

Novel Interpolation Method of Multi-DFT-Bins for Frequency Estimation of Signal with Parameter Step Change

Kai Wang, Shan Liu, Lanlan Wang, Janusz Smulko and He Wen

Abstract—The IpDFT(Interpolation Discrete Fourier Transform) method is one of the most commonly used non-parametric methods. However, when a parameter(frequency, amplitude or phase) step changes in the DFT period, the DFT coefficients will be distorted seriously, resulting in the large estimation error of the IpDFT method. Hence, it is a key challenge to find an IpDFT method, which not only can eliminate the effect of the step-changed symbol, but also can sufficiently eliminate the fence effect and the spectrum leakage. In this paper, an IpDFT based method is proposed to estimate the frequency of the single tone signal with the step-changed parameters in the sampling signal sequence. The relationship between the DFT bins and the step changed parameters is given by several linear equations. At most six different DFT bins are used to eliminate the effect of symbol.

Index Terms—Interpolation Discrete Fourier Transform (DFT); frequency estimation; the single tone signal; step-changed parameter; RF conformance test.

I. INTRODUCTION

WITH the development of signal processing technology, frequency estimation plays an increasingly important role in this field. A large number of parametric methods (time-domain methods) the non-parametric methods (frequency-domain methods) have emerged in the past few decades. For example, the ML (Maximum likelihood) method [1], [2], the Wavelet method [3], the filtering method [4], [5], PLL (Phase locked loop) method [6], prony method [7], Taylor Fourier Filter method [8], [9] and the IpDFT method (the interpolation Discrete Fourier Transform).

The IpDFT method and its improvements, such as the e-IpDFT [10] and i-IpDFT [11], which can be simplified by the Fast Fourier Transform (FFT), is currently one kind of the most commonly used non-parametric methods [12], [13]. The DFT-based estimators are fast and very simple to apply [14], [15]. For the single tone signal, the IpDFT methods have completely solved the estimation error caused by the fence effect.

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With two or three DFT bins, various IpDFT methods [16]–[18] are proposed to estimate the frequency of the signal. For the real-value harmonic signal, the most important challenge is how to compensate for spectrum leakages, which heavily damages the estimation accuracy [19]–[22]. Many variants of DFT, such as Sliding DFT [23], Taylor Fourier Filter method [8], [9], Iterative IpDFT [24] and Frequency Shifting Filtering [25], have been reported to estimate the phasor parameters. All of these methods try to compensate for spectrum leakages, which are caused by the off-nominal frequency operation of the system.

In the IpDFT method, windowing techniques such as the maximum decay sidelobe window [26], [27] and the triangular self-convolution window [28] are also used to reduce harmonic and inter-harmonic interference. Besides windowing techniques, another method is to consider the contribution of each frequency component in the spectrum and to solve the spectrum leakage problem theoretically. [29], [30] completely eliminate the influence of spectrum leakage by a three-point IpDFT based on MDW. Furthermore, the authors of [30] apply the algorithm of [29] to the harmonic signal. Other authors [31], [32] propose an IpDFT based on a scaling factor to estimate the dominant chatter frequency. Based on the rectangular window, [33]–[35] introduce an efficient two-point/three-point IpDFT of the sinusoid signal with or without the damping factor. [36], [37] combine the CLS (complex-value least squares) criteria and SDFT (Smart DFT) to obtain higher algorithm accuracy.

The default signal model in IpDFT methods is the time invariant signal, which means the amplitude, phase and frequency of the signal remain unchanged during the sampling period. However, the time-varying signals also have research significance in real-world scenarios. For example, the basic of RF conformance testing system is a fast and accurate frequency offset estimation method for the wireless systems with different modulation signals, such as quadrature amplitude modulation signal (QAM), phase-shift keying signal (PSK) and frequency-shift keying signal (FSK) [38], [39]. QAM signal, PSK signal and FSK signal can be regarded as carrier signals whose amplitude, phase and frequency vary with the transmission information during different sampling cycles [40], [41]. In the power systems, the precise and rapid frequency estimation is also necessary in PMUs (phasor measurement units) [42], [43]. The standards of phasors estimation provide detailed test requirements for the time-varying signal, including step change in amplitude, phase and frequency

[44]–[46]. However, the research of IpDFT based frequency estimation for the time-varying signal frequency estimation is still in the preliminary stage. Others [47] propose a low complexity IpDFT method to estimate frequency offset for M-QAM coherent optical systems. A novel IpDFT frequency estimation method of single tone signal with bit transition is proposed in [48].

The signals with step-changed parameters are the most important and basic signal modes in the wireless system [40], [41] or the power system [44]–[46]. The basic algorithms, which are utilized in the RF conformance test [38], [39] or in the phasor measurement units [42], [43], are also used for the frequency estimation in different signal modes. A rapid and accurate estimation can greatly ensure reliable performance of the test system. However, when a parameter step changes in the DFT period, the DFT coefficients will be distorted seriously, resulting in the large estimation error of the IpDFT method. Hence, it is a key challenge to find an IpDFT method, which not only can eliminate the effect of the step-changed symbol, but also can sufficiently eliminate the fence effect and the spectrum leakage. In this paper, an IpDFT based method is proposed to estimate the frequency of the single tone signal with the step-changed parameter in the sampling signal sequence. The relationship between the DFT bins and the step-changed parameters is given by several linear equations. At most six different DFT bins are used to eliminate the effect of the step-changed parameters. The results of simulation and experiment confirm that the proposed algorithm can achieve frequency estimation with high accuracy.

II. FREQUENCY ESTIMATION METHOD

A single tone signal with the step-changed parameters can be described as

$$\begin{aligned} x(n) &= s(n) + q(n) \\ &= A_m \exp(\varphi_m) \exp(j\omega_m n) + q(n) \end{aligned} \quad (1)$$

where A_m , φ_m and $\omega_m = 2\pi f_m T_s$ are the m -th ($m \in [0, M - 1]$) unknown parameters of the amplitude, the phase and the angular frequency. $q(n)$ is the additive white Gaussian noise (AWGN) with variance σ_q^2 . f_m is the signal frequency and T_s is the sampling time.

When we obtain an N -sample time sequence $x(n)$, we rewrite the symbols of the m -th jump as U_m and ω_m where $\omega_m = 2\pi l_m / N = 2\pi(k_m + \delta_m) / N$, l_m is the m -th acquired signal cycles. $\delta_m \in [-0.5, 0.5]$ and $k_m \in [0, 1, \dots, \frac{N}{2} - 1]$ are the fractional-part and the integer-part of l_m . The N -point DFT of $x(n)$ is:

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} s(n) W_N^{nk} + \sum_{n=0}^{N-1} q(n) W_N^{nk} \\ &= S(k) + Q(k) \end{aligned} \quad (2)$$

where $W_N^k = e^{-j(\frac{2\pi}{N})k}$, $S(k)$ and $Q(k)$ are the DFTs of $s(n)$ and $q(n)$.

A. the case when the frequency, the amplitude and the phase all step change

In this part, we only consider the case of one step-change in an N -length signal. Let the parameters(frequency, amplitude or phase) step change on the L -th sample. The angular frequency before the step-change is denoted as ω_1 and the angular frequency after the step-change is denoted as ω_2 :

$$s(n) = \begin{cases} U_1 e^{j\omega_1 n}, & n = 0, 1, \dots, L-1 \\ U_2 e^{j\omega_2 n}, & n = L+1, L+2, \dots, N-1 \end{cases} \quad (3)$$

where $U_1 = A_1 \exp(\varphi_1)$ and $U_2 = A_2 \exp(\varphi_2)$ are any two different unknown parameters of amplitude and phase combination. The DFT result at slot k changes to:

$$\begin{aligned} S(k) &= \sum_{n=0}^{L-1} A_1 e^{j\varphi_1} e^{j\omega_1 n} W_N^{nk} + \sum_{n=L}^{N-1} A_2 e^{j\varphi_2} e^{j\omega_2 n} W_N^{nk} \\ &= A_1 e^{j\varphi_1} \frac{1 - (e^{j\omega_1} W_N^k)^L}{1 - e^{j\omega_1} W_N^k} + A_2 e^{j(\varphi_2 + L\omega_2)} W_N^{kL} \frac{1 - (e^{j\omega_2} W_N^k)^{N-L}}{1 - e^{j\omega_2} W_N^k} \\ &= A_1 e^{j\varphi_1} \frac{1 - (e^{j\omega_1} W_N^k)^L}{1 - e^{j\omega_1} W_N^k} + A_2 e^{j\varphi_2'} W_N^{kL} \frac{1 - (e^{j\omega_2} W_N^k)^{N-L}}{1 - e^{j\omega_2} W_N^k} \\ &= \frac{(p_1 + p_2) + (q_1 + q_2) W_N^k + (u_1 + u_2) W_N^{kL} + (v_1 + v_2) W_N^{k(L+1)}}{1 - (\lambda_1 + \lambda_2) W_N^k + \lambda_1 \lambda_2 W_N^{2k}} \end{aligned} \quad (4)$$

