

# 1 Shear band evolution phenomena in direct shear test modelled with DEM

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5  
6 **Abstract:** *A direct shear test is widely used in the geotechnical engineering field. It is an easy and*  
7 *quick test to measure the shear properties of soil. This test often replaces more expensive and difficult*  
8 *tri-axial shear test. Despite that the direct shear test is known and used for a long time, it is still not*  
9 *well investigated at the grain scale. This paper deals with the micro behaviour of the cohesionless sand*  
10 *inside the direct shear box during a geotechnical test. Numerical calculations were performed with*  
11 *open-source discrete code YADE. The full 3D model was used, in order to eliminate some non-physical*  
12 *phenomena. The numerical stress-strain curve was directly compared with the laboratory results and*  
13 *good agreement was obtained. The focus was on the shape of the localized zone, the properties of this*  
14 *zone and its behaviour. The movement of the individual grains was studied. In our analysis, the stresses*  
15 *were also taken into consideration. The conclusions were validated on tests with different initial void*  
16 *ratio (dense or loose material) and with different vertical pressure.*

17  
18 **Key-words:** DEM, shear localization, grain-scale, direct shear

## 19 20 1.0 Introduction

21  
22 Strain localization is one of the most important issues in granular media. The safety of engineering  
23 structures or landslides lies in the deep understanding of this phenomena. The first study on the  
24 stability of clay slopes was done by Collin [1] one hundred and seventy years ago. From an engineering  
25 point of view, the simple laboratory tests of granular media, such as oedometer, triaxial or direct shear,  
26 are most common and relevant. Usually, due to the cost and time involved, the direct shear test is  
27 performed to evaluate the soil properties. The oedometer test provides only basic parameters (not  
28 sufficient) and the tri-axial test is usually too expensive and time-consuming.

29 The direct shear test is known and applied for nearly one hundred years. The first modern shear box  
30 was designed by Casagrande at Harvard in 1932, and displacement control was introduced by Gilboy at  
31 MIT in 1936, which allowed the study of post-peak behaviour [2]. The advantages of the direct shear  
32 test are the simplicity of set-up and equipment used, and the ability of performing the experiment under

1 different saturation, drainage, and consolidation conditions [3-9]. The most important disadvantages are  
2 the size effect (boundary effect) and the strong inhomogeneity [10-13]. Deformation and stress  
3 distribution along the shear plane are highly non-uniform and the principal stresses are not known.

4 Despite the long history, there still exists a lack of knowledge concerning the micro-behaviour of the  
5 localization. Direct shear has widely been studied at the macroscopic level, using continuum models  
6 such as elasto-plasticity [14,15] or hypoplasticity [16].

7 However, for strongly discontinues, heterogeneous and non-linear granular material the discrete  
8 simulations are becoming more popular and widely used [17-24]. There are also many studies on direct  
9 shear with interface in 2D [25-32] or 3D [33-38]. In addition to numerical models, laboratory tests with  
10 PIV measurement were performed [39-41].

11 Though a large amount of the research was done, the formulation of the localization is not yet well  
12 recognized. Most of the papers focused on the macroscopic results or on mesoscopic, but in the residual  
13 regime. In [19], the parameters in direct shear under 3D conditions were studied, however, only 11 700  
14 elements were used. This gives about 30 spheres in height, which could affect the localization zone.  
15 Moreover, only the post-peak behaviour was studied. A large number of spherical elements (as in our  
16 studies) were used in [20, 21], but the grain-level studies are lacking. More works focus on the  
17 interfaces [25-41], where localization forms in the area adjacent to the surface. Moreover, the studies  
18 were limited to the post-peak behaviour. In [23] the analysis of the shear localization was performed  
19 from the beginning of the test. It was limited to 2D calculations and horizontal displacements were  
20 examined only. The shear localization shape was studied in 3D calculations in [24]. The maps of  
21 displacements and shear stresses were presented (only in the residual part).

22 This article focuses mainly on the formulation of the localization at the grain level in direct shear test.  
23 This paper is a second part of the work presented in [18]. More details of the tests can be found there.  
24 Now, the emphasis was placed on displacements and rotation of grains and stresses inside the shear box  
25 in the pre-peak regime. The full 3D study was done with a real diameter of cohesionless sand, due to  
26 catch the localization phenomena. A deep understanding of the localization formulation can help to  
27 better interpret laboratory results and ensure higher safety of engineering structures (since more  
8 complex calculations are often based on soil parameters obtained from such tests).

9 First, the DEM model used in our approach was briefly introduced in Section 2. Then, the calibration  
0 and validation procedure was described and macroscopic results were compared with laboratory ones  
1 (Section 3). In Section 4, the sand-level results were presented, with special attention paid to the shear  
2 localization formulation. Gradients of displacements, rotations or stresses were used. Finally, a



1 conclusion was drawn in Section 5.

2 The novel points of these studies are: 1) full 3D DEM calculations, 2) real mean grain diameter of sand  
3 (a large number of elements along the height), 3) pre-peak results study at sand grain level, 4)  
4 geometric and stress studies based on the grain scale and 5) gradients analysis.

5

## 6 **2.0 DEM method**

7

8 For discrete calculations, the open-source platform YADE [42-44] was used. This model was widely  
9 used in geotechnical problem with localization [18,38,45-47]. In DEM the soil structure is composed  
10 of discontinuous grains (spheres) [48] and its main advantage is the high quality output describing the  
11 behaviour between them. In our approach, the linear contact model and spheres with contact moments  
12 were used to reduce the calculation time. Although the non-linear contact is more relevant [49,50],  
13 the differences were found negligible (less than 10%) [51].

14 In our calculations, only spherical elements were used. To realistically capture the particle behaviour,  
15 some rolling resistance was introduced. Such a model simulates the roughness of the particles, which  
16 is related to the non-perfect shape of the sand grains. However, this approach has several limitations,  
17 i.e. void ratio and mean coordination number may produce less realistic results in comparison with  
18 real-shape grains [52]. It was proved that the particle shape strongly affects the void-based fabric [53].  
19 In future, work clusters of grain [54] or so-called poly-superellipsoids [55] can be used.

20 The model was described in details in [17,18,38,42-47]. Below only some basic equations are listed:

21

$$22 \quad \vec{F}_n = K_n U \vec{N}, \quad (1)$$

22

$$23 \quad \vec{F}_s = \vec{F}_{s,prev} + \Delta \vec{F}_s, \quad \text{with} \quad \Delta \vec{F}_s = K_s \Delta \vec{X}_s, \quad (2)$$

23

$$24 \quad K_n = E_c \frac{2R_A R_B}{R_A + R_B} \quad \text{and} \quad K_s = \nu_c E_c \frac{2R_A R_B}{R_A + R_B}, \quad (3)$$

24

$$25 \quad \Delta M = K_r \Delta \vec{\omega} \quad \text{with} \quad K_r = \beta K_s R_A R_B, \quad (4)$$

25

$$6 \quad M - \eta \frac{R_A + R_B}{2} F_n \leq 0, \quad (5)$$

6

7 where  $U$  is an overlap (penetration depth) between spheres in contact ( $U > 0$  denotes contact,  $U = 0$  if  
8 there is no contact),  $\vec{N}$  is a normal vector at the contact point and  $\Delta \vec{X}_s$  is the incremental tangential  
9 displacement.  $K_n$  and  $K_s$  are the normal and tangential stiffness, correlated with a modulus of elastic of

1 grain contact ( $E_c$ ), grain radius of spheres  $A$  and  $B$  in contact ( $R_A$  and  $R_B$ ) and stiffness ratio ( $\nu_c$ ). The  
2 rolling moments  $\Delta M$  depends on angular increment rotation  $\Delta \vec{\omega}$ , dimensionless rolling stiffness  
3 coefficient  $\beta$  and spheres radii. The limit of the rolling resistance is controlled by the second  
4 dimensionless coefficient  $\eta$ . The Coulomb condition  $|F_s| \leq \mu F_n$  requires an incremental evaluation of  $F_s$   
5 every time step, which leads to some amount of slip each time one the equalities  $F_s = \pm \mu F_n$ . Parameter  
6  $\mu$  denotes friction coefficient between elements.

7 The mechanical responses of the model are presented in Fig.1.

## 8 9 **2.0 Model calibration and validation**

10  
11 The cohesionless sand was calibrated in the tri-axial test and directly compared with the experimental  
12 work of Wu [56]. Satisfactory agreement was found for both the vertical normal stress  $\sigma_l$  and the  
13 volumetric strain  $\varepsilon_v$  versus vertical normal strain  $\varepsilon_l$ . Numerical tests were compared for different initial  
14 pressures ( $\sigma_0 = 50, 200$  and  $500$  kPa). The detailed calibration was presented in [18]. The main  
15 parameters are listed in Table 1. It is worth mentioning that cohesionless sand was calibrated for the 10  
16 times scaled sieve curve ( $d_{50}$  was equal to  $5$  mm, in contrast to the real sand with  $d_{50}$  equal to  $0.5$  mm).  
17 No shear band was present (in our calibration tri-axial test with rigid walls), thus the larger elements  
18 had a negligible influence on the macroscopic results [17]. Moreover, for the real diameter of the  
19 grains, the test would consist of more than  $8\,000\,000$  elements, which is far beyond our possibilities.

20  
21 In this paper, the direct shear tests were performed with the same parameters as in [18] (calibrated in  
22 tri-axial tests). The numerical tests were simulated under full 3D conditions. In this test, the real sieve  
23 curve of Karlsruhe sand was taken into account ( $d_{50} = 0.5$  mm) due to the localization studies. Thus, the  
24 single test consisted of more than  $55\,000$  spheres. The model setup is presented in Figure 2. The length  
25 and height of the direct shear box were equal to  $60$  mm and  $25$  mm, respectively, as in the laboratory  
26 tests. It was proven, that the depth of the sample has a negligible effect on the results of the direct shear  
27 tests [18,45]. Therefore, it was reduced to  $5$  mm in order to shorten the computation time. As for the  
28 boundary condition issue, the size of a test sample must be  $6$  to  $10$  times larger than the particle size  
29 of the material being tested to avoid boundary effects [57]. The constant initial pressure  $\sigma_0$  was  
30 applied to the top box. After the equilibrium state was obtained, the top box moved right ( $u_x$ ) with  
31 constant velocity. The bottom part of the box was fixed throughout the calculations. A small gap

(Fig.2), corresponding to the maximum particle diameter, was created between two boxes to prevent the spheres from locking (as in the experiment). The leakage of the spheres through the gap was negligible and lower than 0.1% of the whole amount of the elements (less than 100 grains). The tests were performed under quasi-static conditions (the inertial number  $I$  was kept below  $1e-4$  [58]).

First, the validation of the model was checked. Note, that the DEM parameters were taken exactly the same as for calibration in the triaxial test (Tab.1). The numerical results of the sample with initial porosity  $e_0 = 0.63$  and initial vertical pressure  $\sigma_0 = 200$  kPa were compared with the experimental study of Salazar [59]. The comparison is shown in Fig. 3. Both the stress and volumetric strain were in a good agreement. The stress peak was slightly higher in the experiment (about 2.5%) and the residuum value was about 10% higher. The volumetric strain showed similar behaviour, with a smaller residuum value for the experiments. This was caused by the use of spheres instead of clumps, which provide artificial dilatancy of the material [18, 60].

It should be noted, that the material parameters were calibrated on the Karlsruhe sand in triaxial experiments, thus perfect agreement could not be obtained here. The agreement can be improved if a real grain shape would be used [61,62].

Next, the influence of the initial porosity of the sample was investigated. Samples with porosities of  $e_0 = 0.53, 0.63$  and  $0.75$  were generated under a constant pressure  $\sigma_0 = 200$  kPa. The different porosities were chosen to capture the spheres' behaviour under various conditions (dense, medium dense and loose, respectively). In typical DEM calculations, it is difficult to obtain void ratios outside the range  $e_0=0.53\div 0.75$  for the spherical grains. The internal friction angle  $\varphi_w$  and volumetric strain  $\varepsilon_v$  versus horizontal displacement  $u_x$  are presented in Fig. 4. As the void ratio increases, the internal friction angle decreases (no peak is observed for the loose specimen), but the residual value remains similar. When the initial density of the samples was dense or medium dense, dilatancy occurred almost from the beginning of the test, as in the loose sample only contraction appeared. The behaviour of the sand in the numerical calculations was similar to reality.

