

1 Development of cluster analysis methodology for identification of model rainfall 2 hyetographs and its application at an urban precipitation field scale

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4 Karol Mikołajewski^{1,2}, Marek Ruman^{2,*}, Klaudia Kosek^{3,*}, Marcin Glixelli⁴, Paulina
5 Dzimińska⁵, Piotr Ziętara⁴, Paweł Licznar⁵

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7 ¹RETENCJAPL Sp. z o.o., Gdańsk 80-868, Poland; karol.mikolajewski@retencja.pl

8 ²Faculty of Natural Sciences, University of Silesia in Katowice, Sosnowiec 41-200, Poland;

9 marek.ruman@us.edu.pl

10 ³Faculty of Civil and Environmental Engineering, Gdansk University of Technology, Gdańsk 80-233, Poland;

11 klaudia.kosek@pg.edu.pl

12 ⁴Krakow Water, Kraków 30-106, Poland; marcin.glixelli@wodociagi.krakow.pl (M.G.),

13 piotr.zietara@wodociagi.krakow.pl (P.Z.)

14 ⁵Faculty of Environmental Engineering, Wrocław University of Science and Technology, Wrocław 50-377,

15 Poland; dziminska.paulina@gmail.com (P.D.), pawel.licznar@pwr.edu.pl (P.L)

16 *Corresponding authors: marek.ruman@us.edu.pl (M.R.); klaudia.kosek@pg.edu.pl (K.K.)

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19 **Abstract:** Despite growing access to precipitation time series records at a high temporal scale,
20 in hydrology, and particularly urban hydrology, engineers still design and model drainage
21 systems using scenarios of rainfall temporal distributions predefined by means of model
22 hyetographs. This creates the need for the availability of credible statistical methods for the
23 development and verification of already locally applied model hyetographs. The methodology
24 development for identification of similar rainfall models is also important from the point of
25 view of systems controlling stormwater runoff structure in real time, particularly those based
26 on artificial intelligence. This paper presents a complete methodology of division of storm
27 rainfalls sets into rainfalls clusters with similar temporal distributions, allowing for the final

28 identification of local model hyetographs clusters. The methodology is based on cluster
29 analysis, including the hierarchical agglomeration method and k-means clustering. The
30 innovativeness of the postulated methodology involves: the objectivization of clusters
31 determination number based on the analysis of total within sum of squares (wss) and the
32 Caliński and Harabasz Index (CHIndex), verification of the internal coherence and external
33 isolation of clusters based on the bootmean parameter, and the designated clusters profiling.
34 The methodology is demonstrated at a scale of a large urban precipitation field of Kraków city
35 on a total set of 1806 storm rainfalls from 25 rain gauges. The obtained results confirm the
36 usefulness and repeatability of the developed methodology regarding storm rainfall clusters
37 division, and identification of model hyetographs in particular clusters, at a scale of an entire
38 city. The applied methodology can be successfully transferred on a global scale and applied in
39 large urban agglomerations around the world.

40

41 **Keywords:** precipitation modelling, storm rainfalls, cluster analysis, classification quality
42 assessment indices

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44 **1. Introduction**

45 Conducting calculations of non-stationary surface runoff of stormwaters and their
46 further transformation in the stormwater system, including detention and retention in water
47 bodies, requires the scenarios availability of rainfall temporal distribution. Precipitation
48 scenarios employed by hydrological models usually determine the values of outflow volumes
49 observed in the calculation results, the dynamics of their changes in time, and changes in the
50 water volume retained in various elements of the drainage system. Due to historical conditions,
51 and particularly calculation limitations, for decades, only the simplest models naturally

52 employing the simplest rainfall scenarios could be implemented in practice in hydrology at its
53 various scales, and particularly in urban hydrology. An extreme example can be the rational
54 model, still commonly used today in designing urban and road drainage systems. It employs
55 block rainfall, i.e., a scenario of rainfall with a specified duration and constant intensity
56 determined based on the local IDF (Intensity-Duration-Frequency) model for the adopted
57 frequency of its occurrence (Kundzewicz and Licznar, 2021(2022)). The block model is
58 obviously a very specific rainfall scenario. Unlike in that model, as observed for natural rainfall,
59 hydrological analyses usually assume the necessity of taking into consideration the temporal
60 variability of point rainfall intensity, usually described in the graphic form by means of
61 hyetographs. Adopting hyetographs in design aimed at reflecting rainfall temporal distributions
62 analogical to the temporal courses of locally recorded rainfalls, and implicitly also potential
63 rainfalls that may occur in the future. This has encountered numerous obstacles since the very
64 beginning, resulting not only from the stochastic (random) nature of rainfall, but also from the
65 multifractal nature of rainfall still unknown at the time. It is currently already evidenced that
66 even simple time series of rainfall records from single rain gauges have a multifractal structure
67 escaping the principles of simple Euclidean geometry (de Lima, 1998; Deidda et al., 1999;
68 Licznar, 2009). Due to this, and due to the limited availability of research material (sets of
69 precise rainfall records at a high temporal resolution), the historical methodology of
70 hyetographs representative development for modelling, customarily called model hyetographs,
71 had to be based on generalisation and simplification, and assumptions that should be currently
72 rejected or subject to suitable validation. All this creates the need for undertaking new research
73 in the scope of identification of model hyetographs.

74 Interest in model hyetographs certainly increased with first attempts of transition from
75 stationary methods of calculation of urban stormwater systems towards non-stationary methods
76 of simulation of their operation during stormwater runoff. It became possible due to the



77 implementation of digital hydrodynamic models of sewage networks, such as e.g., programme
78 SWMM (Storm Water Management Model) (Nix, 1994). Initially, however, due to the limited
79 computing power of the available computers, focus was only placed on flows simulations in the
80 underground canal system for single rainfalls. The model of stormwater runoff transformation
81 in the sewage system was only combined with the hydrological surface runoff model, describing
82 stormwaters inflow to network nodes (manholes and inlets) from the catchment area, and the
83 stormwater system effect. The interaction of the stormwater system with the rainfall receiver
84 was only possible to reflect by means of a suitable threshold condition on the network outlet.
85 Rainfall introduced to the first generation of stormwater systems hydrodynamic models was a
86 hyetograph of either a subjectively selected actual storm rainfall recorded by a pluviograph, or
87 an artificial model hyetograph (often called model rainfall).

88 With rapid improvement of the computing power of computers, but without excessive
89 complication of the algorithms of hydrodynamic models, it became possible to conduct
90 simulations for entire series of local storm rainfalls. Their use enabled actual implementation
91 of long-term simulations. After statistical processing, their final results could provide the basis
92 for the probabilistic verification of the drainage system functioning in terms of stormwater
93 system damming up frequency (Gires et al., 2012, 2013; Licznar et al., 2008), as well as e.g.,
94 the necessary volume of retention reservoirs of stormwaters (Licznar, 2013). This new reality
95 seemingly appeared to bring an end to the application of model hyetographs, at least in the area
96 of urban hydrology. Moreover, with the installation of urban precipitation monitoring networks
97 equipped with new generation electronic rain gauges, high temporal resolution rainfall series
98 became much more available. In simulations of large stormwater systems, particularly in terms
99 of development of their Real Time Control, records of rainfalls from entire networks of rain
100 gauges began to be applied, or even spatial data from weather radars (Jakubiak et al., 2014).
101 Afterwards at operational phase of RTC systems above mentioned observational rainfall data



102 sources are often coupled with numerical weather predictions. It allows proper short-term
103 prediction of extreme rainfall events, indispensable for urban flood alert broadcasting and
104 selection of best scenarios of next RTC strategies. Here, there is a vivid need for effective
105 whether pattern recognition algorithms, based on machine learning technique, as for example
106 support vector machine implemented by Nayak and Ghosh (2013) in case of Mumbai, India.
107 Finally, in response to the stochastic nature of precipitation processes, it was determined that it
108 is the most credible to supply hydrodynamic models with not so much historical precipitation
109 data (i.e., implementations of local precipitation processes that already took place) as much
110 richer synthetic data generated by means of local precipitation generators (Molnar et al., 2006).
111 Owing to progress in the scope of multiplicative random cascades, it became possible to
112 develop generators not only for generating rainfall series (1-D) (Güntner et al., 2001; Hingray
113 and Ben Haha, 2005; Licznar et al., 2011a, 2011b), but also spatial data (2-D) (Over and Gupta,
114 1994; Rupp et al., 2012), or even spatiotemporal data on rainfall (3-D) (Deidda, 2000).

115 Despite all the aforementioned conditions, the end of application of model hyetographs
116 in urban hydrology has not arrived yet. To a certain extent this probably results from the
117 conservatism of the engineers themselves, preferring design and drainage systems modelling
118 based on simple and long-familiar model hyetographs. This facilitates their work e.g., in terms
119 of time and equipment requirements necessary for conducting hydrodynamic simulations, and
120 in accordance with the conservative provisions of rarely modified technical standards (Schmitt,
121 2000). Paradoxically, however, the application of model hyetographs is currently not limited to
122 simple engineering works. They prove useful in the most advanced consulting works. Such
123 works employ a completely qualitatively new approach to modelling stormwater networks,
124 called integrated modelling. The software used for the purpose is much more advanced, because
125 it combines three models: the hydrological streams model (receivers of stormwaters), the
126 hydrodynamic stormwater network model, and the hydrological surface runoff model. The first



127 two models are 1-dimensional, whereas the surface runoff model is a 2-dimensional model with
128 an additional fill parameter. Naturally, the latter must be coupled with the digital terrain model
129 (DTM). Although the application of integrated models brings numerous benefits, e.g., in the
130 form of the possibility of tracing floodings on the surface of the DTM, it requires the use of
131 complicated software launched on equipment with high computing power. Also in this case,
132 simulations of larger drainage systems prove time-consuming and prone to numerical
133 instabilities. As a result, simulation of runoff from the drainage system ceases to be feasible for
134 tens or even hundreds of scenarios of storm rainfalls. In practice, complicated simulations on
135 an integrated model can be only conducted for certain characteristic precipitation. This explains
136 the return to the concept of the model hyetograph application or a narrow group of model
137 hyetographs in the case of time-consuming simulations. It therefore remains an important issue
138 to improve the methodology that could efficiently and objectively determine what model
139 hyetographs would reflect local rainfall distributions variable in time in a satisfactory way.

140 In the case of the methodology modernisation of the model hyetographs development,
141 it should be remembered that it began forming in the situation of strongly limited access to high
142 temporal resolution rainfall records and statistical tools supporting their processes. Rainfall was
143 recorded by means of simple rain gauges, e.g., pluviographs, and the records in the analogue
144 form were difficult to process. Their processing usually involved a review of the records with
145 designation of maximum rainfall with different durations and structure, and determination of
146 the resulting empirical DDF (Depth-Duration-Frequency) or IDF (Intensity-Duration-
147 Frequency) dependencies. Due to this, it was proposed to use the information contained in DDF
148 and IDF dependencies as the starting point for constructing model hyetographs. Next to the
149 rectangular shape of block rainfall mentioned in the introduction, different authors
150 recommended simple modifications of the shape of hyetographs. For example, Sifald (1973),
151 analysing the hydraulic operation of the stormwater network, postulated a hyetograph with a

152 trapeze shape. At the scale of small catchments, Yen and Chow (1980) proposed transformation
153 of rainfall depth read for the predefined duration and frequency from the DDF model into a
154 triangular hyetograph. Probably due to the common use of the unit hydrograph method in the
155 contemporary hydrology aimed at providing an estimate of direct runoff hydrographs resulting
156 from given excess rainfall hyetograph, hyetographs with possibly simple shapes were eagerly
157 used, composed of regular geometric figures. For example, in urban hydrology, Desbordes
158 (1978) implemented the application of a hyetograph composed of “three” triangles. Peyron et
159 al. (2002) attempted to simplify the shape through replacing two triangles representing the start
160 and end impulses with rectangular courses. At the same time, in the conditions of Taiwan, Lee
161 and Ho (2002) postulated the application of a model hyetograph with a shape built from two
162 triangles. Even earlier, independently from the aforementioned papers, a concept appeared for
163 the structure of the model hyetograph to be completely based on the reading of the entire IDF
164 curve. The most classic example of the approach can be the continuous hyetograph for the city
165 of Chicago developed by Keifer and Chu (1957). Another representative of this concept can be
166 Euler type II model hyetograph (Schmitt, 2000), frequently encountered in the hydrodynamic
167 modelling practice in Poland and Germany. The underlying idea of this type of hyetographs
168 structures assumes a single artificial rainfall scenario including maximum point intensities for
169 the entire hierarchy of partial times. It appears to be at variance with the observation of nature,
170 where all maximums of point intensities are usually not recorded in a single rainfall at a
171 specified level of frequency for durations lower and equal to total duration. Nonetheless, relying
172 on the Euler type II hyetograph theoretically offers engineers the possibility to test the operation
173 of the drainage system in a single simulation for all maximum point intensities simultaneously
174 at a specified level of frequency of occurrence. Engineers are accustomed to using IDF curves,
175 and their transformation into a Euler type II model hyetograph requires only simple algebra
176 operations. In the case of analyses of larger drainage systems, it is even simpler for engineers



177 in Poland and Germany to reach for a hyetograph recommended in the guidelines of DVWK
178 (1984). Its development is based on the distribution of rainfall depth read from the DDF model
179 to three rectangles with different intensity levels.

180 With time, the methodology of model hyetographs determination could be improved
181 owing to the availability of increasingly richer measurement data from rain gauge networks. It
182 became feasible to determine model hyetographs based on the large rainfall datasets analysis.
183 The precursor of such an approach was Huff (1967), who analysed data from 49 rain gauges
184 and 12 years (1955-1966) from the state of Illinois in the USA. Because the rainfalls retrieved
185 from the records had different total durations and depths, he proposed normalisation of
186 hyetographs, and application of dimensionless hyetographs. This permitted comparison of a
187 dataset of 261 rainfalls that Huff (1967) classified according to whether the greatest percentage
188 of cumulative rainfall occurred in the first, second, third, or fourth quarter of the storm duration.
189 The resulting model Huff mass curves found numerous applications in hydrology. The
190 analogical methodology of temporal variability analysis of rainfall distributions was not only
191 repeated in further research by Huff himself (1970, 1990), but also by many other scientists,
192 e.g., Pani and Haragan (1981), Bonta and Rao (1987), Bonta (2004), Terranova and Iaquina
193 (2011), Elfeki et al. (2014), Pan et al. (2017). Hydrologists in the USA also commonly apply
194 model hyetographs defined by means of dimensionless mass curves recommended by the SCS
195 (Soil Conservation Service) (McCuen, 1986). The increasingly rapidly growing digital
196 databases of rainfalls records from electronic rain gauges, and the progress in the scope of data
197 mining techniques nowadays allows for the continuation of the trend determined by Huff
198 (1967), and model hyetographs determination based on the actual local rainfall analysis.

199 Access to a large data base from 25 electronic weight rain gauges from the municipal
200 precipitation monitoring network of Warsaw (Poland) has become the impulse for the
201 verification whether the Euler type II model hyetograph applied in practice, developed based

202 on the IDF curve, corresponds with temporal distributions of actual rainfalls. For this purpose,
203 out of approximately 20-year precipitation series, Licznar and Szeląg (2014) desinated a total
204 of 669 storm rainfalls. The set was then divided into subsets with increasing durations,
205 expressed in minutes: [0-45], (45-60], (60-90], (90-120], (120-180], (180-240], (240-300],
206 (300-360], (360-420]. Each of the subsets was moreover supplemented by an additional Euler
207 precipitation model (type II) developed based on the local IDF curve. All subsets were then
208 analysed with the application of the hierarchical agglomeration method. Based on the obtained
209 dendrograms, Licznar and Szeląg (2014) observed that precipitation recorded in Warsaw, even
210 those with approximate durations, have evidently differing hyetographs. Moreover, in each of
211 the subsets, Euler model precipitation (type II) was an extreme outlier in the structure of the
212 dendrogram with the highest bond distance towards all actual rainfalls. Results of the study
213 have become an impulse for the search for a method of designation of more representative local
214 model hyetographs. For this purpose, Licznar et al. (2017) proposed the application of cluster
215 analysis tools in the form of not only the hierarchical agglomeration method, but also non-
216 hierarchical k-means clustering. The methodology also found applications for example in
217 research by Licznar (2018) and Wartalska et al. (2020). Nonetheless, it leaves evident gaps. Its
218 greatest weakness is completely subjective a priori adoption of number k of clusters, i.e., the
219 number of final rainfall models. Two further missing components of the methodology include
220 the objective assessment of the classification results and cluster profiling. They are increasingly
221 important, because in the context of new challenges in urban hydrology, striving for ‘smart city’
222 solutions involves not so much searching for model hyetographs themselves as the possibility
223 of fast and efficient search of similar temporal models of rainfalls. Runoff control systems
224 employing artificial intelligence aim at the implementation of the most effective strategy of the
225 forecasted scenario control of stormwater runoff, adopted based on the already implemented in
226 nature and recorded precipitation phenomenon and runoff caused by rainfall with possibly



227 similar rainfall temporal distribution in reference to local rainfall forecast from the nowcasting
228 system. The final question to be answered from the engineering practice point of view is: to
229 what extent the application of the cluster analysis brings results repeatable at the natural spatial
230 scale of extensive municipal drainage systems? Can for example model hyetographs developed
231 based on rainfall series from nearby rain gauges be treated as credible for analyses of
232 precipitation-runoff phenomena in the territory of the entire city?