where $\lambda_1 = e^{j\omega_1}$, $\lambda_2 = e^{j\omega_2}$, $p_1 = A_1 e^{j\varphi_1}$, $p_2 = -A_2 e^{j\varphi_2'} e^{j\omega_2 L}$, $q_1 = -A_1 e^{j\varphi_1} e^{j\omega_2}$, $q_2 = A_2 e^{j\varphi_2'} e^{j(\omega_1 + \omega_2)(N-L)}$, $u_1 = -A_1 e^{j\varphi_1} e^{j\omega_1 L}$, $u_2 = A_2 e^{j\varphi_2'}$, $v_1 = A_1 e^{j\varphi_1} e^{j(\omega_2 + \omega_1)L}$, $v_2 = -A_2 e^{j\varphi_2'} e^{j\omega_1}$ and $\varphi_2' = \varphi_2 + L\omega_2$. The coarse frequency estimation can be performed as: $\hat{k}_0 = \arg \max_{k \in \{0, 1, \dots, N-1\}} (|X(k)|)$. When we get any six different DFT bins $\mathbf{X}(k) = [X(k_1), X(k_2), \dots, X(k_6)]^T$, we can get the following linear equations according to (4):

$$\mathbf{X}_k = \mathbf{W}_k \boldsymbol{\eta} \quad (5)$$

where $\boldsymbol{\eta}$ and \mathbf{W}_k can be written as:

$$\boldsymbol{\eta} = [p_1 + p_2, q_1 + q_2, u_1 + u_2, v_1 + v_2, \lambda_1 + \lambda_2, \lambda_1 \lambda_2]^T \quad (6)$$

$$\mathbf{W}_k = \begin{bmatrix} 1 & W_N^{k_1} & W_N^{k_1 L} & W_N^{k_1(L+1)} & X(k_1) W_N^{k_1} & -X(k_1) W_N^{2k_1} \\ 1 & W_N^{k_2} & W_N^{k_2 L} & W_N^{k_2(L+1)} & X(k_2) W_N^{k_2} & -X(k_2) W_N^{2k_2} \\ 1 & W_N^{k_3} & W_N^{k_3 L} & W_N^{k_3(L+1)} & X(k_3) W_N^{k_3} & -X(k_3) W_N^{2k_3} \\ 1 & W_N^{k_4} & W_N^{k_4 L} & W_N^{k_4(L+1)} & X(k_4) W_N^{k_4} & -X(k_4) W_N^{2k_4} \\ 1 & W_N^{k_5} & W_N^{k_5 L} & W_N^{k_5(L+1)} & X(k_5) W_N^{k_5} & -X(k_5) W_N^{2k_5} \\ 1 & W_N^{k_6} & W_N^{k_6 L} & W_N^{k_6(L+1)} & X(k_6) W_N^{k_6} & -X(k_6) W_N^{2k_6} \end{bmatrix} \quad (7)$$

According to Eq.(5), $\boldsymbol{\eta}$ can be estimated as:

$$\hat{\boldsymbol{\eta}} = \mathbf{W}_k^{-1} \mathbf{X}_k \quad (8)$$

where $\hat{\cdot}$ is the estimated value of (\cdot) . Let $\hat{\lambda}_1 + \hat{\lambda}_2 = a$ and $\hat{\lambda}_1 \hat{\lambda}_2 = b$, where a and b are calculated from Eq. (6). ω_1 and

149 ω_2 can be estimated as:

$$\begin{cases} \hat{\omega}_1 = \frac{\text{Im}(\ln(a + \sqrt{a^2 - 4b}))}{2} \\ \hat{\omega}_2 = \frac{\text{Im}(\ln(a - \sqrt{a^2 - 4b}))}{2} \end{cases} \quad (9)$$

150
151 Although any of the six different DFT bins can be used in
152 our method, it is recommended to use the DFT bins with the
153 six largest magnitudes to obtain the best estimation results in
154 practical applications.

155 *B. the case when the amplitude and the phase step change,*
156 *but the frequency keeps unchange*

157 In this part, we keep the frequency constant, and let the
158 amplitude and the phase step change during the sampling
159 signal sequence. In order to distinguish the frequency step
160 change of part A, ω_0 is chosen to represent the signal angular
161 frequency. Eq.(4) can be simplified as:

$$\begin{aligned} S(k) &= \sum_{n=0}^{L-1} A_1 e^{j\varphi_1} e^{j\omega_0 n} W_N^{nk} + \sum_{n=L}^{N-1} A_2 e^{j\varphi_2} e^{j\omega_0 n} W_N^{nk} \\ &= A_1 e^{j\varphi_1} \frac{1 - (e^{j\omega_0} W_N^k)^L}{1 - e^{j\omega_0} W_N^k} + A_2 e^{j\varphi_2} W_N^{kL} \frac{1 - (e^{j\omega_0} W_N^k)^{N-L}}{1 - e^{j\omega_0} W_N^k} \quad (10) \\ &= \frac{(\mu_1 - v_2) + (\mu_2 - v_1) W_N^{kL}}{1 - \lambda_0 W_N^k} \end{aligned}$$

162 where $\mu_1 = A_1 e^{j\varphi_1}$, $\mu_2 = A_2 e^{j\varphi_2}$, $v_1 = A_1 e^{j\varphi_1} (\lambda_0)^L$,
163 $v_2 = A_2 e^{j\varphi_2} (\lambda_0)^{N-L}$ and $\lambda_0 = e^{j\omega_0}$. When we get any
164 three different DFT bins $\mathbf{X}_k = [X(k_1), X(k_2), X(k_3)]^T$, $\eta =$
165 $[\mu_1 - v_2, \mu_2 - v_1, \lambda_0]^T$ can be estimated as:

$$\hat{\eta} = \mathbf{W}_k^{-1} \mathbf{X}_k \quad (11)$$

166 where

$$\mathbf{W}_k = \begin{bmatrix} 1 & W_N^{k_1 L} & X(k_1) W_N^{k_1} \\ 1 & W_N^{k_2 L} & X(k_2) W_N^{k_2} \\ 1 & W_N^{k_3 L} & X(k_3) W_N^{k_3} \end{bmatrix} \quad (12)$$

167 ω_0 can be estimated as:

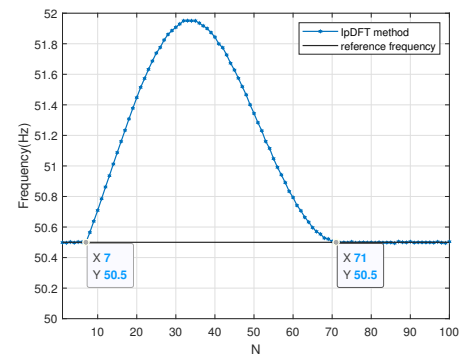
$$\hat{\omega}_0 = \text{Im}(\ln \hat{\lambda}_0) \quad (13)$$

169 In this condition, the DFT bin with the largest magnitude
170 and its adjacent DFT bins: $X(k_0)$, $X(k_0 + 1)$, $X(k_0 - 1)$ are
171 recommended in practical applications for better estimation
172 results.

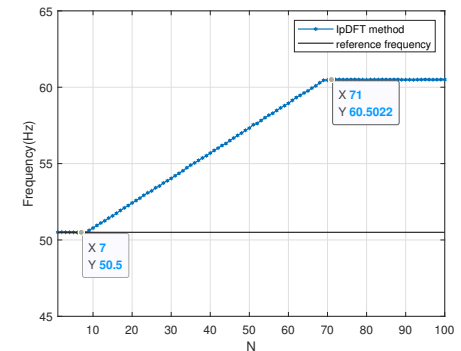
173 *C. Estimation Method of the location of bit transition L*

174 The actual location of bit transition L is important in the
175 proposed algorithm according to Eq. (4) and Eq. (10). In the
176 calculation process, if the value of L is wrong, it will cause
177 the frequency estimation error. In Refs. [47], [48], L is set to a
178 known value by default. However, it is not always an easy job
179 because it is simply impossible to know the transition time
180 beforehand in any measurement. If L is unknown, we can
181 use a sliding window based time-frequency estimation to get

182 the value of L . Assuming the signal of length $N = 128$, the
183 step-change of parameter occurs at an unknown point L . We
184 choose a rectangular window with a window size of $N' = 64$
185 and a step length of 1. By using this rectangular window, we
186 divide the signal into 65 data frames. The frame length and the
187 frame shift of each data frame are 64 and 1, respectively. For
188 the any m -th data frame: $\{x(m - 63), x(m - 1), \dots, x(m)\}$, the
189 IpDFT method given in [16] is used to estimate the
190 frequency $\omega(m)$ of the m -th data frame. The estimation results
191 for the step-changed signals when SNR=40dB are shown in
192 Figure 1. As shown in Fig. 1(a), the amplitude and the phase
193 jump point appears at 7-th point and 71-st point, which means
194 the location of bit transition L is 7. We can also use $71-64=7$
195 to obtain the same result. As shown in Fig. 1(b), the frequency
196 jump point appears at the point as same as the Fig. 1(a) and
197 it obtains the same result.



