

# Prediction of Ship Resonant Rolling - Related Dangerous Zones with Regard to the Equivalent Metacentric Height Governing Natural Frequency of Roll

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**ABSTRACT:** Potentially dangerous zones corresponding to dynamical stability phenomena, possibly encountered by ships sailing in rough sea, are estimated nowadays with the use of the method recommended by IMO in the guidance coded MSC.1/Circ.1228. In this IMO method the parameter governing the natural period of roll is the initial metacentric height. Some earlier studies revealed that the initial metacentric height which is commonly in use on-board ships for the purpose of performing the MSC.1/Circ.1228-recommended calculations, may significantly vary from the so called equivalent metacentric height obtained for large amplitudes of ship's roll. In the light of such ascertainment, the paper deals with resultant resonance roll zones locations with regard to the equivalent metacentric height concept remaining appropriate for large amplitudes of roll. The noteworthy transfer of the resonance zones location is disclosed which reflects the distinct configurations of potentially dangerous ship's course and speed configurations than could be predicted on the basis the initial metacentric height.

## 1 INTRODUCTION

The concept of e-navigation tends to comprise and address the range of objectives related to the wide spectrum of shipping challenges. According to the proposed e-navigation strategy „*e-navigation is expected to provide digital information and infrastructure for the benefit of maritime safety, security and protection of the marine environment, reducing the administrative burden and increasing the efficiency of maritime trade and transport*” (IMO 2005). The declared efforts in the fields of safety, security and increase in the efficiency of maritime transport are inherently associated to many issues and ship stability among them. The stability matters need to be taken into account both during port operations and sea voyage as well. Even though large amplitudes of ship roll are highly unlikely during cargo handling in a port (Krata *et al.* 2013, Krata 2015), dangerous excessive rolling may

happen at seaway (Kobylnski & Kastner 2003). Thus, the ship stability, among others, ought to be addressed by e-navigation solutions.

In 2014 IMO has prepared e-navigation Strategy Implementation Plan - SIP (IMO 2014). The SIP includes tasks defined for five prioritized solutions required for a successful e-navigation implementation and deployment and the task no. S 4.1.5 was formulated as “*Routing and filtering of information on board (weather, intended route, etc.)*”. This shows that weather routing solutions taking into account safety issues resulting from dynamical stability phenomena are considered by IMO as e-navigation tools benefiting in increasing safety and security of shipping. Numerous works are also recently published which aim at combination of ship stability performance and weather routing (Decó & Frangopol 2013; Dong *et al.* 2016).

Weather routing applications commonly take into account some hazards resulting from the ineluctable rolling of ships which may happen at seaway (Krata & Szlapczyńska 2011, Ami seaware, Amaron). They are designed to help navigators to calculate the ship courses prone to arouse the synchronous roll and the parametric resonance, since under the resonance conditions the ship rolling may develop excessively causing the direct threat to the ship, cargo and crew (Kobylnski & Kastner 2003). One could expect that the commonly accepted ship stability standards are effective solution for stability-related accidents. Unfortunately, the decades of experience revealed numerous incidents resulted from dynamic phenomena taking place in rough sea which are not covered by the contemporary stability standards described in the IMO IS Code (IMO 2008). One of the significant respond was publishing in 1995 the IMO guidance MSC/Circ.707 and in later years developed and revised to MSC.1/Circ.1228, published in 2007. This *Revised guidance to the master for avoiding dangerous situations in adverse weather and sea conditions* is intended to give some help to ship masters when sailing in adverse sea conditions. This publication contains a straightforward set of remarks and advices regarding the avoidance of following dangerous dynamical phenomena at sea like surf-riding and broaching-to, reduction of intact stability when riding on a wave crest amidships, synchronous rolling and parametric roll motions (IMO 2007). The last two are strictly related to the natural period of ship's roll and its possible tuning with the encountered wave period.

The notion of the natural period of roll plays a vital role in the resonance determination procedure. According to the guidance IMO MSC.1/Circ.1228 navigators are instructed to obtain the natural period of roll by use of stop watch in calm seas at each departure, which is usually hardly feasible, or alternatively on the basis of IMO recommended simplified formula, referred later in this paper as formula (3). Unfortunately, both methods are not very reliable. The natural period of roll for large amplitudes may considerably differ from observed in calm seas (small roll amplitudes) and the calculated one when the formula (3) is applied (amplitudes close to zero). Therefore the conditions allowing for building up an excessive roll motion due to parametric or synchronous rolling may be significantly different than expected by navigators nowadays (Wawrzynski & Krata 2016a, 2016b). Thus, a new reliable method for the resonant rolling conditions prediction was proposed (Wawrzynski & Krata 2016a). The considered problem refers to the scope of influence of the equivalent metacentric height application on the locations of resonance zones.

The rest of the paper is organized as follows. Section 2 introduces the background of the ship roll motion. Section 3 discusses results of the research, e.g. locations of the resonance zones with regard to the equivalent metacentric height. Finally, Section 4 presents concise summary and conclusions.