The influence of the initial pressure was also checked (only for medium dense samples). The internal friction angle and volumetric strain are plotted in Fig. 5. The highest internal friction angle peak and the highest stiffness were obtained for lower pressure (the residual state was the same for all specimens). Volumetric strain was the lowest for the highest pressure (with a slight contractancy at the

1 beginning). Again, the results were consistent with the general knowledge of the granular media  
2 behaviour.

3 The final localization zone was similar in all specimens (with the exception of the initially loose one  
4 [18]). The small differences were found only in the shear zone thickness [18]. The exemplary shear  
5 zone in the residual state based on grain rotation is presented in Fig. 6. The shear zone was found to be  
6 a horizontal straight line between two boxes. Its thickness varied between  $18-20 \times d_{50}$ , which was in  
7 agreement with other studies [41, 63, 64].

8  
9 However, the direct shear test was not homogeneous from the beginning. Not only did the material  
10 begin to shear in the middle part, but the entire specimen was mobilized. To our best knowledge, there  
11 is a gap in the study about the phenomena of the shear zone creation in a direct shear test. The next  
12 section deals with the initiation and formulation of the localization.

### 14 3.0 Shear band evolution results

15  
16 So far, the research work was mainly focused on the macroscopic behaviour or sometimes on the final  
17 shape of the localization zone. There is a lack of knowledge about how the localization develops (some  
18 information is given in [24]). Usually, the shear localization is considered as a straight line (with some  
19 small fluctuations, especially when a rough interface is introduced). However, this is not the case in the  
20 direct shear. Localization tries to find the best path between the horizontal line (enforced by the boxes)  
21 and the inclined line (enforced by the natural shear angle for the granular material). This section shows  
22 the development of the localization zone in the first phase of the tests.

23 The figures in this articles always show initial part of the shear boxes. During the movement of the  
24 upper box, the area of interest was constant. Thus, the figures always show the region between 0 and 6  
25 cm (the initial box size), due to visibility.

26 First, the fluctuations of the displacement are presented in Fig. 7. They were calculated as  $\vec{u}_i - \vec{u}_{avg}$ ,  
27 where  $\vec{u}_i$  is the displacement of  $i$  sphere and  $\vec{u}_{avg}$  is the mean displacement of all spheres in the entire  
8 specimen. The different orientation of the arrows (or/and small values) shows the localization shape.  
9 From the early beginning of the test, the shear localization formed  $s$ -shape. The small values (low  
0 density) of arrows (colours) show the localization zone clearly, while grey colour shows large values of  
1 the fluctuations. Due to visibility, black arrows are presented for every 50 elements. The material  
2 showed a behaviour as for passive and active earth pressure (near the left and right boundaries,

1 respectively). The *s*-shape flattened during the test and the pure horizontal shear appeared well after the  
2 peak (Fig. 7f).

3 Since the average value of the displacement of all elements is taken, the fluctuations of the  
4 displacement are not a perfect tool for microstructure study. This method causes the information about  
5 a single grain to be fuzzy. Moreover, the fluctuations can also be calculated in a different way, which  
6 would affect the results [46,65] (e.g. mean displacement is taken only from some small region). The  
7 fluctuations were calculated only as a basis for futures studies.

8 In this paper, the focus was laid on the gradient values (total or normalized) of several parameters of  
9 the spheres. The gradient changes have been found as the best indicator of the variations inside the  
10 localization. It directly shows the changes in the behaviour of the granular media.

11 Since the results for each individual grain were too chaotic (no conclusions could be drawn), the mean  
12 value was calculated for the cuboidal REV (represent element volume). The REV had an area size of  
13  $5d_{50} \times 5d_{50}$ . In the third dimension, the entire specimen was taken ( $10d_{50}$ ), because the direct shear  
14 problem is uniform in depth. Thus, about 250 grains were taken into account. The REV volume  
15 ( $\sim 15.625 \text{ mm}^3$ ,  $\sim 250$  grains) was chosen arbitrary, to avoid losing the grain level phenomena and to  
16 prevent chaotic behaviour of individual grains. The size of REV was already successfully used in  
17 [18,46,61]. The REV was then moved by  $d_{50}$  to obtain a better resolution of maps. All the maps are  
18 presented for  $u_x = 0.1$  (beginning of the test), 0.5, 1.0, 1.5, 2.0 (peak), 5.0 (residual state) and 7.0 mm  
19 (end of the test). The study concentrated on the pre-peak behaviour (development of the shear zone).

20 First, the simple change in the position of the grains was studied (Fig.8). It was done separately for  
21 both directions – horizontal ( $u_x'$ ) and vertical ( $u_y'$ ). The scale was defined differently for each  
22 parameter, due to its visibility. The emphasis was on the differences in the shape, not in the values. For  
23 the horizontal displacement, the blue colour corresponds to the lack of movement and the red colour  
24 corresponds to the largest displacement (Fig.8A). The white line shows the mean value, which was in  
25 agreement with the fluctuations. The mean value line also shows the place, where localization may  
26 appear. The same *s*-shaped behaviour was found, what was expected. With an increase of  $u_x$  the *s*-shape  
27 become flatter (up to the peak Fig. 8Ae) and in the residual state it was perfectly horizontal.

8 However, the behaviour was different when the displacement in the vertical direction was analysed  
9 (Fig.8B). Here, the red colour corresponds to the highest upwards movement and the blue colour  
0 corresponds to the highest downwards movement of spheres (maximum scale on both directions was  
1 the same). Thus, the white colour shows no vertical displacement. The concentration of the movement  
2 occurred close to both side boundaries. On the left side, where the displacement was applied, grains

1 moved down. The initial movement was related to the contraction caused by the shear stress. On the  
2 contrary, on the right side of the specimen, the grains moved up. For the vertical displacements, the *s*-  
3 shape did not occur. The mean values (white line) run from the upper-left corner to the bottom-right  
4 one. During the test, it flattened a bit, however, even in the residual state, it was not a straight line.  
5 Such behaviour was in contrast to the one presented with the horizontal displacement maps.  
6 Throughout the test, displacement concentrations had occurred near the boundaries. It was found quite  
7 surprising, that the vertical displacement had a completely different behaviour than the horizontal one.

8  
9 For a deeper study, the gradient of displacement was calculated (Fig.9). The value of the gradient was  
10 normalized by the maximum value in each vertical strip (strip had a width equal to REV). Thus, the red  
11 colour corresponds to the highest positive increment and the blue colour corresponds to the highest  
12 negative increment in the considered vertical strip. The gradient was calculated from the bottom to the  
13 top of the shear box .

14 First, the gradient of the displacement in the horizontal direction ( $u_x^*$ ) was studied (Fig.9A). At the  
15 beginning of the test, the strong concentration appeared close to the boundaries discontinuity (near the  
16 gap between the boxes). At the left side, the inclination of the concentration was oriented downwards,  
17 differing from the right side, where it was directed upwards. The middle part of the specimen was  
18 rather chaotic (Fig.9Aa). The gradient's extremum was developed on both side of the shear box. In the  
19 middle part of the sample, maximum values created small lines, rather than a pronounced single one.  
20 Sometimes even two or three extreme values were found in one vertical strip. It shows, that the  
21 localization was not created as a uniform one, but consisted of many independent blocks. The  
22 extreme's points on the right side were still not connected to the middle ones (Fig. 9Ab). After shear  
23 displacement  $u_x \geq 1.00$  mm the gradient of the horizontal displacement started to formulated one, single  
24 line, however, still it was not straight and had some minor variations (Fig. 9Ac – left part). During the  
25 test, the extremum line flattened to the completely plain one (Fig. 9Ag). At the residual state the  
26 maximum gradient was not identical in the entire specimen and some fluctuations appeared (Fig. 9Af).  
27 A conclusion is that a lot of small shear patterns are established during the test and the maximum value  
8 may appear at different heights. It seems like granular material sought for a proper path to shear, but the  
9 construction of the shear box forced it to form a straight horizontal localization at the mid-height of the  
0 specimen.

1 Even more interesting was a maximum gradient calculated from vertical displacements ( $u_y^*$ ) (Fig. 9B).  
2 After initially chaotic behaviour, the maximum of the gradient occurred close to the boundaries



1 discontinuity, as in the gradient's map of the horizontal direction. Moreover, a strong negative gradient  
2 was visible in the bottom-left and the top-right corners of the shear box. The maximum value inside the  
3 specimen showed a strong diagonal line not connected to the boundaries (Fig. 9Bc). Note, that  
4 inclination in this case was in contrast to the *s*-shape (although, at the beginning the *s*-shape can be  
5 found in blue colour). The behaviour of the sand was similar to the landslide slip more than to the  
6 direct shear. The angle inclination was about 33° (the natural slope angle for sand). After  $u_x \geq 1.50$  mm  
7 the maximum gradient line started to connect each other, with some small noises at the top-left and  
8 bottom-right parts (Fig. 9Bd,e). Even in the residual state, the line was not straight and still, some  
9 branching was visible (Fig. 9Bf,g). The bottom-left part was strong fluctuated during the entire test.  
10 It's worth noting, that the neutral gradient's (white line between red and blue areas) formed similar *s*-  
11 shaped curve. Inside that area, grains experienced highest vertical displacements (on the left side  
12 oriented downwards and on the right side oriented upwards), connected to the dilatancy of the material.

13  
14 To prove, that the final localization zone was composed of several time-depended micro localizations,  
15 the gradient of velocities is plotted ( $v_x$  and  $v_y$ ) (Fig.10).The momentary behaviour is presented (velocity  
16 in a given time-step) instead of cumulative values. The behaviour was similar as for gradient in  
17 displacement, but much more fluctuated. The shape of the maximum gradient was more curved but  
18 much thinner. For example for  $u_x=1.00$  mm, the strong *s*-shape was observed for the gradient of  
19 horizontal velocities (Fig. 10Ad) in contrast to the gradient of horizontal displacement (Fig. 9Ad). Note  
20 also, that in the residual state (Fig. 10Af,g) the maximum values changed their place and shape. Again,  
21 some discontinuities were observed. For the vertical velocities ( $v_y$ ), the differences were even more  
22 significant. During the whole test, the velocities' gradient strongly varied. Up to the peak, the shape  
23 was quite similar to the gradient of displacements. Next, chaotic behaviour appeared, with maximum  
24 and minimum values placed in the middle height of the specimen (Fig10Be-g). The extreme values  
25 (before the peak in the horizontal direction and during the entire test in the vertical direction) were  
26 found not always in the middle height of the specimen, where localization was expected. It was highly  
27 non-uniform.

8  
9 The gradient of the cumulative rotation and contact number was also studied (Fig. 11). Both values,  
0 rotation and contact number, are some of the most common methods to determine shear localization in  
1 DEM (or more, in granular materials). However, since the gradient was calculated, quite chaotic results  
2 were obtained before the peak for both values. After the peak ( $u_x \geq 1.50$  mm) the shear localization was



1 more clear (Fig. 11Ad-g). No special phenomena can be found. The grain rotation was found as a result  
 2 of the shearing, not as a cause.

3 For the contact's number gradient, the contractancy can be found in the top-left and the bottom-right  
 4 parts of the specimen (Fig. 11Bb-d). Finally, the minimum of the gradient created a horizontal line in  
 5 the middle of the specimen's height (Fig. 11Bf,g).

6  
 7 Up to now, the focus was laid on element behaviour as displacement, velocities or rolling. It was  
 8 proven that the first indicator of localization appears in horizontal displacements (it was most visible  
 9 from the beginning). The vertical displacements showed completely different behaviour during almost  
 10 the entire test. Moreover, the rolling of spheres showed some indicators of localization rather late, and  
 11 pure shear can be found at residual state only. It was proven, that the spheres rolling is not a perfect tool  
 12 to mark the localization, at least in the early stage of shearing.

13  
 14 Finally, internal stresses were calculated and studied. The stresses were calculated by the well-known  
 15 Love-Weber formula [66], [67]

$$16 \quad \sigma_{ij} = \frac{1}{V^p} \sum_{c=1}^N x_i^c f_j^c, \quad (6)$$

17 where  $N$  – the number of contact points,  $x_i^c$  - the  $i^{\text{th}}$  component of branch vector jointing from the  
 18 centre of mass of the particle to the contact point 'c',  $f_j^c$  - the  $j^{\text{th}}$  component of the total force at the  
 19 contact point 'c' and  $V^p$  - the cell volume. The values of  $\sigma_{ij}$  were again calculated from an averaging  
 20 cell (REV) of the size  $5d_{50} \times 5d_{50} \times 10d_{50}$  being moved by  $d_{50}$ .