(a)



(b)

Fig. 1: The sliding window based time-frequency estimation (a) the step-change of the amplitude and the phase; (b) the step-change of the frequency

198 The sliding window method can be utilized as a
199 coarse estimation of the location L . Furthermore, we
200 will analyze the effect of transition time error in the
201 frequency estimation and discuss its impact in this paper.
202 We denote the estimated location by the coarse estimation
203 as L' and then we can get a set L consisting of L' and
204 its neighbors: $L = \{\dots, L' - 2, L' - 1, L', L' + 1, L' + 2, \dots\}$.
205 With the elements in set L in turn, we estimate
206 ω according to the method proposed in section
207 II and denote the estimated values as: $\omega(L) =$

TABLE I: MSEs of $\hat{\omega}$ with different sampling points in different conditions(unit: dB)

| L | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
|--|----------|----------|----------|-----------|-----------|----------|----------|----------|----------|----------|----------|
| Condition: the step-change of the amplitude and the phase | | | | | | | | | | | |
| noiseless | -68.6849 | -74.9372 | -85.5932 | -101.7746 | -320.2516 | -82.0556 | -72.7839 | -66.9066 | -62.5740 | -59.1483 | -56.3269 |
| SNR=40dB | -68.5969 | -74.9061 | -83.1398 | -85.5546 | -86.8574 | -80.4095 | -72.7481 | -66.9098 | -62.5128 | -59.1237 | -56.3227 |
| SNR=15dB | -61.1405 | -61.2445 | -62.1591 | -61.3460 | -60.5947 | -61.1215 | -61.2578 | -60.4537 | -59.6906 | -57.9072 | -54.7129 |
| Condition: the step-change of the frequency | | | | | | | | | | | |
| noiseless | -37.6932 | -40.5259 | -44.3341 | -50.5133 | -254.3888 | -50.1606 | -43.4749 | -38.9035 | -35.0644 | -31.6950 | -28.7563 |
| SNR=40dB | -37.6995 | -40.5325 | -44.3040 | -50.5135 | -76.8406 | -50.1302 | -43.4625 | -38.9028 | -35.0705 | -31.6894 | -28.7464 |
| SNR=15dB | -36.9852 | -38.0281 | -42.3912 | -40.3862 | -41.1338 | -44.3340 | -42.7269 | -38.7947 | -34.9241 | -31.823 | -28.7188 |

208 $\{\dots, \omega(L' - 2), \omega(L' - 1), \omega(L'), \omega(L' + 1), \omega(L' + 2), \dots\}$.
 209 In Tab. I, we calculate the MSEs (The mean square error)
 210 between the actual frequency of signal ω and the estimated
 211 frequencies with different location $\omega(L)$ in noiseless/ noisy
 212 condition.

213 There are two factors that affect the estimation accuracy of
 214 our method. One is the value of SNR. The other is the value
 215 of L' . In the low noise environment, the influence of the value
 216 of L' is more important. As shown in Tab. I, we can accurately
 217 estimate ω when the value of L' is right($L' = L$) When the
 218 value of L' is wrong($L' \neq L$), our method is a biased estimation
 219 algorithm. The estimation error will gradually increase with
 220 the increasing of the difference between L and L' . In the high-
 221 noise environment, the influence of SNR is more important.
 222 The influence of the error L' , whose value is slightly different
 223 from the real value L , can be ignored in the estimation process.
 224 We can get the estimated values with little differences even we
 225 use the wrong L' when SNR=15dB.

226 Furthermore, we calculate the differences between two
 227 adjacent estimated frequencies according to Eq. (14). The
 228 results in the noiseless and the low noise condition are shown
 229 in Figure 2 and Figure 3. We set $L = 64$, $N = 128$
 230 and $L = \{60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70\}$. As shown in
 231 Fig.2, when the value of L' is equal to the actual value L the
 232 difference between $\omega(L)$ and adjacent estimated frequencies
 233 $\omega(L - 1)$ is the maximum.

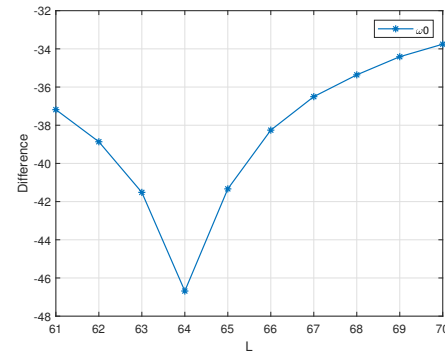
$$\text{diff}(i) = 10\log_{10} |\omega(i) - \omega(i - 1)| (i \in L) \quad (14)$$

234 In this paper, our main research purpose is to reveal the
 235 relation between the DFT bins and the system parameters of
 236 the signal with step-changed parameters, which allows us to
 237 better understand the essence of discrete Fourier transform.
 238 The research on the estimation of the location L is only at
 239 early stage. So far, we haven't given the explicit expression
 240 between the DFT bins and the location L in this paper. In our
 241 future work, we will focus on this issue for further research
 242 and discussion.

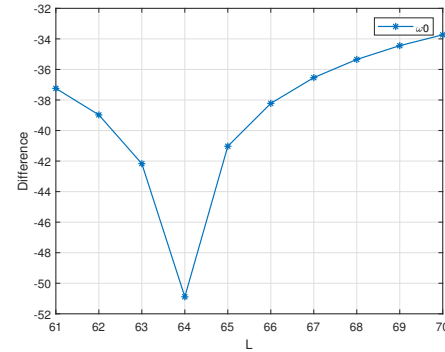
III. PERFORMANCE OF PROPOSED METHOD

243 In this section, we focused on analyzing the estimation per-
 244 formance of the algorithm without any practical application.
 245 The normalized angular frequency ω_i is chosen in this part,
 246 where $\omega_i = 2\pi l_i / N = 2\pi f_i / f_s$ ($i = 0, 1$ or 2).

247 We compare the estimated performance of the considered al-
 248 gorithm through simulation results. The considered algorithms
 249 are the XJX method in Ref. [47], the UFE method in Ref. [48],
 250 the BFEL method in Ref. [48], the ITE-FOE method in Ref.



(a)



(b)

Fig. 2: the difference between two adjacent estimated frequencies when the amplitude and the phase step change a) SNR=40dB; b) noiseless

[49], the DIFF-FOE method in Ref. [50]. Cramer-Rao lower bounds (CRLB) is shown in Eq.(14). 252 253

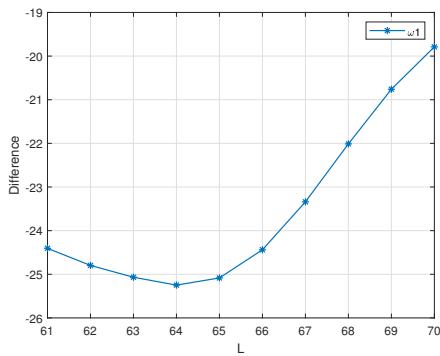
$$\text{CRLB}(\omega_0) \geq \frac{12\sigma^2}{A^2N(N^2 - 1)} \quad (15)$$

254 where A is the amplitude of the signal, N is the signal length
 255 and σ^2 is the variance of the additive white Gaussian noise.

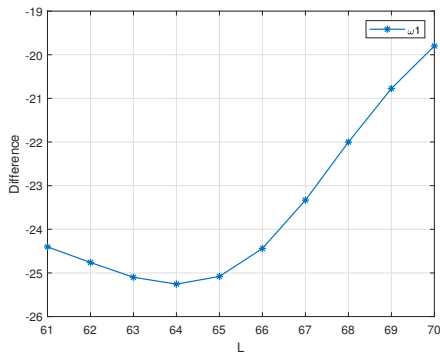
256 The observed sample length N is 128. For each parameter,
 257 3000 runs are performed to evaluate the statistical properties.
 258 The mean square error (MSE) is used to evaluate the perfor-
 259 mance of the proposed estimator and other algorithms, which
 260 is given by:

$$\text{MSE} = 10\log_{10} \left(\frac{1}{M} \sum_{i=1}^M (\hat{\omega}(i) - \omega)^2 \right) \quad (16)$$

261 where $\hat{\omega}(i)$ presents the estimated frequency of the i -th inde-
 262 pendent simulation.



(a)



(b)

Fig. 3: the difference between two adjacent estimated frequencies when the amplitude and the phase step change a) SNR=40dB; b) noiseless

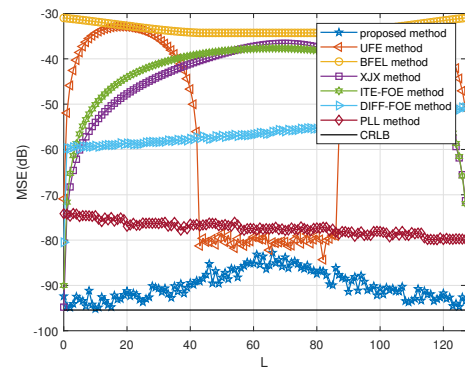


Fig. 4: MSEs of $\hat{\omega}_0$ versus L with $l_0=1.06$ and SNR=40dB

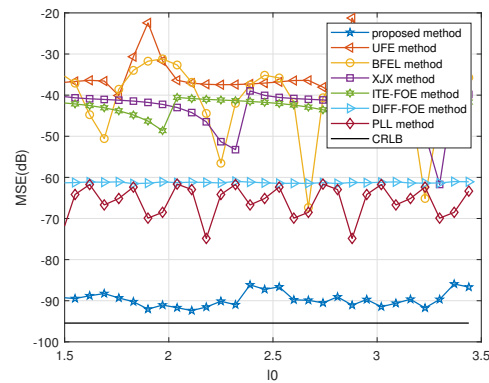


Fig. 5: MSEs of $\hat{\omega}_0$ versus l_0 with SNR=40dB and $L=32$

263 *A. the case when the amplitude and the phase step change,*
 264 *but the frequency keeps unchanged*

265 Firstly, let's consider the case when the amplitude and the
 266 phase step change, but the frequency keeps unchanged. The
 267 jumping point L of our algorithm can be applied to $[0, N-1]$.
 268 When $L = 0$ or $N - 1$, the signal actually degenerates into a
 269 single-frequency complex exponential signal, and our method
 270 actually degenerates into an IpDFT method in [16]–[18]. We
 271 set $U_1 = 0.9 \exp(10\pi/180)$ and $U_2 = \exp(80\pi/180)$. Figure 4
 272 depicts the relationships between L and MSEs in the noisy
 273 environment. Set $l_0=1.06$ and let L change from 0 to 127.
 274 As shown in Fig. 4, our method achieves better performance.
 275 However, the UFE method only performs well when the jump
 276 point L is in the interval $[N/3, 2N/3]$. Furthermore, let's
 277 take $L=32$ as an example to analyze the performances of our
 278 method with different l_0 or SNR.

