Relations between magnetosonic perturbations as an indicator of a magnetosonic exciter and equilibrium parameters of a plasma

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Summary

The thermodynamic relations between perturbation of pressure and perturbation of mass density and between components of velocity which specify a magnetosonic wave, are theoretically studied. A planar flow with the wave vector forming a constant angle with the equilibrium magnetic field, is investigated. The theory considers deviation from adiabaticity of a flow due to some kind of heating-cooling function and thermal conduction of a plasma. It considers also weak impact of nonlinearity. The thermodynamic relations and corresponding diagrams reveal hysteretic character of irreversible processes in a plasma flow and may indicate damping and nonlinear parameters of a flow. They may indicate also the geometry of a flow, the equilibrium parameters of a plasma, and specify a magnetosonic source. The harmonic and impulsive exciters are discussed in this connection.

Keywords Acoustic activity, Acoustic hysteresis, Nonlinear magnetohydrodynamics

1 Introduction

The irreversible phenomena during the wave processes in a medium reveal hysteretic behavior. The history dependent processes are usual in viscous fluid flows and resemble (though much less studied) elastic behavior of solids and phenomena in ferromagnetic and ferroelectric materials. Pictorial images of cycles of periodic perturbations in the strain-stress diagrams in solids with hysteretic nonlinearity are usually represented by loops [1, 2]. The similar loops appear in variation of pressure $\Leftrightarrow$ variation of density diagrams in the thermoconducting fluid flows [3, 4, 5]. Hedberg, Rudenko probably were first who pointed out the importance of hysteretic diagrams in the fluid flows with various damping mechanisms. In this study, we include into consideration the links of components of velocity and prove their applicability to indicate the equilibrium properties of a plasma and conditions of a flow on a par with the pressure $\Leftrightarrow$ density graphs. They determine streamlines and trajectories which also reveal hysteretic character.

Thermal conduction is responsible for hysteretic behavior in the relation between acoustic pressure and acoustic density in Newtonian fluid flows. Hedberg, Rudenko attributed this responsibility improperly to the total attenuation factor which involves shear and bulk viscosity. The mechanical damping does not have impact on the relation between acoustic pressure and acoustic density, but contributes to irreversible losses in energy and momentum in a fluid flow [5, 6, 7, 8]. It contributes also to the links between components of velocity. Thermodynamic relations which specify the wave modes contain also nonlinear terms making the wave motion isentropic in the leading order. As usual, the leading-order quadratic nonlinearity is considered.

The flows with an external energy supply are specified by some heating-cooling function which incorporates inflow of energy and radiative losses. Presence of the heating-cooling function diversifies scenarios of a flow, and in particular, hysteretic behavior and diagrams in the thermodynamic planes [9]. The damping mechanisms may be enhanced by the heating-cooling function or overbalanced by it [10, 11, 12]. The case of overbalancing is the acoustically active case when wave perturbations enhance taking energy from the background. This leads to unusual direction of diagrams in the variation of density $\Leftrightarrow$ variation of pressure plane and streamlines. The variety of hysteretic behavior is diverse due to coexistance of fast and slow magnetosonic modes, variety of the heating-cooling functions, variable angle between the equilibrium magnetic field and the wave vector and variable ratio of acoustic and magnetic equilibrium pressures (that is, plasma-β). We consider periodic and impulsive magnetosonic exciters and hysteretic behavior specific for links of excess pressure and density and components of velocity in Sec. 3, 4.

The main idea of this study is to attract attention to the history-dependent links of the wave perturbations in magnetohydrodynamcs. They may indicate the geometry and equilibrium parameters of a flow, a kind of magnetosonic mode, to specify an exciter and to be useful in conclusions concerning damping mechanisms and peculiarities of the heating-cooling function. The first steps on this way were done in Ref.[9] in regards to $p \Leftrightarrow \rho$ diagrams with account for excitation of the entropy mode [13, 14]. In this study, we continue studies of the diagrams in the plane p, ρ and pay attention to the links between the velocity components in the light of determining the flow parameters.

2 Relations specifying magnetosonic perturbations

The set of MHD equations for perfectly conducting fully ionized gas consists of conservation of mass, momentum equation, energy balance equation, and electrodynamic equations. It contains in general some heating-cooling function L which may depend on pressure p and density ρ of a plasma. This function equals zero in equilibrium thermodynamic state (p_0,ρ_0). A plasma is in essence thermoconducting. We make use of conditions and the simple geometry of a plasma's flow (e.g., Chin, Nakariakov et al. [15, 16]): the wave vector of a planar flow is directed in accordance to the axis z and forms a constant angle θ ($0 \leq \theta \leq \pi$) with the straight equilibrium magnetic field $\vec{B}_0$ which belongs to the (x, z) plane, so as

$$B_{0,x} = B_0 \sin(\theta), \quad B_{0,y} = 0, \quad B_{0,z} = B_0 \cos(\theta).$$