21  
 22 First, the gradients of normal stresses  $\sigma_{xx}$  (horizontal) and  $\sigma_{yy}$  (vertical) are presented in Fig. 12. The  
 23 highest changes in gradients of the normal horizontal stresses were observed at the left part (blue  
 24 colour) and the right part (red colour). The highest concentrations occurred near the moving wall (top-  
 25 left) and blocked wall (bottom-right) during the pre-peak regime. The horizontal lines were visible  
 26 above (blue) and below (red) the mid-part of the specimen in Fig. 12Ae. However, some oblique lines  
 27 also begin to develop. In the post-peak part (Fig.12Af, g), the horizontal gradient lines connected to  
 8 each other, creating a diagonal one. Although the oblique line was most significant, the horizontal  
 9 stresses had a non-homogenous aspect. Still, the main changes in horizontal stresses connect top-left  
 0 and bottom-right part of the box, what was in agreement with the force-chain network [18]. The  
 1 displacement and stresses are not constant in the shear plane, but passive and active pressure zones  
 2 appeared on the left and right side, respectively. This is in accordance with work [22-24]. However, the

1 final shear band was not parallel or perpendicular to the stresses paths in this case. It is consistent with  
2 recent literature, which shows that the maximal force chains are diagonal to the shear zone [20], [34],  
3 [38].

4 The changes in gradients for vertical stresses were different from the horizontal ones. After small shear  
5 displacement ( $u_x=0.50$  mm) up to the end of the test, the behaviour was similar. The vertical stresses  
6 show a lot of diagonal lines with gradient's extrema (Fig.12B). The inclinations were about  $60^\circ$ , which  
7 corresponds to the internal friction angle of the sand. This behaviour occurred throughout the entire  
8 test, independent to the shear localization development. This behaviour was not observed in the  
9 literature, yet.

10  
11 Finally, the gradients of shear stresses ( $\sigma_{xy}$  and  $\sigma_{yx}$ ) are plotted (Fig. 13). From the early beginning, the  
12 maximum shear stresses occurred in the middle part of the shear box. With the height of the specimen,  
13 the shear stress increases up to the middle part and then decreases (Fig.13a-d). Such behaviour was  
14 forced by the construction of the shear box, which enforced shear localization to formulate in the  
15 middle part. However, as the test continued, the shear stresses started to deviate from the straight  
16 horizontal line due to granular media nature (13b-e). Note, that there were multiple shear locations. The  
17 inclinations of shear zones were about  $30-45^\circ$  and were smaller than the internal friction angle. Finally,  
18 after the peak, the changes in gradients were oblique, parallel to the gradient of the horizontal stresses.  
19 The shear stresses showed an inclination between the internal friction angle of sand and completely  
20 horizontal line forced by the shear box.

21  
22 In this article, only one specimen was instigated ( $e_0=0.60$ ,  $\sigma_0=200$  kPa). However, similar behaviour  
23 was observed for different initial stresses ( $\sigma_0=50$  kPa or  $500$  kPa) (Fig. 14). Here, only gradients of  
24 displacement and stresses for  $u_x=1.00$  mm were plotted.

25  
26 The different behaviour was found only for the loose specimen ( $e_0 = 0.75$ ). The granular media  
27 behaviour was there even more non-uniform. The behaviour was strongly affected by passive and  
8 active earth pressure boundary conditions and the shear localization was barely visible in the pre-peak  
9 regime (Fig. 15A). In the residual part, all parameters were more chaotic than for initially dense or  
0 medium dense specimen. Even the horizontal gradient of displacement did not show a straight line  
1 (Fig. 15Ba). The middle part of the specimen showed strong fluctuations through the entire test. The  
2 localization zone was more disturbed by the sand properties than by the shear box structure. Moreover,

1 the vertical displacement shows similar behaviour as the horizontal one (*s*-shape) in contrast to the  
2 medium dense specimen. The *s*-shape was more curved during the entire test and even in the residual  
3 state (Fig. 15B) the localization was not straight.

#### 5 **4.0 Conclusions**

6  
7 The DEM method is a powerful tool to study the micro behaviour of granular media at the grains' level.  
8 The main advantages of this article are fully 3D calculations, the real size of the sand grain, and pre-  
9 peak analysis based on geometric and stresses at grain level. After calibration, the numerical parameters  
10 can be used for different tests with a good agreement with experiments. However, better compatibility  
11 can be achieved with the use of real grain shape. The increase of computer power allows also for more  
12 and more complex calculations. The largest advantage of the discrete numerical calculations is that all  
13 micro-level phenomena can be found, in contrast to laboratory tests, where it is rather complicated,  
14 time-consuming and expensive.

15 The main conclusions from this paper are summarized below:

16 - the best indicator for the localization zone was found as horizontal displacement. It shows the  
17 localization from the very beginning of the test. However, in the pre-peak regime, the localization is not  
18 a straight line, as it would be expected. The place of the shear band is related to the internal friction  
19 angle of the sand and the construction of the direct shear box.

20 - The vertical displacement is unrelated to a horizontal one and shows completely different behaviour  
21 during the shear test (it is more chaotic). Its shape depends more on sand parameters (contractancy)  
22 than on shear box structure.

23 - The gradient of velocities shows that the shear band consist of a lot of small micro-scale localizations,  
24 which are variable over time. Those localizations after cumulation create a final shear band.

25 - The rotation of the spheres shows the localization rather late. It is more an effect of the shear – not the  
26 cause. A similar conclusion can be found in [18].

27 - The vertical stresses show the highest values between top-left and bottom-right walls of the shear box.  
8 The localization is not perpendicular or parallel to these stresses. The vertical stresses have a lot of  
9 single lines, with high inclinations above 60°. That corresponds to the internal friction angle of the  
0 sand. The shear stresses show behaviour between the shear angle of sand and horizontal shear between  
1 two boxes.

2 - In the direct shear, the peak and residual friction angle values are a bit higher (few degrees) than in

1 tri-axial tests, due to the shear band shape (in direct shear test the shape is forced by the construction of  
2 the boxes, while in tri-axial it is a result of the granular media parameters only).

3 - In loose specimen, all above conclusions are even more relevant.  
4

5 In future work, the real microstructure of granular media will be investigated (real shape and positions  
6 of the grains, based on the micro-tomography images). The results will be compared to own laboratory  
7 tests (with DIC maps). Also, cohesion between grains can be introduced since it can cause different  
8 behaviour. The same analysis can also be used for more complex geotechnical tests as biaxial or  
9 triaxial, where the position of the shear localization is not known from the beginning (in contrast to the  
10 direct shear test). Later on the real engineering problems (at a small scale, yet) can be analysed i.e.  
11 active/passive earth pressure or silo outflow.  
12

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17 *zone between granular material and corrugated sheet in DEM and MPM codes*” financed by the  
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19

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# List of tables and figures

**Table 1.** Material micro-parameters for discrete simulations

**Fig.1:** Mechanical response of (a) tangential (b) normal and (c) rolling contact model laws [42]

**Fig. 2:** Model set-up for numerical simulations of direct shear test with the zoom on the gap between boxes

**Fig. 3:** Stress ratio  $\sigma_0/\tau_x$  and strain  $\varepsilon_v$  versus horizontal displacement  $u_x$  from direct shear test for: a) numerical simulations and b) laboratory test of Salazar et al [59] ( $e_0 = 0.63$  and  $\sigma_0 = 200$  kPa)

**Fig. 4:** Internal friction angle  $\varphi_w$  and volumetric strain  $\varepsilon_v$  versus horizontal displacement  $u_x$  from discrete simulations of direct shear test for different initial void ratios: a)  $e_0 = 0.53$ , b)  $e_0 = 0.63$  and c)  $e_0 = 0.75$  ( $\sigma_0 = 200$  kPa)

**Fig. 5:** Internal friction angle  $\varphi_w$  and volumetric strain  $\varepsilon_v$  versus horizontal displacement  $u_x$  from discrete simulations of direct shear test for different vertical load: a)  $\sigma_0 = 50$  kPa, b)  $\sigma_0 = 200$  kPa and c)  $\sigma_0 = 500$  kPa ( $e_0 = 0.63$ )

**Fig. 6:** Exemplary distribution of sphere rotation at the final stage of the direct shear test (red colour – clockwise rotations, blue colour – anti-clockwise rotations) (*colour online*)

**Fig. 7:** Evolution of displacements fluctuations ( $\vec{u}_i - \vec{u}_{avg}$ ) in the entire specimen for: a)  $u_d = 0.10$  mm, b)  $u_d = 0.50$  mm, c)  $u_d = 1.00$  mm, d)  $u_d = 1.50$  mm, e)  $u_d = 2.00$  mm and f)  $u_d = 7.00$  mm ( $e_0=0.63$ ,  $\sigma_0=200$  kPa,  $d_{50}=0.5$  mm) (arrows are multiplied by  $1e03$  due to readability) (grey and red colour correspond to the maximal and minimal fluctuations' value, respectively) (black arrows shows every 50 elements only)



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**Fig.8:** Evolution of displacements: A)  $u_x'$  (horizontal) and B)  $u_y'$  (vertical) in the entire specimen for: a)  $u_x=0.1$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.5$  mm, e)  $u_x=2.0$  mm, f)  $u_x=5.0$  mm and g)  $u_x=7.0$  mm (red colour denotes positive displacement, blue colour denotes negative displacement) (colour online)

**Fig.9:** Evolution of the gradient of displacements: A)  $u_x^*$  (horizontal) and B)  $u_y^*$  (vertical) in the entire specimen for: a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  $u_x=7.00$  mm (blue colour denotes increase, red colour denotes decrease) (colour online)

**Fig.10:** Evolution of gradient of velocities: A)  $v_x$  (horizontal) and B)  $v_y$  (vertical) in the entire specimen for: a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  $u_x=7.00$  mm (red colour denotes increase, blue colour denotes decrease) (colour online)

**Fig.11:** Evolution of gradient of A) rolling and B) contact number in the entire specimen for: a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  $u_x=7.00$  mm (red colour denotes increase, blue colour denotes decrease) (colour online)

**Fig.12:** Evolution of gradient of stresses A)  $\sigma_{xx}$  and B)  $\sigma_{yy}$  in the entire specimen for: a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  $u_x=7.00$  mm (red colour denotes increase, blue colour denotes decrease) (colour online)

**Fig.13:** Evolution of gradient of stresses A)  $\sigma_{yx}$  and B)  $\sigma_{xy}$  in the entire specimen for: a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  $u_x=7.00$  mm (red colour denotes increase, blue colour denotes decrease) (colour online)

**Fig.14:** Maps of gradient for specimen with initial pressure equal: A)  $\sigma_0 = 50$  kPa and B)  $\sigma_0 = 500$  kPa for entire specimen for  $u_x=1.00$  mm: a) horizontal displacement ( $u_x^*$ ) b) vertical displacement ( $u_y^*$ ), c) horizontal stresses ( $\sigma_{xx}$ ), d) vertical stresses ( $\sigma_{yy}$ ), e) shear stresses ( $\sigma_{xy}$ ) and f) shear stresses ( $\sigma_{yx}$ ) (red colour denotes increase, blue colour denotes decrease) (colour online)

1 **Fig.15:** Maps of gradient for initial loose specimen ( $e_0=0.75$ ) for entire specimen for A)  $u_d=1.00$  mm  
2 (pre-peak) and B)  $u_d=5.00$  mm (residual part): a) horizontal displacement ( $u_x^*$ ) b) vertical displacement  
3 ( $u_y^*$ ), c) horizontal stresses ( $\sigma_{xx}$ ), d) vertical stresses ( $\sigma_{yy}$ ), e) shear stresses ( $\sigma_{xy}$ ) and f) shear stresses  
4 ( $\sigma_{yx}$ ) (red colour denotes increase, blue colour denotes decrease) (*colour online*)

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**Table 1.** Material micro-parameters for discrete simulations