279 Figure 5 depicts the relationships between l_0 and MSEs
 280 when SNR=40dB and $L=32$. Then, let l_0 change from 0.5
 281 to 3.5. As shown in Fig. 5, our method can achieve the
 282 minimum MSE -90dB. However, the minimum MSE of UFE
 283 is only -35dB, which means the performance of our method
 284 is much better than that of UFE method when SNR=40dB
 285 and $L=32$. Fig. 6 depicts the relationships between SNR and
 286 MSEs when $L=32$. Set $l_0=1.06$ and let SNR change from 0dB
 287 to 40dB. The MSEs of our method decrease linearly when
 288 SNR increases, which means that our methods can estimate

the frequency correctly when amplitude and phase step change
 on L -th sample. However, UFE method can only estimate the
 frequency correctly when $L \in [N/3, 2N/3]$. In order to verify
 the effect of different sampling points N on the estimated
 frequency, set $l_0=1.06$ and SNR=40dB. The MSE of frequency
 is shown in Table II, and the results show that our method has
 better effect under different sampling points N when $L=32$.

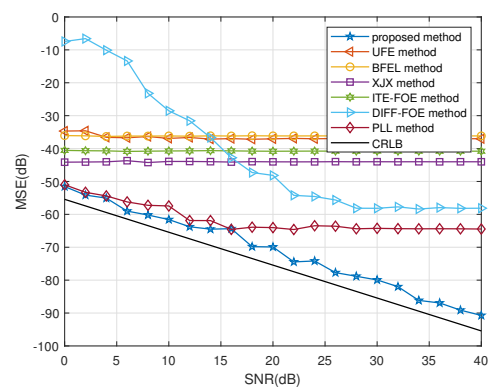


Fig. 6: MSEs of $\hat{\omega}_0$ versus SNR with $l_0=1.06$ and $L=32$

Let's consider the case when the signal is distorted by the
 DC offset or the harmonics. The signal distorted by the DC

TABLE II: MSEs of $\hat{\omega}_0$ with different sampling points N when $L = 32$ (unit: dB)

| | | Proposed Method | UFE Method | BFEL Method | XJX Method | ITE-DFT | DIFF-DFT | PLL |
|--------|----------|-----------------|------------|-------------|------------|---------|----------|--------|
| $L=32$ | $N=128$ | -83.53 | -36.27 | -34.19 | -42.45 | -39.39 | -56.38 | -76.34 |
| | $N=256$ | -92.98 | -38.16 | -38.59 | -55.05 | -50.63 | -63.20 | -81.16 |
| | $N=512$ | -101.51 | -46.13 | -43.72 | -67.28 | -62.74 | -69.79 | -86.23 |
| | $N=1024$ | -108.92 | -56.42 | -49.34 | -79.42 | -75.21 | -76.25 | -91.78 |

TABLE III: Condition of fundamental and harmonics

| Harmonic | 1st | 3rd | 5th | 7th | 9th | 11th | 13th | 15th |
|----------------|-----|--------|--------|--------|--------|--------|--------|--------|
| Amplitude(rms) | 0 | -26.02 | -33.97 | -42.14 | -53.18 | -66.22 | -78.26 | -90.30 |

offset can be expressed as:

$$x(n) = s(n) + q(n) \tag{17}$$

$$= A_m \exp(\varphi_m) \exp(j\omega_m n) + A_{dc} + q(n)$$

where $A_{dc} = 1$. Let $X_{dc}(k)$ be the DFT of the signal A_{dc} . When $k > 0$, the value of $X_{dc}(k) = 0$. In practical applications, if we do not use $X_{dc}(0)$ in estimation process, we can reduce the DC offset interference. The performances distorted by the DC offset are shown in Fig. 7. Set $L=32$ and $SNR = 40$ dB. Our method can correctly estimate the frequencies no matter the DC offset distorted or not.

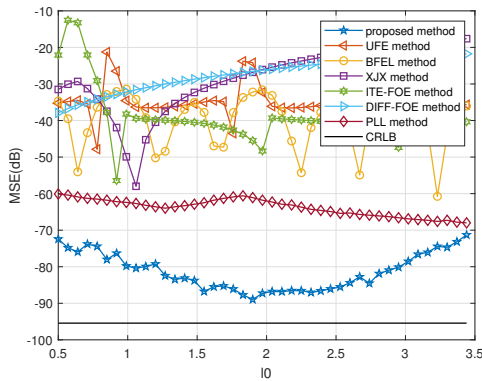


Fig. 7: MSEs of $\hat{\omega}_0$ versus l_0 with $SNR=40$ dB, $A_{dc} = 1$ and $L=32$

The signal distorted by noise and the 2-8 harmonics can be expressed as:

$$x(n) = s(n) + q(n) \tag{18}$$

$$= A_m \exp(\varphi_m) \exp(j\omega_m n) + \sum_{h=2}^8 A_p \exp(j\omega_h n + \varphi_h) + q(n)$$

where A_h are shown in Table III [42], [43]. φ_h is the initial phase which are selected at random in the range $[0, 2\pi)$. Due to the influence of harmonics, the overall estimation effect of our algorithm becomes less satisfactory as shown in Fig. 8.

B. the case when the frequency, the amplitude and the phase all step change

Let's consider the case when the frequency, the amplitude and the phase all step change when $U_1 = 0.9 \exp(10\pi/180)$ and $U_2 = \exp(80\pi/180)$. Set $l_1 = l_2 + 0.5$. As shown in Fig. 9, our method achieve the minimum MSE -75dB when $L=64$ and $SNR=40$ dB. Fig. 10 illustrates that the MSEs of our

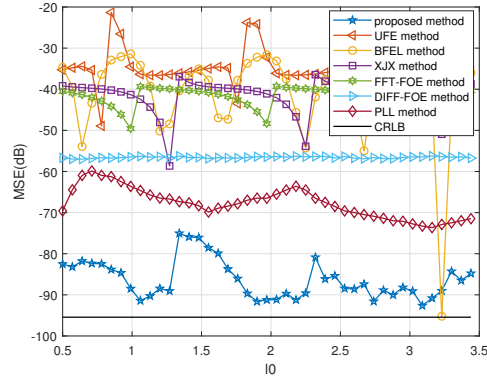


Fig. 8: MSEs of $\hat{\omega}_0$ versus l_0 distorted by 2-7 harmonics with $SNR=40$ dB and $L=32$

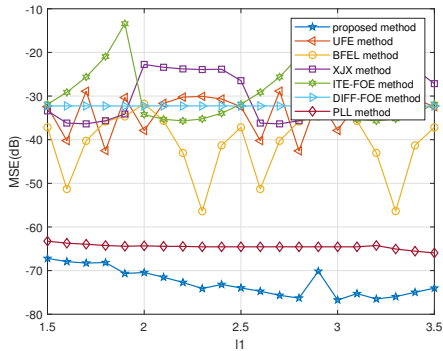
method decrease linearly when SNR increases, which means only our methods can estimate the frequency accurately with a step change in the frequency of the signal. As shown in Fig. 11, when $40 < L < N - 3$, our method can estimate l_1 accurately. When $2 < L < 80$, our method can estimate l_2 accurately. Besides, our method achieves the minimum MSE -75dB when $L=64$ is in the interval $[50, 100]$. Therefore, our method has a better performance when L is in the interval $[3, N-4]$ in the case that the frequency step change. The reason for L taking this interval is as follows. In our algorithm, the signal sequence with step-changed frequency is equivalent to a signal sequence formed by splicing two single-frequency complex exponential signals with different frequencies. For single-frequency complex exponential signals, at least two consecutive samples of $s(n)$ and $s(n+1)$ can accurately estimate the corresponding frequency. In other words, for a single-frequency complex exponential signal, at least two samples are required to fully describe it. Our algorithm considers two single-frequency complex exponential signals jointly. Simulation shows that for each single-frequency complex exponential signal, at least three consecutive samples are required to fully describe its complete information. Therefore, the variation range of L is actually $[3, N - 4]$. When $L < 3$, the frequency of ω_1 cannot be estimated; when $L > N - 4$, the frequency of ω_2 cannot be estimated.

