Malikan M., Wiczenbach T., Eremeyev V.A. (2022) Flexomagneticity in Functionally Graded Nanostructures. In: Altenbach H., Eremeyev V.A., Galybin A., Vasiliev A. (eds) Advanced Materials Modelling for Mechanical, Medical and Biological Applications. Advanced Structured Materials, vol 155. Springer, Cham. https://doi.org/10.1007/978-3-030-81705-3_17

Flexomagneticity in functionally graded nanostructures

Mohammad Malikan, Tomasz Wiczenbach, and Victor A. Eremeyev

Abstract Functionally graded structures have shown the perspective of materials in a higher efficient and consistent manner. This study reports a short investigation by concentrating on the flexomagnetic response of a functionally graded piezomagnetic nano actuator, keeping in mind that the converse magnetic effect is only taken into evaluation. The rule of mixture assuming exponential composition of properties along with the thickness is developed for the ferromagnetic bulk. Nonlocal effects are assigned to the model, respecting Eringen's hypothesis. The derived equations deserve to be analytically solved. Therefore, numerical results are generated for fully fixed ends. It is denoted that the functionality grading feature of a magnetic nanobeam can magnify the flexomagnetic effect leading to high-performance nano sensors/actuators.

Keywords: flexomagneticity, nonlocal elasticity, FGM, nanostructures.

Nomenclature

σ_{xx}	Stress component	k	Material property variation
τ_{xz}	Shear stress	I_z	Area moment of inertia
ξ_{xxz}	Hyper stress	u	Axial displacement of the mid-plane
η_{xxz}	Hyper strain	w	Transverse displacement of the mid-plane
ε_{xx}	Strain component	ϕ	Rotation of beam nodes around the y axis
γ_{xz}	Shear strain	q_{31}	Component of the third-order piezomagnetic tensor
E	Elasticity modulus	g_{31}	Component of the sixth-order gradient elasticity tensor
G	Shear modulus	f_{31}	Component of fourth-order flexomagnetic tensor
u_1	Displacement along x	a_{33}	Component of the second-order magnetic permeability tensor
u_3	Displacement along z	A	Area of the cross-section of the beam
ν	Poisson's ratio	N_x	Axial stress resultant
L	Length of the beam	M_x	Moment stress resultant
b	Width of the beam	Q_x	Shear stress resultant
z	Thickness coordinate	T_{xxz}	Hyper stress resultant
h	Thickness of the beam	ψ	Magnetic potential
k_s	Shear correction factor		

Mohammad Malikan[1] . Tomasz Wiczenbach[1] . Victor A. Eremeyev[1,2,3] (✉)

[1]Department of Mechanics of Materials and Structures, Faculty of Civil and Environmental Engineering, Gdansk University of Technology, 80-233, Gdansk, Poland

[2]Don State Technical University, Gagarina sq., 1, Rostov on Don 3440 0 0, Russia

[3]DICAAR, Università degli Studi di Cagliari, Via Marengo, 2, 09123, Cagliari, Italy

Emails: mohammad.malikan@pg.edu.pl, Tomasz.Wiczenbach@pg.edu.pl, victor.eremeev@pg.edu.pl

1 Introduction

The development of technology leads to the discovery of new, more complex properties of materials. One of the recently discovered ones is flexomagnetism (FM). This phenomenon is currently being investigated by researchers under the influence of static and dynamic states. This effect is based on a strain gradient, which can be called the inverse flexomagnetic effect. The flexo-effect can be observed in direct impact by the presence of an exterior magnetic field gradient. FM effect can occur in crystalline structures and all types of materials, see Fahrner (2005); Lukashev and Sabirianov (2010); Pereira et al (2012); Zhang et al (2012); Zhou et al (2014); Moosavi et al (2017); Eliseev et al (2019); Kabychenkov and Lisovskii (2019); Eliseev et al (2009).

A special case of the composite material is functionally graded materials (FGMs). A characteristic feature of FG materials is a continuous and smooth transition between the various components, see Hadj Mostefa et al (2018); Loh et al (2018); Mahamood et al (2012); Vasiliev et al (2017); Malikan and Eremeyev (2020a); Liu et al (2021); Volkov et al (2019). For instance, between a ceramic material with a very high thermal resistance (low thermal conductivity and expansion coefficient) and a metal with good heat conduction (high thermal conductivity and expansion coefficient), there is a transition phase (interface), in which a smooth, continuous transition, eliminating the formation of microdamage or delamination of the material at the interface is obtained. The smooth transition is achieved by changing the volumetric fraction of the individual components of the composite, causing the effective thermomechanical properties to change from one piece (e.g., metal or metal alloy) to the other part (e.g. ceramic material). In the case of thermal barrier coating (TBC), the thickness of the FGM interface is small compared to the entire volume of the material.