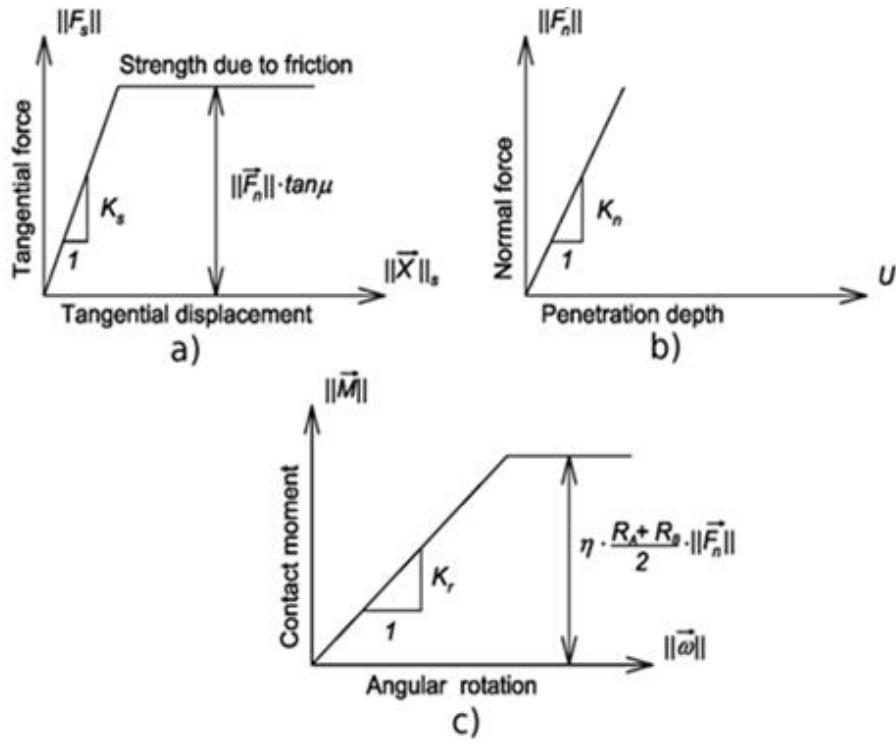
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Material micro-parameters	Value
Modulus of elasticity of grain contact $E_C$ (MPa)	300
Normal/tangential stiffness ratio of grain contact $\nu_C$ (-)	0.3
Inter-particle friction angle $\mu$ (°)	18
Rolling stiffness coefficient $\beta$ (-)	0.7
Moment limit coefficient $\eta$ (-)	0.4

**TABLE 1**

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3 **Fig.1:** Mechanical response of (a) tangential (b) normal and (c) rolling contact model laws [42]

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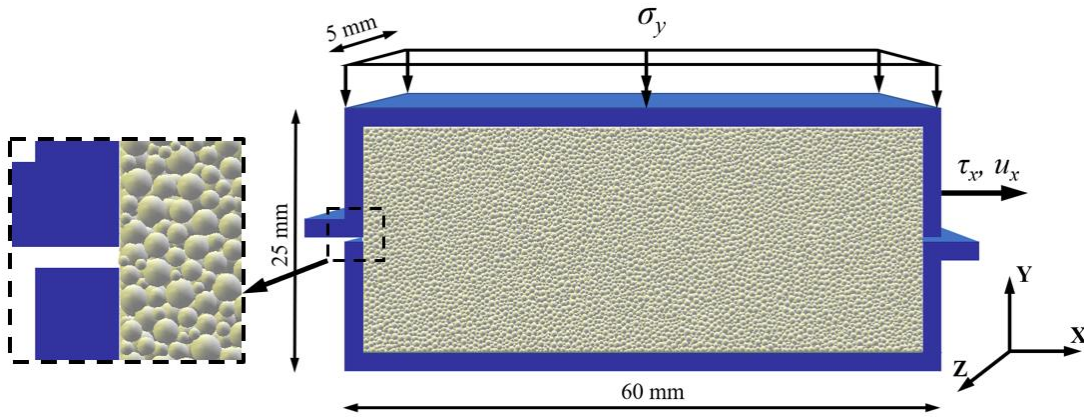
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**FIGURE 1**

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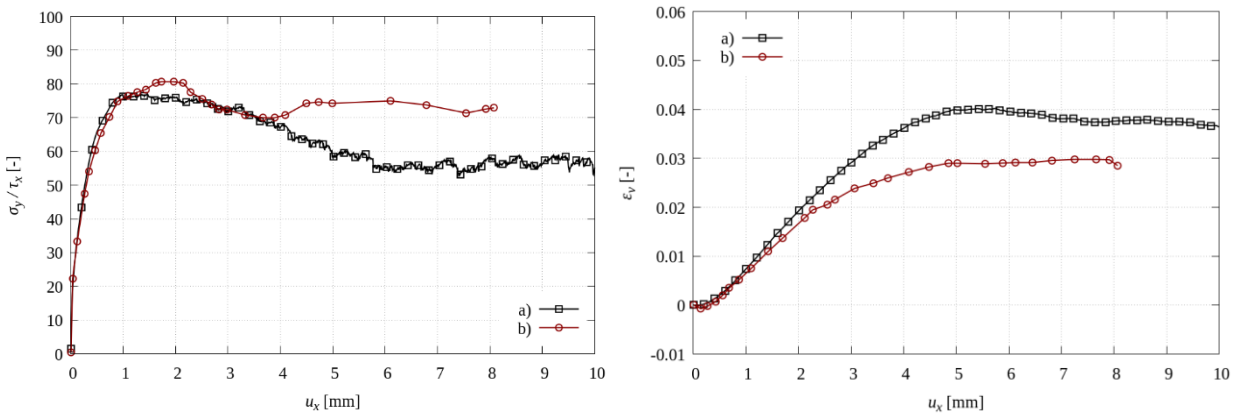
3 **Fig. 2:** Model set-up for numerical simulations of direct shear test with the zoom on the gap between  
4 boxes

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**FIGURE 2**



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9 **Fig. 3:** Stress ratio  $\sigma_0/\tau_x$  and strain  $\epsilon_v$  versus horizontal displacement  $u_x$  from direct shear test for: a)  
10 numerical simulations and b) laboratory test of Salazar et al [59] ( $e_0 = 0.63$  and  $\sigma_0 = 200$  kPa)

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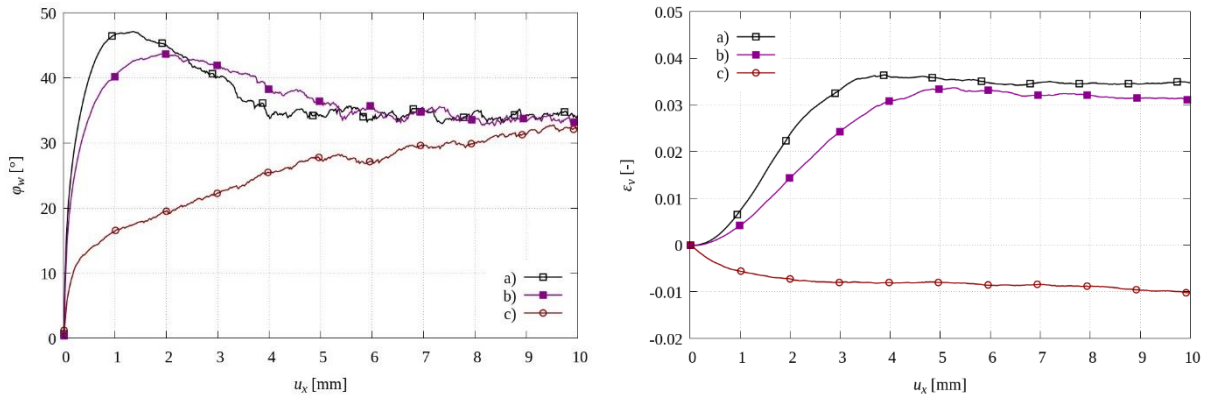
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**FIGURE 3**



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3 **Fig. 4:** Internal friction angle  $\varphi_w$  and volumetric strain  $\varepsilon_v$  versus horizontal displacement  $u_x$  from  
4 discrete simulations of direct shear test for different initial void ratios: a)  $e_0 = 0.53$ , b)  $e_0 = 0.63$   
5 and c)  $e_0 = 0.75$  ( $\sigma_0 = 200$  kPa)

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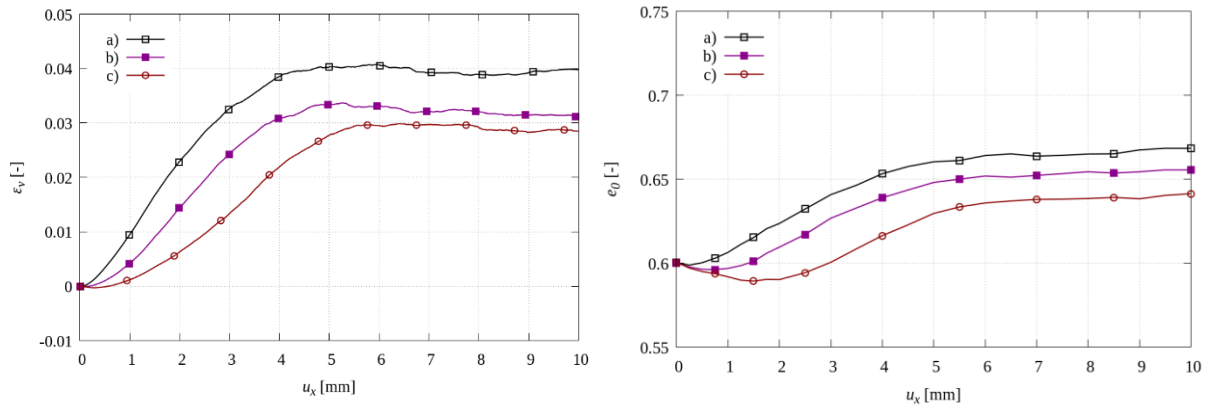
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**FIGURE 4**

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3 **Fig. 5:** Internal friction angle  $\varphi_w$  and volumetric strain  $\varepsilon_v$  versus horizontal displacement  $u_x$  from  
4 discrete simulations of direct shear test for different vertical load: a)  $\sigma_0 = 50$  kPa, b)  $\sigma_0 = 200$  kPa  
5 and c)  $\sigma_0 = 500$  kPa ( $e_0 = 0.63$ )

6

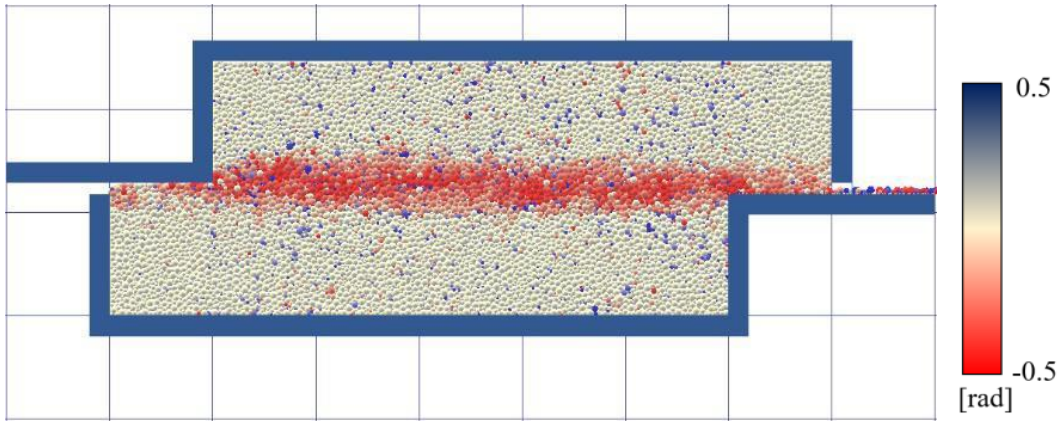
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## FIGURE 5

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3 **Fig. 6:** Exemplary distribution of sphere rotation at the final stage of the direct shear test (red colour –  
4 clockwise rotations, blue colour – anti-clockwise rotations) (*colour online*)