## 2 ROLL MOTION OF THE SHIP MODELED AS THE NONLINEAR DYNAMICAL SYSTEM

The ship rolling under external excitations is commonly described as a nonlinear dynamical system, for instance in the form of formula (1). All crucial components of this equation, e.g. restoring, damping and excitation, may reveal nonlinearities, however the most essential parameter determining the nonlinearity of rolling is the restoring moment. This moment is strictly related to the ship stability which is generally described by the righting arm curve (the GZ curve), although some authors use the initial metacentric height as the stability characteristic. The earlier studies show that the GZ curve and it nonlinearities plays vital role regarding to the resonance mode of roll (Wawrzynski & Krata 2016a, 2016b).

The roll equation applied to solve the ship rolling problem is formulated as follows:

$$\ddot{\phi} + 2\mu \cdot \dot{\phi} + \frac{g}{r_x^2} GZ(\phi) = \xi_w \cos(\omega_e t) \quad (1)$$

where  $\mu$  is the damping coefficient,  $g$  is the gravity acceleration,  $r_x$  is the gyration radius of a ship and added masses (which is assumed to be constant for the sake of simplicity),  $GZ$  is the righting arm,  $\xi_w$  means the exciting moment coefficient and  $\omega_e$  denotes external moment frequency.

The typical phenomenon related to oscillations is the resonance. When nonlinear oscillations take place, resonance frequency is almost constant for very small amplitudes of motion while it depends on the amplitude of the oscillations for large amplitudes which are typical for ship rolling in rough sea. The exemplary cases can be seen in figure 1 where the roll spectra are determined on the basis of the equation (1) with the use of two approaches: first, the linear righting arm which is written in the form  $GZ(\phi)=GM_0 \cdot \phi$  (left plot) and second, the real nonlinear characteristic of the righting arm (right plot). The resonance frequencies trace the maxima of the roll spectra obtained for the increasing value of the excitation moment. It can be clearly seen that the simplified approach (left plot) produces the maxima appearing for one constant value of the roll frequency, while the more accurate approach, taking into account the actual nonlinearity of the GZ curve, results in more complex pattern of the roll spectra maxima track (right plot). The value of the natural roll frequency strictly depends on the amplitude of roll which is omitted nowadays in the IMO recommendations provided in the MSC.1/Circ.1228 guidance for ship masters.

When the linear righting arm characteristic is considered the maxima of roll spectra reflect the so called natural frequency of ship roll which can be easily calculated according to the well known formula provided in the *Intact Stability Code* (IMO 2008):

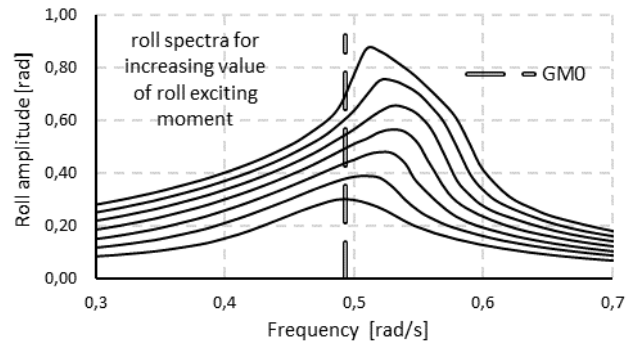
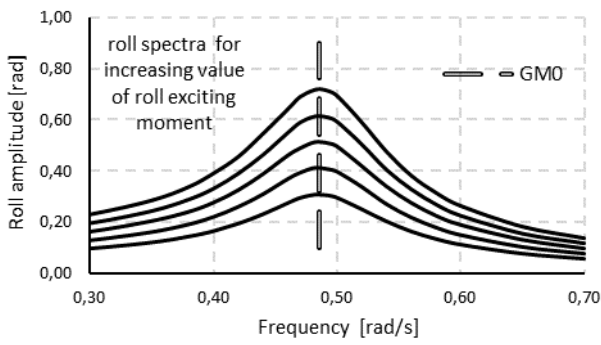


Figure 1. Roll spectra for different values of an exciting moment for the LNG carrier ( $T=12.00$  m,  $GM_0=5.00$  m) calculated with the linear restoring arm (left plot) and actual nonlinear shape of the GZ curve (right plot)