Afterwards, flexomagneticity effect being discovered, several research has been conducted on small scale sensors and actuators and their static or dynamic response subjected to the effect. Within these publications, Zhang et al (2019) concentrated on a nano actuator beam exposed to bending and subjected to the flexomagneticity effect. The static bending equations have been developed following the Euler-Bernoulli beam theory. What is more, the surface elasticity was considered. Diverse boundary conditions were chosen following direct and converse magnetization. According to the obtained results, it could be obtained that the material property, which is the flexomagneticity effect, is size-dependent. Differently, Sidhardh and Ray (2018) investigated the Euler-Bernoulli nanosize beam and clamped-free ends boundary conditions beam subjected to bending with piezo-flexomagnetic effect. Direct and inverse magnetization effects were considered. With the elasticity surface, the examination of the size-dependency was performed for the small beam. The following results present scale-dependent flexomagneticity behaviour. Studies of nanostructures have shown that this effect is very significant for the obtained results, despite the omission of the piezomagneticity.

A comprehensive examination of functionally graded materials with electro-magneto-elastic coupling may be found considering a detailed literature review. Nonetheless, the flexomagnetic phenomena in FGM materials cannot be found in the literature. According to this, the need to investigate the flexomagnetic effect in FGM materials is crucial. Moreover, the impact of flexomagneticity on micro-and nanostructures can be found in recent publications, see Zhang et al (2019); Sidhardh and Ray (2018); Malikan and Eremeyev (2020b); Malikan and Eremeyev (2020c); Malikan et al (2020a); Malikan et al (2020b); Malikan et al (2021); Malikan and Eremeyev (2021). Although research on flexomagnetism has been conducted in recent years, this phenomenon still requires much additional research and raises many questions.

Best of our knowledge, no studies are performed about examining functionally graded materials (FGMs) composed as a nano-actuator beam with the flexomagnetic effect. In this study, we intend to investigate the nanostructured FGM beams containing the flexomagnetic effect. Performed computation to evaluate the small-scale effect following strain gradient elasticity of the nonlocal model. The computed numerical results refer to the semi-analytical method. According to the variations in crucial and significant criteria, the illustrating graphs present a magneto-mechanical model.

2 Mathematical modeling

Let us attach the square nanobeam containing L and h as length and thickness dimensions to a rectangular coordinate system as manifested by Fig. 1.

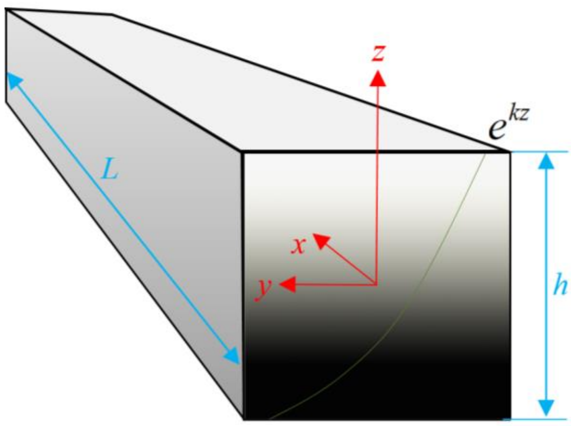

Fig. 1. Hypothetical perspective image of a smart FGM nanobeam in the Cartesian coordinate

The ferromagnetic functionally graded structure with apparent piezomagnetic effect and obscure flexomagnetic influence should concatenate shear deformations to the modeling. Due to this, the Timoshenko beam is preferred in the theoretical model as in Thanh et al (2021)

$$u_1(x,z) = u(x) + (z - z_0)\phi(x) \tag{1a}$$

$$u_3(x,z)=w(x) \tag{1b}$$

In a FGM structure with the physical neutral plane, the neutral surface is not matched to the mid-plane and deviates a bit; however, it could be valid for the case of a geometrical model of FGMs. Physically, the neutral plane does not include any strain and stress during a pure bending deformation. One can describe the position of the neutral surface by, see Thanh et al (2021); Chu et al (2018); Ahmed Hassan et al (2020),

$$z_0=\frac{\int_{-h/2}^{h/2} zE(z)\,dz}{\int_{-h/2}^{h/2} E(z)\,dz} \tag{2}$$

where the shifting between the geometric mid-plane and the physical neutral surface is denoted by z_0.