In order to verify the effect of different sampling points N on the estimated frequency, Set $l_1=1.5$, $l_2=0.5$ and $SNR=40$ dB. The MSE of frequency is shown in Table IV, and the results show that our method has better effect under different sampling points N when $L = \frac{N}{2}$.

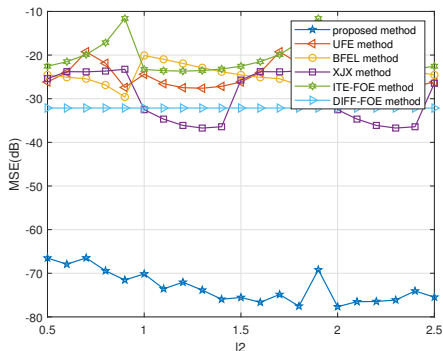
The signal distorted by the DC offset is shown in Fig. 12

TABLE IV: MSEs of $\hat{\omega}_1$ and $\hat{\omega}_2$ with different sampling points N when $L = \frac{N}{2}$ and SNR = 40dB(unit: dB)

| | Proposed Method | UFE Method | BFEL Method | XJX Method | ITE-DFT Method | DIFF-DFT Method | PLL Method |
|--------|-----------------|------------|-------------|------------|----------------|-----------------|------------|
| N=128 | l_1 | -72.54 | -49.51 | -54.64 | -29.56 | -14.93 | -34.65 |
| | l_2 | -70.04 | -26.28 | -26.00 | -23.33 | -16.79 | - |
| N=256 | l_1 | -81.68 | -54.04 | -60.21 | -35.63 | -21.19 | -40.70 |
| | l_2 | -80.39 | -32.40 | -32.04 | -29.37 | -23.13 | -41.71 |
| N=512 | l_1 | -90.63 | -59.33 | -65.94 | -41.67 | -27.34 | -46.74 |
| | l_2 | -89.29 | -38.43 | -38.0 | -35.40 | -29.33 | -47.67 |
| N=1024 | l_1 | -98.73 | -65.08 | -71.84 | -47.70 | -33.43 | -52.74 |
| | l_2 | -98.93 | -44.48 | -44.05 | -41.43 | -35.43 | -53.70 |

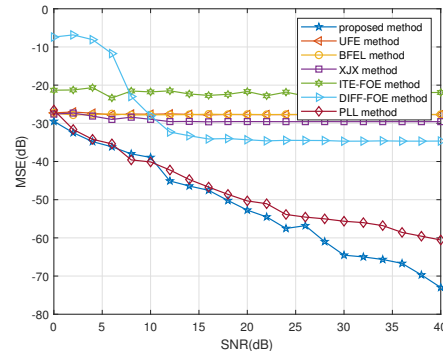


(a)

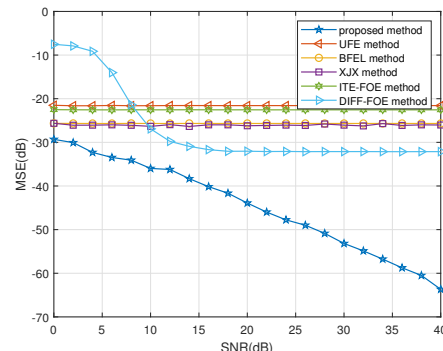


(b)

Fig. 9: The relationships between l_i ($i = 1, 2$) and MSEs of $\hat{\omega}_1$ and $\hat{\omega}_2$ when SNR=40dB and $L = 64$ (a) l_1 ; (b) l_2



(a)



(b)

Fig. 10: The relationships between SNR and MSEs of $\hat{\omega}_1$ and $\hat{\omega}_2$ when $l_1 = 1.5$, $l_2 = 0.5$ and $L = 64$ (a) l_1 ; (b) l_2

and the signal distorted by the harmonics of part A is shown in Fig. 13.

C. Computational Complexity

We give the computational complexity analysis of our method. As shown in Eq. (12), three different DFT bins $X(k)$ are needed to establish the equation for the amplitude or phase jump signal. The evaluation of one DFT sample requires N complex multiplications and $N-1$ complex additions. To compute the frequency parameters, we need a 3×3 complex matrix inverse operation. If the Gauss elimination method is used, the evaluation of 6×6 matrix inversion requires $\sum_{p=1}^6 p \times p$ complex multiplications and $\sum_{p=0}^5 p \times p$ complex additions. Therefore, the

overall complexity of the proposal is $PN + \sum_{p=1}^P p \times p$ complex

multiplications and $P(N-1) + \sum_{p=0}^{P-1} p \times p$ complex additions.

$P=5$ for the case when the frequency step changes and $P=3$ for the case when the amplitude and the phase both step change.

IV. PMU TEST

In the power systems, we select 50.5 Hz as the reference frequency, 5800 Hz as the sampling frequency and the length of the signal sequence is 128 and the specific frequency is denoted as f_i ($i = 0, 1, 2$). Eq. (1) is often used to describe the balanced three-phase voltage signal by means of the Clark transform [51]. In power system, the step changes of amplitude, phase and frequency are the three important conditions which should be considered. Therefore, the basis of

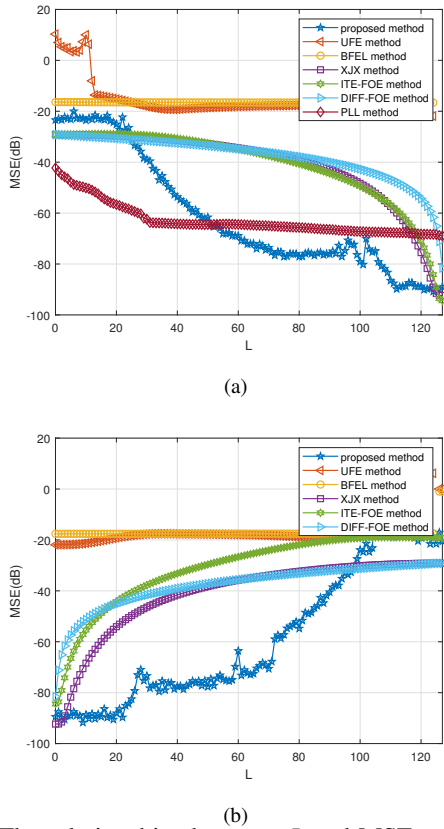


Fig. 11: The relationships between L and MSEs of $\hat{\omega}_1$ and $\hat{\omega}_2$ when $l_1 = 1.5$, $l_2 = 0.5$ and SNR=40dB (a) l_1 ; (b) l_2

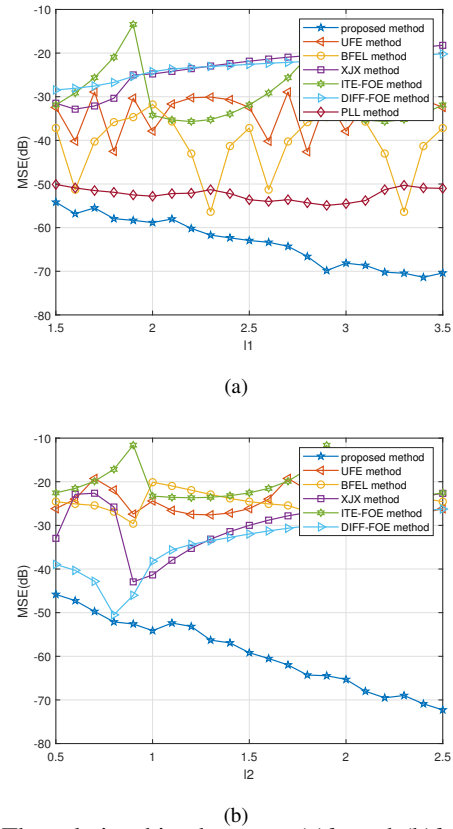


Fig. 12: The relationships between (a) l_1 and (b) l_2 and MSEs of $\hat{\omega}_1$ and $\hat{\omega}_2$ with SNR=40dB, $A_{dc} = 1$ and $L=64$

375 the power quality assessment is a fast and accurate frequency
 376 estimation method under an abrupt signal step. To evaluate the
 377 dynamic response when exposed to an abrupt signal change,
 378 a positive step followed by a reverse step back to the starting
 379 value under various conditions is applied to the amplitude,
 380 phase angle, and frequency of the signal, respectively.

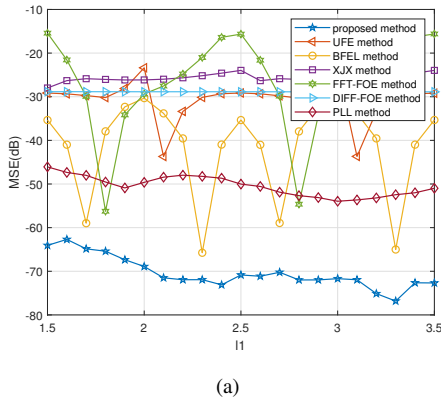
381 A. Results Under the Parameters Step Signal Condition

382 Firstly, let's consider the case when the amplitude and the
 383 phase step change, and the frequency keeps unchanged. Fig.
 384 14 and Fig. 15 show the results of simulation for step change
 385 signals in the magnitude and the phase with SNR=40dB. The
 386 magnitude step size is set to 0.1, and the phase step size is
 387 set to $\pi/18$. Step changes occur at $t = 0.15$ sec and are
 388 released at $t = 0.3$ sec. The proposed algorithm follows the
 389 reference frequency even when the step change occurs and is
 390 released. The PLL algorithm can also estimate the frequency
 391 well under the condition that the frequency keeps constant.
 392 The simulation results of Fig. 14 and Fig. 15 show that the
 393 MSE of the PLL algorithm can reach -76.56dB and -79.78dB,
 394 respectively. Our algorithm mse can roughly reach -95.18dB
 395 and -98.05dB before and after the step change. However, the
 396 UFE algorithm can accurately estimate ω_0 for step change
 397 signals in the magnitude or the phase only when the jump
 398 point L is in the interval [44,86].