Perturbations of all thermodynamic variables depend on time t and co-ordinate z. The system of initial PDE equations takes the form (see also [17, 18, 19]):

$$\begin{gathered}
\frac{\partial \rho}{\partial t}+\vec{\nabla}\cdot(\rho\vec{v})=0,\\
\rho\frac{d\vec{v}}{dt}=-\vec{\nabla}p+\frac{1}{\mu_0}(\vec{\nabla}\times\vec{B})\times\vec{B},\\
\frac{dp}{dt}-\gamma\frac{p}{\rho}\frac{d\rho}{dt}=(\gamma-1)\left[L(p,\rho)+\frac{\chi}{C_P-C_V}\Delta\left(\frac{p}{\rho}\right)\right],\\
\frac{\partial\vec{B}}{\partial t}=\vec{\nabla}\times(\vec{v}\times\vec{B}),\\
\vec{\nabla}\cdot\vec{B}=0,
\end{gathered}\tag{1}$$

where p, ρ, $\vec{v}$, $\vec{B}$ denote hydrostatic pressure, density and velocity of a plasma, the magnetic field, μ_0 is the permeability of free space, χ is the plasma's thermal conduction (supposed to be constant). Δ designates the Laplacian. A plasma is an ideal gas with the adiabatic constant γ which equals a ratio of specific heat capacities under constant pressure and constant volume, $\frac{C_P}{C_V}$. The third equation incorporates the continuity and energy equations and relies to the equation of state of an ideal gas

$$e=\frac{p}{(\gamma-1)\rho}$$

and to its temperature $\frac{p}{(C_P-C_V)\rho}$. We expand all thermodynamic quantities around the equilibrium thermodynamic state as $f(z,t)=f_0+f'(z,t)$. A plasma is static in equilibrium, so as $\vec{v}_0=\vec{0}$. In the one-dimensional geometry, $B'_z=0$ and the number of unknowns reduces from eight to seven. We do not consider mechanical damping and electrical resistivity of a plasma in this study. We disregard also the nonlinear interaction of modes in a flow but consider the nonlinear distortion of a magnetosonic wave and the leading-order nonlinear terms in the links which support adiabaticity of a wave process. The dispersion relations are established by the linearized Eqs(1), if one looks for solution in the form of a sum of planar waves proportional to $\exp(i\omega(k)t-ikz)$ where ω is the wave frequency and k designates the wave number. The leading-order relation specify four magnetosonic modes, if $C_A\neq c_0$, $\theta\neq 0$,

$$\omega=Ck+i\frac{(\gamma-1)(C^2-C_A^2)}{2c_0^2(2C^2-c_0^2-C_A^2)}\left(c_0^2k^2\frac{\chi}{C_P\rho_0}-(c_0^2L_p+L_\rho)\right),\tag{2}$$

where the partial derivatives $L_p=\frac{\partial L}{\partial p}$, $L_\rho=\frac{\partial L}{\partial \rho}$ are evaluated at the equilibrium state (p_0,ρ_0), C is one of four magnetosonic speeds (slow or fast, positive or negative) satisfying equation

$$C^4-C^2(c_0^2+C_A^2)+c_0^2C_{A,z}^2=0,\tag{3}$$

and C_A and c_0

$$C_A=\frac{B_0}{\sqrt{\mu_0\rho_0}},\quad c_0=\sqrt{\frac{\gamma p_0}{\rho_0}}$$

designate the Alfvén speed and the acoustic speed in unmagnetized gas in equilibrium, $C_{A,z}=C_A\cos(\theta)$. The case $\theta=0$, $C=c_0=C_A$ is especial. This case imposes only one magnetosonic mode propagating in the positive direction of axis z which corresponds to the dispersion relation

$$\omega=c_0k+i\frac{(\gamma-1)}{2c_0^2}\left(c_0^2k^2\frac{\chi}{C_P\rho_0}-(c_0^2L_p+L_\rho)\right).\tag{4}$$

We consider weak impact of non-adiabaticity associating with the heating-cooling function and thermal conduction on the wave perturbations, that is, small variation of the wave magnitudes over the characteristic wave length. The dispersion relations and links between specific perturbations in this study are leading-order, that is, they contain terms up to the first powers of L_p, L_ρ, χ. Eqs(2),(3) have been established by Chin, Nakariakov et al. [15, 16]. A small-magnitude flow is acoustically active (magnetosonic perturbations of the wave number k enhance) if

$$c_0^2 L_p + L_\rho > c_0^2 k^2 \frac{\chi}{C_P \rho_0}. \tag{5}$$

This is the condition of acoustical activity corrected with impact of thermal conduction [20, 21, 22]. The dispersion relations Eqs(2),(4) uniquely determine links between specific magnetosonic (ms) perturbations for any mode in a linear flow. To be specific, we consider perturbations propagating in the positive direction of axis z with $C > 0$. The links between components of velocity $v_{ms,x}$, $v_{ms,z}$ and perturbations of pressure p_{ms} and density ρ_{ms}, corrected by the nonlinear terms making a mode nearly isentropic, are