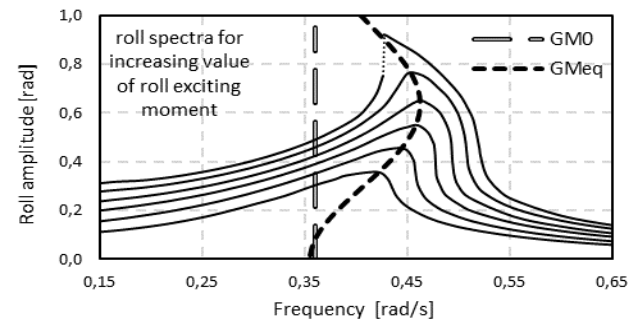
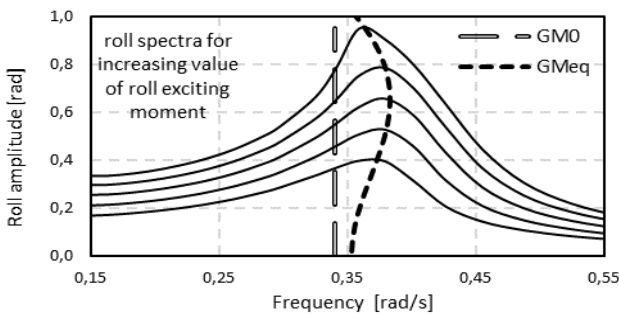


Figure 2. Roll spectra and resonance curves for the 5000 TEU Panamax container ship ( $T=7.50$  m,  $GM_0=1.50$  m) (left plot) and for the general cargo ship ( $T=6.00$  m,  $GM_0=1.00$  m) (right plot)

$$\omega_r = \frac{2\pi}{\tau}$$

where

$$\tau = \frac{2 \cdot c \cdot B}{\sqrt{GM_0}} \quad \text{with the value of } c \text{ coefficient:}$$

$$c = 0.373 + 0.023 \cdot \frac{B}{T} - 0.00043L \quad (3)$$

where  $\tau$  is the natural period of roll,  $GM_0$  is the initial metacentric height,  $c$  is the coefficient describing transverse gyration radius  $r_x$  ( $r_x=c \cdot B$ ),  $B$  is the breadth,  $L$  is the length at the waterline and  $T$  is the mean draft of the ship.

Generally, the formula (3) is in common use on-board for the purpose of IMO MSC.1/Circ.1228 guidance application. In fig. 1 the line of resonance frequency calculated using the  $GM_0$  is marked „GMO” and it can be seen that the maxima of all presented curves coincide with the „GMO” line in left plot while they significantly differs in right plot of figure 1.

The roll period and the resonance frequency, taking the nonlinearity of the GZ curve and the roll amplitude into account, can be calculated according to the method proposed in (Wawrzyński & Krata 2016a). The method is based on the area under the GZ curve and the average inclination of the tangent line to the GZ curve, both calculated from zero up to the roll amplitude. The main formula looks very similar to the IMO’s one:

$$\tau(\phi_A) = \frac{2 \cdot c \cdot B}{\sqrt{GM_{eq}(\phi_A)}} \quad (4)$$

where the value of  $c$  coefficient remains the same as given in the formula (3) and  $GM_{eq}(\phi_A)$  denotes the equivalent metacentric height for a specified roll amplitude  $\phi_A$ .

The formula for the equivalent metacentric height calculation is the following:

$$GM_{eq}(\phi_A) = \frac{\int_0^{\phi_A} GZ(\phi) d\phi}{\phi_A^2} + \frac{GZ_{\phi_A}}{2 \cdot \phi_A} \quad (5)$$

According to formula (5), the value of the equivalent metacentric height  $GM_{eq}$  and as a consequence also the roll period, depend on the roll amplitude. This makes up a noteworthy difference comparing to the original IMO recommended formula.

The verification of this method was performed for seven ships in different loading conditions (total number of analyzed cases add up to 30), and it revealed very good consistency of the roll period predicted by the formulae (4) and (5) with the results of numerical simulations for a wide range of roll amplitudes (Wawrzyński & Krata, 2016b).

Due to the combination with formulae (4) and (5), the roll resonance frequency depends on the amplitude:

$$\omega_r(\phi_A) = \frac{2\pi}{\tau(\phi_A)} \quad (6)$$

Formula (6) combined with formulas (4) and (5) was extensively analyzed (Wawrzyński & Krata 2016b). In the course of those investigations, the final form of the formula (6) was determined:

$$\omega_r^2(\phi_A) = \left( \frac{2\pi}{\tau(\phi_A)} \right)^2 - 2\mu^2 \quad (7)$$

The elaborated formulas allow to calculate the roll resonance frequency as a function of amplitude and further they are called the “ $GM_{eq}$  method”. The influence of the GZ curve nonlinearity affecting the roll characteristics with the increase in roll amplitude can be conveniently presented in the form of the resonance curve which is also called the backbone curve - in the following figures this curve is marked “ $GM_{eq}$ ”. In the research (Wawrzyński & Krata, 2016b) the accuracy of the  $GM_{eq}$  method was examined for different stability characteristics. In most cases, the method reveals very good agreement with the results of rolling simulations. In Fig. 2, the roll spectra for different values of an exciting moment, calculated for two ships, are presented. Each curve shows the roll amplitude versus roll frequency with the maximum reflecting the resonance frequency of oscillations. It can be seen, that the resonance curve calculated by the  $GM_{eq}$  method accurately traces the maxima of all curves. However, for the loading conditions where the simulation results reveal roll amplitude bifurcations the resonance curve obtained from  $GM_{eq}$  method is not very accurate whereas it still performs far better than the IMO recommended formula (3). Although, the bifurcation is known as the relatively difficult problem to modeling and analyzing (Francescutto & Contento 1999). The observed inaccuracy was found in regions with significant jumps of the amplitude.