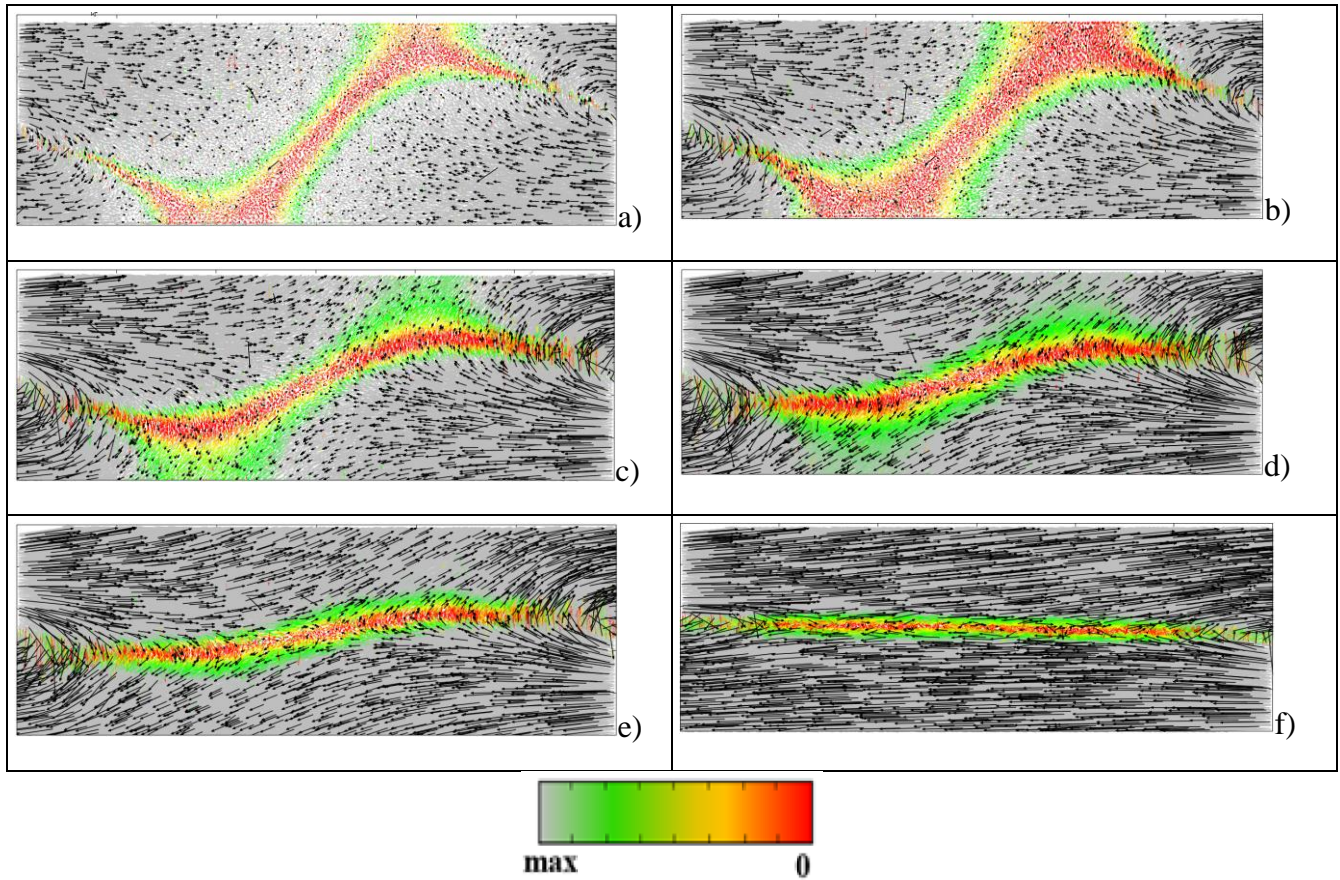
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## FIGURE 6



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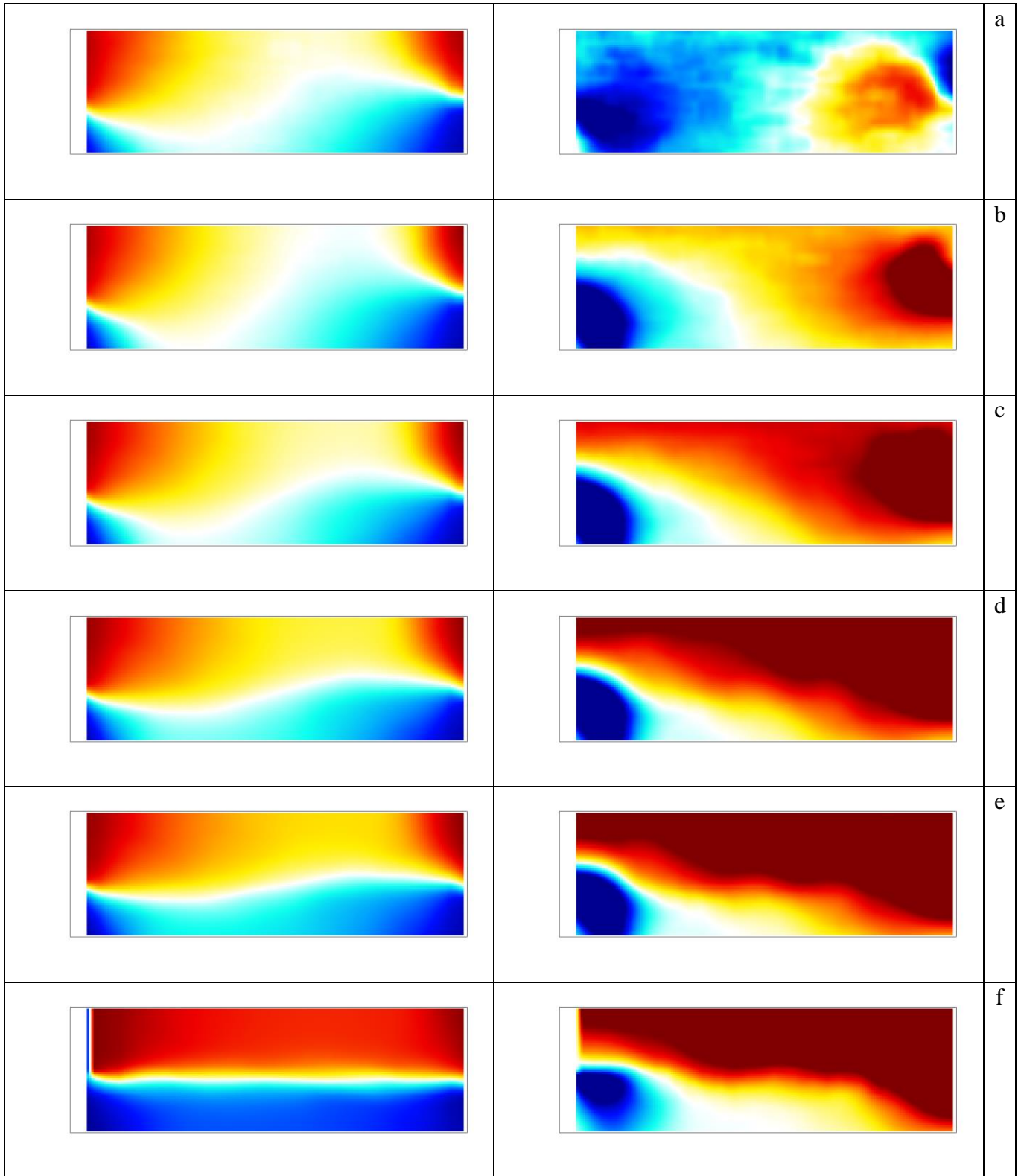
3 **Fig. 7:** Evolution of displacements fluctuations ( $\vec{u}_i - \vec{u}_{avg}$ ) in the entire specimen for: a)  $u_d = 0.10$  mm,  
 4 b)  $u_d = 0.50$  mm, c)  $u_d = 1.00$  mm, d)  $u_d = 1.50$  mm, e)  $u_d = 2.00$  mm and f)  $u_d = 7.00$  mm ( $e_0=0.63$ ,  
 5  $\sigma_0=200$  kPa,  $d_{50}=0.5$  mm) (arrows are multiplied by  $1e03$  due to readability) (grey and red colour  
 6 correspond to the maximal and minimal fluctuations' value, respectively) (black arrows shows every 50  
 7 elements only)

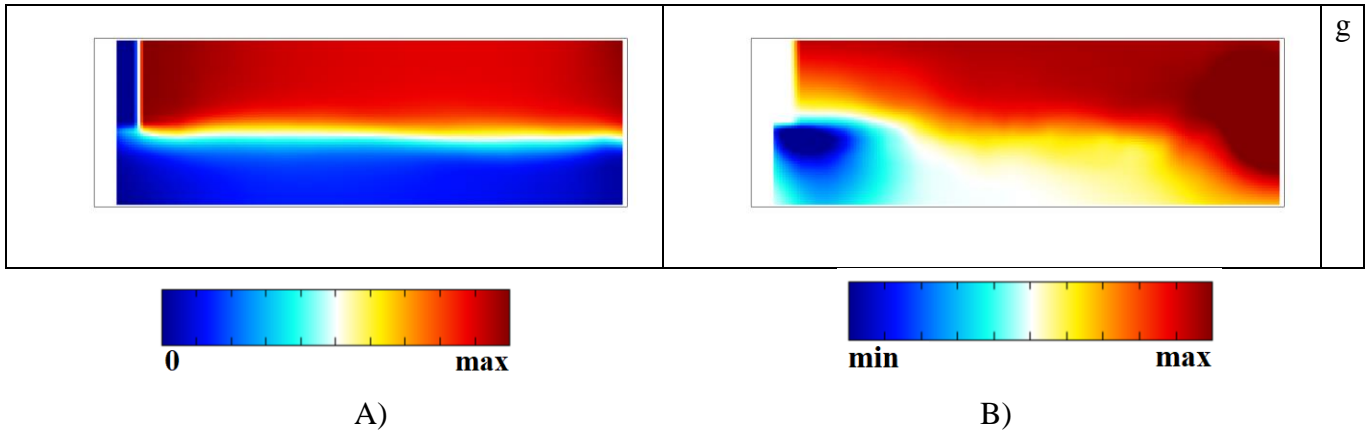
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## FIGURE 7