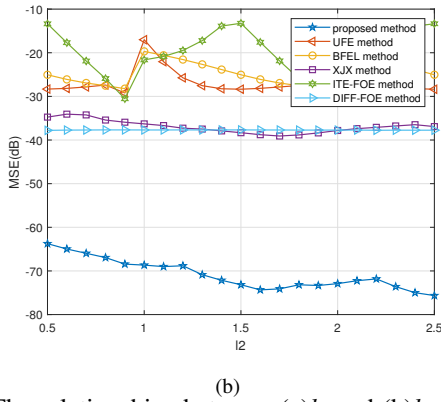
399 Fig. 16 and Fig. 17 show the performances of our method
 400 for the signal with step-changed frequency in the noiseless or

noisy environment. The dynamic response of our algorithm
 needs to be discussed respectively. The reference frequency
 50.5 Hz changes to 60.5 Hz at $t = 1$ sec and is released at
 $t = 2$ sec. When the jump point $L < 88$, our algorithm can
 accurately estimate ω_1 . When the jump point $L > 40$, our
 algorithm can accurately estimate ω_2 . When the jump point
 L is in the interval $L \in [41, 88]$, our algorithm can accurately
 estimate ω_1 and ω_2 at the same time.

For the N -sample signal with the step-changed magnitude
 and phase, the total length of signal with the angular frequency
 ω_0 keeps the length of N unchanged. The information of the
 angular frequency ω_0 always exists in the N sampling points.
 Therefore, when L is the interval $[0, N - 1]$, as shown in
 Fig. 14 and Fig. 16, the estimation accuracy of the algorithm
 remains unchanged. For the N -sample signal with the step-
 changed frequency, the information of ω_1 only contains the
 first L sampling points and the information of ω_2 only contains
 the last $N - L$ sampling points. For the signal with length N ,
 with the increase of the L , the influence of ω_1 in observation
 sequence is greater, and the anti-noise perform of ω_1 is better.
 The smaller L is, the greater the influence of ω_2 in observation
 sequence is, and the better the anti-noise perform ω_2 is.
 Therefore, as shown in Fig. 16, the estimated performance
 of ω_1 will be improved with the increase of L . The estimated
 performance of ω_2 will be improved with the decrease of L .
 The sampling frequency is set to 1800 Hz. In our method,
 rise time occurs when $t = 1.0011s$ ($L = 126$), peak time



(a)



(b)

Fig. 13: The relationships between (a) l_1 and (b) l_2 distorted by 2-7 harmonics and MSEs of $\hat{\omega}_1$ and $\hat{\omega}_2$ with SNR=40dB and $L=64$

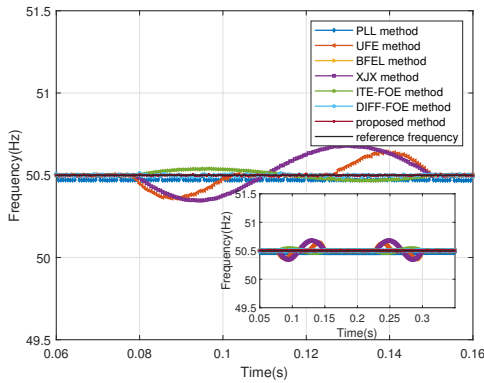


Fig. 14: The estimated frequencies of simulation for step change signals in the magnitude when SNR=40dB.

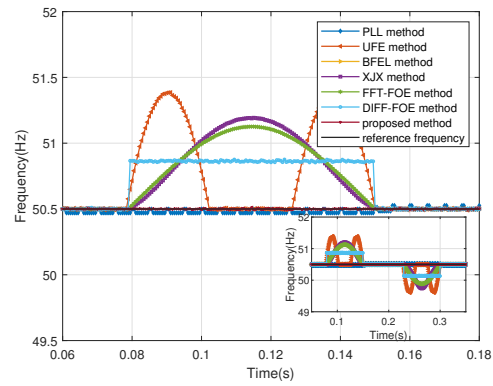


Fig. 15: The estimated frequencies of simulation for step change signals in the phase when SNR=40dB.

value.

It can be observed from Fig. 17 that the minimum MSE of ω_1 and ω_2 can reach -122.58dB and -161.51dB when $L = 3$ in the noiseless environment respectively. Within the allowable error range, there is neither overshoot nor undershoot. When the sampling frequency is set to f_s , the rise time, peak time and adjustment time are all $3/f_s$. To sum up, the PLL can quickly achieve frequency tracking of the step signal in the noisy environment. However, our method has a larger maximum error in the noisy environment. However, it has a considerable advantage that the angular frequencies ω_1 and ω_2 before and after the frequency step change can be calculated simultaneously.

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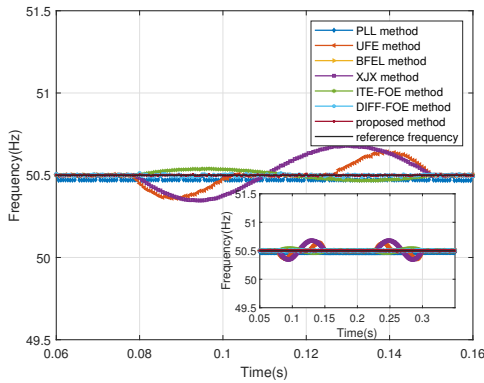


Fig. 16: The estimated frequencies of simulation for step change signals in the frequency when SNR=40dB.

B. Results Under IEEE C37.118.-2014 Standard

This section presents the simulation results of P-Class and M-Class synchrophasor measurements for the power system with 50.5 Hz signal frequency. According to IEEE C37.118.-2014 standard, the estimated performances of various algorithms are compared with an amplitude modulated signal, a phase modulated signal and a ramp signal. Eq. (20) gives the

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$$\frac{X_{\max} - X(\infty)}{X(\infty)} \times 100\% \quad (19)$$

where X_{\max} represents the instantaneous maximum deviation of the adjustment value, and $X(\infty)$ represents the steady-state

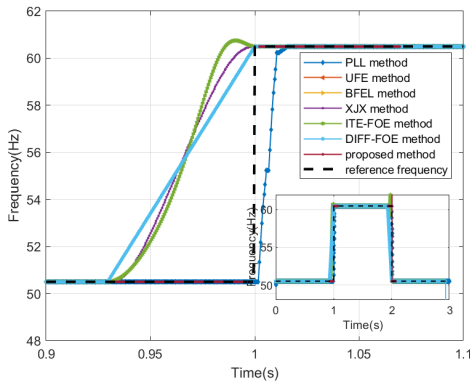


Fig. 17: The estimated frequencies of simulation for step change signals in the frequency under noiseless conditions.

456 amplitude modulated and phase modulated signal and Eq. (21)
457 gives the chirp signal.

$$X_1 = A [1 + \alpha \cos(2\pi f_1 t)] \times \cos [2\pi f_0 t + \beta \cos(2\pi f_1 t - \pi)] \quad (20)$$

$$X_2 = A_1 e^{j(2\pi f_1 t + \pi R_f t^2)} \quad (21)$$

458 The frequency error ($|FE|$), the total vector error (TVE) and
459 rate of change of frequency (ROCOF) are used as the index
460 to evaluate the performance of the considered methods. $|FE|$,
461 TVE and ROCOF are defined as follows:

$$|FE| = |\hat{f} - f| \quad (22)$$

$$\text{TVE} = \sqrt{\frac{(\hat{A}_r - A_r)^2 + ((\hat{A}_i - A_i))^2}{(A_r)^2 + (A_i)^2}} \times 100\% \quad (23)$$

$$\text{ROCOF} = \frac{\hat{f}(n) - \hat{f}(n-1)}{T_{RR}} \quad (24)$$

462 where \hat{A}_r and \hat{A}_i are the real and imaginary parts of the
463 estimated amplitude, A_r and A_i are the real and imaginary
464 parts of the true amplitude, \hat{f} is the estimated frequency and
465 f is the true frequency of the signal. $T_{RR} = 1/f_{RR}$ and f_{RR} is
466 the PMU reporting rate.

467 We will divide this part of simulation into two parts ac-
468 cording to section III. The initial parameters are all set as N
469 $= 128$ and $f_s = 6400$ Hz. The average values and max values
470 of $|FE|$, TVE and ROCOF of all the considered methods will
471 be shown in the following six tables.