233 Considering the overview of the methodology state of rainfall classification in terms of
234 its temporal distribution and model hyetographs identification, the primary objective of this
235 paper is to present the complete cluster analysis methodology supplemented by objective
236 determination of the number of classes, credible assessment of the classification results, and
237 cluster profiling. Another primary objective of the paper is to demonstrate the postulated
238 complete cluster analysis methodology at the scale of a large urban precipitation field, and the
239 resulting answer to the question whether model hyetographs retrieved from records of different
240 rain gauges show mutual compatibility.

241 The pragmatic objective of the study is the complete methodology development of
242 designation of local model hyetographs throughout Poland. In the years 2016-2020, the project
243 of the Polish Atlas of Rains Intensities PANDa was implemented, resulting in a digital base of
244 rainfalls at high temporal resolution (after 30 years of observation from 100 stations), and the
245 national rainfall atlas composed of 12885 local IDF models ascribed to areas designated through
246 the division of Poland with a grid with field dimensions of 5 km per 5 km (Burszta et al.; 2019;
247 Licznar et al., 2020). The PANDa atlas provides the basis for the publicly accessible digital
248 design platform www.waterfolder.com, where you can design and select among others:
249 retention reservoirs of stormwaters, infiltration reservoirs for stormwaters, linear drainage units,
250 gravitational canals of stormwater systems, stormwater pumps and pumping stations, and green
251 roof surfaces. In the scope of the new WaterFolder Connect project, works are currently

252 undertaken aimed at the integration of tools of selection and enabling hydrodynamic
253 simulations of newly designed drainage systems. This will require the designation of local
254 hyetographs for particular areas of Poland. The developed methodology presented in this article
255 is planned to be applied at the scale of the entire country, and then globally.

256 **2. Materials and methods**

257 2.1. Study area

258 Research on objective classification of storm rainfall hyetographs by means of
259 classification quality assessment indices was conducted at a large scale of an urban precipitation
260 field in the territory of Kraków. Kraków is the second largest city in Poland in terms of
261 population (~ 767 000) and surface area (327 km²), located in the south of the country on the
262 Vistula River. The study employed part of the resources of the Polish precipitation data base of
263 the Polish Atlas of Rains Intensities (PANDa) project, and records from the local rain gauge
264 network of the Municipal Water Supply and Sewerage Company (MWSSC) in Kraków.

265 At the initial stage of the PANDa project in the years 2016-2017, a digital base of rainfall
266 series was developed for a total number of 100 rain gauges in Poland. Then it was analysed in
267 terms of occurrence of maximum rainfall intensity (Burszta-Adamiak et al., 2019). The base
268 included among others records from a rain gauge installed in the area of the Kraków–Balice
269 airport (50°04'40", 19°47'42") at a height of 237 m a.s.l., and rain gauge in station Kraków–
270 Wola Justowska (50°03'50", 19°53'25") at a height of 204 m a.s.l. (Figure 1). For station
271 Kraków–Balice, records from the multiannual period 1986-2006 were available, and for station
272 Kraków–Wola Justowska records from the multiannual period 2007-2015. In that set, digital
273 record series from the years 1986-1998 resulted from the digitalisation of pluviograph recording
274 strips. The computer-aided method of conversion of pluviograph recording strips to digital
275 format similar to that proposed by Licznar et al. (2011a) was adopted. Records from a standard

276 unheated pluviograph (200 cm² orifice) covered warm year periods between spring (April or
277 May) and autumn (October or November) when most of storm rainfall events occur in Poland.
278 For later years, i.e., for the multiannual period 1999-2015, all-year records from electronic
279 tipping bucket gauges were already available. On station Kraków–Balice, rainfall was recorded
280 by an electronic rain gauge Aster TPG, and on station Kraków–Wola Justowska, by an
281 electronic rain gauge Met One Instruments 60030. The resolution of recording rainfall depth in
282 the case of the aforementioned devices was 0.1 mm and 0.2 mm, respectively, and their inlet
283 surface was 200 cm². Records of local rainfall provided for the research by the MWSSC came
284 from a network of a total of 23 rain gauges distributed throughout the city (Figure 1).

285

286 **Figure 1.** Location of rain gauges belonging to Waterworks Kraków and the Institute of
287 Meteorology and Water Management

288

289 The entire network was composed of electronic tipping bucket gauges operating during
290 both the summer and winter half-year. In all stations, rain gauges Hobo RG3M were installed,
291 with rainfall record resolution of 0.2 mm and inlet surface of 200 cm². Unfortunately, records
292 of rainfall series from these rain gauges were considerably shorter than in the case of the data
293 base of the PANDa project, covering a period of 30 years (1986-2015). Detailed information
294 regarding periods of rainfall records by particular rain gauges of the measurement network in
295 Kraków is provided in Table 1 in the chapter discussing results obtained in the scope of
296 designation of storm rainfalls. It should be emphasised that all rain gauges used in the analysis
297 were located within the administrative boundaries of the city. The only exception was rain
298 gauge Kraków–Balice located in the direct vicinity of the city boundaries, in the area of the
299 nearby airport. All rainfall series from the entire period of monitoring of the precipitation field,
300 recorded directly in digital form by tipping bucket gauges, as well as those resulting from

301 digitalisation of records on pluviograph record strips, had a uniform temporal resolution of 1
302 minute.

303

304 2.2. Applied methodology

305 Based on the available time series of rainfall records (from the rain gauges of the
306 PANDa project and from the network of MWSSC), storm rainfalls were designated by standard
307 criteria proposed by Schmitt (2000) for identifying storms for urban drainage systems
308 modelling. They are standard criteria applied in Germany and Poland, and used in the already
309 cited paper by Licznar et al. (2011a). The adopted threshold minimum value of the total amount
310 of storm rainfall was 10 mm, and the minimum time interval between single rainfall events was
311 at least 4 hours. The designation of rainfalls in reference to dry periods also employed the
312 minimum value of rainfall depth of 0.1 mm during 5 minutes as the threshold value for the
313 interval to be considered as part of a rainfall event in terms of duration and precipitation amount.
314 The result of designation of storm rainfalls separately from rain gauges of the PANDa project
315 and the MWSSC network were data sets called set No. 1 and No. 2, respectively. Set No. 3
316 analysed in the final part of the research constituted a combination of sets No. 1 and No. 2.

317 Due to the differing durations and total depths of the designated storm rainfalls, further
318 analysis of the sets, i.e., mutual comparison and identification of typical (quantifiable, model)
319 distributions during rainfalls, involved double normalisation of cumulative hyetographs. This
320 procedure was conducted in accordance with the methodology described in the publication
321 (Licznar et al., 2017). It corresponded with the methodology of development of dimensionless
322 hyetographs by Huff (1967). For this purpose, for each storm rainfall, total duration and total
323 rainfall depth was determined. Next, each cumulative storm rainfall hyetograph with known
324 total duration was divided into 100 even time intervals. For each of the subsequent intervals, a
325 corresponding cumulative rainfall increase was determined. Subsequent cumulative

326 precipitation increases were divided by total rainfall depth, obtaining unitary cumulative
327 precipitation increases. As a result, the shape of each of the storm rainfalls was reflected by a
328 hyetograph normalised to a range from 0 to 100% for duration and to a range from 0 to 1 (100%)
329 for rainfall depth.

330 The mutual comparison of shapes of normalised (dimensionless) hyetographs within the
331 analysed sets, and further designation of typical storm hyetographs that could be considered
332 model hyetographs, useful e.g., for modelling of local drainage systems, employed tools for
333 mining large data sets in the form of cluster analysis algorithms. Unlike in the case of earlier
334 attempts to apply cluster analysis for the classification of storm rainfall hyetographs and for
335 identification of model hyetographs (Licznar et al. 2017; Licznar, 2018; Wartalska et al., 2020),
336 this study applied a complete cluster analysis methodology. The complete cluster analysis
337 methodology involves seven stages (Milligan, 1996; Zhou et al., 2014):

- 338 1) Selection of objects and variables;
- 339 2) Selection of the formula of normalisation of variable values;
- 340 3) Selection of distance measure;
- 341 4) Selection of classification method;
- 342 5) Determination of the number of classes;
- 343 6) Assessment of classification results;
- 344 7) Class description (interpretation) and profiling.

345 The implementation of the first two points of the methodology, i.e., selection of objects and
346 their variables, combined with normalisation of their values, was already characterised earlier,
347 and aimed at the development of sets of normalised (dimensionless) hyetographs. Two further
348 stages 3 and 4, covering the selection of classification methods and distance measures, were
349 analogical to those in the already published papers by Licznar et al., 2017, Licznar, 2018,
350 Wartalska et al., 2020. In comparison to these publications, three last missing stages of the

351 cluster analysis were added in this paper, including: objective determination of the number of
352 classes, and assessment of classification results, combined with simplified class profiling. This
353 resulted in a coherent research methodology allowing for the classification of storm rainfall
354 hyetographs, their division into the objectively determined number of clusters, determination
355 of courses of model hyetographs, and their simplified profiling.

356 Cluster analysis tools find broad practical application in collating large data sets. Their
357 implementation in the case of such sets permits the separation of their objects into a certain
358 number of subsets, called clusters, covering mutually similar objects. The requirement of
359 decouplability and sufficiency of the designated clusters is met, i.e., each of the elements
360 belongs to a specific single cluster (Larose, 2005; Stanisiz, 2007). Therefore, the sum of all
361 clusters corresponds with the initial large set of objects, and particular clusters are separate and
362 have no elements in common. In each cluster, objects are approximate, mutually similar, and
363 simultaneously different from objects in other clusters. Depending on the adopted method, the
364 division into clusters can be conducted to an *a priori* determined or undetermined number of
365 clusters. In research in the Kraków polygon, cluster analysis was applied to the division of sets
366 No. 1, No. 2, and No. 3 of dimensionless cumulative hyetographs of storm rainfalls into an
367 undetermined, and then determined number of clusters. For this purpose, the hierarchical
368 agglomeration method and non-hierarchical k-means clustering method were implemented.

369 Agglomeration methods have already found application in research on precipitation, not
370 only in the context of search for similarities in temporal distribution of storm rainfalls for
371 particular locations (Licznar et al., 2017; Wartalska et al., 2020), but also in the case of research
372 on the variability of precipitation conditions within a large municipal precipitation monitoring
373 network (Licznar et al., 2015). In their research conducted on the municipal monitoring network
374 in Warsaw (including 25 rain gauges), comparable to that in Kraków in terms of size, Licznar
375 et al. (2015) successfully implemented agglomeration methods to evidence similarities of

376 empirical distributions of breakdown coefficients (BDCs) of rainfall from rain gauges located
377 in different points in the city, for hierarchies of timescales, corresponding with time from 5 min
378 to 1280 min. Agglomeration methods applied in this type of research aim at combining
379 mutually similar objects through the application of appropriate measures of their mutual
380 distance and the agglomeration method. The starting point is treating each object as a separate
381 cluster. At the subsequent stages of agglomeration, objects most mutually approximate
382 according to the defined measure (i.e., most similar by default) are combined into new clusters,
383 covering objects and clusters resulting from earlier stages, until a single cluster is obtained.
384 Based on the experience of other authors (Licznar et al., 2017; Wartalska et al., 2020), in
385 research in the Kraków polygon, the already verified distance metrics were applied: Euclidean
386 and Euclidean squared distance. The Euclidean distance metric $d(x,y)$ for two objects $x = (x_1,$
387 $\dots, x_r)$ and $y = (y_1, \dots, y_r)$, characterised by r measurement values, is expressed in the following
388 formula (Larose 2005, Stanisiz 2007):

$$389 \quad d(x, y) = \sqrt{\sum_{i=1}^r (x_i - y_i)^2}. \quad (1)$$

390 The Euclidean distance metric has simple and natural interpretation in the case of objects
391 defined by only two or three measurement values ($r = 2$ or $r = 3$), because in that case its
392 equivalent is distance on a plane and in space of two points x and y . Through analogy, in the
393 case of the analysed sets of normalised hyetographs, the Euclidean distance between pairs of
394 hyetographs was determined in space with a considerably higher number of dimensions (for r
395 = 100).

396 Next to adopting a particular mutual distance metric for objects in the measurement
397 space, the objective of combining mutually similar objects and clusters also requires the
398 application of a particular method of their agglomeration. In this case, also based on the already
399 cited papers (Licznar et al., 2017; Wartalska et al., 2020), the popular unweighted pair-group
400 method was applied. In this method, differences in distances between all pairs of elements

401 included to particular clusters are calculated. Values of the averaged differences between all
402 pairs are adopted as the measure of distance between particular clusters. Due to this, it is known
403 which elements of the sets are mutually similar and can be included to shared clusters, and
404 moreover to what extent particular clusters are mutually similar and can be agglomerated into
405 structures of larger clusters. A natural consequence of this is forming on the dendrograms (the
406 resulting diagrams of the set structure in relation to the increasing bond distance, and therefore
407 decreasing similarity between its elements) characteristic ‘chains’ made of similar objects
408 developing increasingly extensive clusters.

409 Considering the primary study objective, i.e., the determination of model hietographs,
410 a more significant research tool was k-mean clustering. The application of this tool permitted
411 separation of the analysed sets into k independent clusters differing to the greatest possible
412 degree. In the case of earlier research on model hietographs (Licznar et al., 2017, Wartalska et
413 al., 2020), the application of the k-means method always involved questionable adopting of k
414 number of clusters subjectively estimated based on the analysis of previously prepared
415 dendrograms. It was assumed that at a certain level of bond distance, the chains of clusters can
416 be cut to obtain several separate subsets of mutually similar dimensionless hietographs. The
417 obvious weakness of such an approach was lack of justification of the adopted level of bond
418 distance at which the dendrogram was divided (cut). Moreover, as not observed in earlier papers
419 (Licznar et al., 2017; Wartalska et al., 2020), simple cut-off of dendrograms at a given level of
420 and distance and obtaining k clusters was not equivalent to the determination of the same
421 number k of independent clusters by means of the k-means clustering method. Particular
422 clusters could include differing subsets and objects as a result of differences in the classification
423 methods between the hierarchical agglomeration method and non-hierarchical method, namely
424 k-means clustering.



425 Unlike in the case of the hierarchical agglomeration method, aimed at combining objects
 426 and subclusters, agglomeration by means of the k-means clustering method aims at fragmenting
 427 the entire set into the *a priori* defined number k of clusters, whereas none of the k clusters is the
 428 subcluster of another cluster. The algorithm of agglomeration by means of the k-means
 429 clustering method therefore involves development of k subclusters, and then moving objects
 430 across them for distances between them within the subclusters to be as small as possible, and
 431 for distances between subclusters to be as large as possible. The moving procedure is repeated
 432 iteratively aiming at the most efficient separation of clusters (Larose, 2005). The final objective
 433 is arriving at a solution in which the designated clusters meet two criteria: that of internal
 434 coherence and external isolation (Gordon, 1999). This task was implemented in the computing
 435 environment of language R due to the resulting calculation difficulties. Their primary source
 436 was the number of classified objects. The number of all divisions of a set of n elements into k
 437 non-empty clusters is escribed by the following formula (Everitt et al., 2001; Gordon, 1999):

$$438 \quad L(n, k) = \frac{1}{k!} \sum_{s=1}^k (-1)^{k-s} \binom{k}{s} s^n, \quad (2)$$

439 where s is the number of class ($s = 1, \dots, k$). In accordance with formula (2), even in the case of
 440 a very small set of 10 objects with their division into 4 non-empty clusters, the number of all
 441 possible divisions is 34 105 ($L(10, 4) = 34105$). For comparison, research has involved division
 442 of large sets of hyetographs including hundreds or even thousands of storm rainfalls.