Regardless the validation of the proposed method presented in (Wawrzyński & Krata, 2016b) this method was applied in irregular wave conditions to the C11 class container ship (Acanfora *et al.* 2017). The advanced nonlinear numerical model called *LaiDyn* was utilized. A set of ship motion simulations was carried out to collect histories of roll of the C11 vessel. The irregular sea modeled with the use of JONSWAP spectrum was applied and numerous sea states and the ship courses were analyzed. The study revealed that the maxima of recorded roll spectra are much closer to the natural frequency of the ship roll obtained according to the proposed method (formula 4 and 5) than the natural roll frequency predicted according to the contemporary formula (3) (Acanfora *et al.* 2017). This study seems to be the strong point speaking in favor of the proposed method.

### 3 EQUIVALENT METACENTRIC HEIGHT VARIATION AND THE RESULTANT RESONANCE ZONES LOCATIONS

As the roll amplitude depended equivalent metacentric height governs the natural frequency of

ship roll, the accuracy of synchronous roll prediction is affected. Thus, the well justified question is related to the realistic extent of the equivalent metacentric height variation. To assess this crucial matter the series of calculations was carried out aiming at the typical loading conditions of numerous cargo vessels. Six different ships were taken into account to cover a variety of ship purpose, their different particulars and loading condition from ballast up to fully loaded ones. The main particulars of considered ships are given in table 1.

The equivalent metacentric height was calculated according to formula (5) for all the vessels listed in Table 1. The wide range of possible amplitudes of roll was taken into account. The performed series of calculations revealed that the equivalent value of the metacentric height may vary significantly according to the amplitude of roll in many cases but not in all of them. Generally, the range of variations depends on the shape of the GZ curve.

The research reveals that in almost half of examined cases the equivalent metacentric height may significantly vary comparing to the initial GM even by 50% which is presented in Fig. 3. Moreover, in one eighth of cases the  $GM_{eq}$  is even twice as large as the initial GM. The  $GM_{eq}$  variations ranging in extreme cases from 169% to 10% which reflects different shapes of GZ curves remaining close to linear in low or wide range of angles of heel, respectively. In the light of the preformed analysis one may state that the formula (3) should be replaced by the formula (4) or (7) to address the problem of accurate prediction of possible synchronous roll of the ship.

Table 1. Main particulars of ships considered in the course of the research.

	Type of vessel considered	length [m]	breadth [m]	draft T [m]		height $GM_0$ [m]	
				min	max	min	max
1	general cargo ship	140.00	22.00	6.00	9.00	0.40	1.00
2	bulk carrier	156.10	25.90	6.50	10.50	1.00	3.50
3	LNG carrier	278.80	42.60	7.50	12.00	1.00	5.00
4	5000 TEU Panamax (old) container ship	283.20	32.20	7.50	13.50	0.50	3.00
5	7500 TEU container ship	285.00	45.60	7.50	12.50	2.00	5.00
6	motor tanker	320.00	58.00	10.00	22.00	5.00	10.00

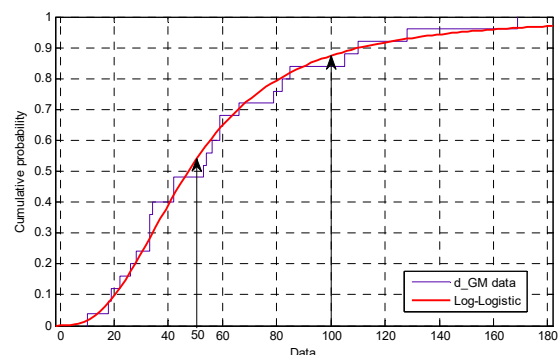


Figure 3. Cumulative distribution function for the equivalent metacentric height variation