472 1) *Amplitude modulation signal*: In the first case when the
473 amplitude and the phase step change when the frequency keeps
474 unchanged, β is set to 0. α is set to -0.1 and 0.1 respectively
475 before and after the step change. 0.2 per unit amplitude
476 variation with 1.0 Hz of frequency is applied to the single-
477 stone signal and the system frequency is set to 50.5 Hz. The
478 average values of $|FE|$, TVE and ROCOF of all the compared
479 methods are shown in Table V-VII. In the second case when
480 the frequency, the amplitude and the phase all step change, the
481 frequency is set to 50.5 Hz and 20.8 Hz respectively before and
482 after the step change. Other parameters remain unchanged in
483 the first case. The estimated results are shown in Table VIII-X.

2) *Phase modulation signal*: In the first case when the
484 amplitude and the phase step change, α is set to 0. β is set
485 to -0.1 and 0.1 respectively before and after the step change.
486 Modulation frequency and system frequency are referred to the
487 amplitude modulated signal shown above. The average values
488 of $|FE|$, TVE and ROCOF of all the compared methods are
489 shown in Table V-VII. In the second case when the frequency,
490 the amplitude and the phase all step change and the frequency
491 is set to 50.5 Hz and 20.8 Hz respectively before and after the
492 step change. Other parameters remain unchanged in the first
493 case. The estimated results are shown in Table VIII-X.
494

3) *Ramp signal*: In the first case when the amplitude and
495 the phase step change, the positive ramp rate is 1.0 Hz/ sec
496 and the negative ramp rate is -1.0 Hz/ sec. The amplitude
497 before and after the step change is set to 1 and 0.9 respectively.
498 The estimated results are shown in Table V-VII. In the second
499 case when the frequency, the amplitude and the phase all step
500 change, and the frequency is set to 50.5 Hz and 20.8 Hz
501 respectively before and after the step change. Other parameters
502 remain unchanged in the first case. The estimated results are
503 shown in Table VIII-X.
504

V. EXPERIMENTAL VERIFICATION

505 In this section, we will demonstrate the advantages of our
506 method in actual conditions. Eq. (1) can be used to describe
507 the modulation signals in the wireless systems, such as quadrature
508 amplitude modulation signal (QAM), phase-shift keying signal
509 (PSK) and frequency-shift keying signal (FSK) [40], [41].
510 QAM signal, PSK signal and FSK signal are the most widely
511 used modulation modes in communication.
512

513 We select the WLAN RF conformance testing, which is
514 the most conformed one in the field of IM. The WLAN RF
515 conformance testing has been widely used in the practical
516 application. The channel environment in the WLAN RF con-
517 formance testing is simple and idealized. Most interferences
518 in the actual wireless communication can be ignored during
519 the testing. Engineers of the RF conformance testing choose
520 the method in Ref. [50], which is also compared in this paper,
521 to estimate the carrier frequency offset(COF).

522 “NI WLAN Analysis” is a wireless RF conformance test
523 system developed by National Instruments [52]. This test
524 system provides the functions of signal generation and analysis
525 for the test application of WLAN 802.11a/b/g/j/p/n/ac/ax/be.
526 One of the most important functions of this test system is
527 to judge whether the test results of the DUT(device under
528 test) meet the test standard. A typical SISO test platform is
529 shown in Fig. 18. Part ③ is the DUT, which is a wireless
530 router supporting 802.11 ax. The output interfaces of the
531 DUT is connected directly to the input interfaces of the RF
532 conformance tester through a green specific cables, which
533 is the part ④ in Fig. 18. The MIMO test scenario can be
534 regarded as several SISO testers working in parallel. Keeping
535 the physical connection form between the DUT and the tester
536 unchanged, we can assemble a MIMO system with several
537 same SISO systems. As the test system should be placed in
538 a microwave anechoic chamber to avoid noise interference,
539 the communication model in the RF test system is simple

TABLE V: Average |FE| (Hz) of all comparative methods

| | Proposed | UFE | BFEL | XJX | ITE-DFT | DIFF-DFT | PLL |
|-------------------------|----------|--------|--------|-------|---------|----------|-------|
| Off-nominal frequencies | 0.016 | 0.512 | 19.189 | 0.256 | 0.189 | 0.053 | 0.021 |
| Harmonic distortions | 0.215 | 10.806 | 8.539 | 6.958 | 6.235 | 5.322 | 0.256 |
| Amplitude modulation | 0.015 | 1.074 | 20.310 | 0.548 | 0.307 | 0.038 | 0.019 |
| Phase modulation | 0.026 | 1.798 | 19.222 | 1.028 | 1.009 | 0.840 | 0.030 |
| Positive ramp | 0.062 | 8.083 | 0.556 | 0.056 | 0.069 | 0.061 | 0.066 |
| Negative ramp | 0.062 | 0.052 | 0.629 | 0.071 | 0.058 | 0.061 | 0.065 |

TABLE VI: Average TVE(%) of all comparative methods

| | Proposed | UFE | BFEL | XJX | ITE-DFT | DIFF-DFT | PLL |
|-------------------------|----------|--------|--------|--------|---------|----------|-------|
| Off-nominal frequencies | 0.783 | 4.420 | 24.861 | 3.974 | 2.543 | 1.179 | 0.831 |
| Harmonic distortions | 0.059 | 0.165 | 0.062 | 0.212 | 0.145 | 0.197 | 0.064 |
| Amplitude modulation | 0.105 | 4.863 | 20.093 | 4.316 | 2.249 | 0.324 | 0.167 |
| Phase modulation | 0.074 | 12.714 | 82.722 | 10.064 | 9.855 | 8.061 | 0.079 |
| Positive ramp | 0.004 | 0.010 | 0.004 | 0.096 | 0.026 | 0.013 | 0.009 |
| Negative ramp | 0.006 | 0.129 | 0.844 | 0.038 | 0.020 | 0.011 | 0.011 |

TABLE VII: ROCOF estimator

| | Proposed | UFE | BFEL | XJX | ITE-DFT | DIFF-DFT | PLL |
|----------------------|----------|-------|-------|-------|---------|----------|-------|
| Amplitude modulation | 0.003 | 0.008 | 0.003 | 0.002 | 0.010 | 0.010 | 0.005 |
| Phase modulation | 0.003 | 0.011 | 0.002 | 0.003 | 0.002 | 0.009 | 0.005 |
| Positive ramp | 0.004 | 0.010 | 0.005 | 0.004 | 0.002 | 0.013 | 0.006 |
| Negative ramp | 0.004 | 0.008 | 0.004 | 0.003 | 0.002 | 0.013 | 0.006 |

540 and idealistic, where the specific cable is used to connect the
 541 communication link. Testers do not need to consider the signal
 542 attenuation and distortion caused by various interferences in
 543 actual wireless communication, such as multipath effects.

544 “NI WLAN Analysis Soft Front Panel” is shown in Fig. 19.
 545 With the zero-IF technique, 2.4G Hz WLAN signal is received
 546 and down converted to baseband signal. After synchroniza-
 547 tion, the I/Q measurement, carrier frequency estimation and
 548 demodulation, the tester calculates the EVM (Error Vector
 549 Magnitude) to evaluate the RF performances of DUT. As
 550 shown in Fig.19, the COF is an important index in RF
 551 conformance test. According to the test requirement, the tester
 552 needs to measure the frequency offset of each subcarrier.
 553 Under this test requirement, only the subcarrier to be tested is
 554 allowed to transmit signals, and the remaining subcarriers are
 555 prohibited from transmitting signal. The tester can control the
 556 DUT to complete the above functions with the AT command.

In summary, the communication model in the RF test system
 is simple and idealistic. The state-of-the-art methods, which
 are not needed to the algorithm engineers of RF conformance
 testing. On the contrary, although they aren’t applicable for
 the actual wireless communication, the methods in Refs. [49],
 [50] and our method are suitable for the RF conformance
 testing. Let’s take 802.11 ax as an example. The transmission
 power and the receiving power used in “NI WLAN Analysis”
 are 10dBm and 20dBm. The line loss is about 3dBm. Then
 bandwidth of 802.11 ax signal is 20 MHz. The sampling fre-
 quency is set $f_s = 1.25 \cdot \text{bandwidth} = 25\text{MHz}$. With the zero-IF
 technique, the 2.4G Hz 802.11ax signal is received and down
 converted to baseband signal. Without the carrier frequency
 offset, the frequency points of subcarriers in baseband are 0Hz,
 20/K Hz, 40M/K Hz....., where K is the total number of
 subcarriers. When frequency offset exists, the frequency point

TABLE VIII: Average |FE| (Hz) of all comparative methods

| | | Proposed | UFE | BFEL | XJX | ITE-FOE | DIFF-FOE | PLL |
|-------------------------|------------|----------|--------|--------|--------|---------|----------|-------|
| Off-nominal frequencies | ω_1 | 0.182 | 18.972 | 19.995 | 9.656 | 35.404 | 8.853 | 0.235 |
| | ω_2 | 0.268 | 25.214 | 24.191 | 39.343 | 65.091 | 20.834 | - |
| Harmonic distortions | ω_1 | 1.082 | 18.167 | 18.976 | 17.047 | 19.062 | 2.232 | 1.131 |
| | ω_2 | 2.232 | 22.386 | 22.243 | 26.322 | 28.546 | 24.327 | - |
| Amplitude modulation | ω_1 | 0.006 | 3.103 | 2.781 | 3.900 | 2.728 | 3.528 | 0.011 |
| | ω_2 | 0.003 | 4.083 | 3.405 | 0.588 | 1.415 | 1.159 | - |
| Phase modulation | ω_1 | 0.016 | 3.860 | 2.101 | 3.838 | 2.534 | 3.919 | 0.021 |
| | ω_2 | 0.011 | 3.326 | 4.086 | 6.526 | 6.222 | 5.768 | - |
| Positive ramp | ω_1 | 0.031 | 0.251 | 0.261 | 0.199 | 0.426 | 0.106 | 0.036 |
| | ω_2 | 0.031 | 0.598 | 0.423 | 0.440 | 0.728 | 0.354 | - |
| Negative ramp | ω_1 | 0.031 | 0.293 | 0.259 | 0.189 | 0.254 | 0.168 | 0.036 |
| | ω_2 | 0.032 | 0.575 | 0.425 | 0.386 | 0.540 | 0.292 | - |