$$v_{ms,x} = \frac{C_{A,z}(c_0^2 - C^2)}{C_{A,x}C^2}\left[v_{ms,z} + \frac{(\gamma-1)C}{(C^4 - c_0^2 C_A^2)}\left((c_0^2 L_p + L_\rho)\int_\infty^z v_{ms,z}dz + c_0^2 \frac{\chi}{C_P\rho_0}\frac{\partial v_{ms,z}}{\partial z}\right) - \right.$$
$$\left. \frac{C(C^4 - 2C^2 c_0^2 + c_0^2(c_0^2 + C_{A,x}^2(\gamma-1))}{2C_{A,x}^2(C^4 - c_0^2 C_{A,z}^2)} v_{ms,z}^2\right], \quad v_{ms,y} = 0, \tag{6}$$
$$p_{ms} = c_0^2\rho_{ms} - \frac{(\gamma-1)}{C}\left((c_0^2 L_p + L_\rho)\int_\infty^z \rho_{ms}dz + c_0^2\frac{\chi}{C_P\rho_0}\frac{\partial \rho_{ms}}{\partial z}\right) + \frac{(\gamma-1)c_0^2}{2\rho_0}\rho_{ms}^2.$$

The case $\theta = 0$, $C = c_0$ is specified by zero transversal components of velocity,

$$v_{ms,x} = v_{ms,y} = 0. \tag{7}$$

The links of magnetosonic perturbations have been derived in [23]. The link between components of velocity $v_{ms,x}(v_{ms,z})$ determines streamlines and trajectories in the planar motion in the x, z plane. Both thermodynamic relations $v_{ms,x}(v_{ms,z})$, $p_{ms}(\rho_{ms})$ reflect the magnetoacoustic hysteretic behavior. The linear terms in the links (6) follow from the dispersion relation, and the nonlinear terms support leading-order isentropicity of the wave mode. The links between p_{ms} and ρ_{ms} generalize the equation of state involving terms responsible for deviation from adiabaticity. Hedberg, Rudenko [3] determined the relation in a Newtonian flow as "constitutive equation" instead of "equation of state" implying an instantaneous relation of variables.

3 Hysteresis curves for periodic and impulsive exciters

We do not consider impact of the secondary entropy mode excited in the field of intense sound in the context of hysteretic diagrams $p_{ms}(\rho_{ms})$. The perturbation in density associating with this mode ρ_{ent} is of order λM^2, where λ is a generic small parameter responsible for deviation from adiabaticity due to impact of the heating-cooling function and thermal conduction, and M is the Mach number, while the nonlinear wave terms are more by an order of magnitude, M^2. The Mach number M is the ratio of the magnitude of a plasma velocity and the magnetosonic speed. Hence, ρ_{ent} has a small impact on the hysteretic curves over characteristic period of perturbation but may accumulate in time. Impact of magnetosonic heating (cooling) on the hysteretic behavior has been considered in Ref.[9].

3.1 Harmonic magnetosonic pressure

The first example is the harmonic magnetosonic dimensionless pressure in the form

$$p_{ms} = Mc_0^2\rho_0 \sin(T - Z),$$

where $T = \omega t$ is the dimensionless time (ω designates the magnetosonic frequency), and $Z = \frac{\omega z}{C}$ is the dimensionless coordinate. We consider perturbations at a transducer situated at $Z = 0$:

$$P = \frac{p_{ms}}{Mc_0^2\rho_0} = \sin(T). \tag{8}$$

The dimensionless perturbation in density in accordance to Eqs(6) sounds as

$$R = \frac{\rho_{ms}}{M\rho_0} = \sin(T) + A\cos(T) - B\sin^2(T), \tag{9}$$

where

$$A = \frac{(\gamma-1)}{\omega C_P\rho_0}\left(\frac{(c_0^2 L_p + L_\rho)C_P\rho_0}{c_0^2} - \frac{\omega^2}{C^2}\chi\right), \quad B = \frac{1}{2}M(\gamma-1) \tag{10}$$

are parameters responsible for deviation from adiabaticity (A may be positive, zero and negative) and for nonlinearity (B is positive). The non-zero A reflects the hysteretic behavior, that is, different behavior if P enlarges or gets smaller in time. $A > 0$ is the case of acoustical activity. The quadratic nonlinearity provided by positive B deforms an elliptic diagram into a crescent with ends adroop down. The hysteretic curve in the plane P, R takes the form

$$(R - P + BP^2)^2 + A^2P^2 = A^2.$$

The width of a curve (a difference between maximum and minimum R at zero P) equals $2|A|$, and the direction of traversing the contour is clockwise if $A > 0$ and counterclockwise if $A < 0$.