443 These calculation challenges were further considerably multiplied during analyses of
 444 measurement sets No. 1, 2, and 3 due to multiple launching of the clustering algorithm with the
 445 application of the bootstrap method. In statistics, bootstrap methods are used to estimate the
 446 distribution of estimation errors by means of multiple sampling with replacement. Their
 447 implementation in the case of research on storm hyetographs from Kraków meant that the
 448 clustering algorithm was performed 150 times, each time for random samples from the analysed
 449 sets. Results obtained in subsequent iterations were compared, allowing for the designation of

450 values of the bootmean parameter. The bootmean parameter was calculated as a mean value of
451 the Jaccard index (Jaccard similarity coefficient) for each of the designated clusters. The
452 Jaccard coefficient itself measures similarity between two sets, and is determined as the ratio
453 of power set of the intersection of sets and power set of sum of these sets. High values of the
454 Jaccard coefficient approximate to 1 strongly suggest perfect repeatability of the separation of
455 objects into clusters. It is assumed that exceeding the threshold of 0.6 for Jaccard coefficients
456 for each of the clusters suggests no occurrence among the designated clusters of clusters with
457 random character, i.e., those including rainfall models deviating from the remaining clusters,
458 but simultaneously not mutually similar. The verification of the above criterion allows drawing
459 conclusions regarding the resulting clusters meeting the criteria of internal coherence and
460 external isolation (Gordon, 1999). The bootmean parameter was used in the study for
461 conducting assessment of results of the classification of particular sets No. 1, 2, and 3.

462 The compute-intensive process of k-means clustering was preceded by the stage of
463 determination of the number of classes. For this purpose, a methodology was adopted analogical
464 to that applied in research on temporal distributions of hourly water uptakes by Dzimińska et
465 al. (2021). The analysis of previously prepared dendrograms provided the basis for the
466 determination of a potential range of the number of clusters that should be considered in the
467 case of division of storm rainfalls in Kraków. The optimal number of clusters was designated
468 from that range based on the analysis of the total within sum of squares (wss) and values of the
469 Caliński and Harabasz Index (CHIndex) for a variable number of clusters. Values of total within
470 sum of squares (wss) and the Caliński and Harabasz Index (CHIndex) were calculated in
471 accordance with the following formulas (Walesiak and Gatnar, 2009):

$$472 \quad wss = \sum_i^k \sum_{x \in C_i} \|x - m_i\|^2, \quad (3)$$

$$473 \quad CHIndex = \frac{SS_B}{SS_W} \cdot \frac{N-k}{k-1}, \quad (4)$$

474 where: k – number of clusters, x – element of a set, C_i – i -th data cluster, m_i – cluster centroid i ,
475 $\|x - m_i\|^2$ - Euclidean distance between two vectors, N – total number of observations
476 (elements of a set), SS_B – total variance between clusters (trace of interclass covariance matrix),
477 SS_W – total internal cluster variance (trace of intraclass covariance matrix). The value of w_{SS}
478 naturally decreases with an increase in the number of clusters k . Nonetheless, the gradient of
479 the decrease evidently decreases after reaching a certain number of clusters considered optimal.
480 In the case of the Caliński and Harabasz Index, the number of clusters is searched, considered
481 optimal, for which its value is the highest. In accordance with formula (4), this means the
482 occurrence of maximisation of the ratio of SS_B and SS_W , i.e., particular clusters differ from one
483 another very significantly, and the elements of the set agglomerated in particular clusters are
484 strongly mutually similar (relatively weakly variable).

485 The last seventh stage of the research covered the description (interpretation) and
486 profiling of the obtained clusters. For all the clusters, their centroids were determined
487 (arithmetic averages calculated from the original values of each variable based on objects
488 developing a given cluster). This provided the basis for obtaining sets of normalised
489 hyetographs for each of the analysed sets No. 1, 2, and 3. They were subject to mutual
490 comparison. Finally, simplified profiling of clusters was conducted. The objective of cluster
491 profiling is the identification of characteristic features of particular clusters allowing for the
492 determination of differences between them. Cluster profiling is conducted based on variables
493 that did not take part in the process of classification of the set of objects. In the case of research
494 on hyetographs of storm rainfalls from Kraków, the available variables that did not participate
495 in the process of classification of the set of objects were total rainfall depths and total rainfall
496 durations. These general rainfall characteristics permitted the determination of distributions of
497 mean precipitation intensities in particular clusters, i.e., simplified profiling of clusters in terms



498 of explanation of differences in the obtained types of model hyetographs from the point of view
499 of intensities of the represented storms.

500

501 **3. Results and discussion**

502 Based on the adopted criteria (Schmitt, 2000), from set No. 1, covering time series of
503 precipitation records from 30 years for rain gauges of the PANDa project, a total of 313 storm
504 rainfalls were designated (Table 1), corresponding to the frequency of their occurrence at a level
505 of approximately 10 storms per year. Analogically, from the larger set No. 2 of precipitation
506 series from 126 years, recorded on 23 rain gauges of the MWSSC network, a total of 1494
507 storm rainfalls were designated (Table 1), corresponding to the frequency of their occurrence
508 at a level of approximately 12 storms annually. The obtained quantities of data sets were in
509 accordance with the expectations and results of previously published research from the territory
510 of Poland. For example, from a set of digitalised pluviography storm records from 38 years
511 from Wrocław (south-western Poland), Licznar et al. (2011a) designated 250 storm rainfalls
512 based on the same criteria, determining their frequency of occurrence at a level of 6.6 times per
513 year. The lower precipitation frequency results from the fact that at each stage, precipitation
514 records covered only periods of 5-6 months with positive air temperatures that allowed for the
515 exposure of non-heated pluviographs. For comparison, in the polygon of the rain gauge network
516 of the city of Warsaw composed of 25 electronic weighing rain gauges recording precipitation
517 throughout the year, the criteria proposed by Schmitt (2000) permitted the designation of a total
518 of 669 storm rainfalls (Licznar and Szelağ, 2014). The latter figure, in the context of records
519 somewhat longer than two years (114 weeks) for each of the rain gauges, translates into the
520 frequency of occurrence of storms equal to 12.3 events per year per single observation point.

521

522 **Table 1.** List of rain gauges belonging to sets No. 1 and 2 with characteristics of designated
523 storms

524

525 Particular rainfalls differed in their courses in time. It is evident based on the example
526 of cumulative normalised (dimensionless) hyetographs of set No. 1 in figure 2. A large majority
527 of rainfalls, both in the case of set No. 1 and No. 2, had hyetographs considerably differing
528 from theoretical hyetographs recommended for application as rainfall scenarios for
529 hydrodynamic modelling of municipal drainage systems, such as: block rainfall, model rainfall
530 according to DVWK, or model rainfall according to Euler type II. This observation also remains
531 in accordance with research of other authors who already previously questioned the justification
532 of drainage systems modelling in Poland commonly being based on model rainfall according
533 to Euler type II (Licznar and Szeląg, 2014; Licznar et al., 2017).

534

535 **Figure 2.** Cumulative dimensionless hyetographs of 313 storm rainfalls from Kraków (set
536 No. 1)

537

538 The designated storm rainfalls in particular sets No. 1 and 2 also differed in terms of
539 total durations and total rainfall depths. In set No. 1, total depths of storm rainfalls were in a
540 range from 10.0 to 105.0 mm, and their durations varied from 24 to 2329 minutes. These
541 parameters in set No. 2 varied within similar ranges from 10.0 to 145.4 mm and from 14 to
542 4576 minutes in the case of total depths and total durations, respectively. These values also
543 showed similar distributions in both analysed sets, as confirmed by histograms included in
544 figure 3. A vast majority of storm rainfalls was characterised by total rainfall depth in a range
545 from 10 mm to 20 mm, and their total durations varied from 2 h to 12 h. The similarity of total
546 distributions of rainfall depths and durations in sets No. 1 and No. 2 presented in figure 3 at

547 least partially justifies the possibility of use of sets of storm rainfalls recorded in stations in city
548 outskirts and belonging to the national precipitation monitoring network in the context of issues
549 regarding urban hydrology at a scale of a city as large as Kraków.

550

551 **Figure 3.** Histograms of rainfall depths and durations for the designated sets of rainfalls from
552 Kraków, diagrams in the top row for set No. 1 (313 rainfalls), diagrams in the bottom row for
553 set No. 2 (1493 rainfalls)

554

555 The analysed sets No. 1 and No. 2 of normalised cumulative precipitation were used for
556 preparing dendrograms presented in figures 4a and 4b, respectively. As demonstrated based on
557 the example of the least abundant set No. 1 in figure 4a, even in the case of number of objects
558 only slightly exceeding 300, their abundance makes the prepared detailed dendrogram largely
559 illegible. Due to this, the same dendrogram and all remaining dendrograms for more abundant
560 sets were prepared for a number of leaf nodes reduced to 30 (Figures 4a, 4b, 5a, and 5b),
561 considerably improving their legibility. In practice, this measure corresponded with cut-off of
562 detailed dendrograms at a level of bond distance of approximately 1.0. On the prepared
563 dendrograms in figures 4a and 4b, it is easy to recognise the characteristic structures in the form
564 of chains connecting similar objects or subclusters of objects occurring for lower bond
565 distances. The obtained image of dendrograms therefore corresponded in terms of quality to
566 dendrograms published in earlier papers regarding the application of agglomeration methods in
567 the analysis of precipitation hyetographs (Licznar and Szeląg, 2014; Licznar et al., 2017,
568 Licznar, 2018; Wartalska et al., 2020). In the case of figure 4b, notice that its structure included
569 a cluster with number 29 with extremely high bond distance (approximately 5.5). Detailed
570 analysis of cluster No. 29 in figure 4b showed that it is composed of only one object.

571

572 **Figure 4 a.** Dendrograms obtained for set No. 1 composed of 313 dimensionless cumulative
573 rainfall hyetographs from Kraków (top panel). Below find the same dendrogram prepared for
574 the reduced number of 30 leaf nodes (bottom panel). In the diagrams, vertical axes show bond
575 distances for particular rainfalls and rainfall clusters. The horizontal axis of the dendrogram on
576 the bottom panel shows numbers of rainfalls in particular clusters; **b.** Dendrogram obtained for
577 set No. 2 composed of 1494 dimensionless cumulative rainfall hyetographs from Kraków. The
578 dendrogram was prepared for the reduced number of 30 leaf nodes. The vertical axis of the
579 diagram shows bond distances for particular rainfall clusters, and the horizontal axis shows
580 their numbers

581

582 **Figure 5 a.** Dendrogram obtained for adjusted set No. 2 composed of 1493 dimensionless
583 cumulative rainfall hyetographs from Kraków. The dendrogram was prepared for the reduced
584 number of 30 leaf nodes. The vertical axis of the diagram shows bond distances for particular
585 rainfall clusters, and the horizontal axis shows their numbers; **b.** Dendrogram obtained for
586 adjusted set No. 3 composed of 1806 dimensionless cumulative rainfall hyetographs from
587 Kraków. The dendrogram was prepared for the reduced number of 30 leaf nodes. The vertical
588 axis of the diagram shows bond distances for particular rainfall clusters, and the horizontal axis
589 shows their numbers

590

591 The object was a hyetograph of a storm rainfall designated from records of rain gauge
592 No. 12 in Płaszów in 2014 (Figure 6). The rainfall had a total depth of 13.2 mm and total
593 duration of 507 min. Although the rainfall event formally meets the adopted criteria of storm
594 rainfall (Schmitt, 2000), it showed very specific course in time (Figure 6). For the first 460 min.
595 of the rainfall, only 0.8 mm of rain was recorded. In the second, considerably shorter, final part
596 of the rainfall lasting 47 min, 12.4 mm of rain was recorded. Considering this very specific

597 course of the rainfall in time, it was excluded from set No. 2, and the analysis was continued
598 for a set of 1493 normalised hyetographs of the remaining rainfall events.

599

600 **Figure 6.** Hyetograph of a rainfall event recorded on station Kraków-Płaszów in 2014

601

602 For the adjusted set No. 2 including 1493 objects, the dendrogram presented in figure 5a
603 was obtained. As a result of elimination of outliers in the form of a single rainfall event from
604 rain gauge No. 12, a dendrogram was obtained with a maximum bond distance at a level
605 somewhat higher than 4.0, comparable like in the case of set No. 1 (Figure 3). Importantly, after
606 the adjustment, the most deviating cluster with number 24 was not composed of a single object,
607 but covered normalised hyetographs for 9 different storm rainfalls (combined into a single
608 cluster for bond distances lower than 1.0). At the final stage of application of hierarchical
609 methods for analysing structures of sets of normalised hyetographs, a dendrogram was also
610 prepared for set No. 3, constituting a combination of sets No. 1 and No. 2, presented in
611 figure 5b. The dendrogram obtained in the case of set No. 3, prepared for 1806 dimensionless
612 storm rainfall hyetographs, had a structure analogical to that of the already discussed
613 dendrograms in figures 4a and 5a. Its maximum bond distance did not exceed 4.0, and it showed
614 characteristic chain connections of similar subclusters of objects, mutually connecting for bond
615 distances of less than 1.0. Irrespective of the similarities indicated herein, the comparison of
616 three dendrograms obtained for sets No. 1, 2 and 3 (Figures 4a, 5a, and 5b) provided no basis
617 for answering the question regarding the appropriate and unquestionable number of classes in
618 the division of normalised hyetographs from Kraków into clusters. To illustrate the problem in
619 a simple way, even if a certain subjective level of bond distance is adopted a priori, e.g., 1.75,
620 then the structure of the dendrogram of sets No. 1, 2 and 3 allows for designating the following
621 mutually divergent numbers of clusters: 11, 9 and 6. Considering the substantial discrepancy in

622 terms of number of clusters in further research, a potential range of the number of clusters k
623 was subject to analysis, covering values from 2 to 20.

624 The results of calculations of the Caliński and Harabasz Index values ($CHIndex$) as well
625 as total within sum of squares (wss) for the number of clusters k within a range from 2 to 20 for
626 set No. 1 are presented in figure 7.

627

628 **Figure 7.** Value of the $CHIndex$ and total within sum of squares (wss), and for a set 1 of 313
629 rainfalls from Kraków, depending on the adopted number of clusters k

630

631 Diagrams of both parameters directly suggest that the optimum number of clusters for
632 set No. 1 should be adopted as equal to 4 ($k = 4$). By theory, wss values naturally decrease with
633 an increase in the number of clusters k , although the decrease gradient evidently decreases after
634 reaching number of clusters $k = 4$. For the same number of four clusters, maximum $CHIndex$
635 value is also observed ($CHIndex=170$). In accordance with formula (4), a high $CHIndex$ value
636 is correlated with maximisation of the ratio of SS_B and SS_W . This means that particular clusters
637 very significantly differ from one another, and elements of the set grouped in particular clusters
638 are strongly similar to one another (relatively weakly variable).

639 The determination of the optimum number of clusters $k = 4$ was followed by the k -
640 means clustering process for set No. 1 of normalised cumulative hyetographs from Kraków.
641 Through the application of the bootstrap method, the following values of the bootmean
642 parameter were obtained for subsequent clusters from 1 to 4: 0.72; 0.65; 0.84, and 0.74.
643 Bootmean parameter values were higher than 0.6 for all four clusters. This evidences that the
644 designated four clusters included no cluster with random character, i.e., one that includes
645 rainfall models deviating from the remaining three clusters, but at the same time not mutually
646 similar. As a result of the k -mean clustering process for set No. 1 including 313 cumulative

647 normalised hyetographs from Kraków, clusters No. 1, 2, 3, and 4 were ascribed 102, 93, 68,
648 and 50 rainfalls, respectively. This corresponded with the share of 35%, 32%, 23%, and 17%,
649 respectively, throughout the set of analysed rainfalls.

650 For the designated clusters, averaged dimensionless cumulative storm rainfall
651 hyetographs were determined, presented in figure 8. The cumulative dimensionless
652 hyetographs were also transformed to the form of storm rainfall hyetographs presented in
653 figure 9.

654

655 **Figure 8.** Diagrams of averaged dimensionless cumulative rainfall hyetographs for four clusters
656 designated by means of k-means clustering for Kraków based on set No. 1 for 313 storm
657 rainfalls

658

659 **Figure 9.** Model dimensionless rainfall hyetographs developed by means of the k-means
660 clustering method for set No. 1 for 313 rainfalls from Kraków. The horizontal axis shows
661 percent increase in rainfall duration, and the vertical axis shows percent shares in total
662 precipitation amount

663

664 The analysis of the obtained model hyetographs shows that the most frequently
665 occurring hyetographs of type 1 and 2 (35% and 32%, respectively) have relatively even values
666 of point rainfall intensity (point rainfall depths for unitary duration intervals of 1/100 of total
667 duration do not exceed 2% total rainfall depth). The rainfalls, however, do not correspond with
668 the simplified block rainfall model, commonly used in designing drainage systems, and even
669 sporadically applied in their hydrodynamic modelling. In the case of numerous unitary duration
670 intervals, point rainfall depths differ from 1%, they are considerably lower, and frequently
671 approximate to 0.5%, or considerably higher, reaching approximately 2% of total rainfall depth.

