of $\alpha_1 - \alpha_2$. Other perturbations specifying the excited mode may be evaluated from Eqs (7), for example, $v_{z,2} \approx -\frac{C_2}{\rho_0}\rho_2$, $p_2 \approx c_0^2 \rho_2$. The directivity property is broken, that is, the excited perturbations do not propagate with their own linear speed. This always happens to the secondary induced perturbations which are determined by the linear specific links (Leble, Perelomova, 2018; Perelomova, 2019b). The conclusion is that the excited perturbation consists of three parts: the first one growing with time in absolute value (this is due to constant compound of $v_{z,1}^2$ for the harmonic signal), the second one propagating with the speed C_1, and the third one propagating with the speed C_2. The enlargement in time undergoes suppressing when the second mode (or other modes which also enhance) is close to be dominant due to nonlinear transfer of momentum and energy. The modes may be redetermined accordingly to directivity properties summarising parts propagating with equal speeds (Perelomova, 2003).

Figure 2 shows $\alpha_1 - \alpha_2$ associating with the fast mode with the positive linear speed C_1 and the se-

a)



b)

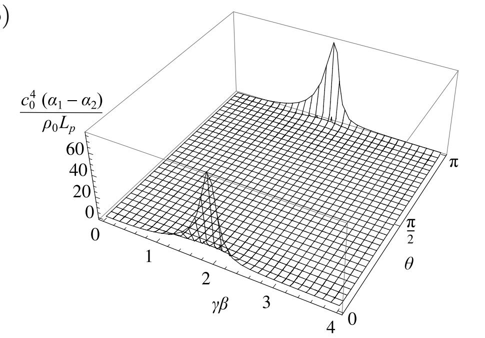


Fig. 2. $\alpha_1 - \alpha_2$, case of dominant fast mode with the linear speed C_1 as functions of plasma-β and θ. $C_2 = -C_1$. Cases of $L(p)$ (a) and $L(\rho)$ (b) and fast magnetoacoustic exciter.

condary mode with $C_2 = -C_1$. Plasma-β is determined as

$$\beta = \frac{2}{\gamma}\frac{c_0^2}{C_A^2}.$$

4.2. Impulsive exciter

The drafts of ρ_2 excited by an impulsive signal

$$v_{z,1} = v_0 \exp(-(\omega(t - z/C_1))^2) \tag{23}$$

in different times are shown in the Fig. 3 (ω denotes the characteristic inverse duration of an impulse). The perturbation has the fronts propagating with the speeds C_1 and C_2. For impulsive signals, there is no part growing with time. It is a kind of a plateau with the constant magnitude forms. The general conclusion is that the reflected mono-polar wave develops. This agrees with the conclusions about features of the reflected wave in the Newtonian flows (Makarov, Ochmann, 1996).

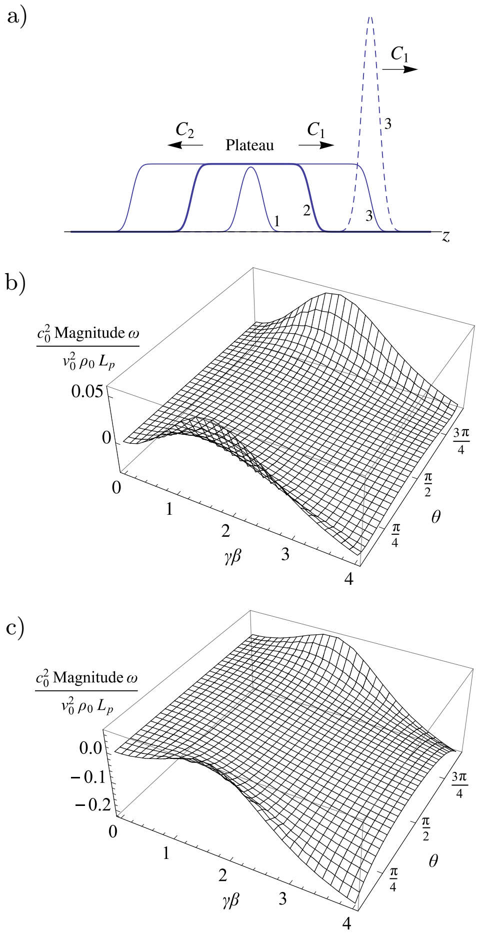


Fig. 3. (a) Drafts of ρ_2 (exaggerated) excited by $v_{z,1} = v_0 \exp(-(\omega t - kz)^2)$ at three consecutive times ordered as 1, 2, 3. An exciting perturbation of density ρ_1 is shown by the dotted line. Below: dimensionless magnitude of the plateau as a function of plasma-β and θ. Cases of $L(p)$ (b) and $L(\rho)$ (c).