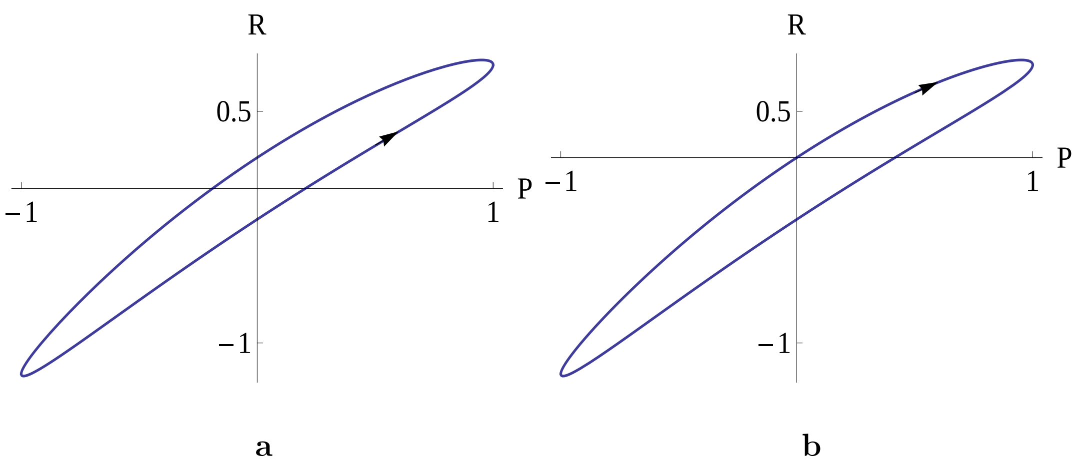


Figure 1. *The exemplary hysteretic curves $P \Leftrightarrow R$ for $B = 0.2$ and $A = -0.2$ (a), $A = 0.2$ (b)*

The dimensionless work done by a plasma element over the cycle equals

$$W = -\int_0^{2\pi} P(T)\frac{dR}{dT}dT = A\pi.$$

The work is determined exclusively by A, and it is positive in acoustically active flows. This corresponds to the cooling of the background, since variation of the internal energy dU over a cycle in the quasi-isentropic processes equals approximately $-W$ and hence is negative in acoustically active flows. A reflects deviation from adiabaticity of a flow due to impact of the heating-cooling function, thermal conduction, and it depends also on the frequency of exciter and C. By means of C, it varies with plasma-β ($\beta = \frac{2}{\gamma}\frac{c_0^2}{C_A^2}$) and θ in accordance to Eq.(3). A variation of temperature over a cycle equals $c_0^2 dU/C_V$.

3.2 A Gaussian impulse

The next kind of an exciter is mono-polar at a transducer,

$$P = \exp(-T^2), \tag{11}$$

where ω denotes the characteristic inverse duration of an impulse. The leading-order form of the dimensionless perturbation of density equals

$$R = \exp(-T^2) - A_1 \frac{\sqrt{\pi}(erf(T) + 1)}{2} + 2A_2 T \exp(-T^2) - B\exp(-2T^2),$$

$$A_1 = \frac{(\gamma - 1)(c_0^2 L_p + L_\rho)}{c_0^2 \omega}, \quad A_2 = \frac{(\gamma - 1)\omega}{C^2 C_P \rho_0}\chi.$$

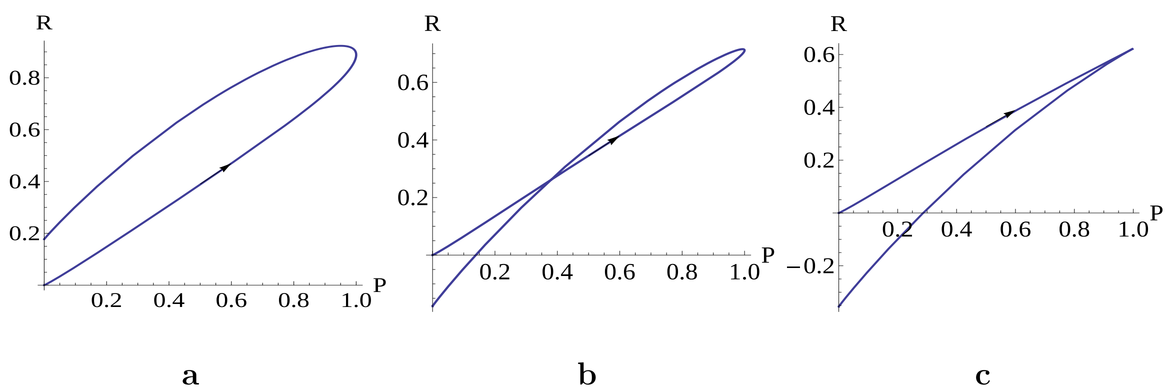


Figure 2. *The exemplary hysteretic curves for* $B = 0.2$ *and* $A_1 = -0.1$, $A_2 = 0.1$ *(a)*, $A_1 = 0.1$, $A_2 = 0.1$ *(b)*, $A_1 = 0.2$, $A_2 = 0.1$ *(c). The case of an impulsive exciter Eq.*(11).