The use of Voight's theorem can define functionally graded properties, also called the rule of mixture, a micromechanics model specifying functionality at any point that generally falls within three functions, namely Sigmoid exponential and power ones. These functions can model the functionality grading property through the thickness or another dimension for a beam/plate. In between those functions, the exponential one is applied in this study as in Atmane et al (2011),

$$P(z)=P_0e^{kz} \tag{3}$$

where the property $P(z)$ can be any effective property at any point of z, P_0 depicts that property at mid-plane (z=0), k shows the index of material property variation in line with the thickness direction. The homogeneous isotropic nanobeam can be described by k=0. In this study, in light of the narrow range of Poisson's ratio for various materials, it is assumed that the value of Poisson's ratio is constant for the FGM.

In the skeleton of Lagrangian strain and based on constituting linearization relevance to stability phenomenon, one writes

$$\varepsilon_{xx}=\frac{du}{dx}+(z-z_0)\frac{d\phi}{dx} \tag{4}$$

$$\gamma_{xz}=\phi+\frac{dw}{dx} \tag{5}$$

$$\eta_{xxz}=\frac{d\varepsilon_{xx}}{dz}=\frac{d\phi}{dx} \tag{6}$$

Lagrange's principle expresses energy formula on the time-independent theme as

$$\delta\int(W+U)=0 \tag{7}$$

in which W and U respectively states thermodynamic work accomplished by external forces and the internal strain energy.

The expanded and variated form of strain energy for a beam is written by

$$\delta U=\int_V\left(\sigma_{xx}\delta\varepsilon_{xx}+\tau_{xz}\delta\gamma_{xz}+\xi_{xxz}\delta\eta_{xxz}-B_z\delta H_z\right)dV \tag{8}$$

subsequent to Eq. (8), there will be risen a few one or two-dimensional relations which can be arranged as follows (one dimensional relates to non-classical boundary conditions and two-dimensional correlates to the equilibrium equations),

$$\delta U^{Mech}=-\int_0^L\left(\frac{dN_x}{dx}\delta u+\frac{dQ_x}{dx}\delta w-Q_x\delta\phi+\frac{dM_x}{dx}\delta\phi+\frac{dT_{xxz}}{dx}\delta\phi\right)dx \tag{9a}$$

$$\delta U^{Mag}=-\int_0^L\int_{-h/2}^{h/2}\frac{dB_z}{dz}\delta\Psi dzdx \tag{9b}$$

$$\delta U^{Mech}=\left(N_x\delta u+Q_x\delta w+M_x\delta\phi+T_{xxz}\delta\phi\right)\Big|_0^L \tag{10a}$$

$$\delta U^{Mag}=\int_0^L\left(B_z\delta\Psi\right)\Big|_{-h/2}^{h/2}dx \tag{10b}$$

in which

$$N_x=\int_{-h/2}^{h/2}\sigma_{xx}dz \tag{11}$$

$$M_x=\int_{-h/2}^{h/2}\sigma_{xx}zdz \tag{12}$$

$$Q_x=k_s\times\int_{-h/2}^{h/2}\tau_{xz}dz \tag{13}$$

$$T_{xxz}=\int_{-h/2}^{h/2}\xi_{xxz}dz \tag{14}$$

Axial compressive load acting on the ends of the nanobeam works on the domain as in Malikan et al (2020c); Malikan and Eremeyev (2020d)

$$W=\frac{1}{2}\int_0^L N_x^0\left(\frac{dw}{dx}\right)^2dx \tag{15}$$

After that, Eq. (15) can be developed by variational technique as

$$\delta W=\int_0^L\left(N_x^0\frac{d\delta w}{dx}\frac{dw}{dx}\right)dx \tag{16}$$

in which the N_x^0 plays the role of buckling force.

It is considered that the magnetic field acts laterally for which one can formulate the following component along the transverse axis,

$$H_z+\frac{d\Psi}{dz}=0 \tag{17}$$

It is deemed that the smart beam is exposed to the reverse flexomagnetic effect. This is mathematically possible concerning the pursuant relation by which one defines a closed-circuit magnetic potential with the utmost potential on the top surface and the least one on the undermost surface of the thickness,

$$\Psi\left(+\frac{h}{2}\right)=\psi,\ \Psi\left(-\frac{h}{2}\right)=0 \tag{18a,b}$$

Let us mingle Eqs. (6, 9b, 10b, 17, and 18) to each other alongside with a few mathematical attempts, then one can obtain the relations of magnetic potential in the thickness direction and the vertical magnetic field,

$$\Psi=\frac{q_{31}(z)}{2a_{33}(z)}\left((z-z_0)^2-\frac{h^2}{4}\right)\frac{d\phi}{dx}+\frac{\psi}{h}\left((z-z_0)+\frac{h}{2}\right) \tag{19}$$

$$H_z=-(z-z_0)\frac{q_{31}(z)}{a_{33}(z)}\frac{d\phi}{dx}-\frac{\psi}{h} \tag{20}$$