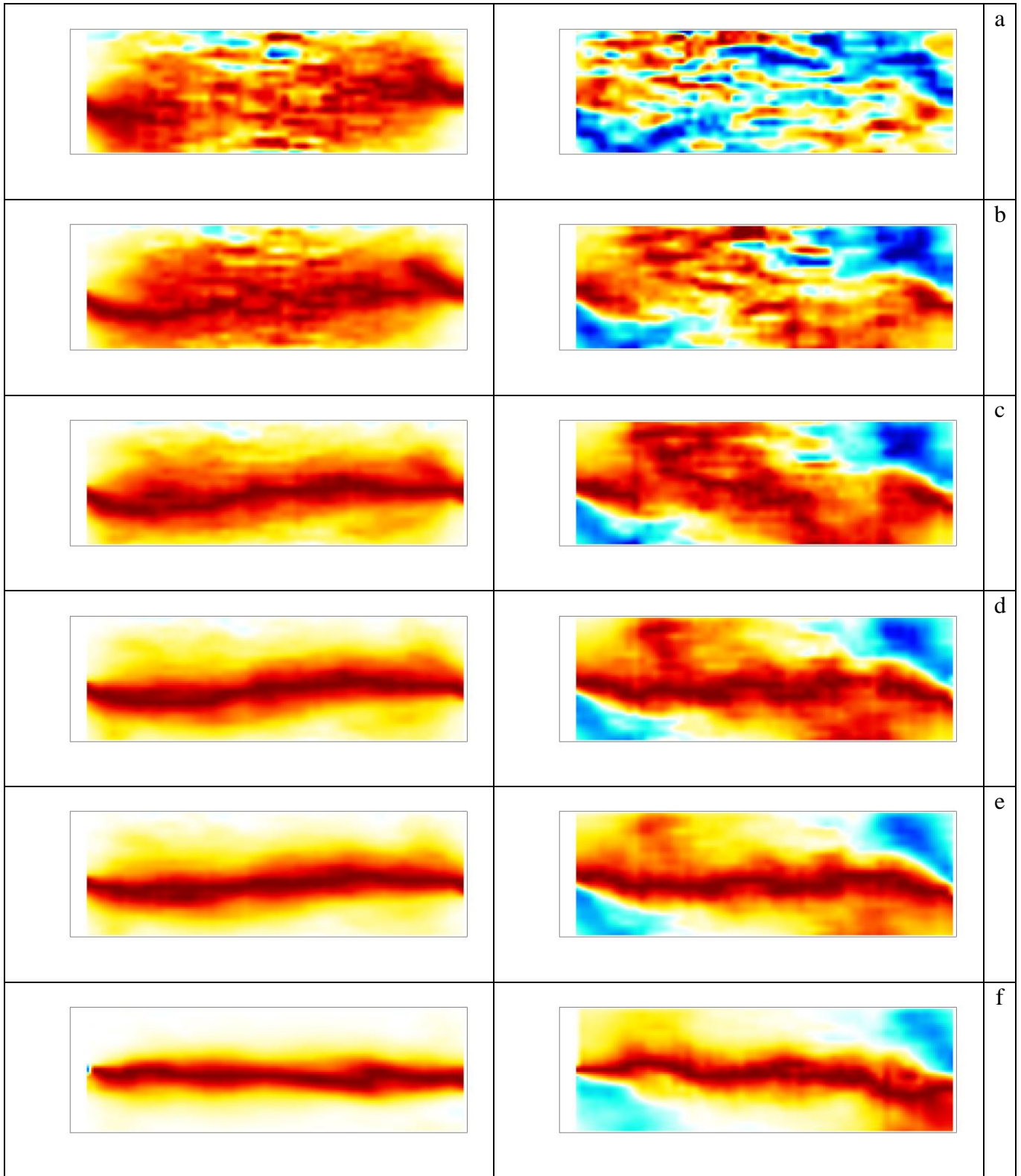


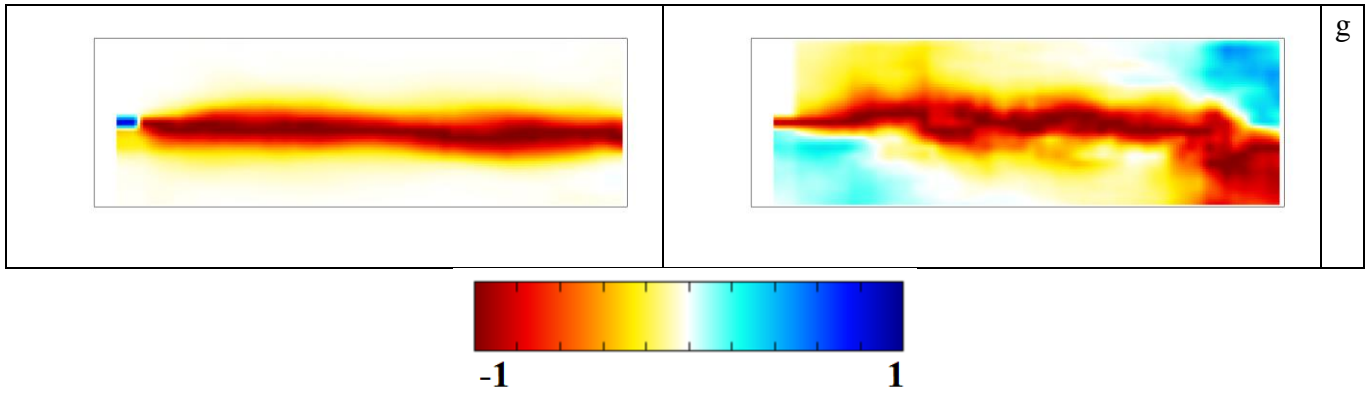


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**Fig.8:** Evolution of displacements: A)  $u_x'$  (horizontal) and B)  $u_y'$  (vertical) in the entire specimen for:  
a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  
 $u_x=7.00$  mm (red colour denotes positive displacement, blue colour denotes negative displacement)  
*(colour online)*

**FIGURE 8**





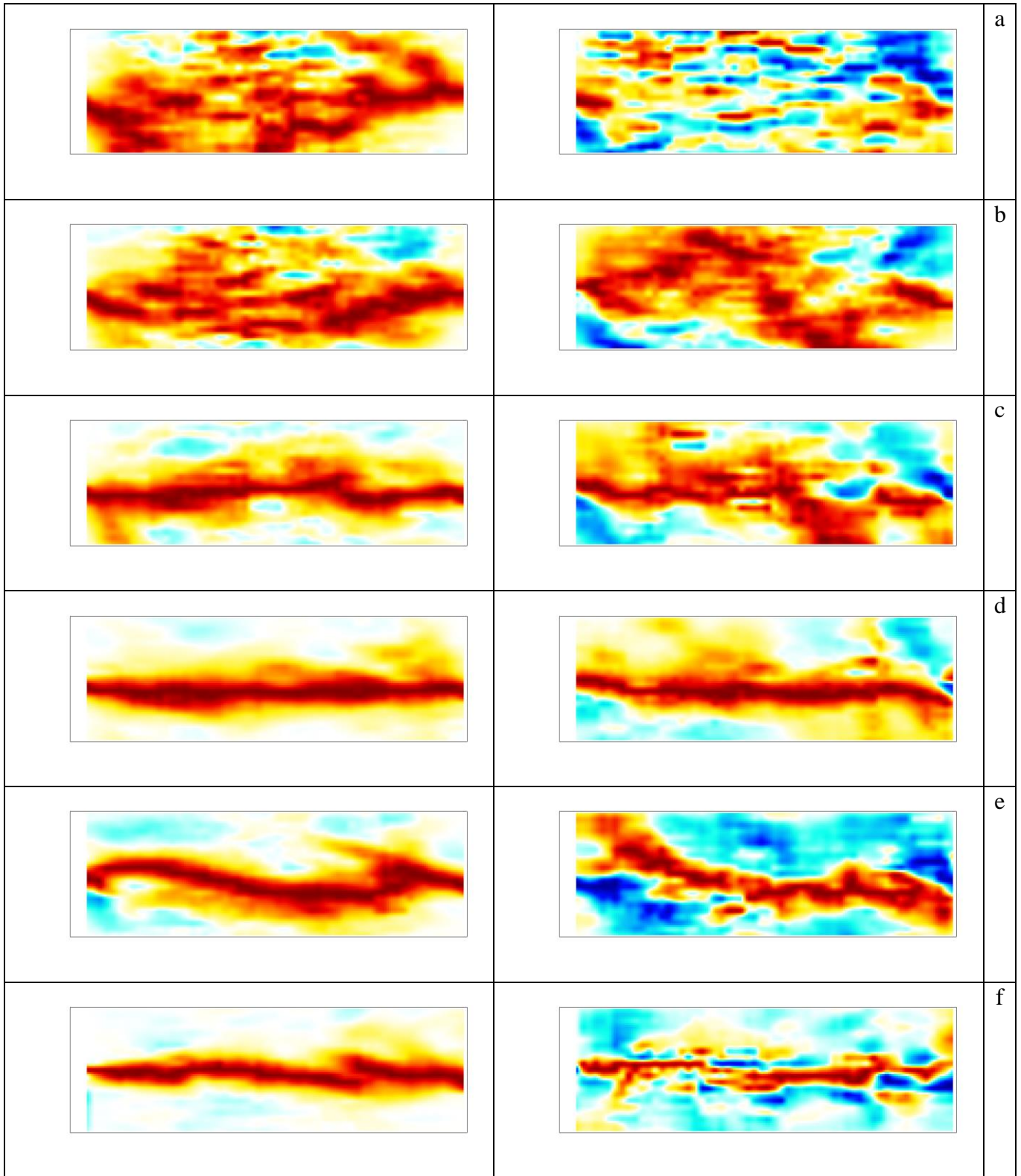
A)

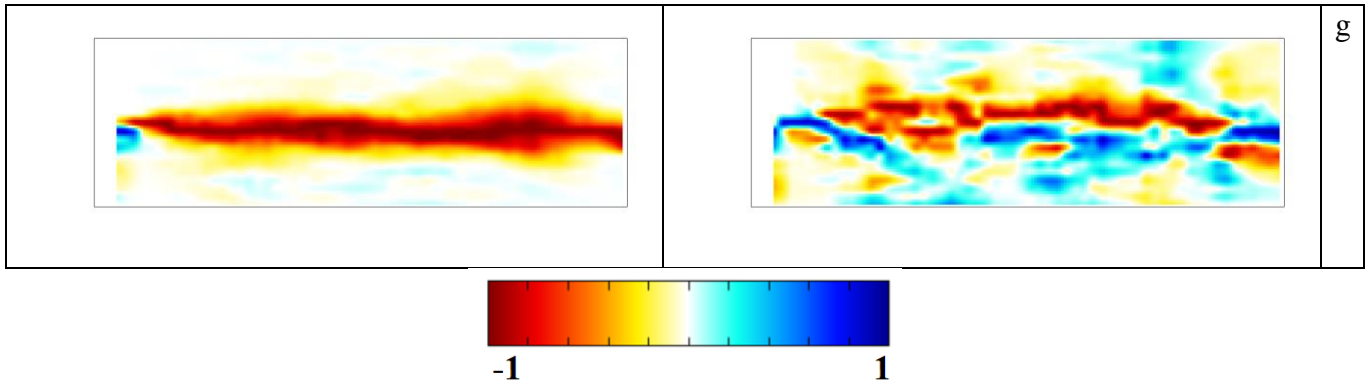
B)

**Fig.9:** Evolution of the gradient of displacements: A)  $u_x^*$  (horizontal) and B)  $u_y^*$  (vertical) in the entire specimen for: a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  $u_x=7.00$  mm (blue colour denotes increase, red colour denotes decrease) (*colour online*)