672 The substantially more seldom occurring model hyetograph for cluster 3 has a shape very
673 generally approximate to model rainfall according to Euler type II recommended by Schmitt
674 (2000) for modelling stormwater systems. Unlike in the case of rainfall model according to
675 Euler type II, the greatest rainfall accumulation occurs not in 1/3 of rainfall duration, but already
676 in its initial part, during the first 10% of the duration of the entire rainfall. Moreover, this
677 accumulation has no character of a very sharp peak (point rainfall depths for unitary duration
678 intervals of 1/100 of total duration do not exceed 4% of total rainfall depth). The most seldom
679 occurring hyetograph shape is that determined in the case of cluster 4. With a high degree of
680 generalisation, the hyetograph can be treated as a mirror reflection of the model hyetograph of
681 cluster 3. This is not strictly accurate, because the rainfall accumulation occurring in the final
682 part of the hyetograph is very obscure. It is observed in the final interval covering approximately
683 30% of the entire rainfall, and point rainfall depths for unitary duration intervals of 1/100 of
684 total duration do not exceed 2.5% of total rainfall depth in that case. Moreover, at the very
685 beginning of the model hyetograph for cluster 4, a small rainfall peak occurs, with no equivalent
686 in the final part of the model hyetograph for cluster 3. Referring to the discussed results obtained
687 in the analysis of set No. 1 from Kraków, it is worth emphasising that the obtained courses of
688 cumulative model hyetographs in figure 8 were very approximate to the cumulative model
689 hyetographs obtained by Licznar (2018) for another rain gauge from Poland, as a result of
690 application of clustering of a set of 213 storm rainfalls by means of the k-means clustering
691 method for a subjectively adopted number of 4 clusters. The obtained results also remain in
692 accordance with earlier studies from the territory of Poland that question the justification of
693 common application of the synthetic rainfall model according to Euler type II due to its
694 deviation from the vast majority of scenarios of temporal course of actual storm rainfalls
695 (Licznar and Szelaąg, 2014; Licznar et al., 2017).



696 Diagrams of the dependencies of the Caliński and Harabasz (*CHIndex*) index values and
697 total within sum of squares (*wss*) on the number of clusters *k* developed for set No. 2 are
698 presented in figure 10.

699

700 **Figure 10.** Value of the CHIndex and total within sum of squares (*wss*), and for set No. 2 of
701 1493 rainfalls from Kraków, depending on the adopted number of clusters *k*

702

703 Analogically to set No. 1, they provide the basis for the determination that the optimum
704 number of clusters for set No. 2 should be adopted as equal to 4 ($k = 4$). For four clusters ($k =$
705 4), an evident peak of the CHIndex is observed ($CHIndex=801$), and the curve of total within
706 sum of squares (*wss*) flattens out to below 2000 after a rapid decrease from a level of
707 approximately 5000. The cited CHIndex and *wss* values cannot be referred to values obtained
708 in the case of set No. 1 (Figure 7). Orders of magnitude in both cases are different, as results
709 from different abundance of sets No. 1 and No. 2, and as accounted for by the structure of
710 formulas (3) and (4).

711 The process of k-means clustering of set No. 2 of normalised cumulative hyetographs
712 from Kraków into four clusters ended with the designation of clusters meeting the criteria of
713 internal coherence and external isolation (Gordon, 1999). The confirmation of the above was
714 obtaining values of the bootmean parameter substantially exceeding the threshold of 0.6 for
715 each of the clusters. The parameter reached: 0.93; 0.91; 0.96, and 0.91, respectively for clusters
716 from 1 to 4. As the final result of clustering, subsequent clusters were ascribed, respectively:
717 510, 506, 232, and 245 storm rainfalls. This corresponded with the respective share of 34%,
718 34%, 16%, and 16% in the entire population of analysed rainfalls in set No. 2. The cited percent
719 share of particular clusters in set No. 2 proved approximate, like in the case of the previously
720 discussed set No. 1. For particular clusters, averaged cumulative storm rainfall hyetographs

721 were also determined, presented in figure 11. The diagrams of averaged normalised cumulative
722 hyetographs in the figure pointed to very high compatibility in terms of temporal distribution
723 with the previously discussed results for set No. 1 (Figure 8).

724

725 **Figure 11.** Diagrams of averaged dimensionless cumulative rainfall hyetographs for four
726 clusters designated by means of the k-means clustering method for Kraków based on set No. 2
727 for 1493 storm rainfalls

728

729 The evident divergence of results of clustering of storm rainfall hyetographs for sets
730 No. 1 and No. 2 became an impulse for undertaking analysis of set No. 3, constituting a
731 combination of the aforementioned sets. As expected, the obtained results proved to be virtually
732 identical to results obtained previously for sets No. 1 and No. 2. Curves of variability of values
733 of the Caliński and Harabasz index (CHIndex) and total within sum of squares (wss) developed
734 for this large set of 1806 storm rainfalls are presented in figure 12, and raise no doubts as for
735 their interpretation. The evident maximum of the CHIndex value ($CHIndex=1105$) is obtained
736 for four clusters ($k = 4$). For four clusters, a very steep gradient of decrease in wss values from
737 more than 5500 to approximately 2000 is also rapidly levelled almost to zero. Clustering by
738 means of the k-means method of hyetographs included in set No. 3 ended with creating four
739 clusters meeting the criteria of internal coherence and external isolation. The obtained
740 bootmean parameter values were practically approximate to 1.0, and for subsequent clusters
741 they reached: 0.94; 0.93; 0.96, and 0.94, respectively. Clusters No. 1, 2, 3, and 4 were ascribed:
742 613, 605, 288, and 300 rainfalls, respectively, corresponding to a respective share of 34%, 33%,
743 16%, and 17% in the entire set No. 3.

744

745 **Figure 12.** Value of the CHIndex and total within sum of squares (wss), and for a set 3 of 1806
746 rainfalls from Kraków, depending on the adopted number k of clusters

747

748 For particular clusters of set No. 3, averaged hyetographs of dimensionless cumulative
749 storm rainfalls were also determined, collectively presented in figure 13.

750

751 **Figure 13.** Diagrams of averaged dimensionless cumulative rainfall hyetographs for four
752 clusters designated by means of the k-means clustering method for Kraków based on set No. 3
753 for 1806 storm rainfalls. For comparison, the diagram also shows Median Time Distributions
754 of Heavy Storm Rainfall at a Point developed by Huff (1990)

755

756 The figure also presents dimensionless hyetographs developed by Huff (1990),
757 constituting the subject of a separate discussion. Pursuant to the expectations, the diagram in
758 figure 13 proved strongly similar to the diagrams of model hyetographs for clusters from 1 to 4
759 determined for sets No. 1 and No. 2 (Figures 8 and 11). For better assessment of the scale of
760 similarity of the model hyetographs obtained for particular clusters in the case of clustering sets
761 No. 1, No. 2, and No. 3, figure 14 was additionally prepared.

762

763 **Figure 14.** Comparison of averaged dimensionless cumulative rainfall hyetographs for four
764 clusters designated by means of the k-means clustering method based on sets No. 1, 2, and 3:
765 313, 1493, and 1806 storm rainfalls recorded in Kraków. The horizontal axes present percent
766 increase in rainfall duration, and the vertical axes show percent shares in total rainfall depth

767

768 The similarity of averaged normalised cumulative rainfall hyetographs obtained for all
769 three sets of storm rainfalls from Kraków within particular clusters is unquestionable. In the

770 case of sets No. 2 and No. 3, the diagrams of normalised cumulative hyetographs even overlap.
771 In the case of cluster No. 4, hyetographs resulting from the analysis of all three sets overlap.
772 This provides the basis for the presumption that the developed methodology of application of
773 cluster analysis for the designation of model hyetographs shows repeatability in the scope of
774 obtained results within the urban precipitation field. In engineering practice, it can be therefore
775 used for the determination of local sets of model hyetographs based on the analysis of
776 approximately 30-year-long rainfall series even from single rain gauges located near city
777 boundaries. They will be able to supply computer models for simulation of drainage systems in
778 the centre of a city as large as Kraków with satisfactory precision.

779 For a better understanding of the obtained results, particularly including the
780 determination of characteristic features of particular clusters that differentiate them from one
781 another, their profiling was performed. Such profiling is called simplified, because it is only
782 based on general variables describing storm rainfalls in the form of their total depths, total
783 durations, and mean intensities. For this purpose, table 2 presents mean values of these variables
784 within particular clusters for sets No. 1 and No. 2. Figures 15a and 15b present histograms of
785 the frequency of occurrence of storm rainfalls with different mean intensities within different
786 clusters, respectively for sets No. 1 and No. 2. Values of rainfall intensity in diagrams in
787 figures 15a and 15b and in table 2 are expressed in a standard unit of $\text{dm}^3/(\text{s}\cdot\text{ha})$ applied in urban
788 hydrology.

789

790 **Table 2.** Mean values of total depths, durations, and intensities of storm rainfalls included in
791 particular clusters for sets No. 1 and No. 2 from Kraków

792

793

794 **Figure 15 a.** Histograms of mean rainfall intensities for four designated clusters in rainfall set
795 No. 1 from Kraków (313 rainfalls); **b.** Histograms of mean rainfall intensities for four
796 designated clusters in rainfall set No. 2 from Kraków (1493 rainfalls)

797

798 Data included in table 2 and in diagrams in figures 15a and 15b suggest that rainfalls
799 classified to cluster 3 usually showed mean intensities higher than those of rainfalls included in
800 other clusters. For the majority of storm rainfalls included in cluster 3 in sets No. 1 and No. 2,
801 mean intensities in sets No. 1 and No. 2 were usually within the range from 5 to 20 $\text{dm}^3/(\text{s}\cdot\text{ha})$,
802 and their mean value within sets No. 1 and No. 2 exceeded 14 $\text{dm}^3/(\text{s}\cdot\text{ha})$. High mean rainfall
803 intensities in cluster 3 did not result from high total rainfall depths, but from evidently shorter
804 durations. In set No. 2, mean rainfall depths for cluster 3 were even evidently lower than for
805 the three remaining clusters. Mean rainfall durations for cluster 3, however, were approximately
806 6 h, whereas for the remaining clusters they were considerably longer, within a range from
807 approximately 8 h to 12 h. This suggests that cluster $n = 3$ included short but very intensive
808 rainfalls, probably with convection genesis. This hypothesis would also explain the greatest
809 variability of point rainfall depths for unitary duration intervals shown on the model hyetograph
810 of cluster 3 in figure 9. Completely different profiling results were obtained for cluster 2 which
811 had a considerably more equalised course of the model hyetograph (Figure 9). Rainfalls
812 classified to this cluster had not only longer durations within the clusters in particular sets No. 1
813 and No. 2, but usually also very low intensities within a range from 0 to 5 $\text{dm}^3/(\text{s}\cdot\text{ha})$. This
814 encourages a hypothesis that this cluster primarily included frontal rainfalls with long durations,
815 but low and considerably more even in time intensities. The research hypotheses stated here
816 regarding the division of storm rainfalls into clusters by rainfall genesis should be verified in
817 further research. It will however require access to synoptic records permitting more precise
818 profiling of the designated clusters.

819 In the summary of the entire discussion of results, it is also necessary to refer to the
820 classic papers by Huff (1967, 1990). Although the research on Dimensionless, Cumulative
821 Rainfall Hyetographs was implemented in different climate conditions (Illinois, USA), with a
822 different approach to the identification of single rainfalls, and with the application of a
823 considerably simpler method of their classification to the first, second, third, or fourth quartile
824 (depending on whether the highest percent of cumulative rainfall occurred in the first, second,
825 third, or fourth quarter of its duration, respectively), relatively high correspondence of the
826 courses of medians is observed (50th-percentile) between dimensionless hyetograph curves
827 derived from point rainfall values derived by Huff (1967, 1990) and model hyetographs in
828 figures 8, 11 and 13. In the case of hyetographs from Kraków, cluster 3 corresponds with first-
829 quartile storms, cluster 1 with second-quartile storms, cluster 2 with third-quartile storms, and
830 cluster 4 with fourth-quartile storms. Further analogies can be sought in general characteristics
831 of particular quartiles. For designing and modelling drainage systems, Huff (1990)
832 recommended the application of first-quartile storm hyetographs for time scales of about 6
833 hours or less, and second-quartile storm hyetographs for time scales of about 6 to 12 hours.
834 These recommendations overlap with mean durations determined for clusters No. 3 and No. 1
835 in sets No. 1 and 2, respectively (Table 2). Nonetheless, at a closer investigation of the study
836 by Huff (1967), differences are observed in terms of frequencies of occurrence of rainfalls with
837 the adopted model hyetographs. In the case of research from Illinois, the relative frequencies of
838 the storms were 30, 36, 19, and 15 percent for the first, second, third, and fourth quartiles,
839 respectively. They are not in accordance with frequencies of 16, 34, 33, and 15 percent obtained
840 for the analogical model hyetographs designated from set No. 3, corresponding to subsequent
841 clusters No. 3, 1, 2, and 4, respectively. The determined divergence, however, does not
842 undermine results from Kraków, because the analogical divergence has already been signalled
843 and discussed by Pani and Haragan (1981). Analysing a set of 117 rainfalls from Texas (USA),

844 recorded in months with the highest probability of occurrence of convection rainfalls, the
845 authors obtained a median (50th-percentile) dimensionless hyetograph curve with shapes fully
846 corresponding with results by Huff (1967), but the determined relative frequencies of the storms
847 were 13, 41, 32, and 14 percent for the first, second, third, and fourth quartiles, respectively.
848 The latter frequencies are considerably more approximate to results from Kraków, despite
849 obvious differences in the location of both research polygons and approach to processing
850 rainfall records and determination of model hyetographs. The qualitative compatibility of study
851 results from Kraków with classic papers by Huff (1967, 1990) and Pani and Haragan (1981) is
852 an additional premise confirming the accuracy of the methodology of identification of model
853 hyetographs of storm rainfalls based on complete cluster analysis and quality indices.
854

855 **4. Summary and final conclusions**

856 Progress in the scope of atmospheric precipitation measurements techniques, and the
857 occurring municipal rain gauge networks expansion requires simultaneous modernisation of the
858 methodology of recorded precipitation series processing. A necessary element of such a
859 methodology are certainly modern methods of objective and automatic search of rainfalls
860 groups with similar courses in time that can be described in a general way by means of model
861 hyetographs. The practical application area of such methods can currently exceed processing
862 sets of local model hyetographs, and find implementation in the practical rainwater runoff
863 control systems operation. Due to the growing number of rain gauges and rapidly increasing
864 sets of rainfall records, also in the case of model hyetographs identification, it becomes justified
865 and necessary to reach for data mining tools, primarily including the cluster analysis.

866 This paper is not pioneer in terms of the very idea of the cluster analysis application in storm
867 rainfall hyetographs classification. The application of the cluster analysis in research on

868 temporal distributions of storm rainfalls has already been postulated by Licznar et al. (2017),
869 and then tested for several locations in Poland (Licznar, 2018; Wartalska et al., 2020). The
870 primary objective of the paper was the improvement of the research methodology to meet the
871 requirements of the complete cluster analysis methodology covering seven stages (Milligan,
872 1996), including: selection of objects and variables; formula selection of variable values
873 normalisation; selection of distance measure; selection of the classification method; the number
874 of classes determination; assessment of classification results; class description and profiling.
875 The aforementioned objective covered the three primary detailed objectives involving:
876 objectivization of the number of clusters determination, the internal coherence and external
877 isolation of clusters verification, and profiling of the retrieved clusters.