There are various ways to study nanostructures. Experimental observations are one of the methods of modeling small scale materials, which due to its high cost, other methods such as atomic modeling, and mechanics of continuous environments are proposed. Modeling by continuous media is much less expensive than the previous two methods and includes Eringen's theory of nonlocal elasticity as Dastjerdi et al (2021), nonlocal strain gradient elasticity theory as Malikan et al (2020d), and modified coupling stress theory as Malikan (2017), Skrzat and Eremeyev (2020), etc. as Romano and Barretta (2017) for nanostructures. Therefore, continuous environment modeling can be used to analyze nanostructures such as buckling of these scaled-down structures. Meanwhile, Eringen's theory of nonlocal elasticity has relatively more straightforward governing relations and calculates the nonlocal effect of small scale to nanoscale structures which is given by the expression as in Eringen (1983),

$$\left(1-\mu\frac{d^2}{dx^2}\right)\sigma_{ij}=C_{ijkl}\varepsilon_{ij} \tag{21}$$

where an additional length scale parameter is exhibited that is called nonlocal parameter and its value has been obtained for some nanostructures in the range of zero up to four square nanometers as in Ansari et al (2010). The parameter μ equals $(e_0a)^2$ including e_0 as a nonlocal quantity and a that is a characteristic length of materials.

Afterwards, Eq. (21) is imposed on Eqs. (11-14) leading to the following relations,

$$\left(1-\mu\frac{d^2}{dx^2}\right)\xi_{xxz}=\left(g_{31}(z)+\frac{q_{31}(z)\,f_{31}(z)(z-z_0)}{a_{33}(z)}\right)\frac{d\phi}{dx}+\frac{f_{31}(z)\psi}{h} \tag{22}$$

$$\left(1-\mu\frac{d^2}{dx^2}\right)\sigma_{xx}=E(z)\frac{du}{dx}+(z-z_0)\left(E(z)+\frac{q_{31}^2(z)}{a_{33}(z)}\right)\frac{d\phi}{dx}+\frac{q_{31}(z)\psi}{h} \tag{23}$$

$$\left(1-\mu\frac{d^2}{dx^2}\right)\tau_{xz}=G(z)A\left(\phi+\frac{dw}{dx}\right) \tag{24}$$

Substituting Eq. (21) in Eqs. (11-14) eventuates the nonlocal stress resultants as,

$$\left(1-\mu\frac{d^2}{dx^2}\right)N_x=I_1\frac{du}{dx}+\left(I_2+I_3\right)\frac{d\phi}{dx}+I_4 \tag{25}$$

$$\left(1-\mu\frac{d^2}{dx^2}\right)M_x=I_5\frac{du}{dx}+\left(I_6+I_7\right)\frac{d\phi}{dx}+I_8 \tag{26}$$

$$\left(1-\mu\frac{d^2}{dx^2}\right)Q_x=H_{44}\left(\phi+\frac{dw}{dx}\right) \tag{27}$$

$$\left(1-\mu\frac{d^2}{dx^2}\right)T_{xxz}=\left(I_9+I_{10}\right)\frac{d\phi}{dx}+I_{11} \tag{28}$$

where the determined parameters can be organized as,

$$\begin{gathered}\left\{I_1,I_2\right\}=\int_{-h/2}^{h/2}E(z)\left\{1,(z-z_0)\right\}dz,\ I_3=\int_{-h/2}^{h/2}(z-z_0)\frac{q_{31}^2(z)}{a_{33}(z)}dz,\ I_4=\int_{-h/2}^{h/2}\frac{\psi q_{31}(z)}{h}dz,\\ \left\{I_5,I_6\right\}=\int_{-h/2}^{h/2}E(z)\left\{(z-z_0),(z-z_0)^2\right\}dz,\ I_7=\int_{-h/2}^{h/2}(z-z_0)^2\frac{q_{31}^2(z)}{a_{33}(z)}dz,\ I_8=\int_{-h/2}^{h/2}(z-z_0)\frac{\psi q_{31}(z)}{h}dz,\\ I_9=\int_{-h/2}^{h/2}g_{31}(z)dz,\ I_{10}=\int_{-h/2}^{h/2}(z-z_0)\frac{q_{31}(z)f_{31}(z)}{a_{33}(z)}dz,\ I_{11}=\int_{-h/2}^{h/2}\frac{\psi f_{31}(z)}{h}dz,\ H_{44}=k_s\int_{-h/2}^{h/2}G(z)Adz\end{gathered} \tag{29}$$