Figure 3 explains the general scheme of excitation of the secondary mode and shows the dimensionless magnitudes of a plateau for the case $C_2 = -C_1$, impulsive exciter (23), fast dominant mode, and cases of $L(p)$ and $L(\rho)$. The magnitude of a plateau equals

$$\frac{(\alpha_1 - \alpha_2)\sqrt{\pi}}{2\sqrt{2}\omega} v_0^2.$$

Hence, it is determined by the difference $\alpha_1 - \alpha_2$ and reminds the plots in Fig. 2. We pay attention to the middle range of angles θ.

5. Concluding remarks

The main result of this study is the nonlinear instantaneous dynamic equation Eq. (17), which describes excitation of an excess density in the magnetoacoustic mode by other magnetoacoustic mode which is dominant, that is, which perturbations are much larger than that of other wave and non-wave modes. This is the nonlinear phenomenon which takes place in the flows with destroyed adiabaticity, for example, due to attenuation of any kind including Newtonian one. We consider impact of the heating-cooling function exclusively. The dominance may be broken in the course of nonlinear evolution due to transfer of energy and momentum of the dominant mode into other wave and non-wave modes.

The impact of Newtonian attenuation and thermal conduction in the context of interaction of modes in a planar flow has been considered by Perelomova (2019b). In particular, it has been discovered that production of the reflected wave perturbations is determined by the total attenuation, not by mechanical viscosity and thermal conduction individually. In contrast, excitation of the entropy mode depends on the mechanical viscosity and thermal conduction individually. It turns out that the individual impact of thermal conduction (in the linear part of inhomogeneous diffusion equation) influences only weakly the shape of excited entropy perturbations and they are determined mainly by the total attenuation. One may expect that taking into account the total attenuation in the plasma flows will correct the conclusions. This would alter the condition of acoustical activity of a flow and introduce additional attenuation proportional to $\frac{\partial^2 v_{z,1}}{\partial z^2}$ (which depends on the spectrum of the dominant signal), links of specific perturbations and magnetoacoustic forces describing interaction of modes. Thermal conduction and Newtonian attenuation are important in the case of the high-frequency exciters.

The dynamic equation which governs perturbations in the secondary wave mode, is extracted from the general system of equations by means of projecting. The projecting eliminates perturbations of foreign modes in the linear part of equations and distributes the coupling terms between equations for the different modes properly. These terms form the forces responsible for nonlinear interaction of modes. The procedure deals with instantaneous values and leads to the system of instantaneous dynamic equations which may be simplified in view of dominance of some mode. Equation (17) is a result of projecting into some magnetoacoustic mode (ordered as second) in the case of other exciting dominant magnetoacoustic mode (ordered as first). Its right-hand side (18) represents the magnetoacoustic force manifesting two origins of the phenomenon, nonlinearity and non-adiabaticity due to the heating-cooling function. Equation (17) is valid for both periodic and aperiodic dominant magnetoacoustic wave. The theory is limited by conditions of weak nonlinearity of a flow and weak attenuation (or enhancement) of wave modes over their characteristic duration.

There is no restriction concerning density flux of the magnetic field in this study. The nonlinear interaction of MHD modes is determined by a number of factors. These are: the equilibrium magnetic density flux, the thermodynamic state of a gas and an angle between the magnetic density flux and the wave vector, the kind of the heating-cooling function and the type of the exciter. The interacting waves may be fast or slow and may propagate in one or different directions. The longitudinal velocity in the dominant wave which contributes to the magnetosonic force should satisfy corresponding dynamic equation, Eq. (12), with the nonlinear term which reflects its self-interaction. This leads to enrichment of the spectrum of harmonics at the transducer exciter with higher order harmonics (Subsec. 3.1). Dynamics of the main and higher order harmonics strongly depends on the kind of the heating-cooling function. In the studies of excited perturbations, we make use of solution to Eq. (12) without accounting for nonlinearity and non-adiabaticity, that is, the solution to a simple wave equation in a form of the running wave. The nonlinear interactions may be of a special interest in the plasma's applications since they take place in flows with finite magnitudes of perturbations. The nonlinear effects, even weak, accumulate over time, leading to distortion of the wave form, formation of solitary waves and shock fronts (Geffen, 1963; Sharma *et al.*, 1981; Petviashvili, Pokhotelov, 1992; Ballai, 2006). The secondary modes in turn have impact on the propagation of the dominant waves.