878 Another substantial objective of the paper was to demonstrate the developed methodology at
879 the scale of a large precipitation field in Poland. This objective covered two detailed objectives,
880 namely testing the methodology on a large measurement set, and equally importantly, analysing
881 its repeatability at the scale of a large urban precipitation field. The basic question was also to
882 what extent model hyetographs developed based on records from nearby rain gauges (located
883 e.g., at an airport or in suburbs) correspond with the shapes of model hyetographs for rain
884 gauges of the urban rain monitoring network. Owing to the collaboration with the Municipal
885 Water Supply and Sewerage Company (MWSSC) in Kraków, Poland, it was possible to apply
886 the developed complete cluster analysis methodology to a large measurement set of 1806 storm
887 rainfalls (Set No. 3), composed of set No. 1 – 313 storm rainfalls designated from two nearby
888 rain gauges belonging to the countrywide network of IMGW, and set No. 2 – 1493 storm
889 rainfalls designated from 23 rain gauges belonging to the municipal rain monitoring network
890 of MWSSC. Three applications of the complete cluster analysis methodology for sets No. 1,
891 No. 2, and No. 3 permitted its thorough testing, designation of a set of model hyetographs for



892 practical application in modelling of drainage systems in Kraków, and drawing the following
893 final conclusions:

- 894 1) The complete methodology of the storm rainfall hyetographs cluster analysis should
895 cover tools for both hierarchical and non-hierarchical analysis of the sets structure.
896 Before the application of the cluster of normalised (dimensionless) hyetographs, key in
897 terms of the final products, i.e., model hyetographs, by means of the k-means method,
898 the hierarchical agglomeration method should be applied to prepare dendrograms of
899 similarity of temporal courses of rainfalls in the analysed sets. The dendrograms should
900 be subject to expert analysis not only in terms of determination of a potential number of
901 clusters in the analysed sets, but more importantly the identification of particularly
902 peculiar rainfall patterns. Like in the case of set No. 2 and rainfall recorded by rain
903 gauge No. 12 in Płaszów in 2014, such records should be removed before the division
904 of the set using a predefined number k of clusters by means of non-hierarchical methods;
- 905 2) The diagrams analysis of correlation of values of the Caliński and Harabasz index
906 (*CHIndex*) and total within sum of squares (*wss*) with number k of clusters permits
907 completely objective determination of the correct number of clusters for which the
908 division of storm rainfall sets should be performed from the similarity point of view of
909 their normalised (dimensionless) hyetographs. For the accurate, optimal number of
910 clusters, maximisation of *CHIndex* values is observed combined with an evident
911 decrease in the gradient of the decrease in *wss* values. In the case of all three analysed
912 sets of storm rainfalls from Kraków, based on analyses of *CHIndex* and *wss* values, the
913 adopted optimum number of clusters was four ($k = 4$), and the choice was positively
914 verified in all further research through obtaining clusters meeting the requirements of
915 internal coherence and external isolation;



916 3) The fundamental element for the credibility of the obtained results of storm rainfalls
917 divisions and model hyetographs identification is the assessment of classification
918 results. The study conducted on three sets of rainfalls from Kraków justified the repeated
919 launching of the clustering algorithm with the application of the bootstrap method.
920 Although this undoubtedly complicates the computing algorithms and prolongs the time
921 of calculations, it allows for calculating the bootmean parameter corresponding to the
922 mean value of the Jaccard Index (Jaccard similarity coefficient) for each of the
923 designated clusters. The bootmean parameter permits drawing objective conclusions on
924 whether particular clusters meet the criteria of internal coherence and external isolation
925 (Gordon, 1999). For all clusters designated from three sets of rainfalls from Kraków,
926 the bootmean parameter usually exceeded the adopted threshold of 0.6, confirming that
927 the designated subclusters included no clusters with random character, i.e., those
928 including rainfall patterns deviating from the remaining clusters, but also evidently
929 mutually different. Relatively lowest values of the bootmean parameter were obtained
930 for the least abundant set No. 1 (in a range from 0.65 to 0.84), whereas for
931 approximately five or six times more abundant sets No. 2 and 3, they were higher than
932 0.9, or even approximate to 1.0. The latter observation suggests that the k-means
933 clustering method is predestined for the analysis of very large sets, and is more reliable
934 in their case;

935 4) The developed complex cluster analysis methodology for the division of sets of storm
936 rainfalls and identification of model hyetographs implemented in the case of all three
937 sets of storm rainfalls from Kraków generated coherent final results. For each of the
938 three sets, the optimum number of clusters was four, and the resulting averaged
939 normalised cumulative hyetographs for particular clusters showed no mutual differences
940 within the three analysed sets. The coherence of the obtained results also concerned the



941 frequency of storm rainfalls occurrence included to particular clusters. For all the three
942 sets, storm rainfalls were distributed in proportions of approximately: 1/3, 1/3, 1/6, and
943 1/6 for clusters No. 1, 2, 3, and 4, respectively. All the aforementioned observations
944 suggest the possibility of development of model hyetographs based on multiannual
945 records from suburban stations (e.g., from the rain gauge at the nearby airport), and like
946 in the case of set No. 1, their application in practice throughout the city in the case of
947 lack of the possibility of hyetographs development based on records from the territory
948 of the city itself (based on set No. 2).

949 5) The obtained set of model hyetographs for Kraków does not include hyetographs with
950 a shape corresponding to that of synthetic hyetographs developed based on IDF
951 (Intensity-Duration-Frequency) or DDF (Depth-Duration-Frequency) models, adopted
952 *a priori* for hydrodynamic modelling of drainage systems in Poland, such as: model
953 rainfall according to Euler type II, block rainfall, or model rainfall according to DVWK.
954 Nonetheless, the shapes of the developed hyetographs point to high similarity to classic
955 medians (50th-percentile) of dimensionless hyetograph curves derived from point
956 rainfall values derived by Huff (1967, 1990) and Pani and Haragan (1981). In results
957 from Kraków, model hyetographs for subsequent clusters No. 3, 1, 2, and 4 correspond
958 to medians (50th-percentile) of dimensionless hyetographs for: first-quartile storms,
959 second-quartile storms, third-quartile storms, and fourth-quartile storms. Profiling of
960 clusters results of storm rainfalls from Kraków also remain in complete accordance with
961 earlier research by Huff (1990) according to which first-quartile storms (storms from
962 cluster 3) usually correspond with time scales of about 6 hours or less, whereas second-
963 quartile storms (storms from cluster 1) usually have longer durations within a range
964 from 6 to 12 hours.

965



966 The development of a complex methodology of storm rainfall hyetographs analysis and
967 its successful testing in a polygon of a large rain gauge network in Kraków offers the possibility
968 of its implementation at a considerably broader scale in the scope of implementation of the
969 WaterFolder Connect project. The practical objective here is to develop credible sets of local
970 model hyetographs in a network of 100 rain gauges in Poland for the purpose of their later use
971 in practice for supplying a digital platform dedicated for designing and modelling drainage
972 systems throughout Poland. Further research, however, must be also undertaken due to new
973 hypotheses that appeared as a result of the study. Firstly, the research hypothesis assuming the
974 correlation of the division of storm rainfalls into particular clusters with the genesis of rainfalls
975 needs to be verified. Moreover, in the context of the determined lack of variability of model
976 hyetographs at the single urban precipitation field scale, it is important to verify the thesis on
977 the regionalisation of study results possibility from more mutually distant rain gauges, and the
978 practical application of common sets of model hyetographs in larger areas of the country. In the
979 future, the developed methodology of hyetograph analysis could be also implemented in other
980 countries around the globe.

981

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991

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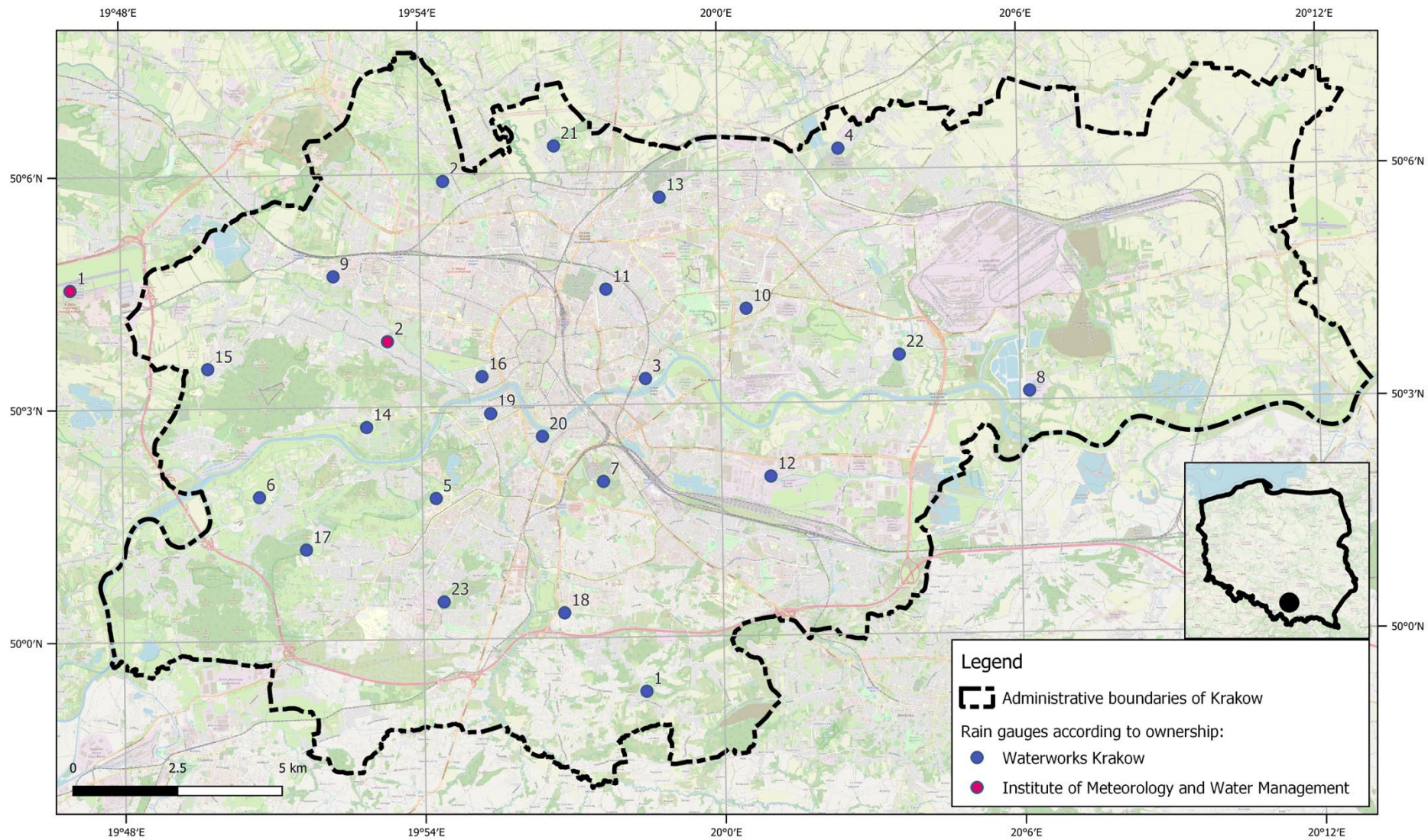


Figure 1. Location of rain gauges belonging to Waterworks Kraków and the Institute of Meteorology and Water Management

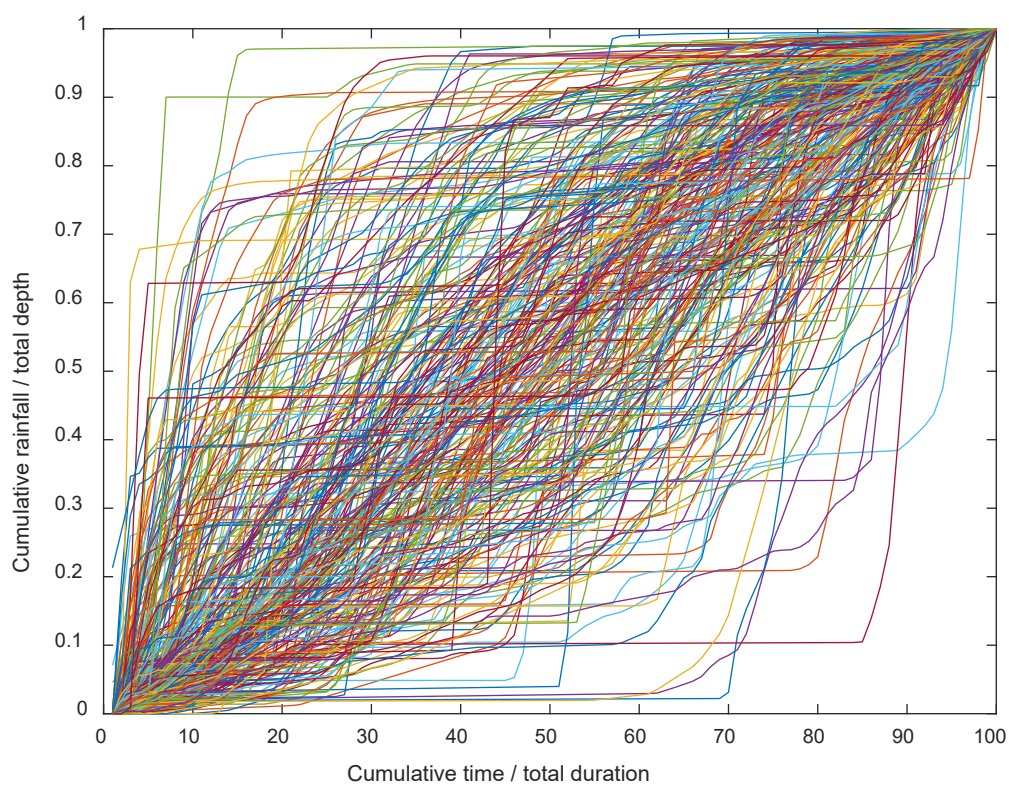


Figure 2. Cumulative dimensionless hyetographs of 313 storm rainfalls from Kraków (set No. 1)