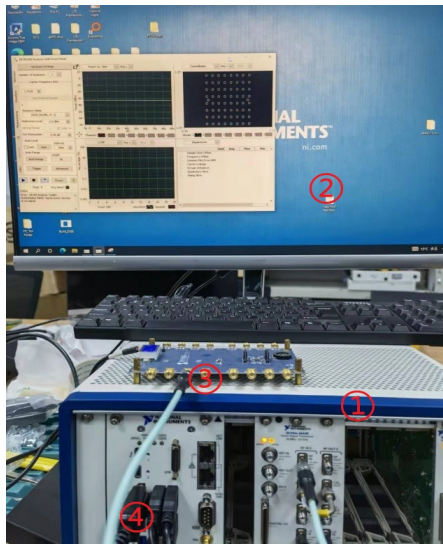


Fig. 18: The test system ①test platform device; ②screen; ③the DUT(device under test); ④the input interfaces through cables.

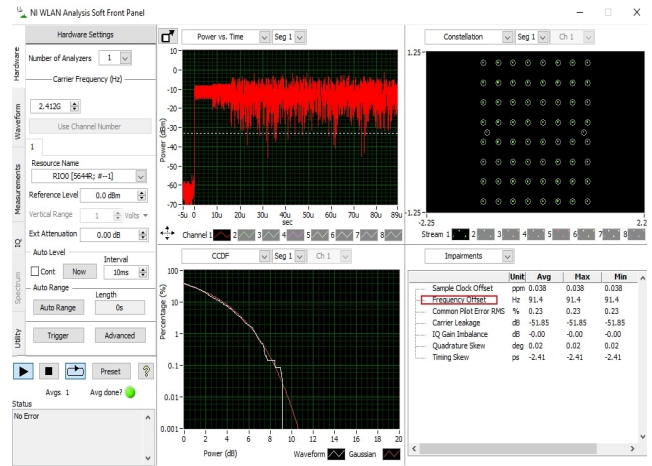


Fig. 19: NI WLAN Analysis Soft Front Panel.

574 of k -th subcarrier is $\Delta f_k + k \frac{20}{K}$ MHz, where Δf_k is the frequency
 575 offset and its value is no more than 100Hz. We select the
 576 0-th subcarrier as the research object and set the frequency
 577 offset as 91.4Hz. The 64-QAM signals are sent by the DUT
 578 and measured by “NI WLAN Analysis”. We cooperate with
 579 algorithm engineers of NI to give a performance comparison
 580 in the case that the two adjacent symbols jump, when N
 581 = 128 and $L = 64$. The comparison results are shown in
 582 Table XI. Our method can accurately estimate the frequency
 583 in the two-symbol combination scenario. When the difference
 584 between the two symbols is large (such as $U_1 = 1 + j$ and

$U_1 = 1 + 7j$), the method proposed in [50] can still estimate
 the actual carrier. In addition, if we select the first 64 sample
 points of the signal, the signal is a single carrier signal with
 the constant amplitude, frequency and phase. At this time,
 the carrier can be accurately estimated by Refs. [49], [50].
 The MSEs of the two algorithms in noise-free are -312dB
 and -337dB. The MSEs of the two algorithms in “NI WLAN
 Analysis” are -87dB and -85dB. Therefore, in the noisy state,
 if we select the 64-point sampling point corresponding to a
 symbol for the estimation in Refs. [49], [50], the estimation
 results are respectively close to and slightly worse than the
 estimation result obtained by our algorithm with 128 adjacent
 sampling points of two different symbols.

TABLE IX: Average TVE(%) of all comparative methods

| | | Proposed | UFE | BFEL | XJX | ITE-FOE | DIFF-FOE | PLL |
|-------------------------|------------|----------|--------|--------|--------|---------|----------|-------|
| Off-nominal frequencies | ω_1 | 0.313 | 12.225 | 10.102 | 19.697 | 28.236 | 17.040 | 0.358 |
| | ω_2 | 0.155 | 1.082 | 0.502 | 0.609 | 5.366 | 0.757 | - |
| Harmonic distortions | ω_1 | 2.531 | 14.057 | 11.203 | 18.011 | 21.272 | 12.599 | 2.582 |
| | ω_2 | 2.543 | 14.172 | 13.543 | 19.432 | 24.212 | 13.122 | - |
| Amplitude modulation | ω_1 | 0.116 | 12.077 | 10.000 | 21.449 | 17.429 | 19.048 | 0.162 |
| | ω_2 | 0.078 | 35.462 | 7.456 | 46.934 | 16.657 | 22.939 | - |
| Phase modulation | ω_1 | 1.224 | 16.028 | 10.000 | 19.072 | 19.848 | 21.520 | 1.269 |
| | ω_2 | 1.843 | 16.553 | 19.037 | 31.641 | 22.159 | 25.062 | - |
| Positive ramp | ω_1 | 0.010 | 1.240 | 0.830 | 0.337 | 5.369 | 0.567 | 0.015 |
| | ω_2 | 0.154 | 1.082 | 0.502 | 0.609 | 5.366 | 0.756 | - |
| Negative ramp | ω_1 | 0.350 | 1.687 | 0.884 | 0.475 | 0.467 | 0.837 | 0.403 |
| | ω_2 | 0.991 | 1.418 | 0.600 | 0.749 | 0.715 | 0.635 | - |

TABLE X: ROCOF estimator

| | | Proposed | UFE | BFEL | XJX | ITE-FOE | DIFF-FOE | PLL |
|----------------------|------------|----------|--------|--------|--------|---------|----------|-------|
| Amplitude modulation | ω_1 | 0.001 | 0.004 | 0.001 | 0.002 | 0.003 | 0.010 | 0.005 |
| | ω_2 | 0.001 | 0.004 | 0.001 | 0.002 | 0.003 | 0.010 | - |
| Phase modulation | ω_1 | 0.020 | 0.006 | 0.002 | 0.004 | 0.048 | 0.009 | 0.025 |
| | ω_2 | 0.246 | 23.326 | 23.086 | 40.526 | 79.222 | 15.768 | - |
| Positive ramp | ω_1 | 0.043 | 19.579 | 4.201 | 25.795 | 55.027 | 0.016 | 0.048 |
| | ω_2 | 0.637 | 19.580 | 4.201 | 25.795 | 55.027 | 0.016 | - |
| Negative ramp | ω_1 | 0.086 | 22.696 | 4.117 | 24.382 | 32.778 | 0.016 | 0.091 |
| | ω_2 | 0.635 | 22.697 | 4.117 | 24.382 | 32.779 | 0.016 | - |

TABLE XI: MSEs of $\hat{\omega}_0$ with different combinations of 64QAM symbol (unit: dB)

| | Proposed Method | UFE Method | BFEL Method | XJX Method | ITE-DFT Method | DIFF-DFT Method | PLL Method |
|------------------|-----------------|------------|-------------|------------|----------------|-----------------|------------|
| {1 + j, 1 + 3j} | -98.9064 | -90.6185 | -41.8568 | -28.6852 | -36.5555 | -75.7581 | -96.4532 |
| {1 + j, 1 + 5j} | -99.2897 | -95.1879 | -33.8279 | -28.6842 | -34.3535 | -90.5598 | -96.9367 |
| {1 + j, 1 + 7j} | -100.0315 | -96.9839 | -26.2352 | -28.6839 | -33.5013 | -96.5699 | -96.4578 |
| {1 + j, -1 - j} | -94.9775 | -93.7092 | -41.4395 | -30.0236 | -33.4679 | -76.3738 | -94.7346 |
| {1 + j, -1 - 3j} | -98.6781 | -97.5644 | -28.8736 | -29.5114 | -32.8817 | -77.8916 | -95.3346 |
| {1 + j, -1 - 5j} | -96.2039 | -95.8673 | -31.3567 | -28.0228 | -27.5122 | -91.1075 | -96.0328 |
| {1 + j, -1 - 7j} | -99.7640 | -99.6668 | -23.8708 | -29.2829 | -32.5654 | -94.2626 | -96.7843 |

VI. CONCLUSION

The IpDFT algorithm proposed in this paper effectively estimates the frequency of the single tone signal with parameter step change in the sampling signal sequence. The relationship between the DFT bins and the step changed parameters is given by several linear equations. At most six different DFT bins are used to eliminate the effect of the parameter (frequency, amplitude or phase) step change on the L -th sample. The results of simulation and experiment confirm that the proposed algorithm gives precise results. As a future work, we will study an improved method for the multi-frequency signal with multi-symbol step change.

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