The acoustical activity and wave nonlinear phenomena in acoustically active media have counterparts in many flows, among them, in flows of gases with excited vibrational degrees of molecules and chemically reacting gases (Osipov, Uvarov, 1992; Molevich, 2001; Leble, Perelomova, 2018). That is why the analytical methods and results may find application in similar problems of fluid flows. The nonlinear dynamics of a plasma is the most complex case in view of presence of the magnetic field which introduces additional modes and types of intermode interactions. The general conclusion is that the excited wave perturbations include parts propagating with different speeds, that is, with the linear speed of the dominant mode and the own linear speed of the secondary mode. In the case of impulsive exciters, the secondary perturbations take the form of a plateu impulse with the fronts propagating with different speeds.

Appendix

Table 1. The components of α_1, α_2 (Eq. (18)) proportional to L_ρ and L_p in the case $C_2 = -C_1$.

α_1	
L_ρ	$\left(\frac{(\gamma-1)(2C_1^{12}(2C_1^2+C_{A,z}^2)+c_0^8(6C_1^2C_{A,z}^4-2C_1^4C_{A,z}^2)+c_0^2C_1^8(C_{A,z}^4(1-14\gamma)-C_1^4(7+6\gamma)+2C_1^2C_{A,z}^2(10\gamma-11))}{16C_1^2(C_1^4-c_0^2C_{A,z}^2)^4} + \frac{(\gamma-1)(c_0^6(2C_1^8+C_{A,z}^8(7-9\gamma)+C_1^6C_{A,z}^2(2-3\gamma)+C_1^4C_{A,z}^4(\gamma-23)+C_1^2C_{A,z}^6(11\gamma-8))}{16C_1^2(C_1^4-c_0^2C_{A,z}^2)^4} + \frac{(\gamma-1)c_0^4C_1^2(C_1^2C_{A,z}^4(48-43\gamma)-\gamma C_1^6+9C_{A,z}^6(3\gamma-2)+C_1^4C_{A,z}^2(17\gamma+6))}{16(C_1^4-c_0^2C_{A,z}^2)^4}\right)\rho_0 L_\rho$
L_p	$\left(\frac{(\gamma-1)C_1^4(-2c_0^8+c_0^6(11C_1^2-3C_A^2)+5c_0^2C_1^4(C_A^2-3C_1^2)-C_1^2(C_1^2-C_A^2)((14-5\gamma)C_1^2C_A^4+2(\gamma-4)C_1^4+(\gamma-3)C_A^4))}{8(C_1^4-c_0^2C_{A,z}^2)^3} + \frac{(\gamma-1)C_1^4c_0^2((15-4\gamma)C_1^2C_A^2+2(\gamma-1)C_1^4+(2\gamma-9)C_A^4)}{8(C_1^4-c_0^2C_{A,z}^2)^3}\right)\rho_0 L_p$
α_2	
L_ρ	$\frac{(\gamma-1)(C_1^2-C_{A,z}^2)(C_1^4(2\gamma-1)-c_0^2C_{A,z}^2(\gamma-1)-\gamma c_0^2C_1^2)}{4C_1^2(C_1^4-c_0^2C_{A,z}^2)^2}\rho_0 L_\rho$
L_p	$\frac{(\gamma-1)c_0^2(C_{A,z}^2-C_1^2)((2-3\gamma)C_1^2+(\gamma-1)C_A^2+(2\gamma-1)c_0^2)}{4(C_1^4-c_0^2C_{A,z}^2)^2}\rho_0 L_p$

Table 2. The components of α_1, α_2 (Eq. (18)) proportional to L_ρ and L_p in the case $C_2 = \frac{c_0 C_{A,z}}{C_1}$.