## FIGURE 9





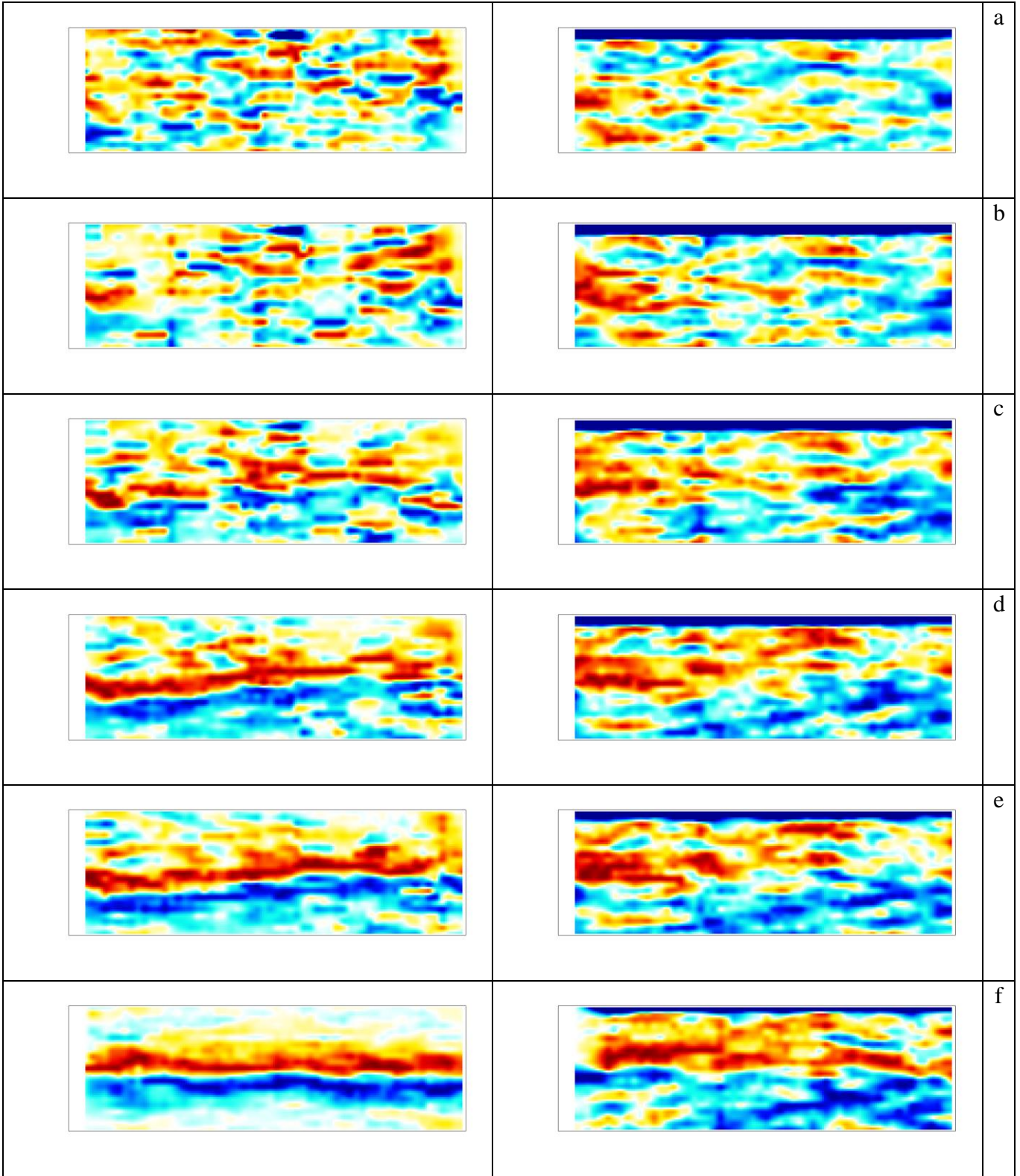


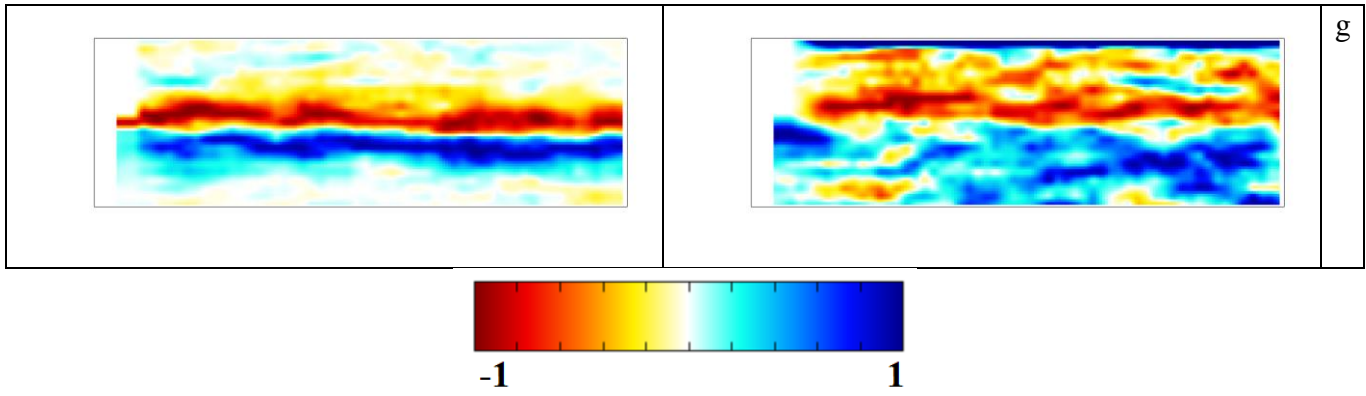
A)

B)

**Fig.10:** Evolution of gradient of velocities: A)  $v_x$  (horizontal) and B)  $v_y$  (vertical) in the entire specimen for: a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  $u_x=7.00$  mm (red colour denotes increase, blue colour denotes decrease) (*colour online*)

## FIGURE 10



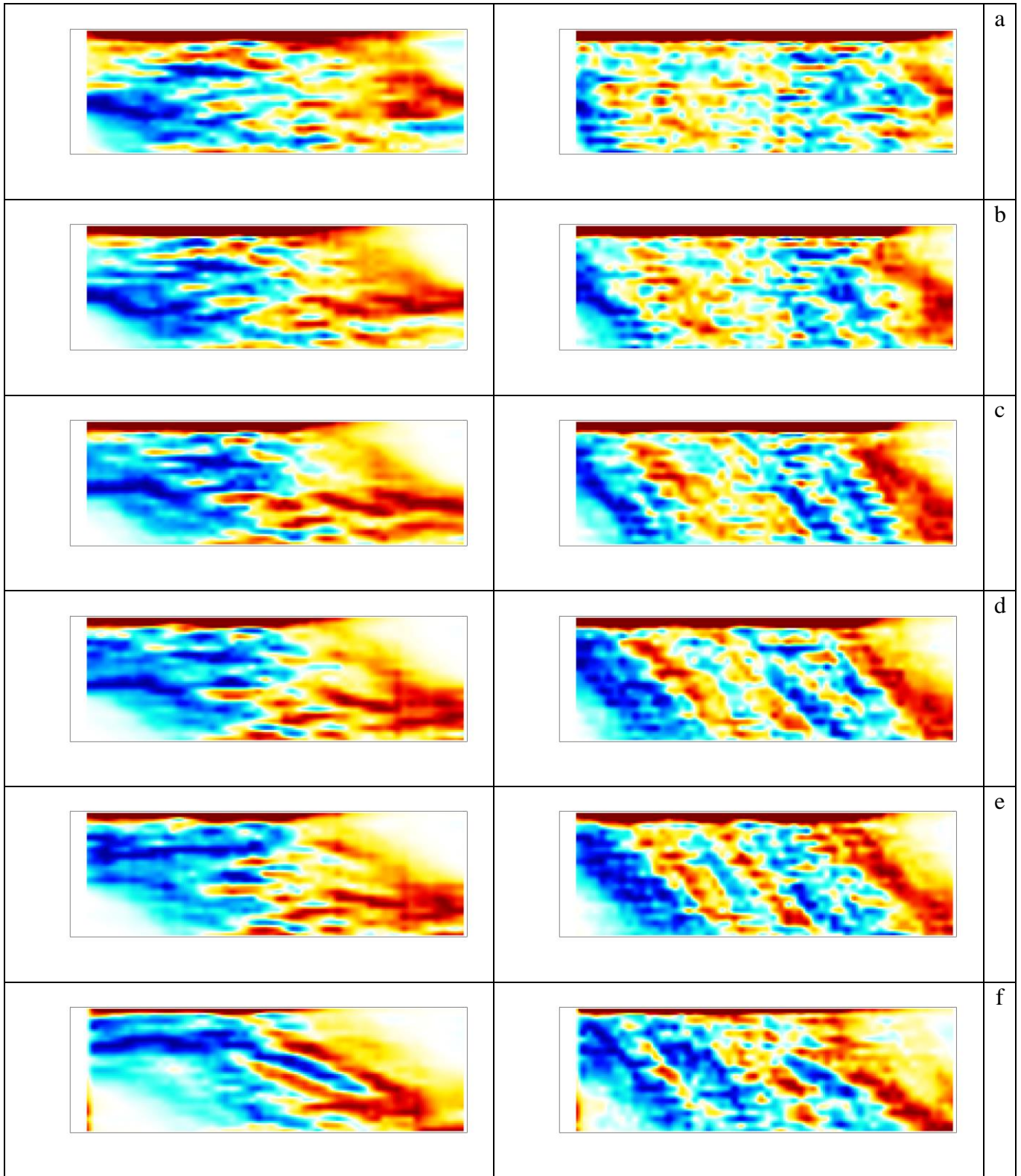


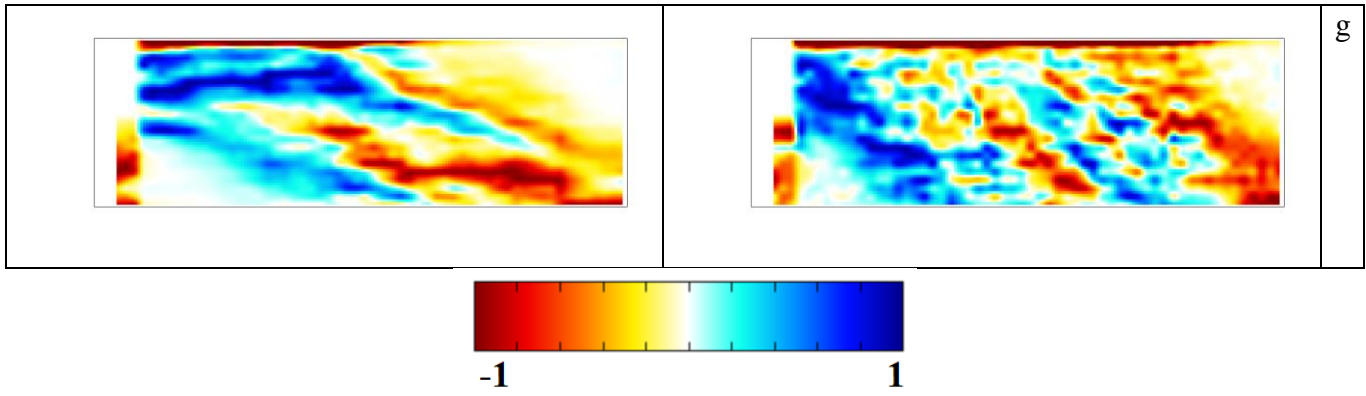
A)

B)

**Fig.11:** Evolution of gradient of A) rolling and B) contact number in the entire specimen for:  
 a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  
 $u_x=7.00$  mm (red colour denotes increase, blue colour denotes decrease) (*colour online*)