The dimensionless work done by a plasma element when T varies from $-\infty$ till ∞, is determined by the summary factor of non-adiabaticity A:

$$W = -\int_{-\infty}^{\infty} P(T)\frac{dR(T)}{dT}dT = (A_1 - A_2)\sqrt{\frac{\pi}{2}} = A\sqrt{\frac{\pi}{2}}.$$

It is positive in acoustically active flows. The residual quantity R_{res} is determined exclusively by A_1,

$$R_{res} = -A_1\sqrt{\pi}.$$

Hence, the residual value does not depend on θ, plasma-β and χ but only on the kind of the heating-cooling function and the characteristic duration of an impulse.

4 Streamlines and trajectories

The velocity field determined by links (6), is not potential due to impact of the magnetic field, $\vec{\nabla} \times \vec{v} \neq 0$. In the absence of magnetic field ($C_A = 0$) or parallel propagation with the speed c_0, $v_{ms,x} = 0$, the velocity field is potential and the streamlines are directed along axis z. Without impact of the heating-cooling function, thermal conduction and nonlinearity, the streamlines are straight lines with tangent of inclination to the axis z, $\frac{C_{A,z}(c_0^2-C^2)}{C_{A,x}C^2}$, which is negative at $\theta < \pi/2$ and positive at $\theta > \pi/2$ for the fast magnetosonic modes (vice versa for the slow modes). Fig.3 shows the leading-order angle between $v_{ms,z}$ and $v_{ms,x}$,

$$\alpha = \arctan\left(\frac{C_{A,z}(c_0^2 - C^2)}{C_{A,x}C^2}\right)$$

for slow and fast magnetosonic modes at different β ($\beta = 2/\gamma$ corresponds to $c_0 = C_A$ with links (7) and demarcates different scenarios of behavior in dependence on θ). In all evaluations, $\gamma = \frac{5}{3}$.

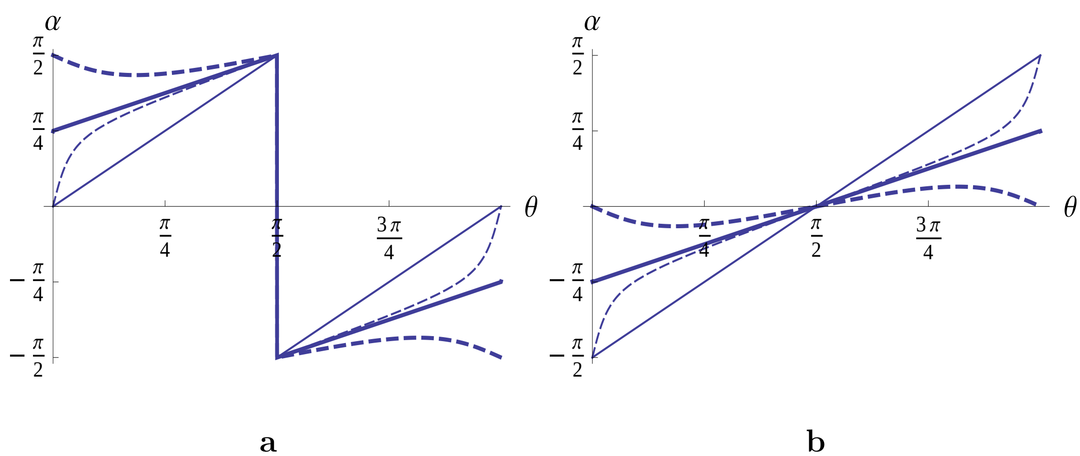


Figure 3. *The leading-order angle between* $v_{ms,z}$ *and* $v_{ms,x}$, $\alpha = \arctan\left(\frac{C_{A,z}(c_0^2-C^2)}{C_{A,x}C^2}\right)$ *for slow (a) and fast magnetosonic modes (b). Thin solid line corresponds to* $\beta = 0$*, thin dotted line corresponds to* $\beta = 1$*, thick solid line corresponds to* $\beta = 2/\gamma$*, and thick dotted line corresponds to* $\beta = 3$.