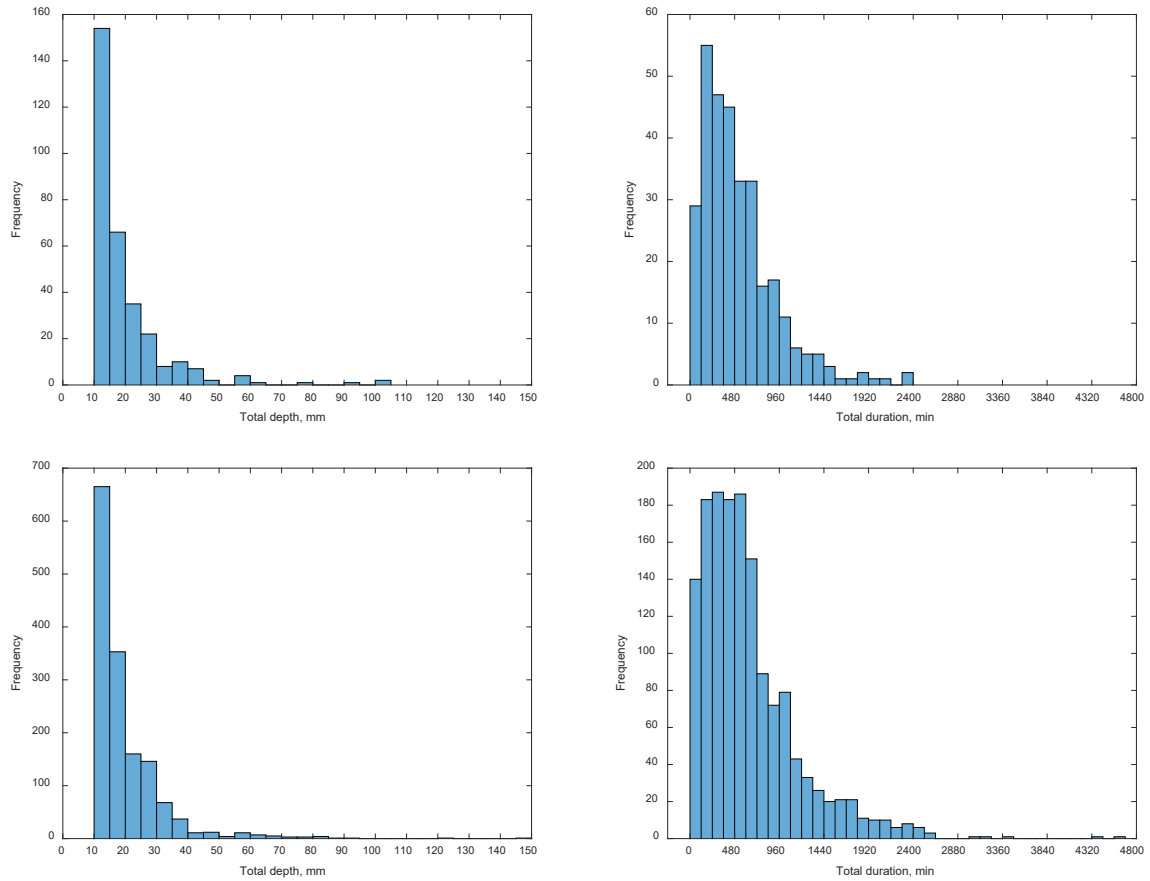
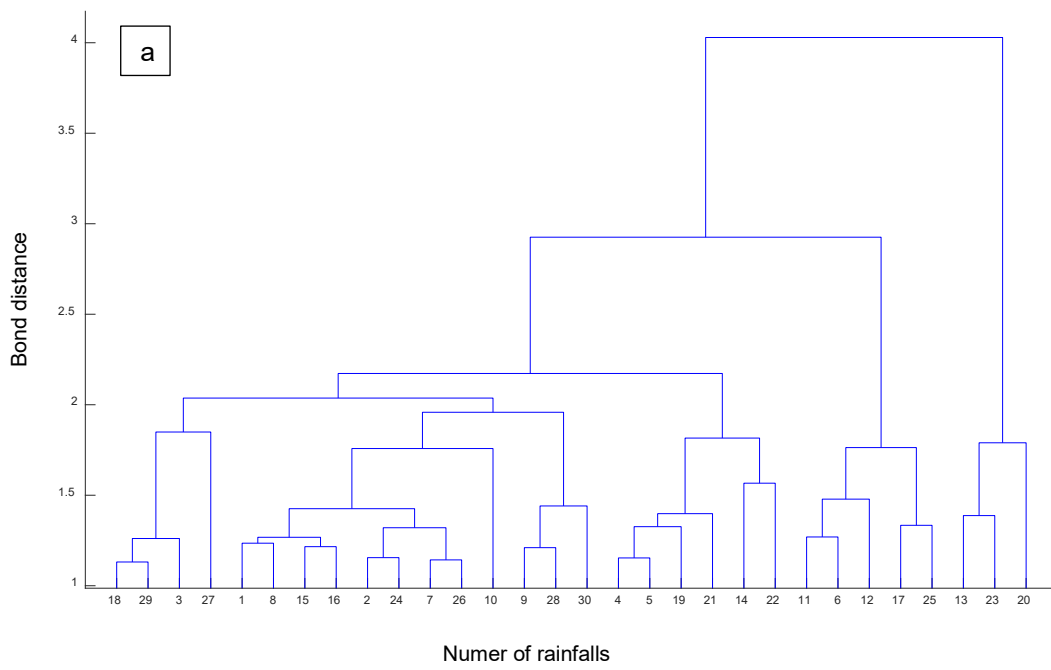
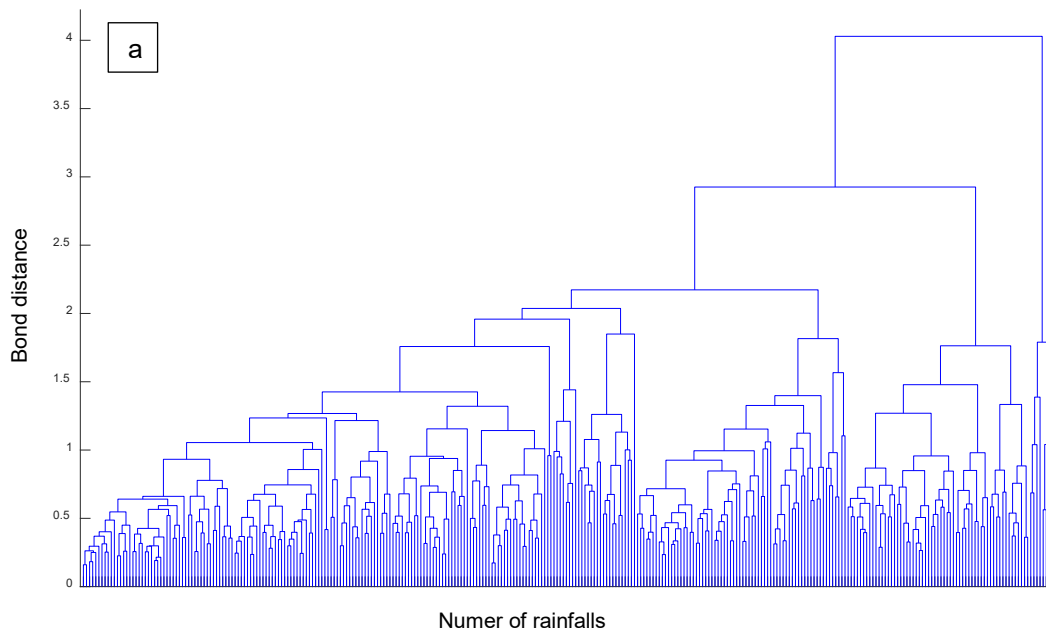


Figure 3. Histograms of rainfall depths and durations for the designated sets of rainfalls from Kraków, diagrams in the top row for set No. 1 (313 rainfalls), diagrams in the bottom row for set No. 2 (1493 rainfalls)



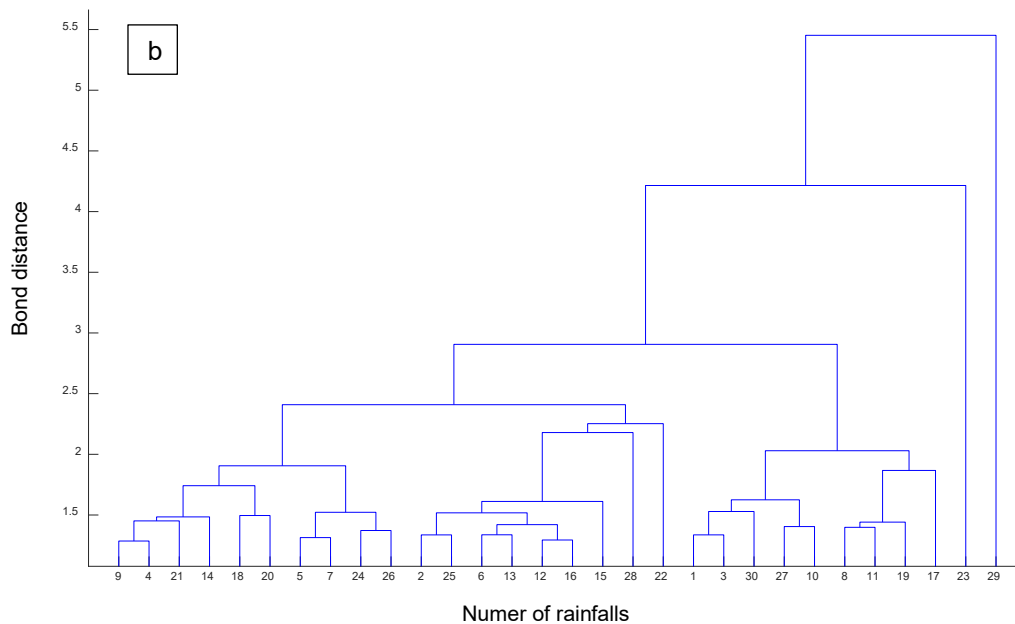


Figure 4 a. Dendrograms obtained for set No. 1 composed of 313 dimensionless cumulative rainfall hyetographs from Kraków (top panel). Below find the same dendrogram prepared for the reduced number of 30 leaf nodes (bottom panel). In the diagrams, vertical axes show bond distances for particular rainfalls and rainfall clusters. The horizontal axis of the dendrogram on the bottom panel shows numbers of rainfalls in particular clusters; **b.** Dendrogram obtained for set No. 2 composed of 1494 dimensionless cumulative rainfall hyetographs from Kraków. The dendrogram was prepared for the reduced number of 30 leaf nodes. The vertical axis of the diagram shows bond distances for particular rainfall clusters, and the horizontal axis shows their numbers

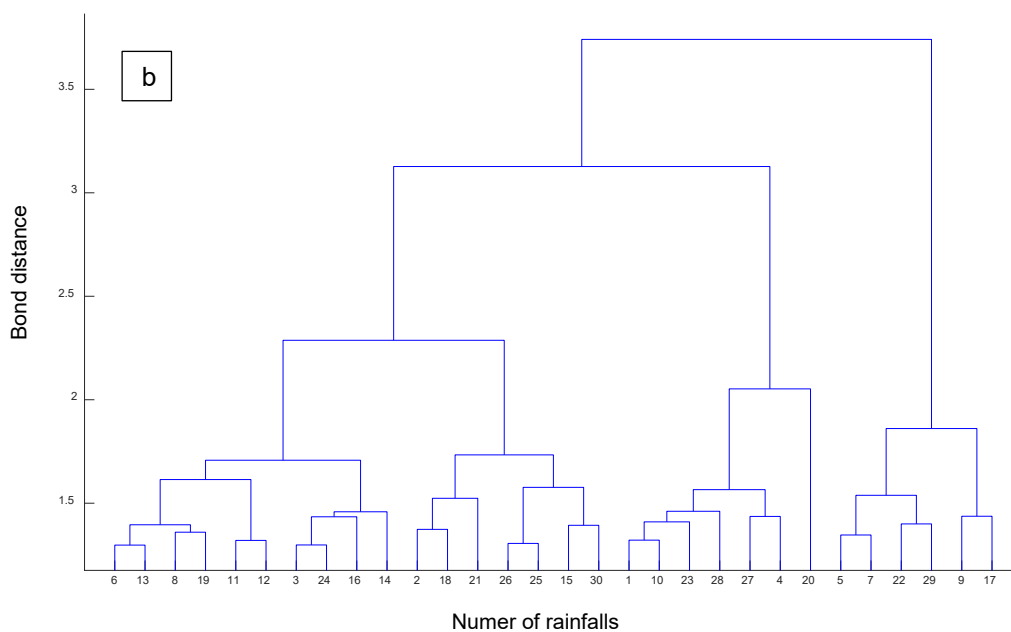
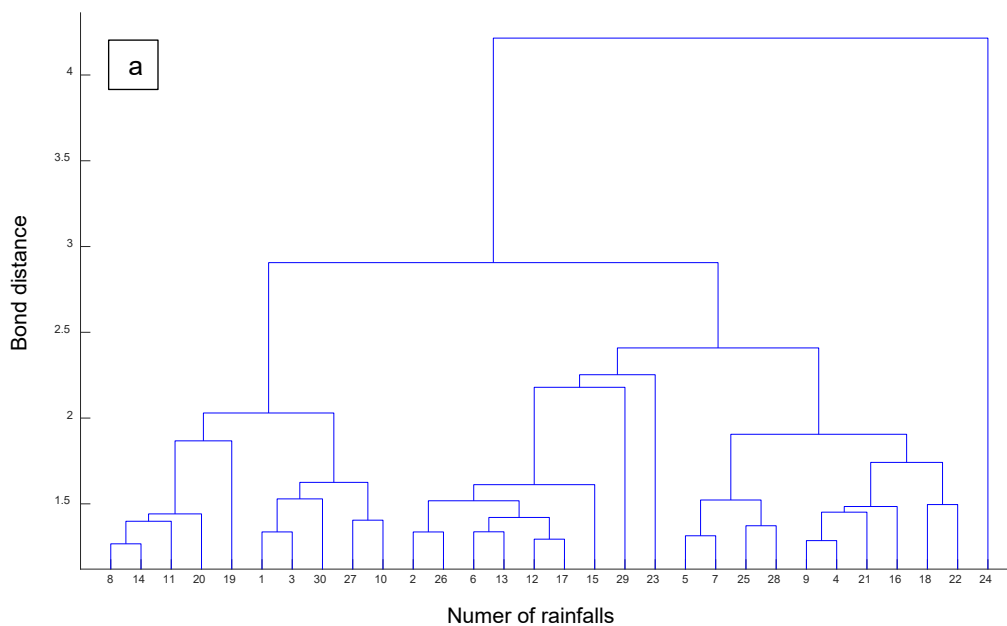


Figure 5 a. Dendrogram obtained for adjusted set No. 2 composed of 1493 dimensionless cumulative rainfall hyetographs from Kraków. The dendrogram was prepared for the reduced number of 30 leaf nodes. The vertical axis of the diagram shows bond distances for particular rainfall clusters, and the horizontal axis shows their numbers; **b.** Dendrogram obtained for adjusted set No. 3 composed of 1806 dimensionless cumulative rainfall hyetographs from Kraków. The dendrogram was prepared for the reduced number of 30 leaf nodes. The vertical

axis of the diagram shows bond distances for particular rainfall clusters, and the horizontal axis shows their numbers

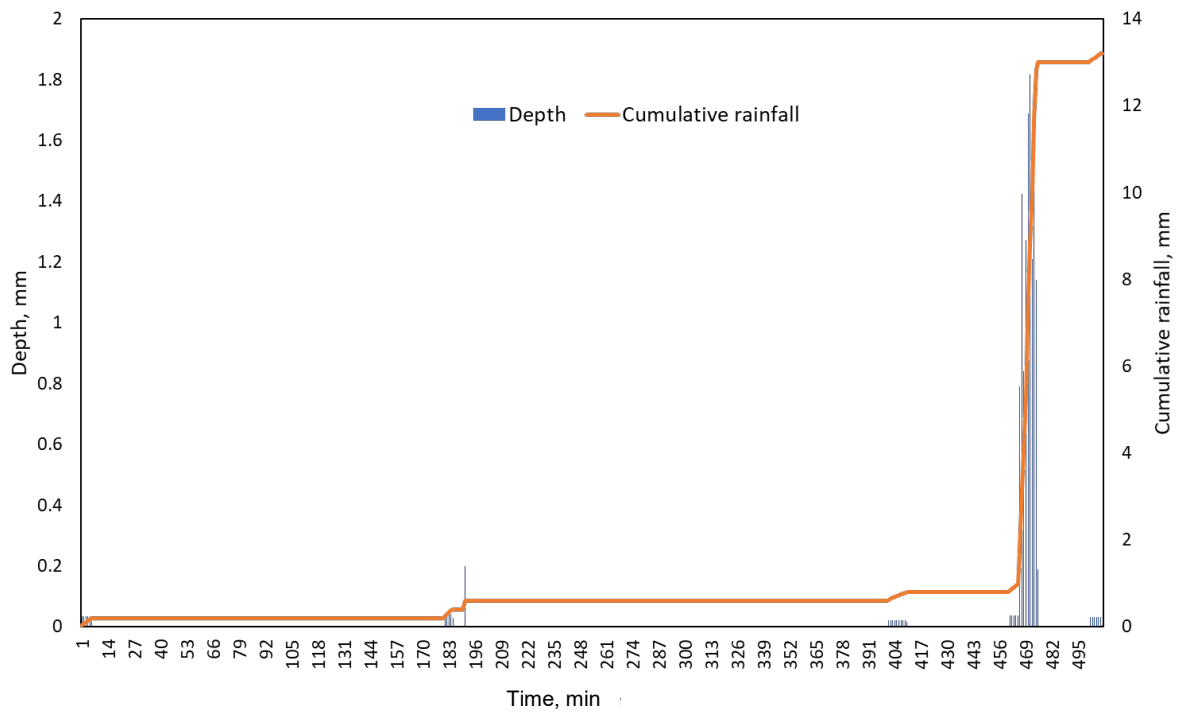


Figure 6. Hyetograph of a rainfall event recorded on station Kraków-Płaszów in 2014

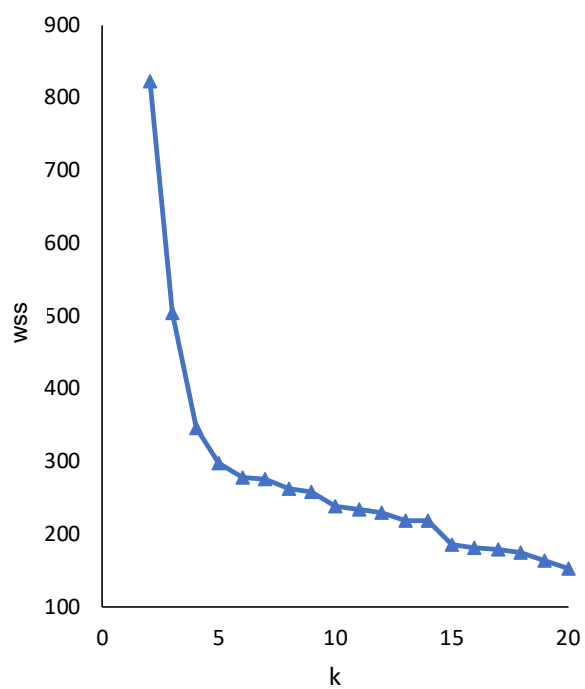
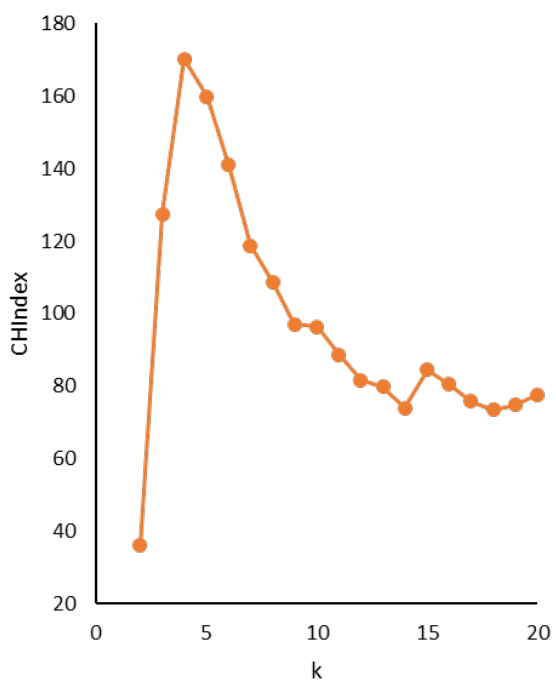