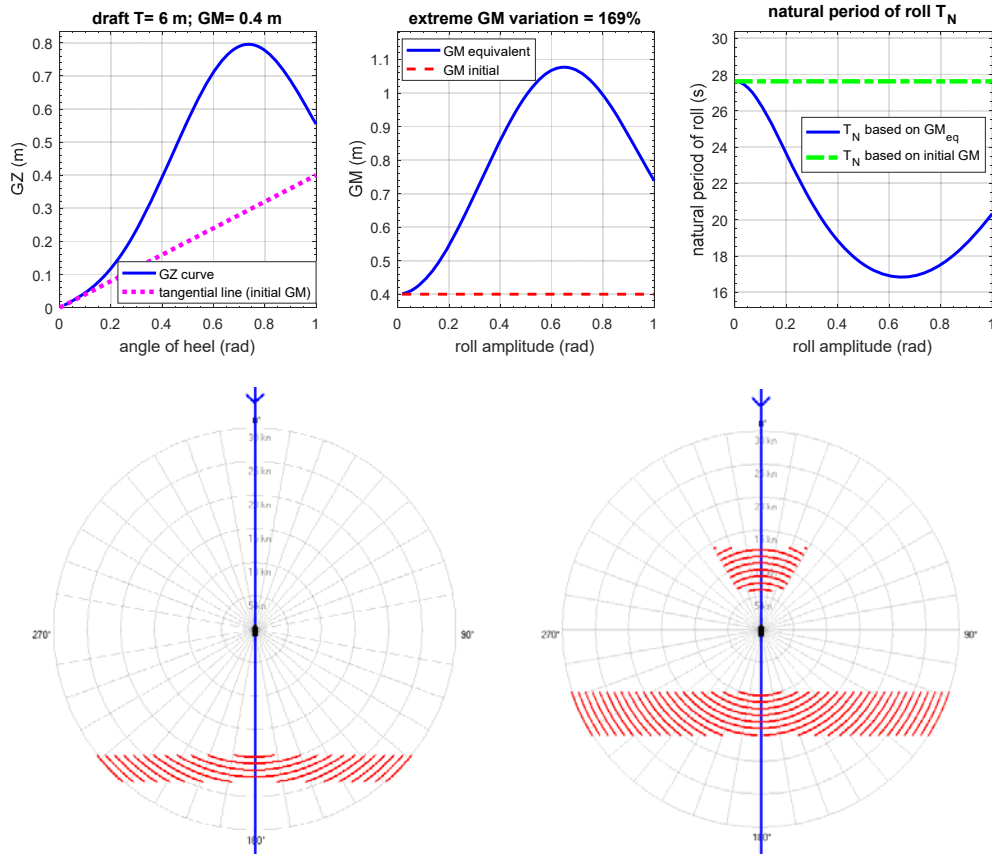


Figure 4. General cargo ship, LBP = 140.00 m, B = 22.00 m. Ship stability characteristics and the natural period of roll calculated according to the formula (5) – upper plots. Location of resonance zoned obtained for the initial metacentric height (lower left plot) and for the equivalent metacentric height (lower right plot)

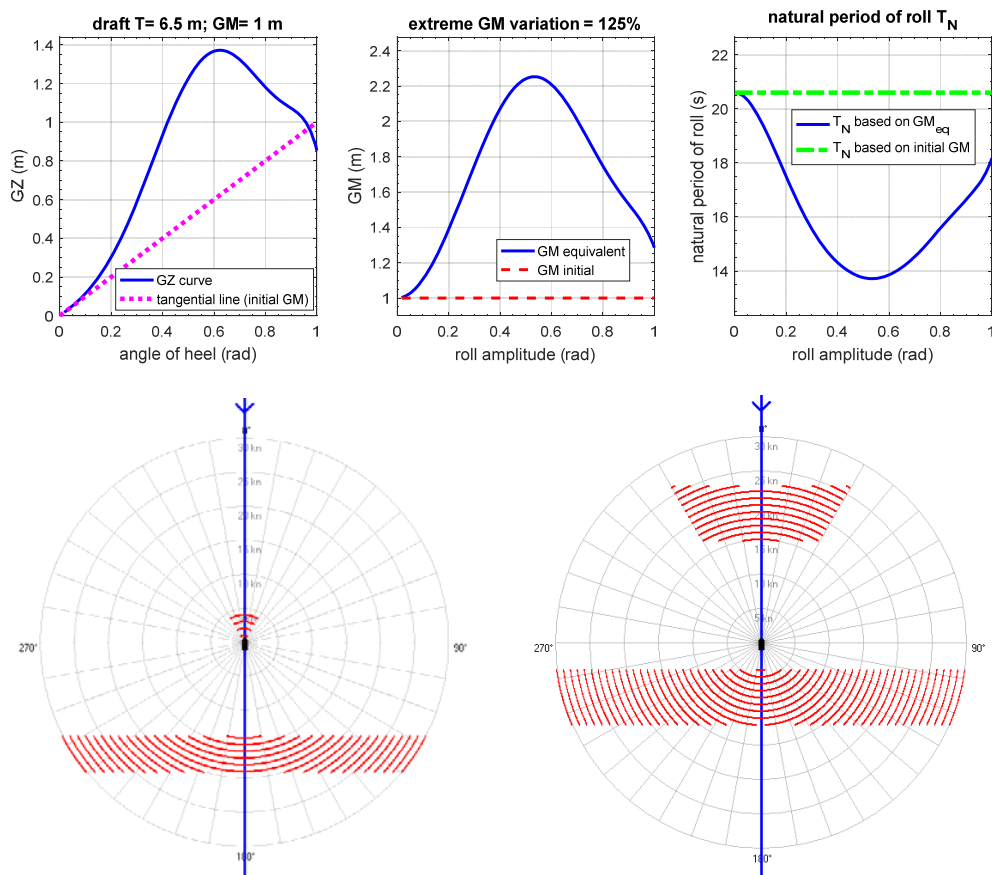


Figure 5. Bulk carrier, LBP = 156.10 m, B = 25.90 m. Ship stability characteristics and the natural period of roll calculated according to the formula (5) – upper plots. Location of resonance zoned obtained for the initial metacentric height (lower left plot) and for the equivalent metacentric height (lower right plot)