The roots of Eq.(3) for $\theta = 0$ are $C = C_A$ and $C = c_0$. The magnetosonic modes is this one which propagates with the speed c_0, that is, the slow mode for $\beta < \frac{2}{\gamma}$ and the fast mode for $\beta > \frac{2}{\gamma}$. For all these modes, $\alpha = 0$ at $\theta = 0$. The dispersion relations are degenerate if $\beta = \frac{2}{\gamma}$ and $c_0 = C_A$. There are three linearly independent modes corresponding to a wave propagating in the positive direction of axis z (in fact, two modes are Alfvén, and one is magnetosonic, and any linear combination of them also represents the wave motion). The degeneration disappears in regards to a magnetosonic mode in the presence of thermal conduction or/and the heating-cooling function. The case $\theta = \frac{\pi}{2}$ is also especial. In this case, the roots of Eq.(3) are $C = 0$ and $C = \sqrt{c_0^2 + C_A^2}$, hence, there is only one (fast) wave mode specified by $v_{ms,x} = 0$. The stationary non-wave mode with $C = 0$ is specified by zero v_z. This formally leads to a variety of α from $-\frac{\pi}{2}$ till $-\frac{\pi}{2}$ in the plots regarding to the slow mode. Deviation from adiabaticity due to the heating-cooling function and thermal conduction makes the slope to vary with time and condition the hysteretic behavior. Also, nonlinearity in the relation (6) corrects the straight streamlines. The curves $v_{ms,x}(v_{ms,z})$ become folded down for any β and θ in the both cases of slow and fast modes.

4.1 Harmonic longitudinal velocity

The dimensionless components of velocity $V_z = \frac{v_{ms,z}}{MC}$, $V_x = \frac{v_{ms,x}}{MC}$ given by (6) are determined in the parametric form by equalities

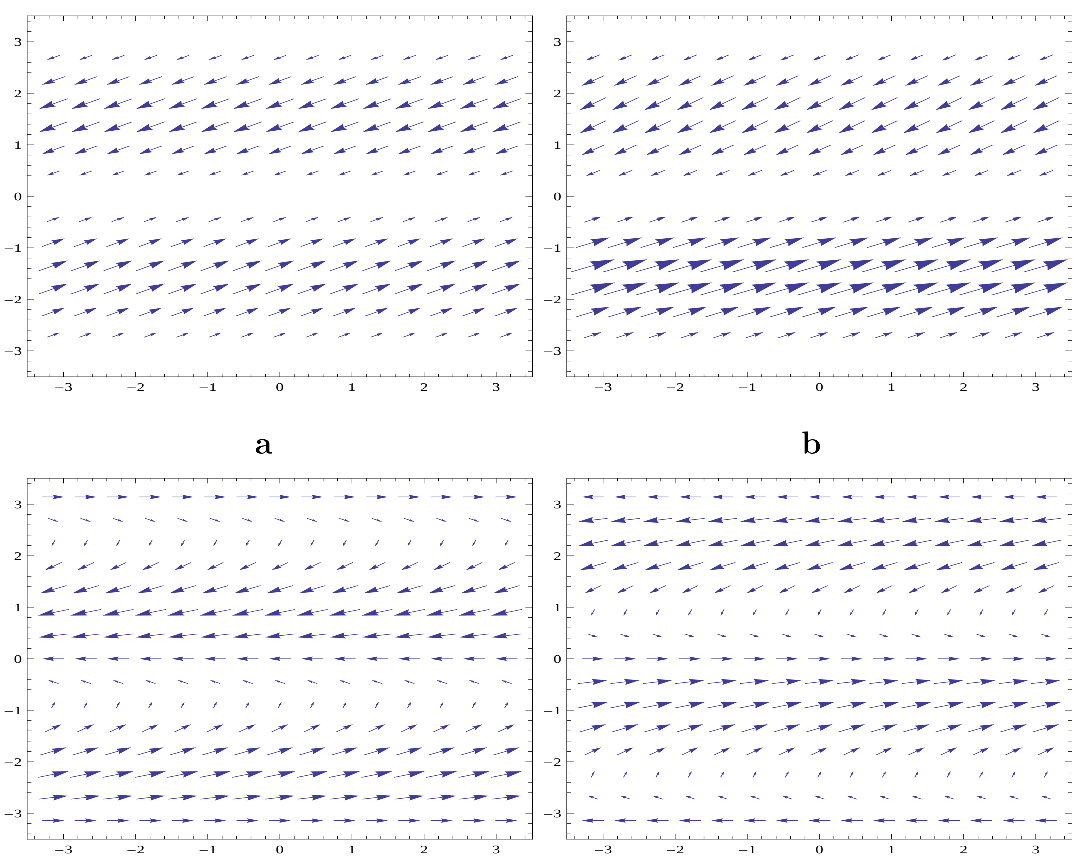
$$V_z = \sin(T - Z), \quad V_x = K_1 \sin(T - Z) + K_2 \cos(T - Z) + K_3 \sin^2(T - Z), \tag{12}$$

$$K_1 = \frac{C_{A,z}(c_0^2 - C^2)}{C_{A,x}C^2},$$

$$K_2 = K_1 \frac{c_0^2 C^2}{(C^4 - c_0^2 C_{A,z}^2)} A,$$

$$K_3 = K_1 \frac{C^2(C^4 - 2C^2 c_0^2 + c_0^2(c_0^2 + C_{A,x}^2(\gamma - 1))}{2C_{A,x}^2(C^4 - c_0^2 C_{A,z}^2)} M.$$

The exemplary hysteretic streamlines in the plane X, Z ($X = \frac{\omega x}{C}$) are plotted in Fig. 4.



a **b**

c **d**

Figure 4. *The exemplary hysteretic streamlines at $T = 0$ in the plane X (horizontal axis), Z (vertical axis). $K_1 = 2$ (a): $K_2 = K_3 = 0$, (b): $K_2 = 0, K_3 = 0.5$ (c): $K_2 = -2, K_3 = 0$, (d): $K_2 = 2, K_3 = 0$. The case of harmonic exciter* (12).