## FIGURE 11



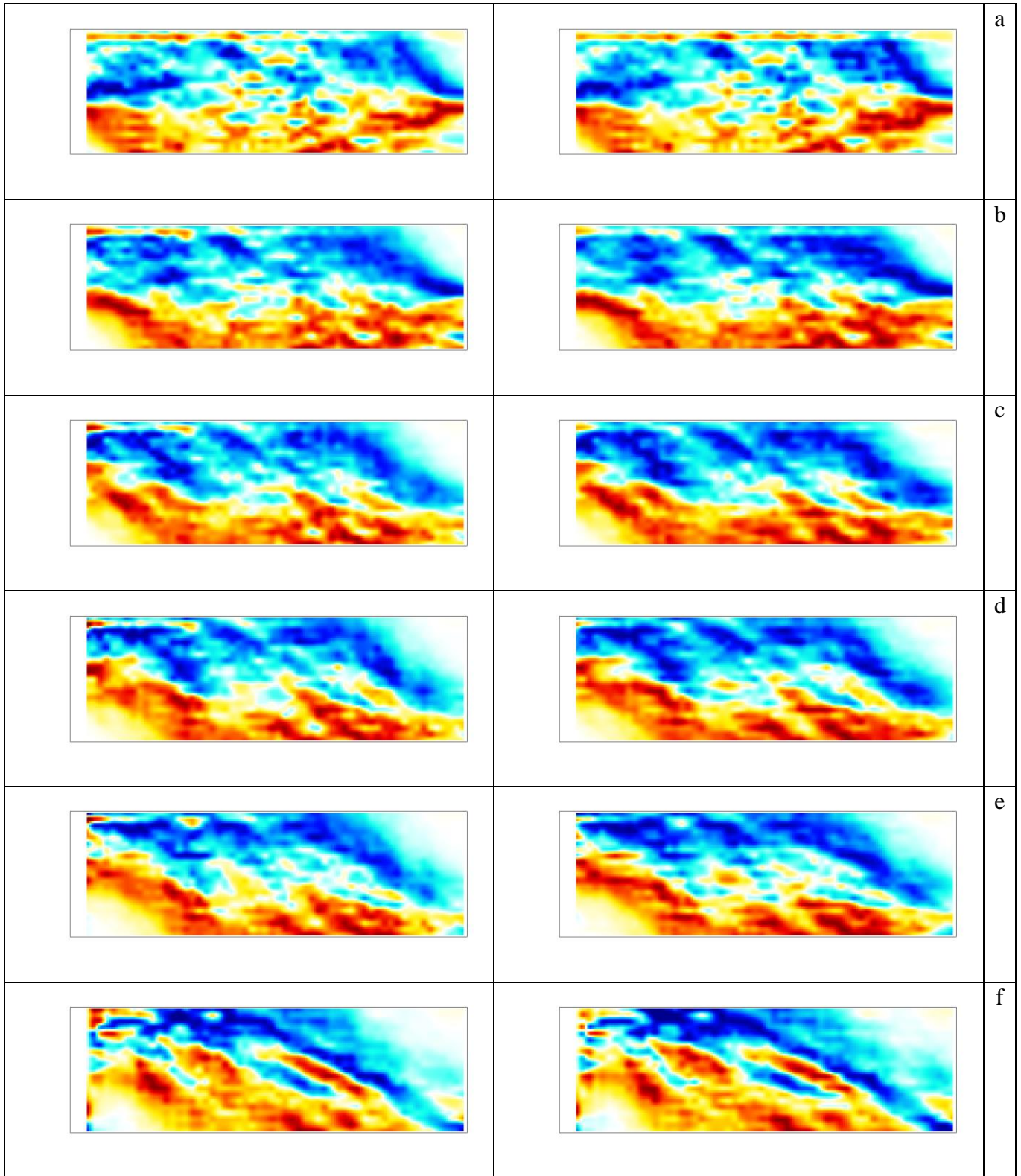


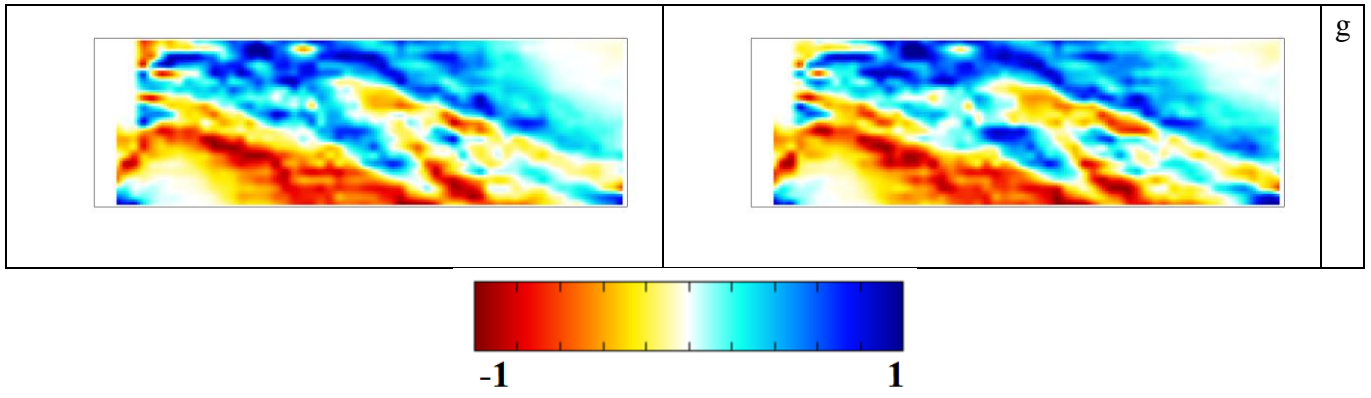
A)

B)

**Fig.12:** Evolution of gradient of stresses A)  $\sigma_{xx}$  and B)  $\sigma_{yy}$  in the entire specimen for: a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  $u_x=7.00$  mm (red colour denotes increase, blue colour denotes decrease) (*colour online*)

## FIGURE 12





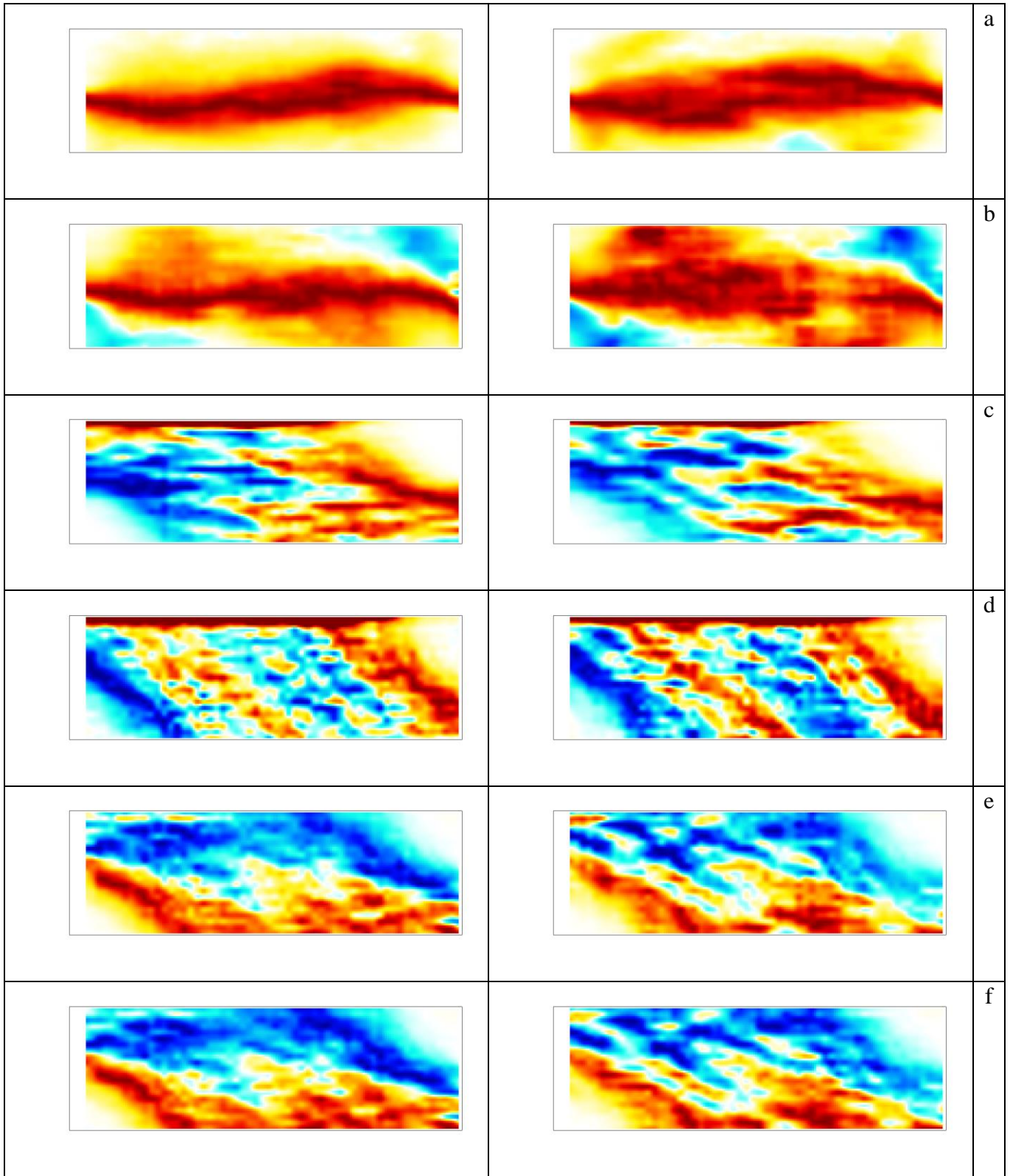
A)

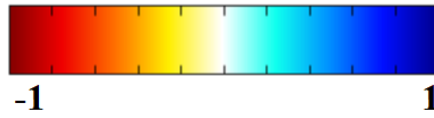
B)

**Fig.13:** Evolution of gradient of stresses A)  $\sigma_{yx}$  and B)  $\sigma_{yx}$  in the entire specimen for: a)  $u_x=0.10$  mm, b)  $u_x=0.50$  mm, c)  $u_x=1.00$  mm, d)  $u_x=1.50$  mm, e)  $u_x=2.00$  mm, f)  $u_x=5.00$  mm and g)  $u_x=7.00$  mm (red colour denotes increase, blue colour denotes decrease) (*colour online*)

## FIGURE 13







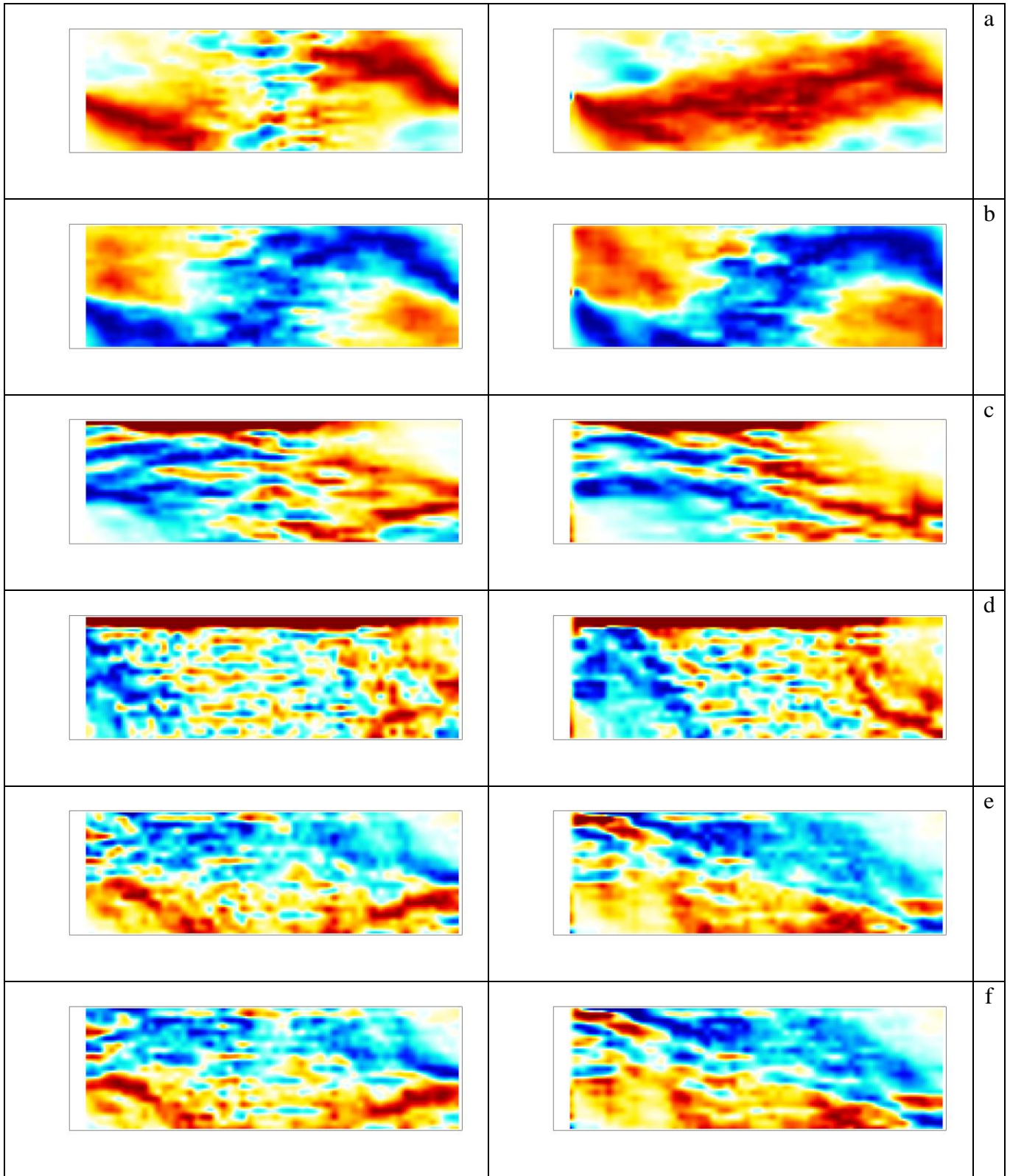
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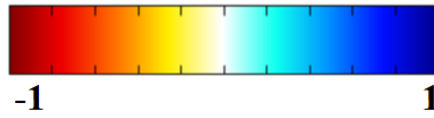
A)

B)

**Fig.14:** Maps of gradient for specimen with initial pressure equal: A)  $\sigma_0 = 50$  kPa and B)  $\sigma_0 = 500$  kPa for entire specimen for  $u_x=1.00$  mm: a) horizontal displacement ( $u_x^*$ ) b) vertical displacement ( $u_y^*$ ), c) horizontal stresses ( $\sigma_{xx}$ ), d) vertical stresses ( $\sigma_{yy}$ ), e) shear stresses ( $\sigma_{xy}$ ) and f) shear stresses ( $\sigma_{yx}$ ) (red colour denotes increase, blue colour denotes decrease) (*colour online*)

**FIGURE 14**





A)

B)

**Fig.15:** Maps of gradient for initial loose specimen ( $e_0=0.75$ ) for entire specimen for A)  $u_d=1.00$  mm (pre-peak) and B)  $u_d=5.00$  mm (residual part): a) horizontal displacement ( $u_x^*$ ) b) vertical displacement ( $u_y^*$ ), c) horizontal stresses ( $\sigma_{xx}$ ), d) vertical stresses ( $\sigma_{yy}$ ), e) shear stresses ( $\sigma_{xy}$ ) and f) shear stresses ( $\sigma_{yx}$ ) (red colour denotes increase, blue colour denotes decrease) (*colour online*)

**FIGURE 15**