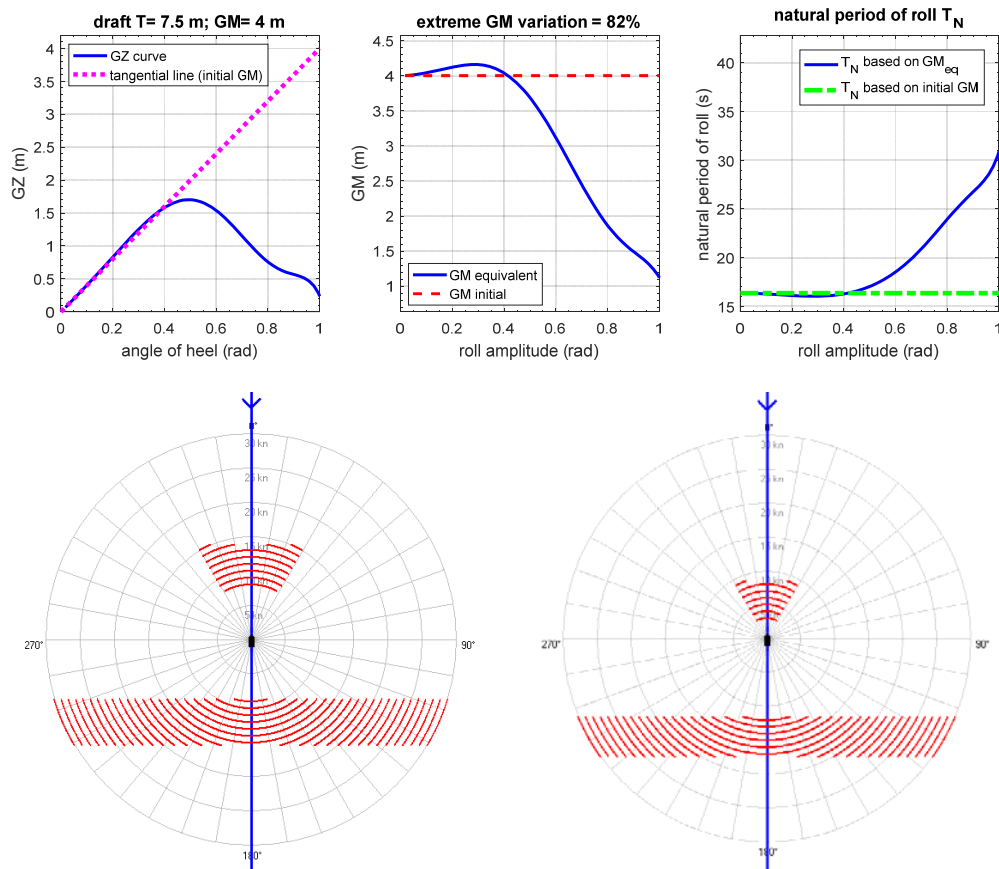


Figure 6. LNG carrier,  $LBP = 278.80$  m,  $B = 42.60$  m. Ship stability characteristics and the natural period of roll calculated according to the formula (5) – upper plots. Location of resonance zoned obtained for the initial metacentric height (lower left plot) and for the equivalent metacentric height (lower right plot)