Figure 7. Value of the CHIndex and total within sum of squares (wss), for a set 1 of 313 rainfalls from Kraków, depending on the adopted number of clusters k

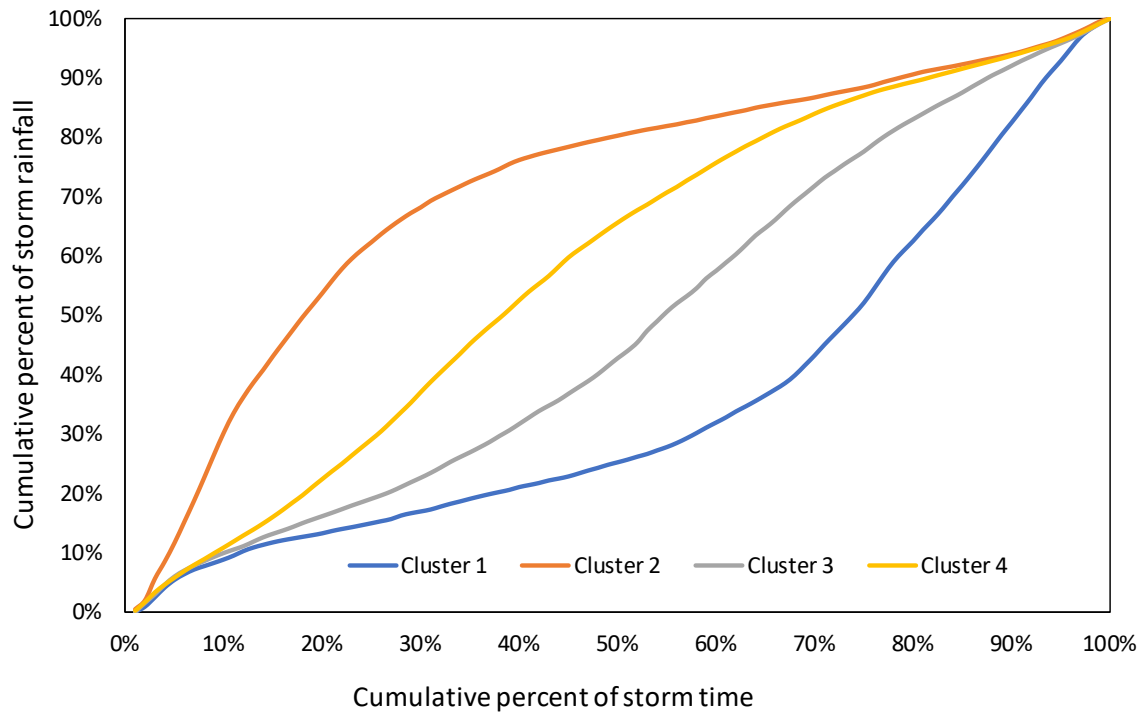


Figure 8. Diagrams of averaged dimensionless cumulative rainfall hyetographs for four clusters designated by means of k-means clustering for Kraków based on set No. 1 for 313 storm rainfalls

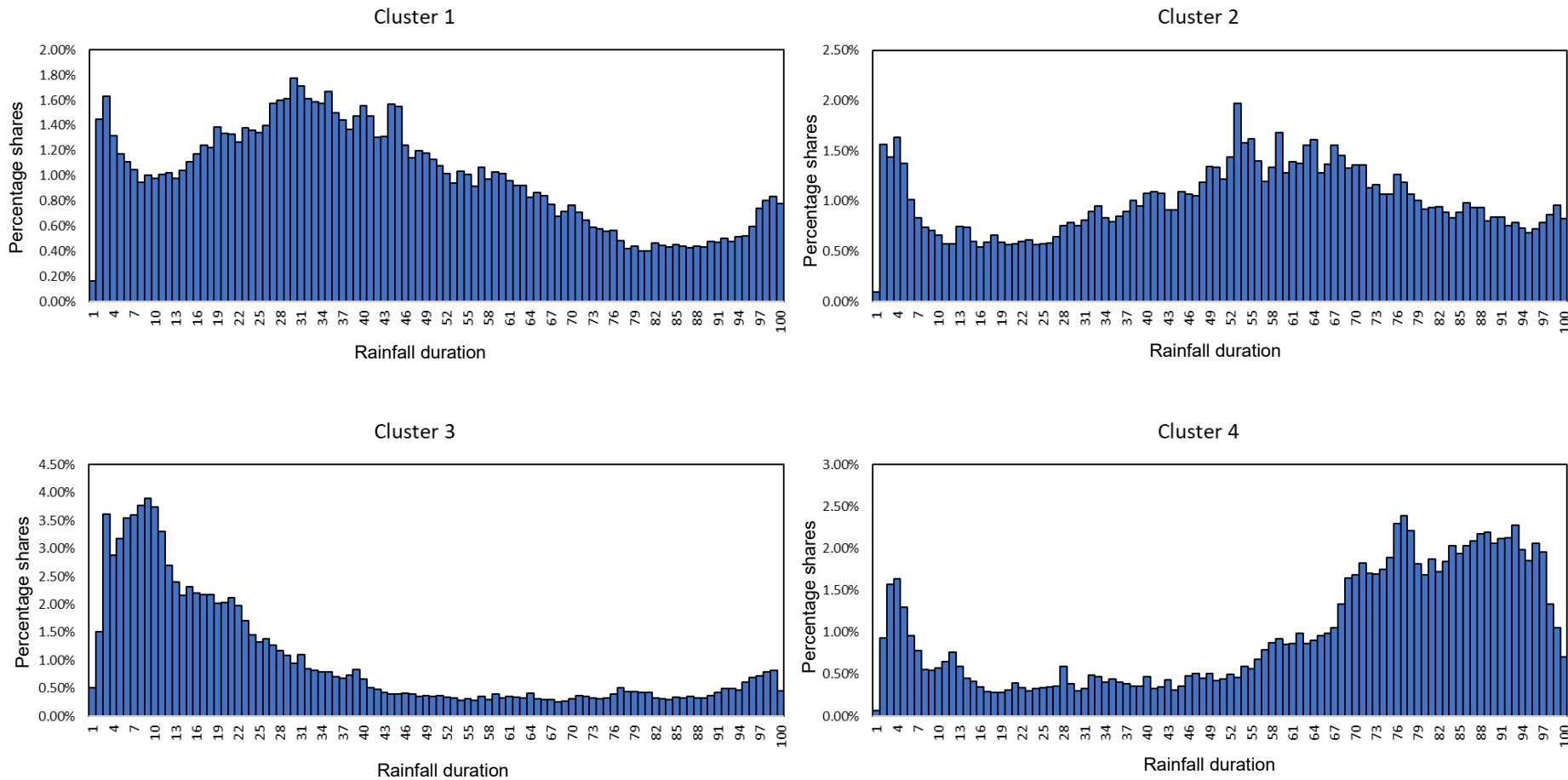


Figure 9. Model dimensionless rainfall hyetographs developed by means of the k-means clustering method for set No. 1 for 313 rainfalls from Kraków. The horizontal axis shows percent increase in rainfall duration, and the vertical axis shows percent shares in total precipitation amount

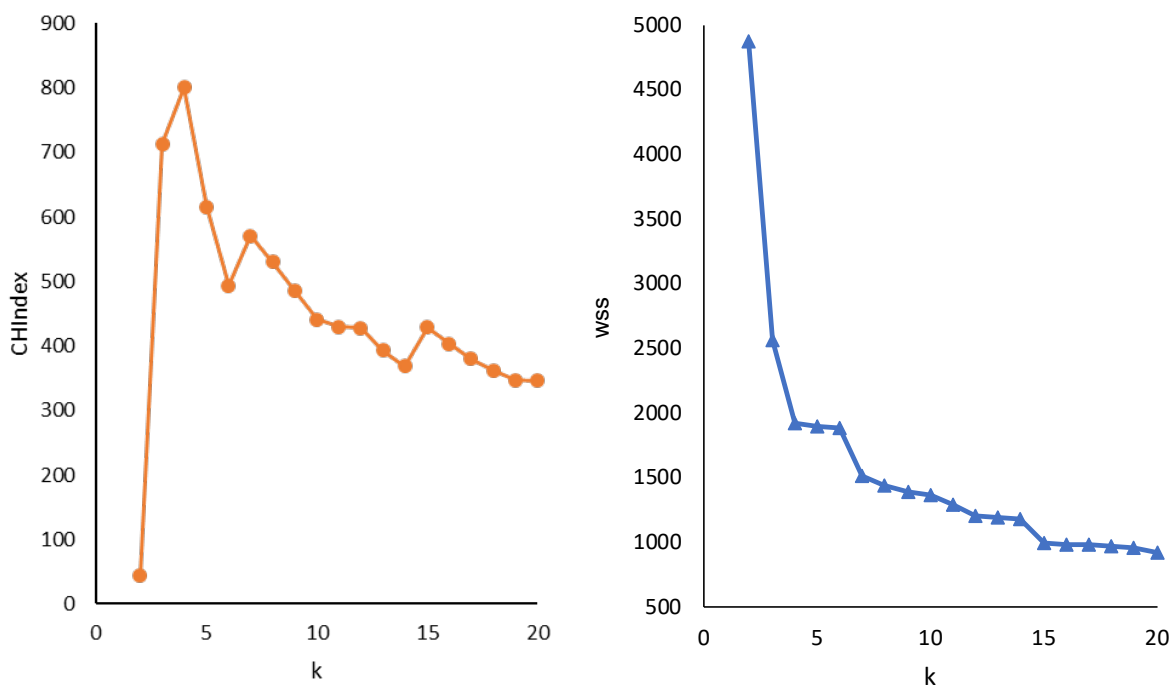


Figure 10. Value of the CHIndex and total within sum of squares (wss), for set No. 2 of 1493 rainfalls from Kraków, depending on the adopted number of clusters k

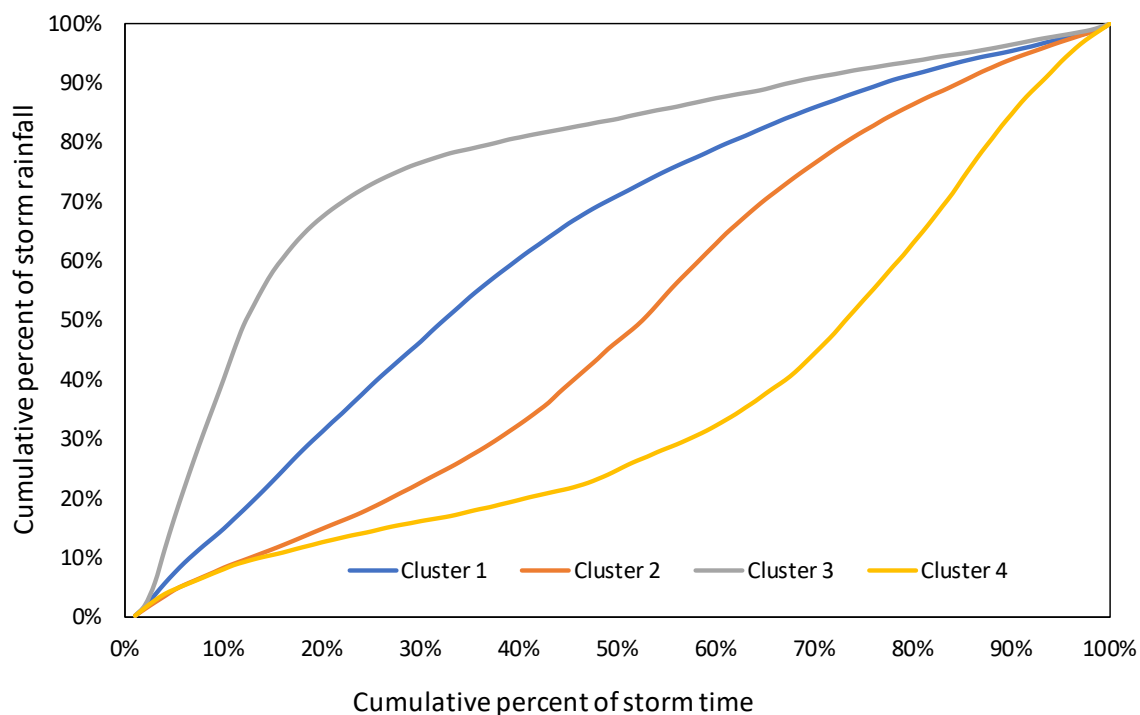


Figure 11. Diagrams of averaged dimensionless cumulative rainfall hyetographs for four clusters designated by means of the k-means clustering method for Kraków based on set No. 2 for 1493 storm rainfalls

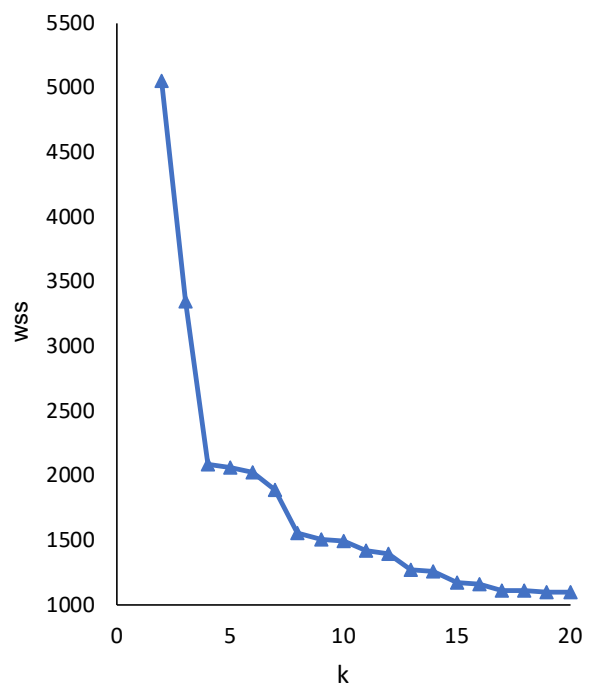
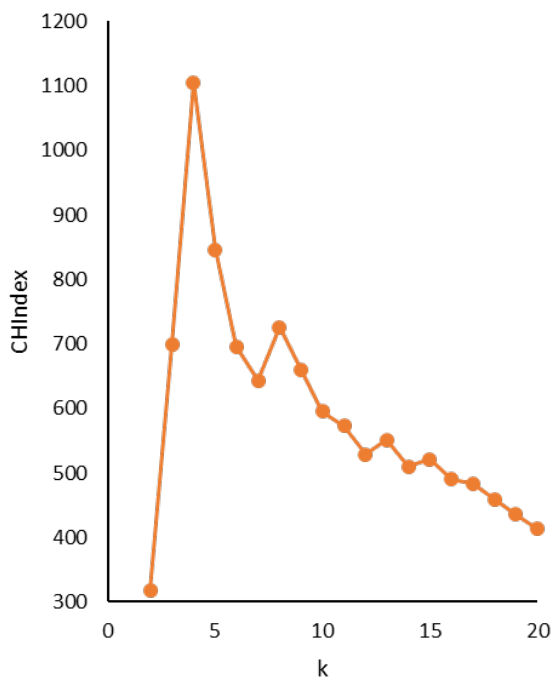


Figure 12. Value of the CHIndex and total within sum of squares (wss), for a set 3 of 1806 rainfalls from Kraków, depending on the adopted number k of clusters