This is the time to pull the governing equations out of the Eqs. (9, 10, and 16) and sort them as

$$\frac{dN_x}{dx}=0 \tag{30}$$

$$\frac{dQ_x}{dx}+N_x^0\frac{d^2w}{dx^2}=0 \tag{31}$$

$$\frac{dM_x}{dx}+\frac{dT_{xxz}}{dx}-Q_x=0 \tag{32}$$

Thereupon, Eqs. (25-28) with the aid of Eqs. (30-32) can be uncomplicated as,

$$N_x=I_1\frac{du}{dx}+\left(I_2+I_3\right)\frac{d\phi}{dx}+I_4 \tag{33}$$

$$M_x=-\mu\left[\left(I_9+I_{10}\right)\frac{d^3\phi}{dx^3}+N_x^0\frac{d^2w}{dx^2}\right]+I_5\frac{du}{dx}+\left(I_6+I_7\right)\frac{d\phi}{dx}+I_8 \tag{34}$$

$$Q_x=-\mu N_x^0\frac{d^3w}{dx^3}+H_{44}\left(\phi+\frac{dw}{dx}\right) \tag{35}$$

$$T_{xxz}=\mu\frac{d^2T_{xxz}}{dx^2}+\left(I_9+I_{10}\right)\frac{d\phi}{dx}+I_{11} \tag{36}$$

Let us recast Eqs. (30-32) based on Eqs. (33-36) as

$$I_1\frac{d^2u}{dx^2}+\left(I_2+I_3\right)\frac{d^2\phi}{dx^2}=0 \tag{37}$$

$$\left(1-\mu\frac{d^2}{dx^2}\right)\left\{N_x^0\frac{d^2w}{dx^2}\right\}+H_{44}\left(\frac{d\phi}{dx}+\frac{d^2w}{dx^2}\right)=0 \tag{38}$$

$$\left(1-\mu\frac{d^2}{dx^2}\right)\left\{\left(I_9+I_{10}\right)\frac{d^2\phi}{dx^2}\right\}+I_5\frac{d^2u}{dx^2}+\left(I_6+I_7\right)\frac{d^2\phi}{dx^2}-H_{44}\left(\phi+\frac{dw}{dx}\right)=0 \tag{39}$$

To compute the buckling load, Eqs. (37-39) which are coupled together shall be solved.

The combination of longitudinal magnetic force together with the axial mechanical compressive load results in the total axial load as

$$N_x^0=P_{cr}+q_{31}\psi \tag{40}$$

3 Solving procedure

Mathematically, we can implement all boundary conditions for a FGM system. However, inspecting the physics of a FGM involving the transition in the neutral plane, Karamanli and Aydogdu (2020) approved that while considering a FGM model consisting of z_0, there is no buckling mode for simple boundary conditions since the beam/plate will bend before bifurcation buckling. It means the buckling will not occur at this boundary condition for the physical model of FGMs. Thus, in what follows, we urge to present clamped-clamped system only brings about an analytical process as

$$u(x)=\sum_{m=1}^{\infty}\frac{d}{dx}\left[\sin^2\left(\frac{m\pi}{L}x\right)\right]\exp\left(i\omega_n t\right) \tag{41}$$

$$w(x)=\sum_{m=1}^{\infty}\sin^2\left(\frac{m\pi}{L}x\right)\exp\left(i\omega_n t\right) \tag{42}$$

$$\phi(x)=\sum_{m=1}^{\infty}\cos^2\left(\frac{m\pi}{L}x\right)\exp\left(i\omega_n t\right) \tag{43}$$

where t is the identification of time in time-dependent problems.

The foregoing series will satisfy the clamped condition ($u=w=0,\ M_x\neq 0$) at both ends for present numerical calculations for axial buckling loads of a square ferromagnetic nanobeam made of a functionally graded compound continued by obtaining residual of Eqs. (37-39),

$$\int_0^L R_1(x)u(x)\,dx=0 \tag{44}$$

$$\int_0^L R_2(x)w(x)dx = 0 \quad (45)$$

$$\int_0^L R_3(x)\phi(x)dx = 0 \quad (46)$$

where $R_i(x)(i=1,..,3)$ are exhibitions of equations' residuals. The next equation computes the critical buckling load

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{Bmatrix} u \\ w \\ \phi \end{Bmatrix} = 0 \quad (47)$$

Then

$$\det\left[K_{ij}\right] = 0 \quad (48)$$

Consequently, Eq. (48) confronts us with a polynomial relation to which the solution will keep going P_{cr}.

4 Results and discussions

Let us here illustrate the influence of functionally graded fabrication on the flexomagnetic effect detailedly upon the conduction of a parametric study. The postulation of size-dependency for functionality composition is also demonstrated. First of all, Table 1 incorporates the required material properties, which will be employed in the analysis. Second, by presenting a scientific interpretation, one observes the importance of FGMs in embossing the flexomagnetic response. Pursuing this, there are two kinds of structures, piezomagnetic FGM shown by FG-PM and piezomagnetic FGM in conjunction with flexomagneticity displayed by FG-PFM.