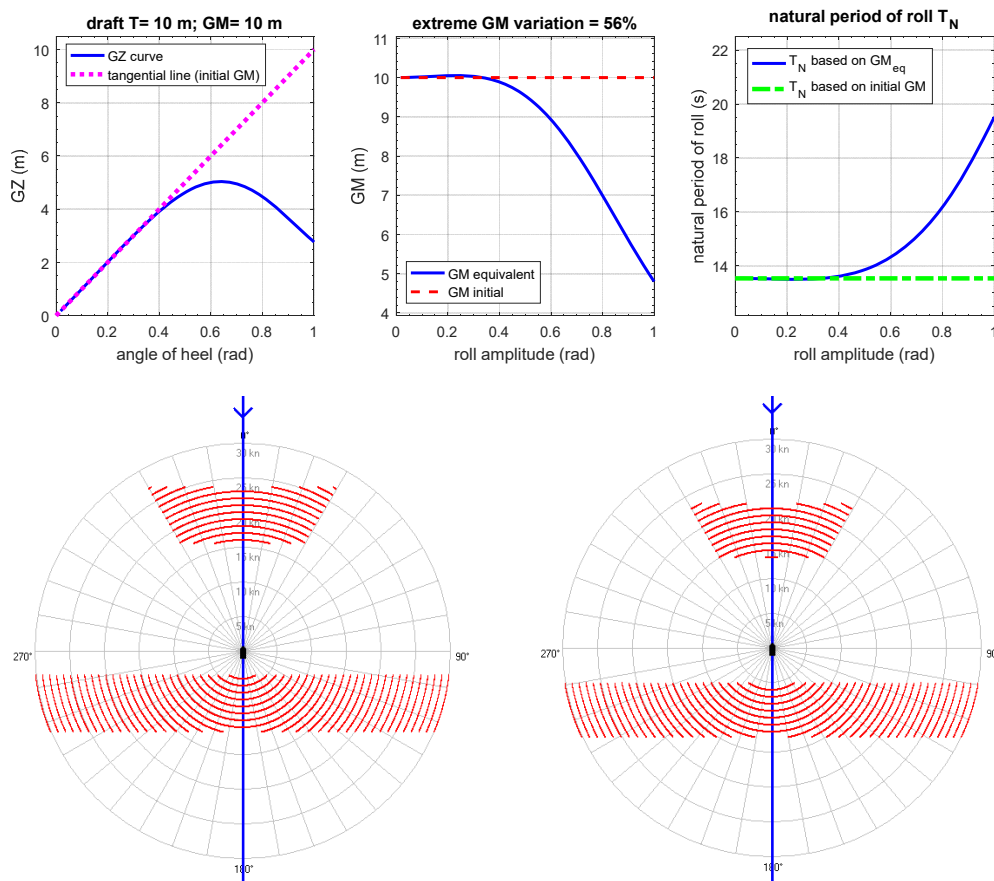


Figure 7. Motor tanker,  $LBP=320.00$  m,  $B=58.00$  m. Ship stability characteristics and the natural period of roll calculated according to the formula (5) – upper plots. Location of resonance zoned obtained for the initial metacentric height (lower left plot) and for the equivalent metacentric height (lower right plot)

The locations of resonance-related dangerous zones established according to the IMO Circ. 1228 recommendations, vary in corresponding extend. The calculations and graphical presentation of resonance zones locations are performed with the use of a software tool described in (Krata & Szlapczyńska 2011). The ships mentioned in Table 1 are considered and the dangerous zones are obtained for the assumed wind force 8B and the sea state reflecting fully developed wave system with the dominant wave period equal to 11 seconds. The sample results of calculations are shown in Fig. 4 to 7.

In case of relatively small difference of the metacentric height, comparing the initial and equivalent value, the resonance zones are located very close (see Fig. 7). This means that regardless the formula enabling the natural period of roll calculation (formula 3 or 4) the configuration of ship's speed and courses to be avoided by the officer of the watch is alike. Such a situation takes place when GZ curve is close to linear for a wide range of angles of heel. Otherwise, the reliable resonance zones obtained on the basis of the equivalent metacentric height can be far from the inaccurate expectations resulting from the initial GM utilization (see Fig. 4 and 5).

The dangerous zones locations shown in Fig. 4 to 7 are established for the dominant wave period equal 11 s. This allows for presentation of the direct dependence of the potentially dangerous courses arrangement on the equivalent metacentric height

obtained for the realistic range of roll amplitudes. According to these graphs one may suspect that for some vessels the resonance problem actually does not exist (see Fig. 6 for instance where the ship speed is insufficient to enter the resonance zone). Such a conclusion would be misleading since the dangerous zones are calculated for one value of the wave period equal 11 seconds. In case of really long ships such a period is related to the wave length significantly shorter than the ship's length. Thus, the dangerous resonance zones would appear for longer waves with correspondingly longer periods. Such calculations are presented in Fig. 8 obtained for the increasing wave period. The largest considered vessel mentioned in Table 1 is taken into account with the draft equal 22 meters and the equivalent metacentric height equal 5.4 meter.

Discussing the sample computing results we can notice that, even despite application of the relatively simple model of resonance zones locations, the practical use of the IMO Circ. 1228 requires a dedicated software available on-board. The number of input data is large enough to exclude the manual calculations. The equivalent metacentric height needs to be computed, the ship characteristics including draft play a vital role and the wave period and as a consequence the wave length varies significantly depending on sea conditions. Thus, the proper decision support tool would be helpful on-board every single vessel to enable a reliable and accurate prediction of the dangerous zones location.