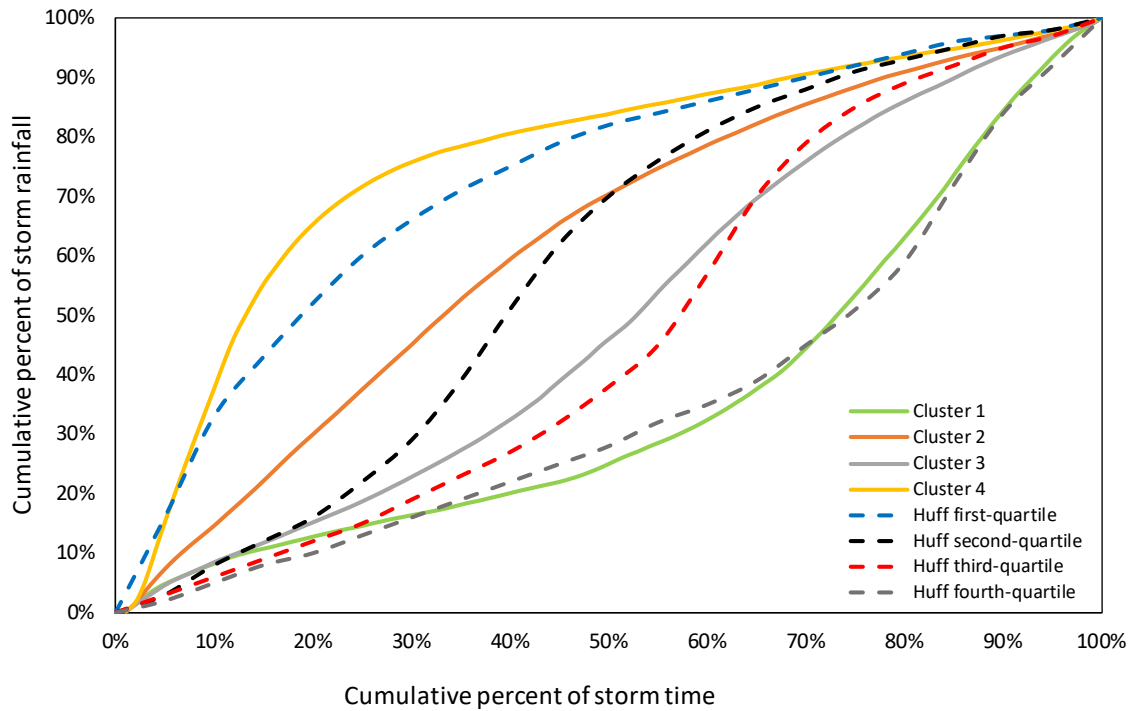
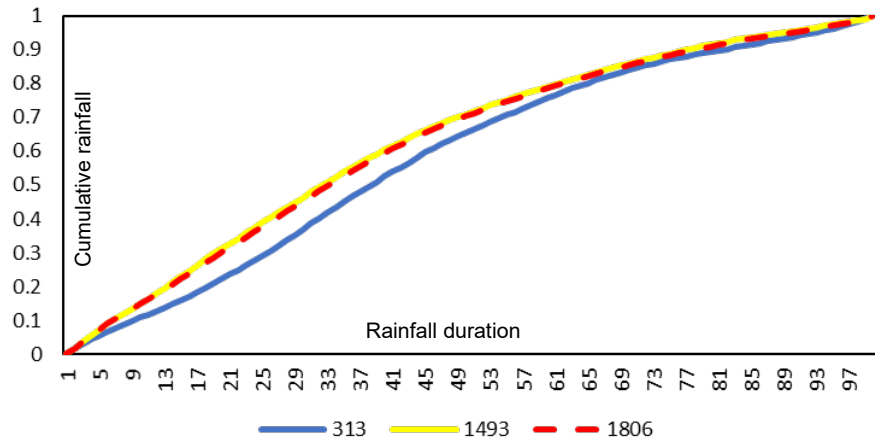
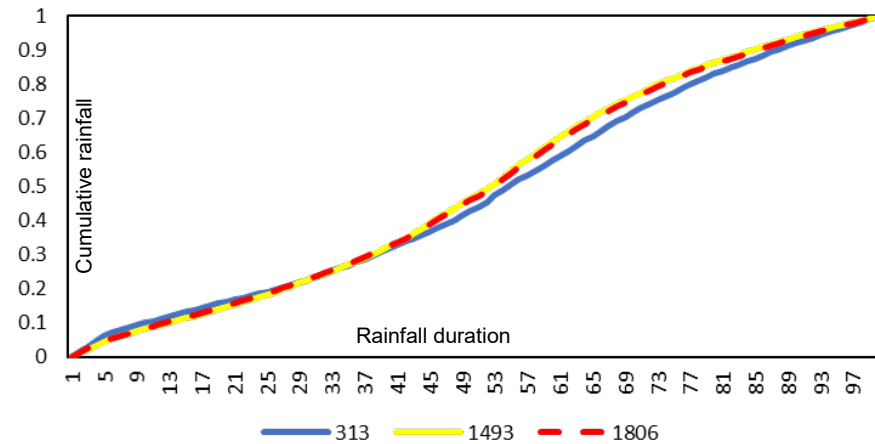


Figure 13. Diagrams of averaged dimensionless cumulative rainfall hyetographs for four clusters designated by means of the k-means clustering method for Kraków based on set No. 3 for 1806 storm rainfalls. For comparison, the diagram also shows Median Time Distributions of Heavy Storm Rainfall at a Point developed by Huff (1990)

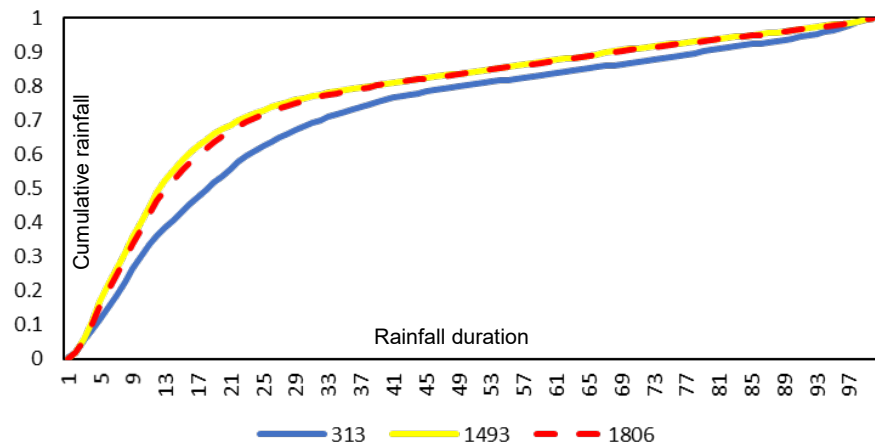
Cluster 1



Cluster 2



Cluster 3



Cluster 4

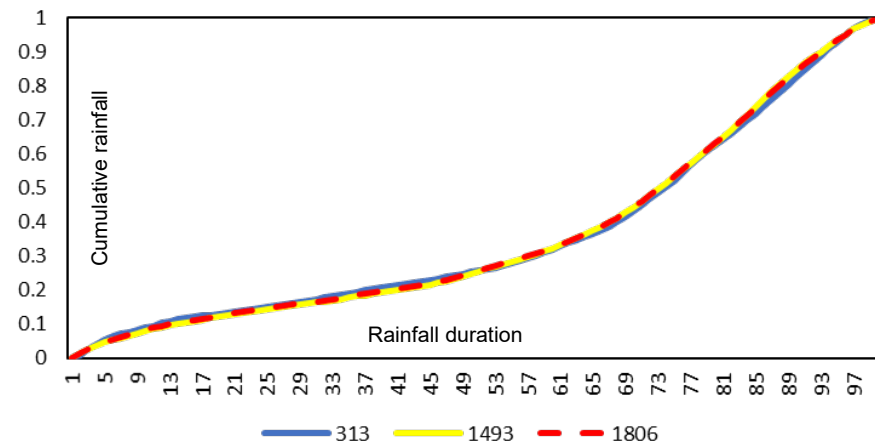
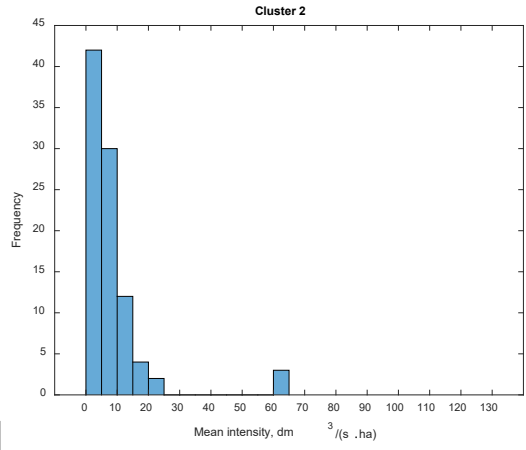
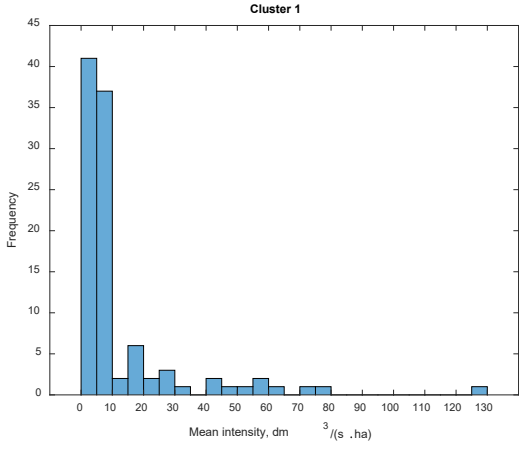
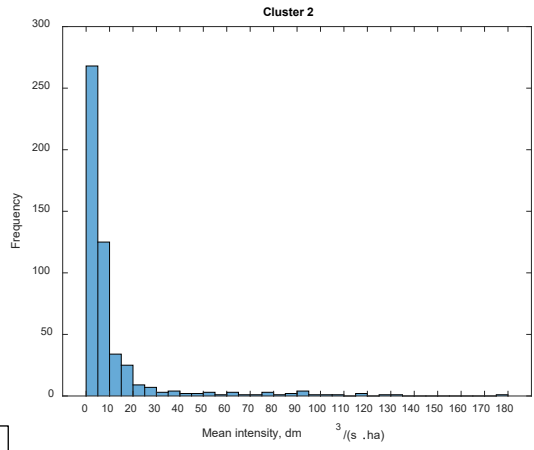
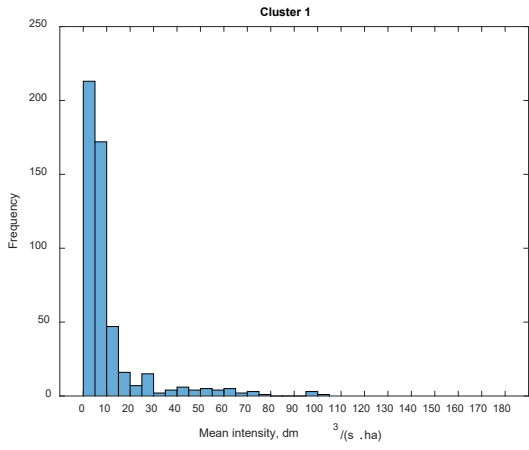
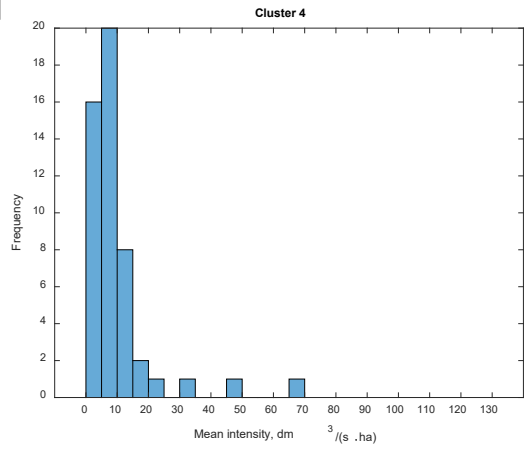
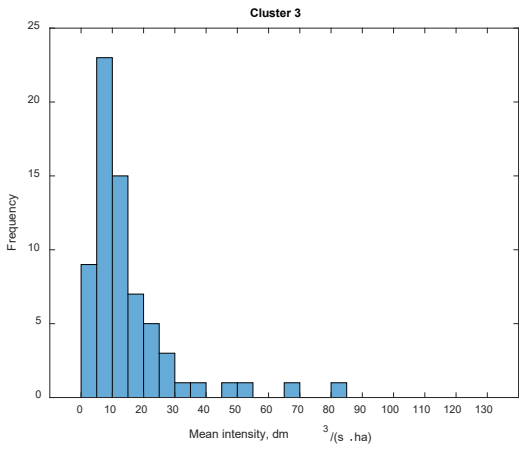


Figure 14. Comparison of averaged dimensionless cumulative rainfall hyetographs for four clusters designated by means of the k-means clustering method based on sets No. 1, 2, and 3: 313, 1493, and 1806 storm rainfalls recorded in Kraków. The horizontal axes present percent increase in rainfall duration, and the vertical axes show cumulative rainfall depth



a



b

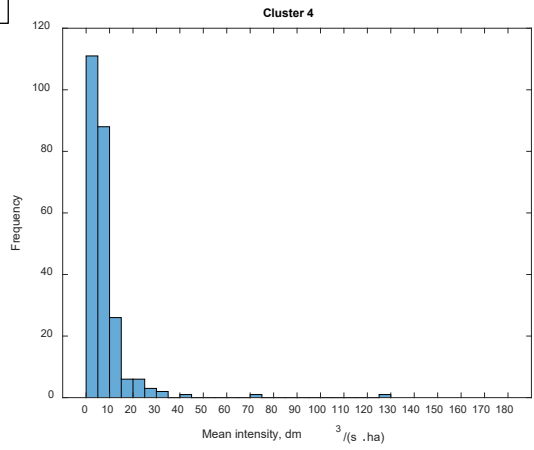
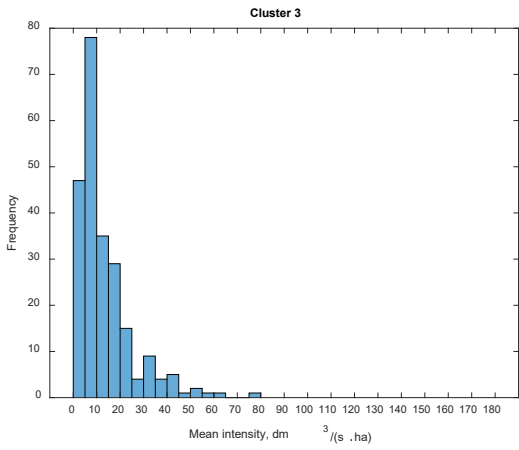


Figure 15 a. Histograms of mean rainfall intensities for four designated clusters in rainfall set No. 1 from Kraków (313 rainfalls); **b.** Histograms of mean rainfall intensities for four designated clusters in rainfall set No. 2 from Kraków (1493 rainfalls)

Table 1. List of rain gauges belonging to sets No. 1 and 2 with characteristics of designated storms

No.	Rain gauge location	Observation period	Number of storm rainfalls	Minimum duration, min.	Maximum duration, min.	Minimum precipitation amount, mm	Maximum precipitation amount, mm
Set No. 1							
1	Kraków – Balice	1986 – 2006	313	24	2329	10.0	105.0
2	Kraków – Wola Justowska	2007 – 2015					
Set No. 2							
1	Bełzy	2016 ÷ 2018	36	89	1639	10.6	74.0
2	Chabrowa	2014 ÷ 2018	41	22	2206	10.2	52.8
3	Miedziana	2010 ÷ 2018	128	20	4359	10.15	121.6
4	Jeziorany	2013 ÷ 2018	86	59	2615	10.0	81.2
5	Kampus UJ	2014 ÷ 2018	67	14	1940	10.2	67.6
6	Kostrze	2013 ÷ 2016	33	49	2377	10.4	43.8
7	Krzemionki	2015 ÷ 2018	59	39	1927	10.0	68.2
8	Kujawy	2013 ÷ 2016	40	32	2258	10.2	93.6
9	Lindego	2008 ÷ 2018	106	21	3015	10.0	60.0
10	Narciarska	2014 ÷ 2018	49	17	2324	10.0	76.0
11	Olsza	2011 ÷ 2012	11	38	1065	10.4	19.4
12	Płaszów	2011 ÷ 2018	100	26	2499	10.0	69.6
13	Reduta	2015 ÷ 2018	49	47	2517	10.0	62.4
14	Rybna	2015 ÷ 2018	43	22	1697	10.2	61.8
15	Rzepichy	2012 ÷ 2018	80	22	2500	10.0	57.2
16	Senatorska	2013 ÷ 2018	81	34	2134	10.2	67.2
17	Skotniki	2009	5	67	839	10.2	24.2
18	Stojałowskiego	2011 ÷ 2018	92	32	3239	10.2	64.8
19	Szwedzka	2008 ÷ 2018	49	21	4576	10.0	145.4
20	Wilga	2013 ÷ 2018	102	25	2518	10.0	73.6
21	Witkowice	2013 ÷ 2018	60	31	2121	10.2	86.0
22	Żagłowa	2012 ÷ 2018	86	22	2405	10.0	82.4



23	Zawiła	2013 ÷ 2018	91	65	2595	10.2	82.2
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Table 2. Mean values of total depths, durations, and intensities of storm rainfalls included in particular clusters for sets No. 1 and No. 2 from Kraków

Cluster	Total depth, mm	Total duration, min	Mean intensity, dm ³ /(s·ha)
Set No. 1			
1	20.0	577	13.0
2	19.3	598	8.5
3	19.5	348	14.7
4	18.8	505	9.7
Set No. 2			
1	19.6	665	10.8
2	20.4	712	11.3
3	17.5	365	14.2
4	19.9	668	7.7