The mechanical and structural properties are used by, see Lu et al (2016); Balsing Rajput et al (2013); Senthil et al (2018)

Table 1. Employed structural properties

$CoFe_2O_4$
E_0=286 GPa
f_{31}=10^{-9} N/A
q_{31}=580.3 N/Am
a_{33}=1.57×10^{-4} N/A^2

The purpose of this short study is to investigate whether functional grading can lead to changes in the flexomagnetic response of materials or not. In fact, it will be more difficult to construct matter in a functional state. However, if it gives us the privilege of having a more significant flexomagnetic effect and greater polarization overall, it will have unique advantages that can persuade designers of smart magnet nanosensors and actuators to turn to functionally graded magnetic nanomaterials. In this section, with the help of Figure 2, this vital issue related to the property of flexomagnetism will be investigated. What can be seen is that the functional grading property has a direct effect on the polarization and will lead to an increase in the flexomagnetic property. As shown, with the increasing the value of the *k-index*, one sees an increase in the distance between the results of PFM and PM, especially for larger values of the nonlocal parameter. Increasing the value of the nonlocal parameter increases this difference to prove that the functionally graded property, in turn, can exhibit size-dependent behavior. Therefore, the fabrication of ferromagnetic nanomaterials as a functional scale, regardless of fabrication complexity, will provide more efficient nanosensors and magnetic nano actuators with greater efficiency and a more substantial flexomagnetic effect.

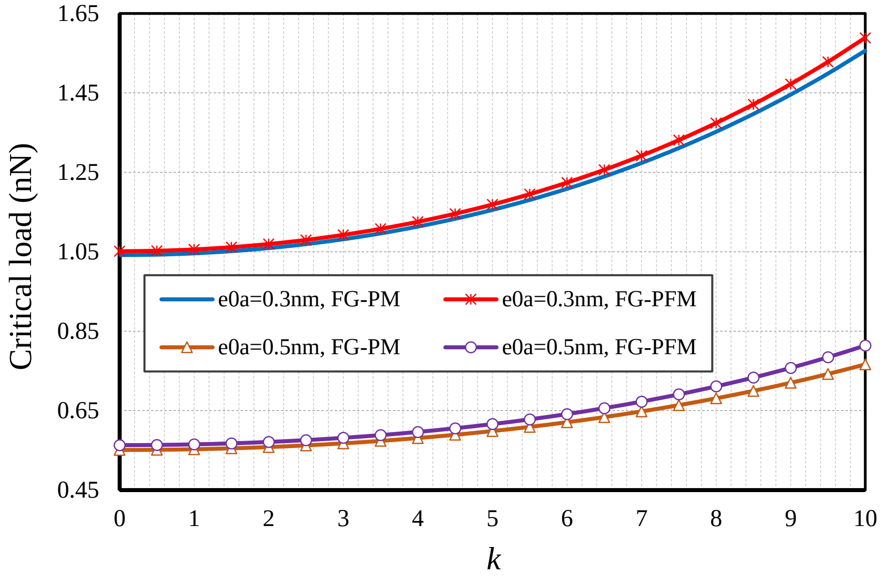

Fig. 2. FGM variation index vs. critical buckling load for various cases (Ψ=1 mA, m=1, L/h=10)

5 Conclusions

Galerkin weighted residual analytical approach has been implemented to compute critical buckling load for a ferroic functionally graded shear deformable structure inclusive of flexomagneticity. Besides, the composition of the material has been presumed as the rule of mixture corresponding to the exponential distribution. The physical neutral surface has been taken into the investigation. The size-dependent behavior has been replaced in the derived linear stability equations using Eringen's nonlocal elasticity differential model to provide nanosize effects. Further, in the framework of performing a short analysis, some new achievements have been established. It was indicated that producing piezomagnetic functionally graded nanostructure creates a more noticeable flexomagnetic effect. Notwithstanding this, the current research is short, and the results will open novel potential studies on ferric functionally graded structures and their application in small-scale sensors and actuators.

Acknowledgements
V.A. Eremeyev acknowledges the support of the Government of the Russian Federation (contract No. 14.Z50.31.0046).

References

Ahmed Hassan, AH & Kurgan, N 2020, 'Bending analysis of thin FGM skew plate resting on Winkler elastic foundation using multi-term extended Kantorovich method', *Engineering Science and Technology, an International Journal*, vol. 23, pp. 788-800.

Ait Atmane, H, Tounsi, A, Ahmed Meftah, S & Abdesselem Belhadj, H 2011, 'Free Vibration Behavior of Exponential Functionally Graded Beams with Varying Cross-section', *Journal of Vibration and Control*, vol. 17, pp. 311.