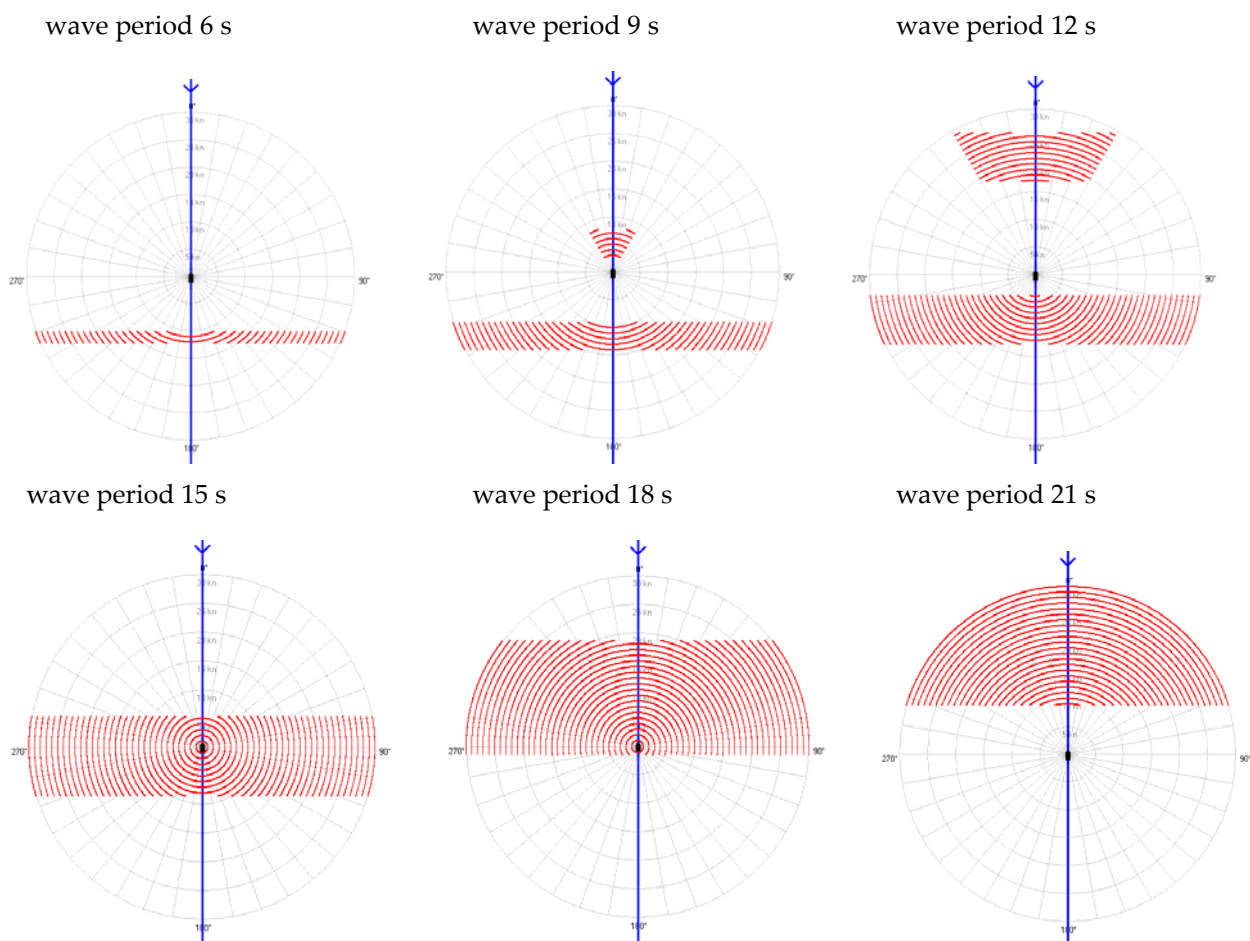


Figure 8. Locations of resonance zones for the largest considered motor tanker ( $LBP=320$  m,  $B=58$  m,  $T=22$  m, initial  $GM=5.4$  m) for increasing wave period equal to: 6 s; 9 s; 12 s; 15 s; 18 s; 21 s

The problem of prediction the dangerous zones locations related to the ship synchronous roll and parametric resonance is discussed in the paper. It is emphasized that the nonlinearity of the GZ curve plays a vital role in roll modeling while the contemporary recommended simplified formula for the natural roll frequency estimation omits this nonlinearity at all. To address the problem the equivalent metacentric height is utilized instead of the initial GM. It is revealed that such modification of the natural roll frequency evaluation method makes it strictly depended on the actual roll amplitude of the ship. This creates the important question related to the significance of the  $GMeq$  in terms of practical shift in dangerous resonance zones location. The study comprises numerous different types of ships sailing in realistic loading conditions.

The conducted research revealed that the inaccuracy of the resonance zone location may be very significant when the GZ curve is strongly nonlinear within the range of the considered roll amplitudes. The contemporary method based on the initial GM suggests the navigator to avoid speed and course configurations far irrelevant in comparison to the reliable modeling based on the equivalent GM. On the other hand when the GZ curve is linear up to relatively large angle of ship's heel, both analyzed methods produce very similar results.

Bearing in mind the wide scope of navigator's interest when on the watch, we suggest to implement a dedicated software designed to support decisions regarding avoidance of the potentially dangerous zones obtained according to the IMO MSC.1/Circ.1228 *Revised guidance to the master for avoiding dangerous situations in adverse weather and sea conditions*. The equivalent metacentric height shall be in use instead of the previous initial metacentric height. Such an approach would facilitate the application of trustworthy set of dynamic constraints to the route planning which is an intention of the e-navigation concept making the marine navigation safer and more efficient.

#### ACKNOWLEDGEMENT

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