Fig.5 shows two cycles of exemplary trajectories. The nonlinearity makes the shape spiral. Without nonlinearity, the trajectories are elliptic. The shift along axis X over a cycle is positive and equals πK_3 for any initial Z. Since K_3 depends on plasma-β, θ and M and differs for slow and fast magnetosonic modes, a shift may indicate the parameters of a flow. The direction of trajectories is conditioned by the sign of summary degree of nonadiabaticity, A. It is counterclockwise in acoustically active flows.

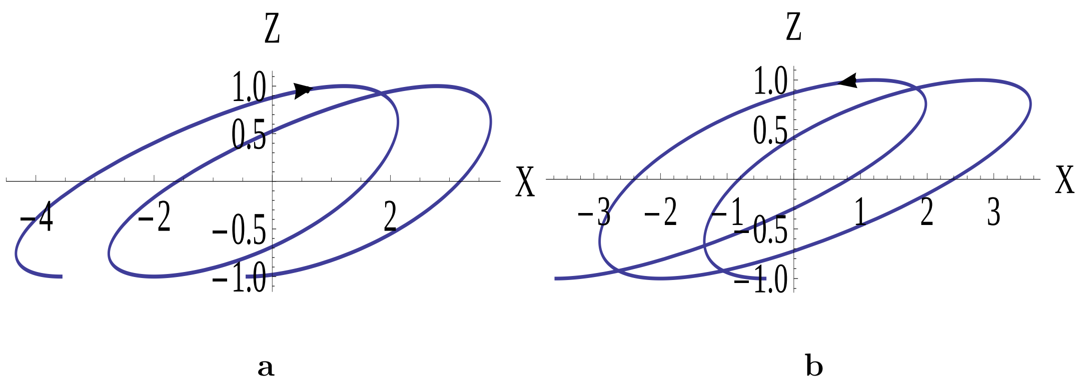


Figure 5. *The trajectories for $K_1 = 2$, $K_3 = 0.5$ and $K_2 = -2$ (a), $K_2 = 2$ (b)*

4.2 Gaussian impulse

The streamlines and trajectories are determined by the following relations

$$V_z = \exp(-(T-Z)^2),$$

$$V_x = K_1 \exp(-(T-Z)^2) - K_1 A_1 \frac{c_0^2 C^2}{(C^4 - c_0^2 C_{A,z}^2)} \frac{\sqrt{\pi}(erf(T-Z)+1)}{2} +$$

$$2K_1 A_2 \frac{c_0^2 C^2}{(C^4 - c_0^2 C_{A,z}^2)} (T-Z)\exp(-(T-Z)^2) + K_3 \exp(-2(T-Z)^2).$$

For any initial point, the difference between vertical coordinates over the total time of evolution, from $T = -\infty$ till $T = \infty$, equals $\sqrt{\pi}$ for this kind of impulse.

5 Concluding remarks

The main aim of this study is to indicate suitability of the relations between specific variables in the MHD wave process to determine the equilibrium properties of a plasma, geometry, conditions of a flow, a kind and frequency of a magnetosonic exciter. All perturbations of the thermodynamic quantities (p', ρ', v_x, v_y, v_z, B'_x, B'_y) are connected by the links which specify a mode of a flow. Among all variety of these links, we pay attention to the relations between p_{ms} and ρ_{ms}, $v_{ms,x}$ and $v_{ms,z}$ (Eqs(6)) in the magnetosonic mode which wave vector forms a constant angle θ with the equilibrium magnetic field. In the geometry of a flow under study, $v_{ms,y} = 0$. The links in the magnetosonic mode are supplemented by the nonlinear terms which make the wave process nearly isentropic. This introduces nonlinear distortions in the diagrams of specific perturbations.