Ansari, R, Sahmani, S & Arash, B 2010, 'Nonlocal plate model for free vibrations of single-layered graphene sheets', *Physics Letters A*, vol. 375, no.1, pp. 53-62.

Balsing Rajput, A, Hazra, S & Nath Ghosh, N 2013, 'Synthesis and characterisation of pure single-phase CoFe2O4 nanopowder via a simple aqueous solution-based EDTA-precursor route', *Journal of Experimental Nanoscience*, vol. 8, pp. 629-639.

Chu, L, Dui, G & Ju, Ch 2018, 'Flexoelectric effect on the bending and vibration responses of functionally graded piezoelectric nanobeams based on general modified strain gradient theory', *Composite Structures*, vol. 186, pp. 39-49.

Sh. Dastjerdi, M. Malikan, R. Dimitri, F. Tornabene, Nonlocal elasticity analysis of moderately thick porous functionally graded plates in a hygro-thermal environment, Composite Structures 255 (2021) 112925. Doi: 10.1016/j.compstruct.2020.112925

Eliseev, EA, Morozovska, AN, Glinchuk, MD & Blinc, R 2009, 'Spontaneous flexoelectric/flexomagnetic effect in nanoferroics', *Physical Review B*, vol. 79, pp. 165433.

Eliseev, EA, Morozovska, AN, Khist, VV & Polinger, V 2019, *effective flexoelectric and flexomagnetic response of ferroics*, Elsevier, Netherlands.

Eringen, AC 1983, 'On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves', *Journal of Applied Physics*, vol. 54, pp. 4703–4710.

Fahrner, W 2005, *Nanotechnology and Nanoelectronics*, Springer, Germany.

Hadj Mostefa, A, Merdaci, S & Mahmoudi, N 2018, *An Overview of Functionally Graded Materials «FGM»*, Springer, cham.

Kabychenkov, AF & Lisovskii, FV 2019, 'Flexomagnetic and flexoantiferromagnetic effects in centrosymmetric antiferromagnetic materials', *Technical Physics*, vol. 64, pp. 980-983.

Karamanli, A & Aydogdu, M 2020, 'Bifurcation buckling conditions of FGM plates with different boundaries', *Composite Structures*, vol. 245, pp. 112325.

Liu, TJ, Yang, F, Yu, H & Aizikovich, SM 2021, 'Axisymmetric adhesive contact problem for functionally graded materials coating based on the linear multi-layered model', *Mechanics Based Design of Structures and Machines*, vol. 49, pp. 41-58.

Loh, GH, Pei, E, Harrison, D & Monzón, MD 2018, 'An overview of functionally graded additive manufacturing', *Additive Manufacturing*, vol. 23, pp. 34–44.

Lu, Z-l, Gao, P-z, Ma, R-x, Xu, J, Wang, Z-h & Rebrov, EV 2016, 'Structural, magnetic and thermal properties of one-dimensional CoFe2O4 microtubes', *Journal of Alloys and Compounds*, vol. 665, 428-434.

Lukashev, P and Sabirianov, RF 2010, 'Flexomagnetic effect in frustrated triangular magnetic structures', *Physical Review B*, vol. 82, pp. 094417.

Mahamood, RM, Akinlabi, ET, Shukla, M & Pityana, SL 2012, *Functionally Graded Material: An overview*, London, UK.

M. Malikan, Electro-mechanical shear buckling of piezoelectric nanoplate using modified couple stress theory based on simplified first order shear deformation theory, Applied Mathematical Modelling 48 (2017) 196-207. https://doi.org/10.1016/j.apm.2017.03.065

M. Malikan, V. A. Eremeyev, A new hyperbolic-polynomial higher-order elasticity theory for mechanics of thick FGM beams with imperfection in the material composition, Composite Structures 249 (2020) 112486. Doi: 10.1016/j.compstruct.2020.112486

M. Malikan, V. A. Eremeyev, Free Vibration of Flexomagnetic Nanostructured Tubes Based on Stress-driven Nonlocal Elasticity, Springer Nature, Switzerland, (2020). https://doi.org/10.1007/978-3-030-47491-1_12

M. Malikan, V. A. Eremeyev, On the geometrically nonlinear vibration of a piezo-flexomagnetic nanotube, Mathematical Methods in the Applied Sciences, (2020). https://doi.org/10.1002/mma.6758

M. Malikan, V. A. Eremeyev, On the Dynamics of a Visco–Piezo–Flexoelectric Nanobeam, Symmetry 12 (2020) 643. https://doi.org/10.3390/sym12040643