	α_1
L_ρ	$\left(\frac{(\gamma-1)(9C_1^{16}(C_1^2-C_{A,z}^2)+2c_0^{11}C_1^2C_{A,z}^3(C_1^2-C_{A,z}^2)+2c_0C_1^{14}C_{A,z}(C_{A,z}^2-C_1^2)-4c_0^{10}C_{A,z}^4(C_{A,z}^4+2C_1^2C_{A,z}^2-C_1^4))}{16c_0^3(C_1^4-c_0^2C_{A,z}^2)^4(C_1^2C_{A,z}-C_{A,z}^3)} - \frac{(\gamma-1)(2c_0^3C_1^{10}C_{A,z}(C_1^2-C_{A,z}^2)((9-5\gamma)C_{A,z}^2+5(\gamma-2)C_1^4)+2c_0^9(C_1^8C_{A,z}-6C_1^4C_{A,z}^5-(\gamma-3)C_1^6C_{A,z}^3-2(\gamma-1)C_{A,z}^9+3\gamma C_1^2C_{A,z}^7)}{16c_0^3(C_1^4-c_0^2C_{A,z}^2)^4(C_1^2C_{A,z}-C_{A,z}^3)} + \frac{(\gamma-1)(c_0^2C_1^{12}(3C_1^4(\gamma-4)-6(\gamma+4)C_1^2C_{A,z}^2+(3\gamma+28)C_{A,z}^4)+2c_0^5C_1^6C_{A,z}(C_1^4-C_{A,z}^2)((13-6\gamma)C_{A,z}^4+(\gamma-2)C_1^2C_{A,z}^2+(5\gamma-9)C_1^4)}{16c_0^3(C_1^4-c_0^2C_{A,z}^2)^4(C_1^2C_{A,z}-C_{A,z}^3)} - \frac{(\gamma-1)(2c_0^7C_1^2C_{A,z}(C_1^2-C_{A,z}^2)(2(5-3\gamma)C_1^2C_{A,z}^4+(\gamma-4)C_1^6-(\gamma-3)C_{A,z}^6-(6\gamma-7)C_{A,z}^4C_{A,z}^2)}{16c_0^3(C_1^4-c_0^2C_{A,z}^2)^4(C_1^2C_{A,z}-C_{A,z}^3)} + \frac{(\gamma-1)(c_0^8C_1^4C_{A,z}^2((2-3\gamma)C_1^4+(29-3\gamma)C_{A,z}^4+(6\gamma+1)C_1^2C_{A,z}^2)+c_0^4C_1^8((2\gamma-1)C_1^6-3(\gamma-19)C_1^4C_{A,z}^2+(12\gamma-11)C_1^2C_{A,z}^4)}{16c_0^3(C_1^4-c_0^2C_{A,z}^2)^4(C_1^2C_{A,z}-C_{A,z}^3)} - \frac{(\gamma-1)c_0^6C_1^4(2(13-6\gamma)C_1^6C_{A,z}^2-4C_{A,z}^8+2(3-5\gamma)C_1^2C_{A,z}^6+(\gamma-2)C_1^8+(22+21\gamma)C_1^4C_{A,z}^4)}{16c_0^3(C_1^4-c_0^2C_{A,z}^2)^4(C_1^2C_{A,z}-C_{A,z}^3)}\right)\rho_0 L_\rho$
L_p	$\left(\frac{(\gamma-1)(2c_0^5C_1^4C_{A,z}(C_1^2-7C_{A,z}^2)+2c_0C_1^8C_{A,z}(C_{A,z}^2-3C_1^2)+c_0^8(C_1^2C_{A,z}^2-C_1^4)+c_0^7(2C_{A,z}^5+4C_1^2C_{A,z}^3-2C_1^4C_A,z))}{8(C_1^4-c_0^2C_{A,z}^2)^3(C_1^2-C_{A,z}^2)} + \frac{(\gamma-1)c_0^3(6C_1^8C_{A,z}+8C_1^6C_{A,z}^3-2C_1^4C_{A,z}^5)+(2\gamma-3)C_1^{10}(C_1^2-C_{A,z}^2)+c_0^6C_1^4((\gamma+2)C_{A,z}^4-(\gamma-4)C_1^4-6C_1^2C_{A,z}^2)}{8(C_1^4-c_0^2C_{A,z}^2)^3(C_1^2-C_{A,z}^2)} + \frac{(\gamma-1)(c_0^4C_1^2(C_1^2-C_{A,z}^2)(4C_1^2C_{A,z}^2+(\gamma+1)C_{A,z}^4+(5\gamma-9)C_1^4)-c_0^2C_1^6(C_1^2-C_{A,z}^2)((7\gamma-12)C_1^2-(\gamma+6)C_{A,z}^2)}{8(C_1^4-c_0^2C_{A,z}^2)^3(C_1^2-C_{A,z}^2)}\right)\rho_0 L_p$
	α_2
L_ρ	$\frac{(\gamma-1)(C_1^2-c_0^2)(c_0C_{A,z}(c_0^2-C_1^2)(C_1^2+C_{A,z}^2)-(\gamma-1)c_0^2(C_1^4-C_{A,z}^4)+2\gamma C_1^4(C_1^2-C_{A,z}^2))}{4c_0^2(C_1^4-c_0^2C_{A,z}^2)^2(C_1^2-C_{A,z}^2)}\rho_0 L_\rho$
L_p	$\frac{(\gamma-1)(C_1^2-c_0^2)(c_0^3C_{A,z}(C_1^2+C_{A,z}^2)-c_0C_1^2C_{A,z}(C_1^2+C_{A,z}^2)-(\gamma-1)c_0^2(C_1^4-C_{A,z}^4)+2\gamma C_1^4(C_1^2-C_{A,z}^2))}{4(C_1^4-c_0^2C_{A,z}^2)^2(C_1^2-C_{A,z}^2)}\rho_0 L_p$

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