The pictorial rendition of $P \Leftrightarrow R$ are in fact diagrams which reflect hysteretic character of irreversible thermodynamic processes. The hysteretic behavior reflects different links between P and R in the domains when P enlarges or gets smaller in time. The leading-order term responsible for a slope of a curve, equals c_0^2 and does not depend on plasma-β and θ. The terms on the right-hand side of Eqs(6) proportional to $(c_0^2 L_p + L_\rho)\int_\infty^z \rho_{ms} dz \approx -C(c_0^2 L_p + L_\rho)\int_{-\infty}^t \rho_{ms} dt$ and $\frac{\chi}{C_P \rho_0}\frac{\partial \rho_{ms}}{\partial z} \approx -\frac{\chi}{C C_P \rho_0}\frac{\partial \rho_{ms}}{\partial t}$ reflect the time-dependent behavior, that is, dependence of current thermodynamic state on the history. The first term which associates with the heating-cooling function, includes an integral operator. Also, relaxation of different species is inherent to some integral operator with a kernel reflecting frequency-dependent absorption [6, 7, 24, 25, 26]. The work done by a plasma element over a period of harmonic excitation is determined by the total parameter responsible for deviation from adiabaticity, A (Eq.(10)). It may take positive, zero or negative value in dependence on the heating-cooling function. This concerns also the work done by an impulsive exciter over the total temporal domain. Positive work associates with the negative variation of the internal energy and temperature of the background. That means magnetosonic cooling of a medium in acoustically active flow and in turn has impact on propagation of wave perturbations due to variation in their speed [27]. The hysteretic curve transversal direction is clockwise in the flows where inflow of energy overbalances losses due to thermal conduction and counterclock-wise in the flows with the summary damping. An impulsive excitation is characterized by a residual perturbation of density associating exclusively with the heating-cooling function. It is positive if $c_0^2 L_p + L_\rho < 0$. The nonlinearity deforms the hysteretic curves but has no impact on the work done by a plasma and a residual value.

The link between the components of velocity $v_{ms,x}$, $v_{ms,z}$ (Eqs(6)) in a flow also diserves attention on a par with $p'(\rho')$. We may conclude that the factors in relation between the components of velocity are much more sensitive to the variations of the equilibrium parameters and conditions of a flow than that in the relation between an excess pressure and density. In particular, while the slope tangent in the $p'(\rho')$ diagram equals in the leading oder c_0^2, the slope tangent in the $v_{ms,x}(v_{ms,z})$ diagram equals in the leading oder $\frac{C_{A,z}(c_0^2 - C^2)}{C_{A,x} C^2}$. It may be confidently used to establish the kind of the wave mode, θ and plasma-β. The direction of trajectories is conditioned by the total degree of non-adiabaticity A. The case $\theta = 0$, $C = c_0$ yields $v_{ms,x} = 0$. Velocity of a plasma may be measured remotely by doppleroscopy.

Briefly summarizing, the links between thermodynamic perturbations and corresponding pictorial images may confidently indicate thermodynamic processes in a medium, also in remote observations. The work done by the fluid element is proportional to the total factor responsible for deviation from adiabaticity and determines an enlargement of the background temperature which is easy for measurement. In turn, this factor depends on the characteristic frequency of magnetosonic perturbations and magnetosonic sound speed C and hence, on the plasma-β and θ. The unusual bypass direction of hysteretic curves and trajectories points an acoustical activity of a flow, and the residual excess density in impulsive excitation depends on the heating-cooling function exclusively. The diagrams may point the characteristic frequency and a kind of exciter and determine a degree of non-adiabaticity. The streamlines are much more sensitive to the variations of the equilibrium parameters of a flow than $P \Leftrightarrow R$ diagrams and also may be useful in reconstruction of thermodynamic properties of a medium. The relations may be useful in remote reconstruction of the magnetosonic source in a medium with the known thermodynamic properties and, on the contrary, in studying of the unknown thermodynamic properties by means of various exciters.

The links between specific perturbations are underestimated in many problems of wave motion. They may be in fact referred to as "constitutive equations" [3]. The integral links between wave perturbations are inherent to many physically significant flows. They appear in flows of spatially inhomogeneous in equilibrium media, in flows of gases with various relaxation processes and in flows with external sources of energy [6, 7, 8, 25]. Generally, an integral link contains frequency-dependent kernel [24, 26]. This is of especial importance in the case of impulsive exciters with the broad frequency spectrum [8, 28]. We pay attention to harmonic and impulsive exciters but do not consider exciters with discontinuities in this study. A saw-tooth wave forms in acoustically active flows if damping mechanisms are comparatively small [12, 16, 29]. It may appear at sudden excitation of a medium or a result of evolution of any initial perturbation. The wave forms with discontinuities experience nonlinear damping [6, 7]. The peculiarities of pure nonlinear damping at the wave fronts compared to Newtonian attenuation in the context of hysteresis have been discussed by Hedberg, Rudenko [3]. We consider a deviation from adiabaticity associating with the heating-cooling function and thermal conduction. The balance of these two effects may lead to unusual features of hysteretic behavior, in particular, to the inverse direction of the hysteretic curves. Involving into consideration mechanical viscosity and electrical resistivity would contribute to relation $v_{ms,x}(v_{ms,z})$ but does not contribute to the $p_{ms}(\rho_{ms})$ link (Eqs(6)) which remains unchanged. The impact of excitation of the entropy mode in the field of magnetosonic waves on the hysteretic behavior, is not considered. It has been discussed in Ref.[9] and have much in common with hysteretic phenomena in other flows which may be acoustically active [4, 5, 8].

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study, which is a purely theoretical one.

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