M. Malikan, V. A. Eremeyev, Flexomagnetic response of buckled piezomagnetic composite nanoplates, Composite Structures 267 (2021) 113932. Doi: 10.1016/j.compstruct.2021.113932

M. Malikan, V. A. Eremeyev, K. K. Żur, Effect of Axial Porosities on Flexomagnetic Response of In-Plane Compressed Piezomagnetic Nanobeams, Symmetry 12 (2020) 1935. https://doi.org/10.3390/sym12121935

M. Malikan, V. A. Eremeyev, H. M. Sedighi, Buckling analysis of a non-concentric double-walled carbon nanotube, Acta Mechanica 231 (2020) 5007–5020. https://doi.org/10.1007/s00707-020-02784-7

M. Malikan, M. Krasheninnikov, V. A. Eremeyev, Torsional stability capacity of a nano-composite shell based on a nonlocal strain gradient shell model under a three-dimensional magnetic field, International Journal of Engineering Science 148 (2020) 103210. https://doi.org/10.1016/j.ijengsci.2019.103210

M. Malikan, N. S. Uglov, V. A. Eremeyev, On instabilities and post-buckling of piezomagnetic and flexomagnetic nanostructures, International Journal of Engineering Science 157 (2020) 103395. https://doi.org/10.1016/j.ijengsci.2020.103395

M. Malikan,T. Wiczenbach, V. A. Eremeyev, On thermal stability of piezo-flexomagnetic microbeams considering different temperature distributions, Continuum Mechanics and Thermodynamics, (2021). https://doi.org/10.1007/s00161-021-00971-y

Moosavi, S, Zakaria, S, Chia, CH, Gan, S, Azahari, NA & Kaco, H 2017, 'Hydrothermal synthesis, magnetic properties and characterization of CoFe2O4 nanocrystals', *Ceramics International*, vol. 43, pp. 7889-7894.

Pereira, C, Pereira, AM, Fernandes, C, Rocha, M, Mendes, R, Fernández-García, MP, Guedes, A, Tavares, PB, Grenèche, J-M, Araújo, JP & Freire, C 2012, 'Superparamagnetic MFe2O4 (M = Fe, Co, Mn) Nanoparticles: Tuning the Particle Size and Magnetic Properties through a Novel One-Step Coprecipitation Route', *Chemistry of Materials*, vol. 24, pp. 1496-1504.

Romano, G & Barretta, R 2017, 'Nonlocal elasticity in nanobeams: the stress-driven integral model', *International Journal of Engineering Science*, vol. 115, pp. 14-27.

Senthil, VP, Gajendiran, J, Gokul Raj, S, Shanmugavel, T, Ramesh Kumar, G & Parthasaradhi Reddy, C 2018, 'Study of structural and magnetic properties of cobalt ferrite (CoFe2O4) nanostructures', *Chemical Physics Letters*, vol. 695, pp. 19-23.

Skrzat, A & Eremeyev, VA 2020, 'On the effective properties of foams in the framework of the couple stress theory', *Continuum Mechanics and Thermodynamics*, vol. 32, pp. 1779–1801.

Sidhardh, S & Ray, MC 2018, 'Flexomagnetic response of nanostructures', *Journal of Applied Physics*, vol. 124, pp. 244101.

Thanh Tran, T, Nguyen, P-C, Pham & Q-H 2021, 'Vibration analysis of FGM plates in thermal environment resting on elastic foundation using ES-MITC3 element and prediction of ANN', *Case Studies in Thermal Engineering*, vol. 24, pp. 100852.

Vasiliev, AS, Volkov, SS, Belov, AA, Litvinchuk, SY & Aizikovich, SM 2017, 'Indentation of a hard transversely isotropic functionally graded coating by a conical indenter', *International Journal of Engineering Science*, vol. 112, pp. 63-75.

Volkov, SS, Vasiliev, AS, Aizikovich, SM, Mitrin, BI 2019, 'Axisymmetric indentation of an electroelastic piezoelectric half-space with functionally graded piezoelectric coating by a circular punch', *Acta Mechanica*, vol. 230, pp. 1289-1302.

Zhang, N, Zheng, Sh & Chen, D 2019, 'Size-dependent static bending of flexomagnetic nanobeams', *Journal of Applied Physics*, vol. 126, pp. 223901.

Zhang, JX, Zeches, RJ, He, Q, Chu, YH & Ramesh, R 2012, 'Nanoscale phase boundaries: a new twist to novel functionalities', *Nanoscale*, vol. 4, pp. 6196-6204.

Zhou, H, Pei, Y & Fang, D 2014, 'Magnetic field tunable small-scale mechanical properties of nickel single crystals measured by nanoindentation technique', *Scientific Reports*, vol. 4, pp. 1-6.